Supersymmetric extensions of the SM

Andrea Romanino
SISSA

Plan

* Introduction and motivations
* From general principles to the most general renormalizable N=1 supersymmetric lagrangian
* The MSSM: definition and analysis
  - Electroweak symmetry breaking
  - Spectrum
  - Phenomenology
* Beyond the MSSM

Supersymmetry:
fermions $\leftrightarrow$ bosons

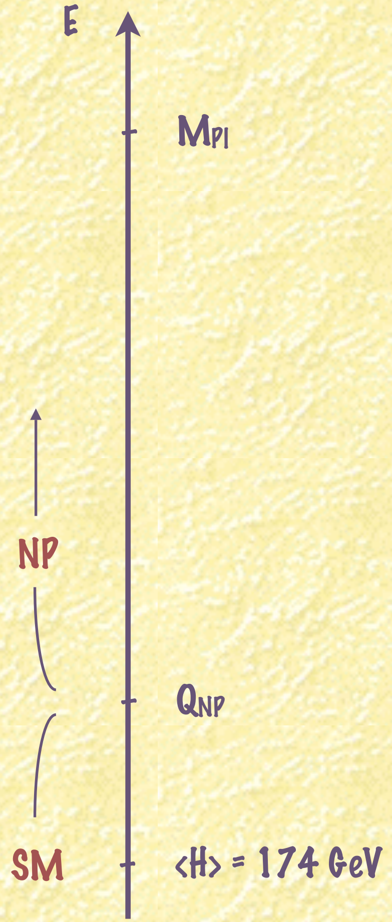
Motivations

- Phenomenological
  - Solves the hierarchy problem
  - Precisely predicts gauge coupling unification
  - Provides a natural DM candidate (needs R_P)
  - See below...
- Theoretical
  - Unification of fermions and bosons
  - Local supersymmetry = supergravity + crucial in string theory
  - Completes the list of possible symmetries of S (under hypotheses)
  - Powerful technical tool

Beyond the Standard Model

* Experimental "problems" of the SM:
  - Gravity
  - Dark matter
  - Baryon asymmetry
* Experimental hints of physics beyond the SM
  - Neutrino masses
  - Quantum number unification
* Theoretical puzzles of the SM:
  - $\langle H \rangle \ll M_{Pl}$
  - Family replication
  - Small Yukawa couplings, pattern of masses and mixings
  - Gauge group, anomaly cancellation, charge quantization, quantum numbers
* Theoretical problems of the SM:
  - Hierarchy or Naturalness problem
  - Cosmological constant problem
  - Strong CP problem

The hierarchy problem as a handle on new physics



* The SM is an effective theory valid below a cut-off Q_{NP}

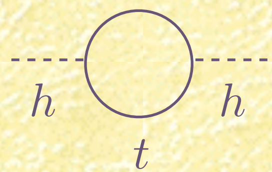
  Where is Q_{NP}? Which physics at Q_{NP}?

* The main guideline is still provided by the naturalness argument (hierarchy problem)

* Which arises if the Higgs exists as a fundamental interacting scalar up to Q_{NP} and $Q_{NP} \gg m_h$

$$\delta m_h^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 Q_{\rm NP}^2 = \begin{cases} m_h^2 \left(\dfrac{Q_{\rm NP}}{0.5\,\mathrm{TeV}}\right)^2 & \text{if } m_h = 115\,\mathrm{GeV} \\ m_h^2 \left(\dfrac{Q_{\rm NP}}{2\,\mathrm{TeV}}\right)^2 & \text{if } m_h = 250\,\mathrm{GeV} \end{cases}$$

More on renormalizability and naturalness

- $\delta m_h^2 \sim \delta m_h^2(\text{top}) \approx$ 

$= 12\,\lambda_t^2 \int \frac{k^3 dk}{8\pi^2}\frac{1}{k^2} + \ldots \xrightarrow{\text{cut-off}} \frac{3G_{\rm F}}{\sqrt{2}\pi^2} m_t^2 Q^2 + \ldots$

- Renormalization: $(m_h^2)_{\rm phys} \approx (m_h^2)_{\rm tree} + \frac{3G_{\rm F}}{\sqrt{2}\pi^2} m_t^2 Q^2, \quad Q \to \infty$

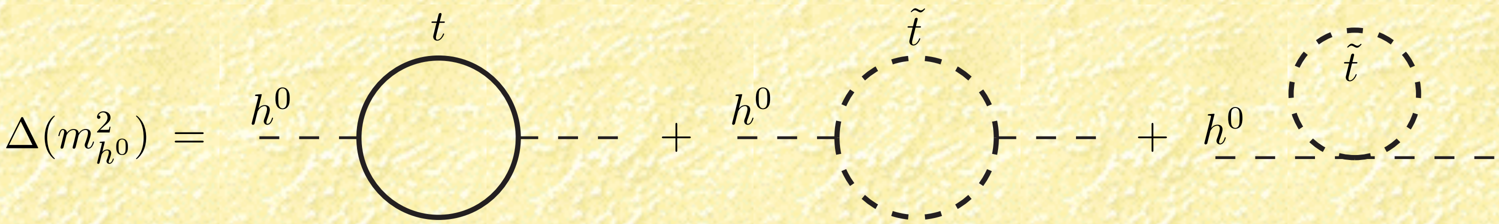
- The naturalness problem arises if Q corresponds to a physical threshold

[Luty, hep-th/0509029]

- (renormalizability might not be a fundamental property of 4D QFT)

