Problem : Point co-group of a k point

The little group of a **k** point of the Brillouin zone is a subgroup of the space group of the solid $\{(S_i, \boldsymbol{\tau}_i)\}$ (S_i is a rotation and $\boldsymbol{\tau}_i$ a translation) that contains all the operations for which the rotational part satisfies the equation:

$$S_i \mathbf{k} = \mathbf{k} + \mathbf{G}_{S_i}, \tag{1}$$

where $\mathbf{G}_{S_i}$ is a reciprocal lattice vector. The point group formed by the operations $\{S_i\}$ that satisfy this relation is called small point group of **k** or point co-group of **k**.

- Consider the **k** points Γ, X, M, R of the Brillouin zone of the cubic lattice. For each of these points find the point co-group of **k** assuming a monatomic solid with one atom per cell.

- Same problem for the **k** points Γ, X, K, W, U, and L of the Brillouin zone of the face-centered cubic lattice.

- Same problem for the **k** points Γ, H, N, P, of the Brillouin zone of the body-centered cubic lattice.

- Same problem for the **k** points Γ, K, M, A, H, L of the Brillouin zone of the hexagonal lattice.

- Write a code that receives in input the rotation matrices of the space group of a solid, the primitive vectors of the reciprocal lattice, and the coordinates of a **k** point and gives as output the list of rotation matrices that belong to the point co-group of **k**. Check the correctness of your code on some of the **k** points analyzed in the previous questions.

Hint: The definition of the **k** points can be found in the file `brillouin_zones.pdf` in the `Quantum ESPRESSO` documentation.