# Photons as quantum simulators: A fast review of a semi-long story...

#### Dimitris G. Angelakis

Earlier work in while DGA was based CQC Cambridge (03-08) in collaboration with:

Jaeyoon Cho (Belfast) Sougato Bose (UCL) Alastair Kay (Cambridge) Stefano Mancini (Camerino) Marcelo Santos (Belo Horizonte) Daniel Burgarth (Oxford) Dieter Jacksh (Oxford)



Centre for Quantum Computation, University of Cambridge



Science Department, Technical University of Crete Current work in my group formed in 2009 in the Centre for Quantum Technologies, Singapore

DGA (Crete and CQT)

LC Kwek (PI CQT) Elica Kyoseva (post-doc) John Goold (postdoc) Dai Li (PhD student) MingXia Huo (PhD student) Two more post-doc positions open

interaction with CQT experimentalists:

Bjorn Hessmo (PI CQT-cold atoms chip and many body CQED)



Centre for Quantum Technologies

## Outlook

- Review of the basic ideas in using photons for quantum simulations-Initial approaches/motivation
- Our work in Cambridge/Crete/Singapore the last three years
- Latest news-Spin charge separation!
- Future research plans

#### A brief review of the basics: I

Photons in doped coupled cavity arrays for simulating many body effects ?



CCAs could be seen as a system "complementary" to optical lattices

 Co-existence of accessibility to individual constituents AND the strong interaction between them
 Allowing simulations of arbitrary network geometries rather than those derivable from superposing periodic lattices (a larger range of possible simulable Hamiltonians)
 Hybrid solid state optical system

#### **Basics II: Possible platforms**



Coupled defects in photonic crystal structures doped with atoms or q-dots. Imamoglu, Vuckovic, Noda,



Coupled open micorcavities Trupke, Hinds,...



Circuit QED Wallraff, Schoelkopf, Girvin,Mooij…

> Coupled toroidal microresonators Vahala, Kippenberg...



# ...motivated the first work on quantum coherent effects with CCAs in 2004...



 Dimitris G. Angelakis, M. Santos, V. Yanopappas, A.K. Ekert, "A proposal for the implementation of quantum gates with photonic-crystal coupled cavity waveguides", (arXiv:quant-ph/0410189) Phys. Lett. A. Vol.362, 377 (2007)

### Two level atoms and photon blockade as effective nonlinearity in CCAs for many body stuff...?



$$H^{free} = \omega_d \sum_{k=1}^{N} a_k^{\dagger} a_k + \omega_0 \sum_k |e\rangle_k \langle e|_k$$
$$H^{int} = g \sum_{k=1}^{N} (a_k^{\dagger} |g\rangle_k \langle e|_k + H.C)$$
$$H^{hop} = A \sum_{k=1}^{N} (a_k^{\dagger} a_{k+1} + H.C)$$

• DGA, M. Santos, S. Bose, "Photon blockade induced Mott transitions and XY spin models in coupled cavity arrays" (arXiv: **June 06**) Phys. Rev. A (Rap. Com.) vol. 76, 031805 (2007).

This model is now known as the Jaynes-Cummings-Hubbard model

#### Mott transition and "Jaynes-Cummings-Hubbard model"

Photon blockade in action: Probability of getting to one/two polariton excitations per site (thick solid line/thick dashed line) as a function of detuning



• DGA, M. Santos, S. Bose, "Photon blockade induced Mott transitions and XY spin models in coupled cavity arrays" (arXiv: June 06) Phys. Rev. A (Rap. Com.) vol. 76, 031805 (2007).

Parallel work by : M. Hartmann, F. Brandao, M. Plenio, (arXiv: June 06), Nat. Phys. 2, 849 (06)

#### Polariton Mott transition and the birth of "Jaynes-Cummings-Hubbard model"



DGA, M. Santos, S. Bose, "Photon blockade induced Mott transitions and XY spin models in coupled cavity arrays" (arXiv: June 06) Phys. Rev. A (Rap. Com.) vol. 76, 031805 (2007).