How supersymmetry solves the hierarchy problem

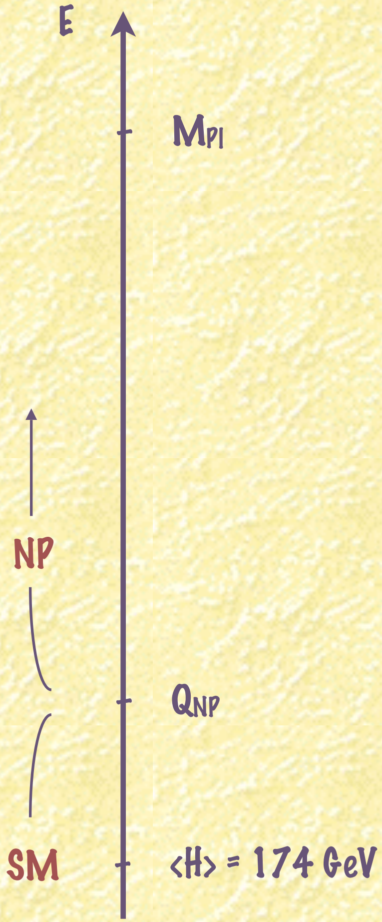
$$\frac{3}{4\pi^2}\lambda_t^2 Q^2 \qquad\qquad -\frac{3}{4\pi^2}\tilde{\lambda}_t Q^2$$



$$\boxed{\lambda_t^2 = \tilde{\lambda}_t}$$

- Note that it is crucial that the coupling are exactly equal. Supersymmetry breaking, if it is not to spoil the solution of the hierarchy problem should maintain this equality

How well supersymmetry solves the hierarchy problem: LEP and the residual hierarchy



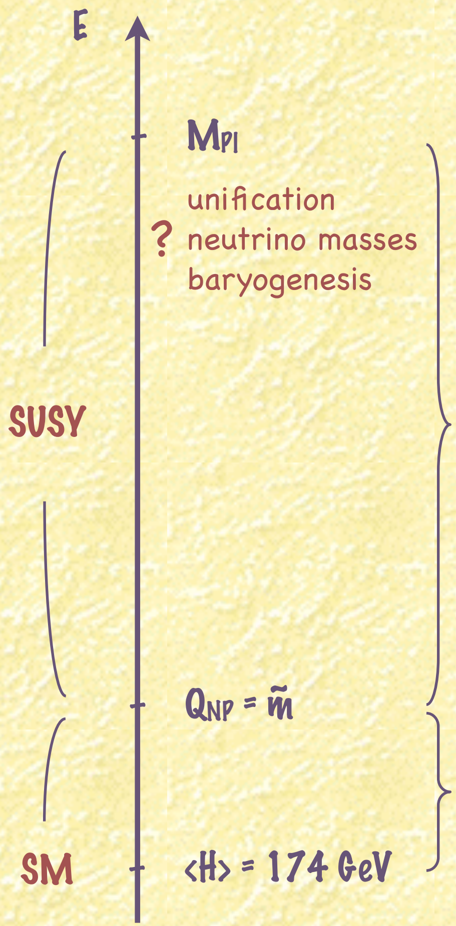
No hint of physics at Q_{NP}:

$$\mathcal{L}_{\rm SM}^{\rm eff}(E < Q_{\rm NP}) = \mathcal{L}_{\rm SM}^{\rm ren} + \sum_i \frac{c_i}{Q_{\rm NP}^2}\mathcal{O}_i + \ldots \qquad \frac{c_i}{Q_{\rm NP}^2} \lesssim \frac{1}{(5\,{\rm TeV})^2}$$

$$Q_{\rm NP} \gtrsim \sqrt{c_i}\cdot 5\,{\rm TeV} \approx \begin{cases} 50\,{\rm TeV} & \text{composite SM fermions} \\ 5\,{\rm TeV} & \text{composite Higgs} \\ 0.5\,{\rm TeV} & \text{1-loop perturbative} \end{cases}$$

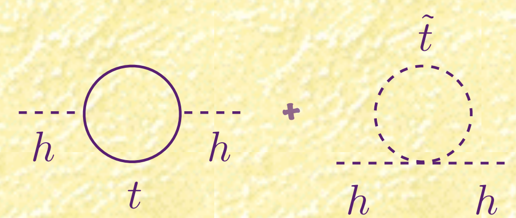
$$\delta m_h^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 Q_{\rm NP}^2 = \begin{cases} m_h^2 \left(\dfrac{Q_{\rm NP}}{0.5\,{\rm TeV}}\right)^2 & \text{if } m_h = 115\,{\rm GeV} \\ m_h^2 \left(\dfrac{Q_{\rm NP}}{2\,{\rm TeV}}\right)^2 & \text{if } m_h = 250\,{\rm GeV} \end{cases}$$

MSSM



$$Q_{\rm NP} \gtrsim \sqrt{c_i} \cdot 5\,\mathrm{TeV} \approx 0.5\,\mathrm{TeV}$$

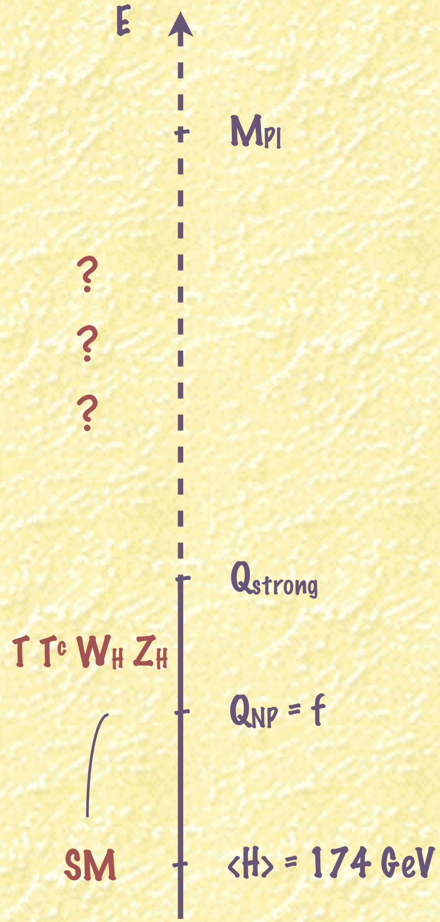
$$\delta m_h^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 Q_{\rm NP}^2 \log \frac{M_{\rm Pl}^2}{Q_{\rm NP}^2}$$



$$\delta m_h^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 Q_{\rm NP}^2 = \begin{cases} m_h^2 \left(\dfrac{Q_{\rm NP}}{0.5\,\mathrm{TeV}}\right)^2 & \text{if } m_h = 115\,\mathrm{GeV} \\ m_h^2 \left(\dfrac{Q_{\rm NP}}{2\,\mathrm{TeV}}\right)^2 & \text{if } m_h = 250\,\mathrm{GeV} \end{cases}$$

