

Programme

Monday 9 July 2018

10:30 **opening**

10:40 Andrea Trombettoni • *Ultracold fermions across the BCS-BEC crossover: optical lattices and Josephson effect*

11:40 Domenico Finco • *A Model Hamiltonian for Feshbach Resonances*

lunch buffet

15:00 Raffaele Carlone • *Non-linear Schrödinger equation in two dimensions with concentrated non-linearities*

16:00 **coffee break**

16:25 Lorenzo Tentarelli • *Non-linear singular perturbations of the fractional Schrödinger equation in dimension one*

Tuesday 10 July 2018

9:30 Ludovic Pricoupenko • *Zero-range modeling: from Efimov physics to quasi-1D atomic systems*

10:30 **coffee break**

10:55 Rodolfo Figari • *Regularised quadratic forms for a three boson system with zero-range interactions*

11:55 Vladimir Lotoreichik • *Mini-course: Shape optimisation and surface interactions – I*

lunch buffet

15:00 Illya Karabash • *Asymptotic distribution of narrow resonances*

16:00 **coffee break**

16:25 Yuriy Golovaty • *1D Schrödinger operators with localized potentials and finite-rank perturbations*

17:25 Giulia Basti • *The three-body problem in dimension one: from short-range to point interactions*

Wednesday 11 July 2018

9:30 Teoman Turgut • *Some singular models in physics*

10:30 **coffee break**

10:55 Matteo Gallone • *Darwin-like perturbations of hydrogenoid Hamiltonians*

11:55 Julian Schmidt • *A direct description of Nelson-type Hamiltonians via boundary conditions*

Thursday 12 July 2018

9:30 Vladimir Lotoreichik • *Mini-course: Shape optimisation and surface interactions – II*

10:30 **coffee break**

10:55 Diego Noja • *The linear KdV equation on a star graph*

11:55 Jussi Behrndt • *Boundary triple techniques for Schrödinger and Dirac operators with singular interactions*

lunch buffet

15:00 Andrea Posilicano • *Direct and inverse scattering for singular perturbations*

16:00 **coffee break**

16:25 Horia Cornean • *Two dimensional Schrödinger operators with point interactions: threshold expansions, zero modes and L_p -boundedness of wave operators*

17:25 Raffaele Scandone • *Smoothing and scattering properties of three-dimensional Schrödinger operators with point interactions*

Friday 13 July 2018

9:30 Pascal Naidon • *A short review of Efimov physics*

10:30 **coffee break**

10:55 Marcel Griesemer • *The 2D Fermi polaron and its high density limit*

11:55 Andrea Ottolini • *Spectral analysis of the 2+1 fermionic trimer with contact interactions*

lunch buffet

15:00 'post mortem' discussion session:

Particle systems with contact interactions, reloaded – the next problems

Giulia Basti • *The three-body problem in dimension one: from short-range to point interactions*

Hamiltonians with point interactions are often used to treat few-particle systems with short-range interactions at low energy which exhibit interesting properties such as the Efimov effect for three particles in dimension three. These Hamiltonians are often defined using the theory of self-adjoint extensions. From a physical point of view a less abstract construction is obtained if one shows that these Hamiltonians are the limit of Hamiltonians with smooth, suitably rescaled two-body potentials. We consider the simpler one-dimensional case for three particles. We study a Hamiltonian with potentials rescaled a $v_\sigma^\epsilon = \epsilon^{-1} v_\sigma(\epsilon x)$, where σ denotes a pair of particles. As $\epsilon \rightarrow 0$ we show the convergence to a limit Hamiltonian formally obtained by replacing the potential v_σ with $\alpha_\sigma \delta_\sigma$, where $\alpha_\sigma = \int v_\sigma$. Joint work with C. Cacciapuoti, D. Finco and A. Teta.

Jussi Behrndt • *Boundary triple techniques for Schrödinger and Dirac operators with singular interactions*

In this lecture we give a brief introduction to the abstract technique of boundary triples and their Weyl functions, and we show how these methods can be applied to analyze singular perturbations of Schrödinger and Dirac operators. The aim of this introductory talk is to illustrate the extension theory approach via boundary and interface conditions and to highlight the role of the Weyl function in spectral theory. We shall also compare the boundary triple method with the traditional form approach to singular perturbations.

Horia Cornean • *Two dimensional Schrödinger operators with point interactions: threshold expansions, zero modes and L^p -boundedness of wave operators*

We study the threshold behaviour of two dimensional Schrödinger operators with finitely many local point interactions. We show that the resolvent can either be continuously extended up to the threshold, in which case we say that the operator is of regular type, or it has singularities associated with s - or p -wave resonances or even with an embedded eigenvalue at zero, for whose existence we give necessary and sufficient conditions. An embedded eigenvalue at zero may appear only if we have at least three centres. When the operator is of regular type we prove that the wave operators are bounded in $L^p(\mathbb{R}^2)$ for all $1 < p < \infty$. With a single centre we always are in the regular type case. This is joint work with A. Michelangeli and K. Yajima.