#### XY spin models with photons



$$P_{k}^{(+)} = |g,0\rangle_{k} \langle 1,+|_{k}$$
$$P_{k}^{(-)} = |g,0\rangle_{k} \langle 1,-|_{k}$$

$$H = A \sum_{k}^{+} P_{k}^{(-)} P_{k+1}^{(-)} + H.C$$

#### XX+YY model with polaritons

• DGA, M. Santos, S. Bose, "Photon blockade induced Mott transitions and XY spin models in coupled cavity arrays" (arXiv: **June 06**) Phys. Rev. A (Rap. Com.) vol. 76, 031805 (2007).

#### ....So Coupled Cavity Arrays was born.. and trying to walk its first steps...



The last three years roughly 120 papers followed based on these initial works, originating from various groups-and us-studying:

Incomplete list of groups currently working in CCAs: Cambridge-Crete/Singapore, Pisa, Imperial/TUM, Stanford, Beijing, Harvard, Belfast, Salerno, Zurich, Cavendish, Camerino,, Queensland...

Effective spin models: XY, XXZ, high-spins, ....

The phase diagram and entanglement...

Fractional Hall states...

Photon propagation in CCAs...

Fermionized/Tonks-Girardeau photon gases...

Driven cavities and non-equilibrium many body effects

# Highlights of our work in CCAs the last two years

- 1. Photon blockade Mott transition and XY spin models
- 2. Creating cluster states and implementing measurement based QC
- 3. Simulating full Heisenberg spin models and high spins
- 4. Heralded generation of two-photon polarization entanglement with coupled cavities
- 5. Driven cavity arrays and steady state quantum correlations
- 6. Fractional Hall Effect
- 7. Efficient implementation of Grover search
- 8. Coherent control of many body properties
- 9. Spin charge separation with light!

## Highlights I: CCAs cluster states and measurement based quantum computing



• Dimitris G. Angelakis, Alastair Kay, "Weaving light-matter qubits into a one way quantum computer", New J. Phys. Vol. 10, 023012 (2008).

#### Highlights II: Effective spin models

1D XXZ spin models and Generalized spin models (hexagonal Kitaev)



Alastair Kay and Dimitris G. Angelakis, "Reproducing spin lattice models in strongly coupled atomcavity systems. Eur. Phys. Lett. 84 (2008) 20001
Jaeyoon Cho, Dimitris G. Angelakis, Sougato Bose, "Simulation of high-spin Heisenberg chains in coupled cavities." Phys. Rev. A 78 062338 (2008).

# Spin lattice models using V systems and two polarizations-no lasers



 $H_{int} = \omega_0 \left( a a^{\dagger} + b b^{\dagger} + |A\rangle \langle A| + |B\rangle \langle B| \right)$  $+ g \left( |A\rangle \langle g| \otimes a + |g\rangle \langle A| \otimes a^{\dagger} \right)$  $+ g \left( |B\rangle \langle g| \otimes b + |g\rangle \langle B| \otimes b^{\dagger} \right),$ 

In the basis  $|\psi, N_A, N_B\rangle$ , we can calculate that the (unnormalised) on-site eigenvectors are

$$\begin{aligned} \left| \Psi_{S,n}^{0} \right\rangle &= \sqrt{S-n} \left| A, n-1, S-n \right\rangle - \sqrt{n} \left| B, n, S-n-1 \right\rangle \\ \Psi_{S,n}^{\pm} \right\rangle &= \sqrt{n} \left| A, n-1, S-n \right\rangle + \sqrt{S-n} \left| B, n, S-n-1 \right\rangle \\ &\pm \sqrt{S} \left| g, n, S-n \right\rangle \end{aligned}$$

with energies  $S\omega_0$  and  $S\omega_0 \pm g\sqrt{S}$  respectively (see Fig. 1).  $N_A$  and  $N_B$  are the number of a and b photons in the cavity, and  $\psi$  is the state of the atom.