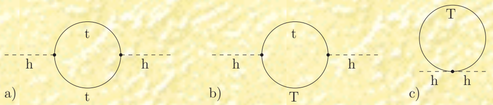
Little Higgs

Higgs mass protected by H(x) → H(x) + c



- Q_{NP}: Q_{strong} → f = global symmetry breaking scale (separate the top loop cutoff from Q_{strong} > 5 TeV)
- Bounds on Q_{NP} from EWPT still worse than MSSM (unless T-parity is used), FT similar
- No dramatic gain but interesting alternative

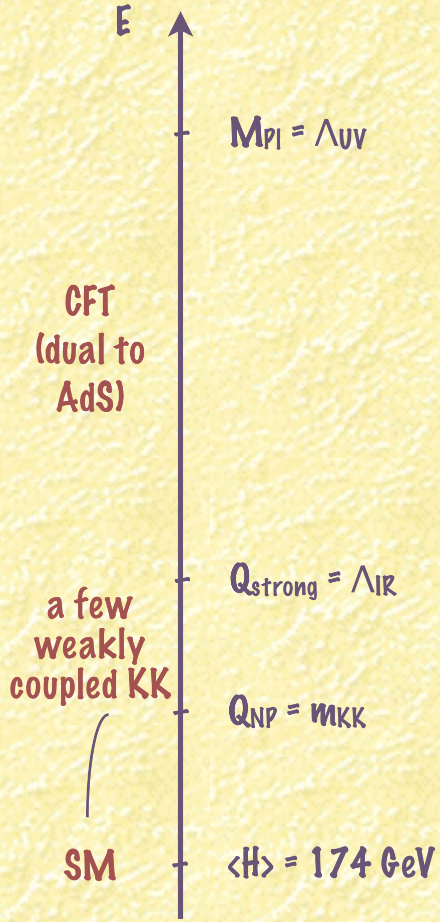
$$\delta m_h^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 Q_{\rm NP}^2 \log \frac{Q_{\rm strong}^2}{Q_{\rm NP}^2} + \text{2-loop}$$

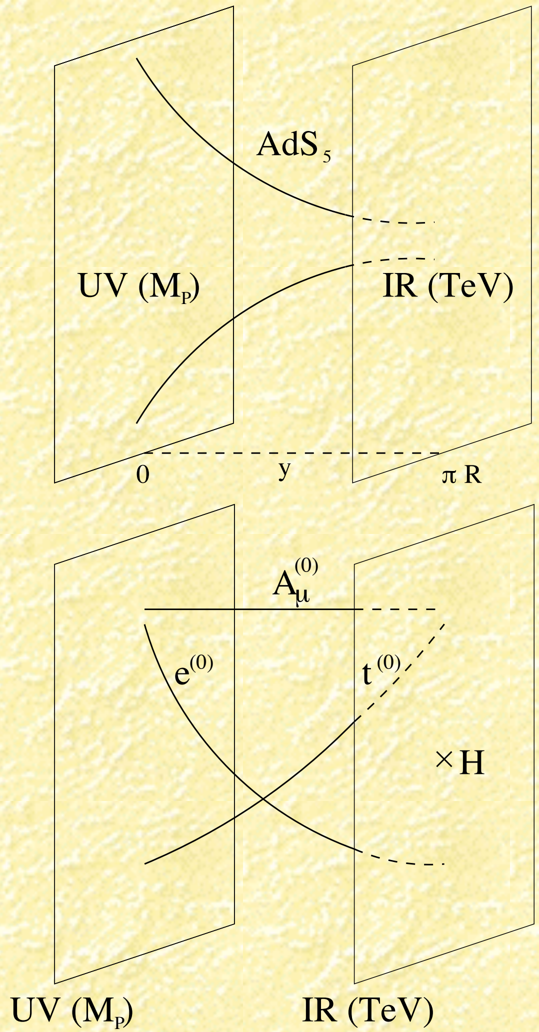


$$\delta m_h^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 Q_{\rm NP}^2$$

[Arkani-Hamed Cohen Georgi 01, Arkani-Hamed Cohen Katz Nelson 02, Arkani-Hamed Cohen Katz Nelson Gregoire Wacker 02]

Warping and composite Higgs





$$m_H \sim M_5 e^{-\pi k R}$$

$$k = \text{curvature}$$

- Breaking of G_{bulk} by bc's: $H = (A_5)_0$, or Little Higgs + UV completion and solution of the hierarchy problem
- m_H protected from Q_{strong} by 5D gauge symmetry, or collective breaking
- UV brane: elementary
  IR brane: composite (H, t_R)
- Q_{strong} > 5 TeV as usual
  m_{KK} > TeV, watch $Z \to b\bar{b}$
- Gauge coupling unification in a novel way (but limited calculability)

[Contino Nomura Pomarol hep-ph/0306259
Agashe Contino Pomarol hep-ph/0412089
hep-ph/0605341]

The cosmological constant problem

$$\delta m_H^2 \propto Q_{\rm SM}^2 \rightarrow Q_{\rm SM} \sim m_H$$

$$\text{SUSY:} \quad \delta m_H^2 \propto \tilde{m}^2 \log \frac{Q_{\rm SUSY}}{\tilde{m}}$$

$$\delta\Lambda \propto Q_x^4 \rightarrow Q_x \sim 10^{-3}\,\text{eV???}$$

$$\text{SUSY:} \quad \delta\Lambda \propto \tilde{m}^2 Q_{\rm SUSY}^2$$

Notations

- η = diag(+---), $(\sigma_\mu) = (\mathbf{1},\sigma_i)$, $\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\epsilon_{\alpha\beta}\,\epsilon^{\beta\gamma} = \delta_\alpha{}^\gamma$