Raffaele Carlone • *Regularised quadratic forms for a three boson system with zero-range interactions*

We consider a two-dimensional non-linear Schrödinger equation with concentrated non-linearity. In both the focusing and defocusing case we prove local well-posedness, i.e., existence and uniqueness of the solution for short times, as well as energy and mass conservation. In addition, we prove that this implies global existence in the defocusing case, irrespective of the power of the nonlinearity, while in the focusing case we prove the existence of blowing-up solutions.

Rodolfo Figari • *Regularised quadratic forms for a three boson system with zero-range interactions*

We investigate different zero range Hamiltonians for a system of three bosons in a Hilbert space of tensorial wave functions. The Hamiltonians are associated to quadratic forms that are proved to be bounded from below.

Domenico Finco • *A model Hamiltonian for Feshbach resonances*

We discuss a two channel model for Feshbach resonances: we provide some results about the number and localization of resonances, and the behaviour of the scattering length near them.

Matteo Gallone • *Darwin-like perturbations of hydrogenoid Hamiltonians*

Schrödinger operators with Coulomb potential provide a good description of the spectrum of hydrogenoid atoms up to small corrections due to the fine structure. A more refined model must include the first relativistic corrections, among which the Darwin term that effectively consists of a point-like perturbation at the origin. In this talk I will discuss the rigorous construction of Darwin-perturbed hydrogenoid Hamiltonians and the associated radial problem of Schrödinger operators with Coulomb interaction on the half line. Using the Krein-Visik-Birman extension theory I will present a very effective classification of their self-adjoint realisations that permits, in particular, to obtain an explicit formula for their eigenvalues.

Yuriy Golovaty • *1D Schrödinger operators with localized potentials and finite-rank perturbations*

Norm resolvent approximation for a wide class of point interactions in one dimension is constructed. To analyse the limiting behaviour of Schrödinger operators with δ' -like potentials as well as localized singular rank-two perturbations, we show that the set of limit operators is quite rich. Depending on parameters of the perturbation, the limit operators are described by both the connected and separated boundary conditions. In particular an approximation for a four-parametric subfamily of all the connected point interactions is constructed.

Marcel Griesemer • *The 2D Fermi polaron and its high density limit*

The Fermi polaron denotes a system composed of an ideal gas of N fermions interacting with an impurity particle. The interaction between impurity and fermions is given in terms of two-body point interactions whose strength is determined by the two-body binding energy. In this talk we describe a generalized Birman-Schwinger principle and its application to the Fermi polaron. We then prove that the so-called polaron energy is an upper bound to the ground state energy and that it is asymptotically correct in the limit of high fermion density. The latter is true at least for infinite mass of the impurity. This is joint work with Ulrich Linden and David Mitrouskas.

Illya Karabash • *Asymptotic distribution of narrow resonances*

For several classes of Schrödinger operators, we study the internal structure of the set of resonances with the aim to define the asymptotic density of physically more relevant narrow resonances. Then it is planned to discuss how these asymptotic densities are connected with the geometry and the symmetries of the resonator. The talk is based on a joint research with S. Albeverio.

Vladimir Lotoreichik • *Mini-course: Shape optimisation and surface interactions*

The mini-course will focus on various important mathematical tools, which are particularly useful in proving geometric optimisation results for the lowest eigenvalue of a self-adjoint Hamiltonian with a surface interaction. Such optimisation results can be viewed as counterparts of the celebrated Faber-Krahn inequality. We intend to discuss the following three methods.

- The method of parallel coordinates.
- The technique of boundary integral equations.
- Rearrangements of functions.

Each method will be illustrated by proving with its help an optimization result either for the Robin Laplacian or for the multi-dimensional Schrödinger operator with a δ -interaction supported on a hyper-surface.

Pascal Naidon • *A short review of Efimov physics*

The Efimov effect is the appearance of an effective scale-invariant long-range force among three particles interacting via short-range pairwise interactions. It is a striking demonstration that, in quantum mechanics, long-range forces may be mediated by particles even though they interact through short-range forces. Since the experimental confirmations of this effect during the last decade, the study of systems related to the Efimov effect has grown into a field known as Efimov physics, bringing interesting experimental and theoretical developments. In this talk, I will review some of these developments, from the point of view of physics, and hopefully inspire new mathematical questions.

Diego Noja • *The linear KdV equation on a star graph*

The linear KdV equation on a star graph will be discussed, as a first and simple model of a network of branching channels. The time independent part of the linear KdV equation is described by the so called Airy operator, a third order differential operator usually studied on a line or a half-line. The main interest is to establish when the Airy operator generates a well defined dynamics, i.e., a contraction semigroup. This property depends on suitable boundary conditions at the vertex of the graph, which will be completely characterised. It turns out that in the special case of unitary dynamics, the graph has to be balanced, i.e., with the same number of ingoing and outgoing edges at the vertex. In both unitary and contraction dynamics, restrictions on admissible boundary conditions occur if conservation of mass (coinciding for this model with the integral of the solution) is further imposed. The above well-posedness results can be considered preliminary to the analysis of nonlinear wave propagation on branching structures. Work in collaboration with Delio Mugnolo and Christian Seifert (Mugnolo D, Noja D, Seifert C, Airy-type evolution equations on star graphs, *Analysis & PDE* 11-7 (2018), 1625-1652).