 $\begin{array}{l} |0\rangle \ = \ (|A,0,0\rangle - |g,1,0\rangle)/\sqrt{2} \\ |1\rangle \ = \ (|B,0,0\rangle - |g,0,1\rangle)/\sqrt{2} \end{array}$ 

A.Kay and D. G. Angelakis, . Eur. Phys. Lett. 84 (2008) 200018 Reproducing spin lattice models in strongly coupled atom-cavity systems

#### Spin lattice models using $\Lambda$ systems using external lasers



Coping with spontaneous emission and dissipation by making interactions off-resonant through the application of a number of **constant** external fields (only virtual photons mediate the interactions now) and also no need for Troter expansion

Simulation of high-spin Heisenberg chains in coupled cavities. Jaeyoon Cho, Dimitris G. Angelakis, Sougato Bose, Phys. Rev. A 78 062338 (2008).

#### Spin lattice models-XXZ



$$H_{hop} = J_a(a_i^{\dagger}a_{i+1} + a_i a_{i+1}^{\dagger}) + J_b(b_i^{\dagger}b_{i+1} + b_i b_{i+1}^{\dagger}),$$

$$H_{\text{eff}} = \sum_{a,b \in \{0,1\}^2} \left| b \right\rangle \left\langle a \right| \sum_{\mu} \frac{\left\langle b \right| H_{hop} \left| \mu \right\rangle \! \left\langle \mu \right| H_{hop} \left| a \right\rangle}{E - E_{\mu}}$$

$$\begin{array}{ll} |0\rangle &=& (|A,0,0\rangle - |g,1,0\rangle)/\sqrt{2} \\ |1\rangle &=& (|B,0,0\rangle - |g,0,1\rangle)/\sqrt{2} \end{array}$$

$$H_{\text{eff}} = -B_z(\mathbb{1} \otimes Z + Z \otimes \mathbb{1}) - \lambda_z Z \otimes Z - \lambda_x(XX + YY)$$

where

$$\kappa = \frac{31}{32g} \left( J_a^2 + J_b^2 \right) \quad B_z = \frac{5}{8g} \left( J_a^2 - J_b^2 \right)$$
$$\lambda_z = \frac{9}{32g} \left( J_a^2 + J_b^2 \right) \quad \lambda_x = \frac{9J_a J_b}{16g}$$

A.Kay and D. G. Angelakis, . Eur. Phys. Lett. 84 (2008) 20001

#### Generalized spin models-Kitaev's hexagonal



A. Kay and D. G. Angelakis. Eur. Phys. Lett. 84 (2008) 20001



#### Higher spins

$$H_{\text{eff}} = -B_z(\mathbb{1} \otimes Z + Z \otimes \mathbb{1}) - \lambda_z Z \otimes Z - \lambda_x (XX + YY)$$

where

$$\kappa = \frac{31}{32g} \left( J_a^2 + J_b^2 \right) \quad B_z = \frac{5}{8g} \left( J_a^2 - J_b^2 \right)$$
$$\lambda_z = \frac{9}{32g} \left( J_a^2 + J_b^2 \right) \quad \lambda_x = \frac{9J_a J_b}{16g}$$

If there is an average of S excitations per ite, then there are S + 1 ground states,  $\Psi_{S;n}^{i}$ , or n = 0 to S, enabling the simulation of spin-S particle. For **qutrits**:

$$J_X = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} / \sqrt{2} \qquad \qquad J_Y = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} / \sqrt{2},$$

$$J_{Z} = -i[J_{X}, J_{Y}] \text{ and}$$

$$\kappa = \frac{124\sqrt{2}}{7g} \left(J_{a}^{2} + J_{b}^{2}\right) \quad B_{z} = \frac{53}{2\sqrt{2}g} \left(J_{a}^{2} - J_{b}^{2}\right)$$

$$\lambda_{z} = \frac{123}{7\sqrt{2}g} \left(J_{a}^{2} + J_{b}^{2}\right) \quad \lambda_{x} = \frac{123\sqrt{2}J_{a}J_{b}}{7g}.$$

### Highlights III: Generating the Fractional Hall state...



Jaeyoon Cho, Dimitris G. Angelakis, Sougato Bose, "Fractional Quantum Hall state in coupled cavities" Phys. Rev. Lett. 101, 246809 (2008)