- Ψ Dirac spinor $\leftrightarrow$ ψ, ψ_c left-handed Weyl spinors: $\Psi = \begin{pmatrix} \epsilon\,\psi_c^* \\ \psi \end{pmatrix}$

$$\underbrace{\Psi_L, \overline{\Psi_R}}_{(0,1/2)} + \underbrace{\Psi_R, \overline{\Psi_L}}_{(1/2,0)} \leftrightarrow \boxed{\psi, \psi_c} + \psi^*, \psi_c^*$$

$$\overline{\Psi_1}\Psi_2 = \psi_1^c\psi_2 + (\psi_1\psi_2^c)^* \quad \overline{\Psi_1}\gamma^\mu\Psi_2 = \psi_1^\dagger\sigma^\mu\psi_2 - (\psi_2^c)^\dagger\sigma^\mu\psi_1^c$$

$$\Psi_L = \begin{pmatrix} 0 \\ \psi \end{pmatrix}, \Psi_R = \begin{pmatrix} \epsilon\,\psi_c^* \\ 0 \end{pmatrix}$$

$$(\psi_1\psi_2 = \psi_2\psi_1 = \psi_{1\alpha}\epsilon^{\alpha\beta}\psi_{2\beta})$$

From general principles to the most general renormalizable N=1 supersymmetric lagrangian

The general supersymmetric algebra

[Sohnius, Phys Rept 128 (1985)
Wess and Bagger, Supersymmetry and supergravity, Univ. Pr. (1992)
Martin, hep-ph/9709356
Nilles, Phys Rept 110 (1984)]

$\mathcal{G}$ = set of symmetry generators G such that

- G S = S G
- G = $\int d\alpha\, d\beta\, G_{ij}(\alpha,\beta) a^\dagger_{i\alpha} a_{j\beta}$ on asymptotic states
- Spin statistics connection holds

($a^\dagger_{i\alpha}$ creates the particle i with quantum numbers α)

THEN

[Coleman Mandula, Phys Rev 159 (1967)
Haag Lopuszanski Sohniius, Nucl. Phys B88 (1975)]

* $\mathcal{G} = \mathcal{B} + \mathcal{F}$, where $\mathcal{B} \ni$ B: b → b, f → f; $\mathcal{F} \ni$ F: b → f, f → b

* $\mathcal{G}$ is a graded Lie algebra: $[\mathcal{B},\mathcal{B}]$, $\{\mathcal{F},\mathcal{F}\} \subseteq \mathcal{B}$, $[\mathcal{B},\mathcal{F}] \subseteq \mathcal{F}$

  $[[G_1,G_2]_\pm,G_3]_\pm + [[G_2,G_3]_\pm,G_1]_\pm \pm [[G_3,G_1]_\pm,G_2]_\pm = 0$ "graded Jacobi identity"

  $G_i \in \mathcal{G}$ bosonic or fermionic, "-" if two are fermions

  $[G_1,G_2]_\pm = [G_1,G_2]$ if G_1 or G_2 bosonic, $= \{G_1,G_2\}$ otherwise

* $\mathcal{B}$ = Poincaré $\oplus$ $\mathcal{L}_{int}$ generators: P_μ $L_{\mu\nu}$ B_r (hermitian)

  $U(a,L)^\dagger B_r\, U(a,L) = B_r$ (or $[P_\mu, B_r] = [L_{\mu\nu}, B_r] = 0$)

  $\mathcal{L}_{int}$ = compact semisimple $\oplus$ abelian

* $\mathcal{F}$ = supersymmetry generators: $Q_{i\alpha}$, $\bar{Q}_{i\alpha} = (Q_{i\alpha})^\dagger$

  i = 1,...,N number of supersymmetries α = 1,2 left-handed Weyl index

  $U(a,L)^\dagger\, Q_{i\alpha}\, U(a,L) = L_\alpha{}^\beta\, Q_{i\beta}$

  $U(g)^\dagger\, Q_{i\alpha}\, U(g) = R(g)_{ij}\, Q_{j\alpha}$

  $\{Q_{i\alpha}, \bar{Q}_{j\beta}\} = 2\,\delta_{ij}\sigma^\mu_{\alpha\beta}P_\mu \quad \{Q_{i\alpha}, Q_{j\beta}\} = 2\,\epsilon_{\alpha\beta}Z_{ij}$

  $Z_{ij} \in \mathcal{L}_{int}$ antisymmetric, $[Z_{ij}$, anything$] = 0$ ("central" charges)

Properties and N=1

* Supersymmetry generators: b ↔ f; #b = #f
* $[P^2, Q_{i\alpha}] = 0 \Rightarrow m_b = m_f$: supersymmetry must be broken
* $\langle\Omega|H|\Omega\rangle \propto \Sigma_{i\alpha} (|Q_{i\alpha}\Omega|^2 + |\bar{Q}_{i\alpha}\Omega|^2) \geq 0$: SSSB ⇔ vacuum energy > 0
* N supersymmetries: massive 1P states have $j \geq N/2$

  massless 1P states have $|j| \geq N/4$ (if odd, $N \rightarrow N+1$)
* $j \leq 2 \Rightarrow N \leq 8$

  $j \leq 1 \Rightarrow N \leq 4$

  chiral gauge theory $\Rightarrow N \leq 1$

N=1 supersymmetry algebra

- $\mathcal{G}$ = Poincaré + Internal group generators + Q_α, $\bar{Q}_\alpha$
- $Q_\alpha \to L_\alpha{}^\beta\, Q_\beta$, $[P_\mu, Q_\alpha] = 0$