Andrea Ottolini • Spectral analysis of the 2+1 fermionic trimer with contact interactions

The 2+1 fermionic trimer with contact interaction is a model for two fermions with unit mass and a different particle of mass m subject to an interaction whose range is essentially zero. In this talk, after a brief overview of the rigorous construction for the Hamiltonian for m above the stability threshold, I will give a complete (mass-dependent) picture of its spectrum: essential spectrum, finiteness of the discrete spectrum, existence/absence of bound states. This is based on a joint work with S.Becker and A. Michelangeli.

Andrea Posilicano • Direct and inverse scattering for singular perturbations

After providing a criterion of asymptotic completeness and a representation of the S-matrix for the scattering couple (A_0, A) of semi-bounded self-adjoint operators such that $\{u \in D(A_0) \cap D(A) : A_0 u = Au\}$ is dense, we give applications to the case in which A_0 corresponds to the free Laplacian in $L^2(\mathbb{R}^n)$ and A describes the Laplacian with self-adjoint boundary conditions on rough compact hyper-surfaces. For such models, some results about the inverse scattering problem are also presented. (Joint work with Andrea Mantile, Laboratoire de Mathématiques CNRS, Reims.)

Ludovic Pricoupenko • Zero-range modeling: from Efimov physics to quasi-1D atomic systems

The zero-range modeling is introduced in the context of ultracold physics. A historical survey of the three-body problem is given from the early works of Skorniakov and Ter Martirosian (STM) to the breakthrough of Efimov. Efimov physics is then investigated in the context of ultracold atoms. It is shown how it appears in the 3-fermions plus one-impurity problem. Finally, the issue of "how Efimov states are modified in the presence of a one-dimensional (1D) atomic waveguide" is addressed. The existence of an excited trimer for an arbitrarily large and positive 1D scattering length permits one to exhibit the difference between quasi-1D and strictly 1D systems.

Raffaele Scandone • Smoothing and scattering properties of three-dimensional Schrödinger operators with point interactions

We discuss a two channel model for Feshbach resonances: we provide some results about the number and localization of resonances, and the behaviour of the scattering length near them. In this talk, after an overview of the low-energy spectral behavior of three-dimensional Schrödinger operators with point interactions, I will discuss their smoothing properties, as well as the L_p -boundness of the corresponding wave operators. As an application, I will present some recent results on the well-posedness of the singular Hartree equation in three dimensions. Based on joint works with G. Dell'Antonio, V. Georgiev, F. Iandoli, A. Michelangeli, A. Olgiati, and K.Yajima.

Julian Schmidt • A direct description of Nelson-type Hamiltonians via boundary conditions

When quantum particles are linearly coupled to a real bosonic field, the standard approach is to apply an ultraviolet-renormalisation method in order to define a Hamiltonian for the system. The domain and the action of the Hamiltonian obtained by this procedure are however hard to describe. In this talk I will present a new way of defining such Hamiltonians on Fock space which avoids these difficulties. In joint work with Jonas Lampart, a direct description of the Hamiltonian without UV-cutoff for a class of models – including the so called Nelson model – has been obtained. After introducing the models and briefly explaining the problem and the standard method, I will construct the Hamiltonian without cutoffs directly by using abstract interior-boundary conditions. These boundary conditions relate sectors with different numbers of bosons but the techniques are very similar to those employed in the theory of many-body point interactions. I will conclude by discussing more recent results and possible future projects.

Lorenzo Tentarelli • Non-linear singular perturbations of the fractional Schrödinger equation in dimension one

The talk presents some recent results on non-linear singular perturbations of delta type of the fractional Schrödinger equation $i \partial_t \psi = (-\Delta)^{s/2} \psi + \beta |\psi|^{2\sigma} \psi \cdot \delta$ with $s \in (\frac{1}{2}, 1]$, $\beta \in \mathbb{R}$, $\sigma > 0$ in dimension one. Precisely, I will discuss local and global well posedness (in a strong sense), conservation laws and existence of blow-up solutions and standing waves. This is a joint work with R. Carlone and D. Finco.

Andrea Trombettoni • *Ultracold fermions across the BCS-BEC crossover: optical lattices and Josephson effect*

After discussing the use of ultracold atoms for quantum simulations, I gave a brief introduction on the main properties of ultracold fermions in the BCS-BEC crossover. I then discuss two applications related to their tunneling properties: the attractive Hubbard model in an optical lattice, focusing on role of anisotropic potentials, and the Josephson dynamics of two ultracold fermionic gases at the unitary limit weakly linked by a controllable barrier.

Teoman Turgut • *Some singular models in physics*

It is well-known that the delta-function potential in two dimensions requires renormalisation: we propose a semi-relativistic model of delta-like interaction on two-dimensional manifolds, which is again singular. By means of heat kernel techniques we show that the model can be renormalised along similar lines, thus we obtain a well-defined resolvent. An extension of this delta-like attraction models concentrated on curves is also proposed. We further obtain some recent results on a many-body version the former model.