#### Recent works in the last 6 months in CQT Singapore and in Crete

- Elica Kyoseva, Dimitris G. Angelakis, LC Kwek, A single-interaction step implementation of a quantum search in coupled micro-cavities arXiv:0908.3308.
- Coherent control of steady state entanglement in driven cavity arrays Dimitris G. Angelakis, Li Dai, LC Kwek, arXiv:0906.2168
- Steady state entanglement between distant hybrid light-matter qubits under classical driving. Dimitris G. Angelakis, Stefano Mancini, Sougato Bose, Eur. Phys. Lett. 85 20007 (2009).
- Simulating spin-charge separation with light. D G. Angelakis, M. Huo. E. Kyoseva, LC Kwek, arXiv:1006:1644

# Quantum search in a "single" interaction step...

•Information in the lowest two hyperfine states of the dopants

•Applying an appropriately tuned global laser, the global reflection operator needed for the quantum search algorithm can be realized in **a** *single physical operation*.



Elica Kyoseva, Dimitris G. Angelakis, LC Kwek, A single-interaction step implementation of a quantum search in coupled micro-cavities arXiv:0908.3308.EPL 2010

### Driven cavity arrays I-Steady state Entanglement



D. G. Angelakis, S. Mancini, S. Bose Steady state entanglement between distant hybrid light-matter qubits under classical driving. Eur. Phys. Lett. 85 20007 (2009).

### Driven cavity arrays II: Coherent control of many body correlations





Three defects coupled to three waveguides pumbed by classical fields

Two defects coupled to three pumped waveguides

Steady state quantum correlations between any pair coherent controlled through the tuning of the phase difference between the driving fields.

Correlations could be probed by looking at the correlations of the emitted photon spectrum DGA, Dai Li, Kwek LC, Preprint: arXiv:0906.2168, EPL 2010

# Alternative systems to look for strongly correlated photon effects: Polaritons in gases

Strongly interacting photons effects could be observed in an "effective easier to implement" coupled cavity system?



Use ideas from polariton propagation and EIT type of effects in atomic gases to look for many body photon correlations

Tonks gas of photons

Explore the quantum simulator abilities of similar system towards a range of condensed matter effects.

DGA, Huo, Kyoseva, Kwek : arXiv:0911.1644

## Luttinger liquid physics with light



DGA, et al: arXiv:1006.1644

### Hamiltonian of the system $H = H^{a} + H^{b}$

$$\begin{split} \mathsf{H}^{\mathsf{x}} &= - \ \hbar n_{z}^{\mathsf{x}} \sum_{i=1}^{2} \int dz \{ -\omega_{33}^{\mathsf{x}} \sigma_{33}^{\mathsf{x}} + (-\omega_{q}^{(i)} + \Delta_{2}^{\mathsf{x}}) \sigma_{22}^{\mathsf{x}} \\ &+ \ (-\omega_{33}^{\mathsf{x}} - \omega_{q}^{(i)} - \Delta_{4}^{\mathsf{x}}) \sigma_{44}^{\mathsf{x}} + g_{i}^{\mathsf{x}} \sqrt{2\pi} (\sigma_{21}^{\mathsf{x}} + \sigma_{43}^{\mathsf{x}}) \times \\ & \left( \hat{E}_{i,+} \mathsf{e}^{\mathsf{i}(k_{qu}^{(i)} z - \omega_{qu}^{(i)} t)} + \hat{E}_{i,-} \mathsf{e}^{\mathsf{i}(-k_{qu}^{(i)} z - \omega_{qu}^{(i)} t)} \right) \\ &+ \ \left( \Omega_{i,+}(t) \mathsf{e}^{\mathsf{i}(k_{cl}^{(i)} z - \omega_{cl}^{(i)} t)} + \Omega_{i,-}(t) \mathsf{e}^{-\mathsf{i}(k_{cl}^{(i)} z + \omega_{cl}^{(i)} t)} \right) \sigma_{23}^{\mathsf{x}} \\ &+ \ \mathsf{H.c.} \} \end{split}$$

$$u_g^{(1)} = 
u^{(1)} \Omega_1^2 / \pi(g_1^a)^2 n_z^a \text{ and } 
u_g^{(2)} = 
u^{(2)} \Omega_2^2 / \pi(g_2^b)^2 n_z^b$$