  $Q_\alpha \to e^\omega\, Q_\alpha$ ("R-symmetry") or invariant under internal symmetries

  $\{Q_\alpha, \bar{Q}_\beta\} = 2\,\sigma^\mu_{\alpha\beta} P_\mu \quad \{Q_\alpha, Q_\beta\} = 0$
- 1 particle supersymmetry multiplets:

m ≠ 0

j	1P multiplets			
0	2	1		
1/2	1	2	1	
1		1	2	1
3/2			1	2
2				1

(j ≥ 1/2)

m = 0

j		
-1		1
-1/2	1	1
0	2	
1/2	1	1
1		1

(|j| ≥ 1/2)

#B = #F

Field multiplets

* **(A, ψ, F)** "scalar" ("chiral") multiplet

  A, F complex scalars, ψ left-handed Weyl spinor

  Off-shell real field DOFs: 4B+4F; on-shell: 2B+2F (F auxiliary)

  $[A] = 1$, $[\psi] = 3/2$, $[F] = 2$

* **(v^μ, λ, D)** massless "vector" ("real") multiplet

  v^μ real vector, λ left-handed Weyl spinor, D real scalar

  Off-shell real field DOFs: 4B+4F; on-shell: 2B+2F (D auxiliary)

  $[v^\mu] = 1$, $[\lambda] = 3/2$, $[D] = 2$

* **$(v^\mu, \lambda, \chi, C, D, N)$** massive vector multiplet

  χ Weyl, C N complex scalars, D N auxiliary

$m \neq 0$

j		
0	2	1
1/2	1	2
1		1

$m = 0$

j		
-1		1
-1/2	1	1
0	2	
1/2	1	1
1		1

Renormalizable N=1 supersymmetric gauge theories

* Specify the gauge group G
* Specify the chiral superfield content $\Phi_i = (A_i, \psi_i, F_i)$ and the representation of G on them: $g \in G$: $\Phi_i \to U(g)_{ij}\, \Phi_j$
* Associate a massless vector superfield to each generator of G: $t_A \leftrightarrow (v^A{}_\mu, \lambda^A, D^A)$ (λ^A, D^A transform with the adjoint, $v^A{}_\mu$ as usual)
* Specify a gauge invariant olomorphic function W(A): the superpotential $[W] = 3$: renormalizability $\Rightarrow W = (\lambda_{ijk}/3) A_i A_j A_k + (\mu_{ij}/2) A_i A_j + m^2{}_i\, A_i$

$$
\begin{aligned}
\mathcal{L}_{\rm susy} = {} & D_\mu A_i^\dagger D^\mu A_i + \psi_i^\dagger i\sigma^\mu D_\mu \psi_i + F_i^\dagger F_i \\
& - \frac{1}{4} v_A^{\mu\nu} v^A_{\mu\nu} + \lambda_A^\dagger i\sigma^\mu D_\mu \lambda_A + \frac{1}{2} D_A^2 \\
& - \frac{1}{2}\partial_i\partial_j W(A)\psi_i\psi_j - \partial_i W(A) F_i + \text{h.c.} \\
& - \left(\sqrt{2} g_A A_i^\dagger T_A^{ij} \lambda^A \psi_j + \text{h.c.}\right) - g_A A_i^\dagger T_A^{ij} D_A A_j \\
& + g_A \xi_A D^A + \theta \text{ term}
\end{aligned}
$$

[In superfield formalism:
$W = W(\phi)$, $[\theta] = -1/2$
$\phi = A + \sqrt{2}\,\psi\theta + F\theta^2$]

F, D non-dynamical
$[F] = [D] = 2$

Eliminating auxiliary fields

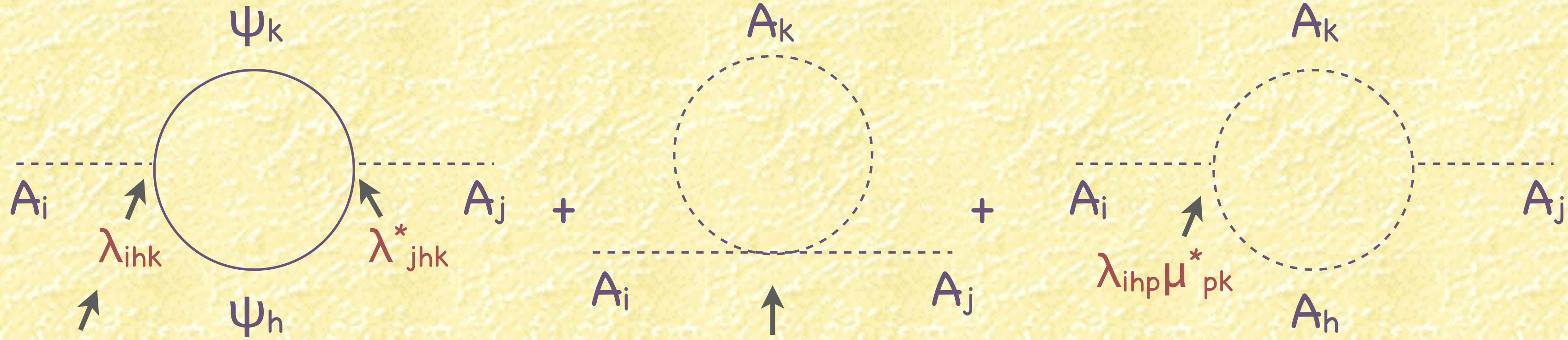
- Equations of motion for F, D: $F_i^\dagger = \partial_i W(A) \quad D_A = g_A A_i^\dagger T_A^{ij} A_j$
- Omitting FY and θ term:

$$\boxed{\begin{aligned} \mathcal{L}_{\text{susy}} &= \text{Kinetic} + \text{gauge for } A_i,\ \psi_i,\ v_A^\mu,\ \lambda_A \\ &\quad - \left(\frac{1}{2}\partial_i\partial_j W(A)\psi_i\psi_j + \sqrt{2}g_A A_i^\dagger T_A^{ij}\lambda^A\psi_j + \text{h.c.}\right) - V(A) \\ V(A) &= F_i^\dagger F_i + \frac{1}{2}D_A^2 \geq 0 \end{aligned}}$$

- Continuous symmetries (commuting with gauge):
  - commuting with supersymmetry: Q(A) = Q(ψ), Q(v_μ) = Q(λ) = 0, **Q(W) = 0**
  - R-symmetries: R(ψ) = R(A)-1, R(v_μ) = 0, R(λ) = 1, **R(W) = 2**

Non renormalization theorem
and the solution of the hierarchy problem

- Second line in L_{susy} does not get perturbative radiative corrections
- First line does, but it is (logarithmic) wave function renormalization
- Example: $W \supseteq -\mu_{ij} A_i A_j \Rightarrow V \supseteq (\mu^\dagger \mu)_{ij} A^\dagger_i A_j$, quadratically divergent?



From $-(1/2)\partial_h \partial_k W(A)\ \psi_h \psi_k$

$W(A) \supseteq (\lambda_{ihk}/3) A_i A_h A_k$

From $F^\dagger_h F_h$,

$F^\dagger_h = \partial_h W(A)$

+ gauge contributions

Proof at all orders uses superfields formalism or Seiberg argument (hep-ph/9309335)

- Interpretation: supersymmetry relates scalar masses to fermion masses, which are protected by chiral symmetry

Explicit (soft) supersymmetry breaking

* $\tilde{m}_e \gtrsim 100$ GeV, not = 0.5 MeV
* Most mechanisms of supersymmetry breaking take place at Q » TeV, give rise to effective, explicit, soft supersymmetry breaking terms at Q = TeV
* "Soft" = do not give rise to quadratic divergences

$$\mathcal{L} = \mathcal{L}_{\text{susy}} + \mathcal{L}_{\text{soft}}$$

$$-\mathcal{L}_{\text{soft}} = m^2_{ij} A^\dagger_i A_j + \left(\frac{M_{AB}}{2} \lambda_A \lambda_B + w(A) + \text{h.c.} \right)$$

[Girardello Grisaru , NPB 194 (1982)]

  - $w(A)$ olomorphic, $w = (a_{ijk}/3) A_i A_j A_k + (b^2_{ij}/2) A_i A_j + c^3_i A_i$
  - All terms in $\mathcal{L}_{\text{soft}}$ proportional to a (supersymmetry breaking) mass scale
  - $(M_{ij})/2\ \psi_i \psi_j$ can be reabsorbed, $w(A, A^\dagger)$, $M_{Ai}\ \lambda_A \psi_i$ give quadratic divergences in the presence of gauge singlets (and very suppressed in explicit models)
* Gaugino masses break R-symmetry

Back

Spontaneous supersymmetry breaking (SSSB)

* SSSB ⇔ V > 0 ⇔ F ≠ 0 or D ≠ 0 $\quad V(A) = F_i^\dagger F_i + \frac{1}{2} D_A^2 \geq 0$

  (if V_{min} = 0, there could still be SSSB in false vacua)

* SSSB should not couple to the SM fields at the renormalizable + tree level:
  - $\mathrm{Tr}(M^2_{s=0}) - 2\,\mathrm{Tr}(M^2_{s=1/2}) + 3\,\mathrm{Tr}(M^2_{s=1}) = 0$ (tree level, canonical kinetic term)
  - no gaugino masses

  [Ferrara Girardello Palumbo, PRD20 (1979)]

* Typically: SSSB in hidden sector at Q_{SSSB} » TeV, communicated to the SM fields by "messengers" at Q_{mess} » Q_{SSSB} (gravity, heavy charged fields, etc)

The MSSM

The Minimal Supersymmetric Standard Model (MSSM)

[Martin, hep-ph/9709356; Drees Godbole Roy, Haber Kane, Phys Rept 117 (1985)]

* "Minimal" = minimal number of fields
* $G = G_{SM} = SU(3)_c \times SU(2)_w \times U(1)_Y$
* Embedding of SM fields in (A,ψ) (chiral) or (v_μ,λ) (vector) multiplets:

SM	g_μ	W_μ	B_μ	q_i	u^c_i	d^c_i	l_i	e^c_i	h
$SU(3)_c$	8	1	1	3	$\bar{3}$	$\bar{3}$	1	1	1
$SU(2)_w$	1	3	1	2	1	1	2	1	2
$U(1)_Y$	0	0	0	1/6	-2/3	1/3	-1/2	1	-1/2

- Gauge bosons ⊆ vector multiplets (with gaugi**nos**)

$$g^A_\mu \to \hat{g}^A \equiv (g^A_\mu, \tilde{g}^A) \qquad \text{(with "gluinos")}$$
$$W^a_\mu \to \hat{W}^a \equiv (W^a_\mu, \tilde{W}^a) \qquad \text{(with "Winos")}$$
$$B_\mu \to \hat{B} \equiv (B_\mu, \tilde{B}) \qquad \text{(with "Binos")}$$

- Fermions ⊆ chiral multiplets (with **s**fermions, **s** for "scalar")

$$\begin{aligned} l_i &\to \hat{l}_i \equiv (\tilde{l}_i, l_i) \\ e^c_i &\to \hat{e}^c_i \equiv (\tilde{e}^c_i, e^c_i) \end{aligned} \quad \text{(with "sleptons")} \qquad \begin{aligned} q_i &\to \hat{q}_i \equiv (\tilde{q}_i, q_i) \\ u^c_i &\to \hat{u}^c_i \equiv (\tilde{u}^c_i, u^c_i) \\ d^c_i &\to \hat{d}^c_i \equiv (\tilde{d}^c_i, d^c_i) \end{aligned} \quad \text{(with "squarks")}$$