#### Group velocities

## Equations of motion

$$\left(\frac{\partial}{\partial t} \pm \nu^{(1)} \frac{\partial}{\partial z}\right) \hat{E}_{1,\pm}(z,t) = \mathrm{i}\sqrt{2\pi} n_z^{\mathrm{a}} g_1^{\mathrm{a}} \left(\sigma_{12,\pm}^{\mathrm{a}}(z,t) + \sigma_{34,\pm}^{\mathrm{a}}(z,t)\right)$$

$$\left(\frac{\partial}{\partial t} \pm \nu^{(2)} \frac{\partial}{\partial z}\right) \hat{E}_{2,\pm}(z,t) = \mathrm{i}\sqrt{2\pi} n_z^{\mathrm{b}} g_2^{\mathrm{b}} \left(\sigma_{12,\pm}^{\mathrm{b}}(z,t) + \sigma_{34,\pm}^{\mathrm{b}}(z,t)\right)$$

Setting  $\Psi_{1,2} = (\Psi_{1,2;+} + \Psi_{1,2;-})/2$  and  $A_{1,2} = (\Psi_{1,2;+} - \Psi_{1,2;-})/2$ 

$$\begin{split} \partial_t \Psi_1 + \nu^{(1)} \partial_z A_1 &= - \sqrt{2\pi} \frac{(g_1^{\rm a})^2}{2\Omega_1^2} n_z^{\rm a} \partial_t \Psi_1 \\ &- \mathrm{i} \frac{(g_1^{\rm a})^2}{\sqrt{2\pi} \Delta_4^{\rm a}} \left( 2\Psi_1^{\dagger} \Psi_1 + A_1^{\dagger} A_1 \right) \Psi_1 \\ &- \mathrm{i} \frac{(g_1^{\rm a})^2 (g_2^{\rm a})^2 \Omega_2^2}{\sqrt{2\pi} (g_1^{\rm b})^2 \Delta_4^{\rm a} \Omega_1^2} \left( \Psi_2^{\dagger} \Psi_2 + A_2^{\dagger} A_2 \right) \Psi_1 \\ &+ \mathrm{noise}, \\ \partial_t A_1 + \nu^{(1)} \partial_z \Psi_1 &= - \mathrm{i} \sqrt{2\pi} \frac{(g_1^{\rm a})^2}{\Delta_2^{\rm a}} n_z^{\rm a} A_1 - \frac{(g_1^{\rm a})^2}{\sqrt{2\pi} \Delta_4^{\rm a}} \Psi_1^{\dagger} \Psi_1 A_1 \\ &+ \mathrm{noise} \end{split}$$

# Two component Lieb Liniger model for polaritons

$$H \;=\; \hbar \int dz \left\{ \sum_i \left[ rac{1}{2m_i} \partial_z \Psi_i^\dagger(z) \partial_z \Psi_i(z) + rac{U_i}{2} 
ho_i^2(z) 
ight] + V_{12} 
ho_1(z) 
ho_2(z) 
ight\}$$

$$\frac{1}{m_1} = -\frac{4\Delta_2^a \nu_g^{(1)}}{\Gamma_{1D}^a n_z^a}$$
 and  $\frac{1}{m_2} = -\frac{4\Delta_2^b \nu_g^{(2)}}{\Gamma_{1D}^b n_z^b}$ 

Effective masses

$$U_1 = rac{\Gamma_{1D} 
u_g^{(1)}}{2\Delta_4^a} \ U_2 = rac{\Gamma_{1D} 
u_g^{(2)}}{2\Delta_4^b}$$