- Higgs ⊆ chiral multiplets (with Higgsi**nos**)

  lepton number conservation: $h \neq \tilde{l}_i$

  anomaly cancellation + fermion masses: $h \to \hat{h}_u \equiv (h_u, \tilde{h}_u) + \hat{h}_d \equiv (h_d, \tilde{h}_d)$

$$\lambda_U u^c q\, h^* + \lambda_D d^c q\, h \to \lambda_U u^c q\, h_u + \lambda_D d^c q\, h_d$$

The MSSM superfield content

MSSM	$\hat{g}_\mu$	$\hat{W}_\mu$	$\hat{B}_\mu$	$\hat{q}_i$	$\hat{u}^c_i$	$\hat{d}^c_i$	$\hat{l}_i$	$\hat{e}^c_i$	$\hat{h}_u$	$\hat{h}_d$
$SU(3)_c$	8	1	1	3	$\bar{3}$	$\bar{3}$	1	1	1	1
$SU(2)_w$	1	3	1	2	1	1	2	1	2	2
$U(1)_Y$	0	0	0	1/6	-2/3	1/3	-1/2	1	1/2	-1/2
		vector					chiral			

SM field content + gauginos, sfermions, Higgsinos (and 1 more Higgs doublet)

"**s**particles" , **s** for "supersymmetric"

The superpotential and R-parity

* The most general renormalizable gauge invariant superpotential:

$$W = \lambda^U_{ij}\hat{u}^c_i\hat{q}_j\hat{h}_u + \lambda^D_{ij}\hat{d}^c_i\hat{q}_j\hat{h}_d + \lambda^E_{ij}\hat{e}^c_i\hat{l}_j\hat{h}_d + \mu\,\hat{h}_u\hat{h}_d$$

→ SM Yukawas
+ Higgs and Higgsino mass
+ more interactions

$$+\lambda_{[ij]k}\hat{l}_i\hat{l}_j\hat{e}^c_k + \lambda'_{kji}\hat{l}_i\hat{q}_j\hat{d}^c_k + \lambda''_{i[jk]}\hat{u}^c_i\hat{d}^c_j\hat{d}^c_k + \mu'_i\hat{l}_i\hat{h}_u$$

→ L and B violation:
proton decay,
neutrino masses

* In the SM: L, B accidentally conserved (welcome)
* In the MSSM: L, B accidentally conserved once matter parity (P_M) or equivalently R-parity (P_R or R_P) is imposed
* P_M = +1 on $\hat{h}_u$, $\hat{h}_d$ (scalar component ∈ SM)

  P_M = −1 on $\hat{q}$, $\hat{u}^c$, $\hat{d}^c$, $\hat{l}$, $\hat{e}^c$ (fermion component ∈ SM)

  $P_M = (-1)^{3(B-L)}$ (remnant of B−L gauge symmetry?), commutes with SUSY
* R_P = +1 on q, u^c, d^c, l, e^c, h_u, h_d (SM fields)

  R_P = −1 on $\tilde{q}$, $\tilde{u}^c$, $\tilde{d}^c$, $\tilde{l}$, $\tilde{e}^c$, $\tilde{h}_u$, $\tilde{h}_d$ (supersymmetric partners)

  $R_P = (-1)^{3(B-L)+2s}$, discrete R-symmetry

Consequences of R_P

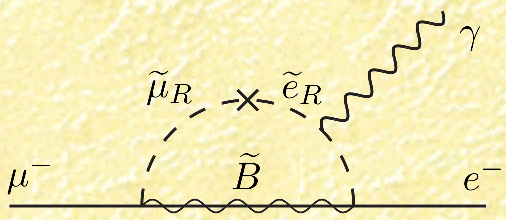
* Constrains the form of W, $\mathcal{L}_{soft}$ (B, L accidentally conserved)

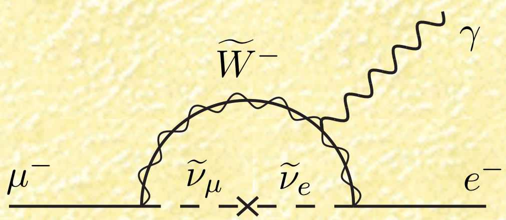
$$
\begin{aligned}
W &= \lambda^U_{ij}\hat{u}^c_i\hat{q}_j\hat{h}_u + \lambda^D_{ij}\hat{d}^c_i\hat{q}_j\hat{h}_d + \lambda^E_{ij}\hat{e}^c_i\hat{l}_j\hat{h}_d + \mu\,\hat{h}_u\hat{h}_d \\
-\mathcal{L}_{\rm soft} &= A^U_{ij}\tilde{u}^c_i\tilde{q}^j h_u + A^D_{ij}\tilde{d}^c_i\tilde{q}^j h_d + A^E_{ij}\tilde{e}^c_i\tilde{l}^j h_d + m^2_{ud}h_u h_d + \text{h.c.} \\
&\quad + (\tilde{m}^2_q)_{ij}\tilde{q}^\dagger_i\tilde{q}_j + (\tilde{m}^2_{u^c})_{ij}(\tilde{u}^c_i)^\dagger\tilde{u}^c_j + (\tilde{m}^2_{d^c})_{ij}(\tilde{d}^c_i)^\dagger\tilde{d}^c_j + (\tilde{m}^2_l)_{ij}\tilde{l}^\dagger_i\tilde{l}_j \\
&\quad + (\tilde{m}^2_{e^c})_{ij}(\tilde{e}^c_i)^\dagger\tilde{e}^c_j + m^2_{h_u}h^\dagger_u h_u + m^2_{h_d}h^\dagger_d h_d \\
&\quad + \frac{M_3}{2}\tilde{g}_A\tilde{g}_A + \frac{M_2}{2}\tilde{W}_a\tilde{W}_a + \frac{M_1}{2}\tilde{B}\tilde{B} + \text{h.c.}
\end{aligned}
$$