Interspecies repulsion

$$V_{12} = \frac{\pi (g_1^a)^2 (g_2^a)^2 \nu_g^{(1)}}{\left(g_2^b\right)^2 \Delta_4^a \nu^{(1)}} + \frac{\pi (g_2^b)^2 \left(g_1^b\right)^2 \nu_g^{(2)}}{\left(g_1^a\right)^2 \Delta_4^b \nu^{(2)}}$$

Intra-species repulsion

## Spin-charge separation

Let us now define the effective parameters  $u_i =$  $\sqrt{\rho_{0,i}U_i/m_i}$  and  $K_i = \sqrt{\pi^2 \rho_{0,i}/(m_i U_i)}$  with  $\rho_{0,i}$  equal to  $n_{ph,i}$ , the photon number of *i*th quantum field in our scheme. It is known from the works by Girardeau and others, that when  $u_1 = u_2 = u$  and  $K_1 = K_2 = K$  the above LL Hamiltonian Eq. 1 can be transformed into new one comprised separately of two parts, the charge part  $H_c$ and the spin part  $H_s$  as  $H = H_c + H_s$  [1]. This allows for the separation of a single excitation into two separate ones each comprised of spins or of charge/density. These can propagate through the liquid with different velocities given by  $u_{c,s} = u\sqrt{1 \pm \frac{(V_1+V_2)K}{\pi u}}$ . In our case, the charge(spin) density corresponds to the sum(difference) of the corresponding polaritons densities  $n_{c,s} = n_1 \pm n_2$ with  $n_i = \langle \Psi_i^{\dagger} \Psi_i \rangle$  and  $\Psi_i = (\Psi_{i;+} + \Psi_{i;-})/2$  the symmetric combinations of the two counter-propagating dark state polaritons generated by each set of atom-field interaction.

# Preparation, driving and measurement



- 1. Preparation:
- a) Two few photon coherent pulsesE1 and E2 enter the fiber from the left
- b) Control fields adiabatically off
- c) Formation and trapping of  $\Psi$ 1,  $\Psi$ 2.

# Preparation, driving and measurement



- 1. Preparation:
- a) Two few photon coherent pulses E1 and E2 enter the fiber from the left
- b) Control fields adiabatically off
- c) Formation and trapping of  $\Psi$ 1,  $\Psi$ 2.
- 2. Driving to the strongly interacting regime:
- a) Intra and inter species repulsions are tuned up though dynamic control of the detunings. Both pairs of control fields are also turned on.
- b) Effective Bragg grating formed trapping the polaritons
- c) Evolution under the LL Hamiltonian
- 3. Measurement----→

# Preparation, driving and measurement



There are two ways to observe the separation in this system:

 By measuring the time evolution of a single excitation as usually predicted for the case of cold atoms proposals as well

In our case the charge (spin) density waves, after the release of the polaritons, will transfer to the sum (difference) of the corresponding time dependent photon intensities for each propagating field

Or by directly measuring the spectral function on the propagating light pulses...

#### Direct optical measurement of the spectral function



The single particle spectral function for our photonic system. The effective spin and charge velocities are  $u_{s}=0.5$  and  $u_{c}=1$  and can be achieved for optical depths around OD=4000, with 10 photons in each pulse initially, and single atom cooperativity for each atomic species of 0.2. Measurement is done though standard quantum optical measurement techniques measuring correlations in the intensities of the output fields and by analyzing the optical spectrum.

# Summary

State of the art in "photonic quantum simulators" in CCAs

A range of "simulable" many body effects in CCAs.

- Mott transition
- Spin models
- Hall effect
- Non equilibrium studies
- Luttinger Liquid and Spin charge separation

#### Future

- Need for realistic studies of specific promising platforms -driven defects in photonic crystals/ Circuit QED/open cavities...new systems (hollow fibers)?
- Explore the versatility of current systems simulation of more complex many body effects

#### Quantum Optics and Many Body Effects - Distributed Q.I.P groups



Front row: L.C. Kwek, L. Dai, Y. Li, CL Ching, E. Kyoseva, M. Huo, E. Tan (secretary) Back row: H. Reslen(visitor), S. Benjamin, D. Browne, H. Cable, Setiawan, D.G. Angelakis