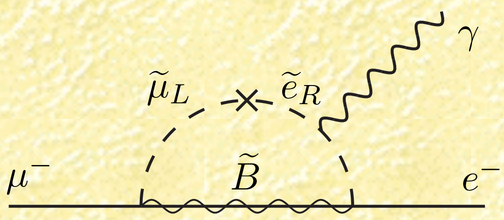
* **MSSM** ≡ G_{SM} + field content above + most general R_P-invariant W, $\mathcal{L}_{soft}$
* Sparticles are produced in pairs
* The Lightest Supersymmetric Particle (LSP) is stable
* R_P = +1 and -1 fermions and scalars do not mix

Parameter counting

* 3 gauge couplings, quantum numbers, θ_{QCD}
* Supersymmetric part: (3x18+2) – (9x5+2–5) = 14 = 9 fermion masses + 4 CKM parameters + 1 Higgs/ino mass = SM – 1 (Higgs coupling predicted)
* With $\mathcal{L}_{soft}$: [3x18+2 (W) + 3x2 (gaugino masses) + 3x18+2 (w) + 5x9+2 (scalar masses)] – [9x5+2 ($U(3)^5 \times U(1)^2$) + 1 (R-symmetry) – 3 (B, L, Y)] = 120 = SM + 105 = 14 + 3 gaugino masses + 3x6+3 sfermion masses + v, tanβ, m_A + 79 mixing and phases
* Too large FCNC and CPV processes in most of the parameter space, e.g.:







The Constrained MSSM (CMSSM)

* Assume that at some scale $M_0 \gg$ TeV the soft term satisfy:
    - $M_1 = M_2 = M_3 \equiv M_{1/2}$ (universal gaugino masses)
    - $A_{U,D,E} = A_0\, \lambda_{U,D,E}$ (A-term proportionality) (also define $m^2_{ud} = B_0\, \mu$)
    - $(\tilde{m}^2_q)_{ij} = (\tilde{m}^2_u)_{ij} = (\tilde{m}^2_d)_{ij} = (\tilde{m}^2_l)_{ij} = (\tilde{m}^2_e)_{ij} = m^2_0\, \delta_{ij}$ (universal scalar masses)
* Motivation:
    - Benchmark model with few parameters and FCNCs under control
    - Minimal supergravity (msugra) gives the CMSSM (with model-dependent A_0-B_0 relation)
* Parameter counting: 106 → 4 dimensionful pars + 2 phases (no new mixing pars, all mixing can be expressed in terms of CKM: an example of Minimal Flavour violation)

Phase convention

Complex soft parameters:	μ	$M_{1/2}$	A_0	$B_0\mu$
R-symmetry:	μ	$M_{1/2}\ e^{2i\omega}$	$A_0\ e^{2i\omega}$	$B_0\mu\ e^{2i\omega}$
Peccei-Quinn symmetry	$\mu\ e^{2i\alpha}$	$M_{1/2}$	A_0	$B_0\mu\ e^{2i\alpha}$

* R-symmetry: $\mathcal{L}_{susy}$ invariant, $R[\lambda\lambda] = 2$, $R[W] = 2 \Rightarrow R[w] = 2$
* Peccey-Quinn: $\hat{h}_{u,d} \to \hat{h}_{u,d}\ e^{i\alpha}$, $PQ(u^c q h_u) = PQ(d^c q h_d) = PQ(e^c l h_d) = 0$
* Standard phase convention: $M_{1/2} > 0$, $B_0\mu = m^2_{ud} > 0$, phases in μ, A_0 also used in the MSSM (provided that the gaugino phases differ by π)
* Constraints from EDMs: $|\sin\varphi_\mu|$, $|\sin\varphi_A| \lesssim 10^{-2}$ (supersymmetric CP "problem")

CP-conserving CMSSM

* Physical parameters (besides gauge, fermion masses and mixings)

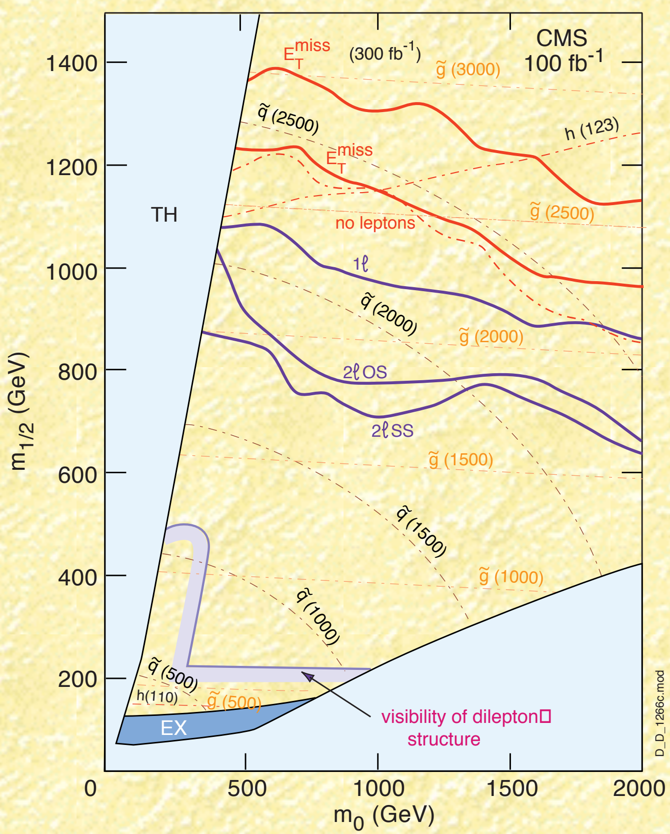
  $-\infty < m^2_0 < \infty$, $-\infty < A_0 < \infty$, $|\mu| > 0$, $M_{1/2} > 0$, $m^2_{ud} > 0$, $\mathrm{sign}(\mu) = \pm 1$

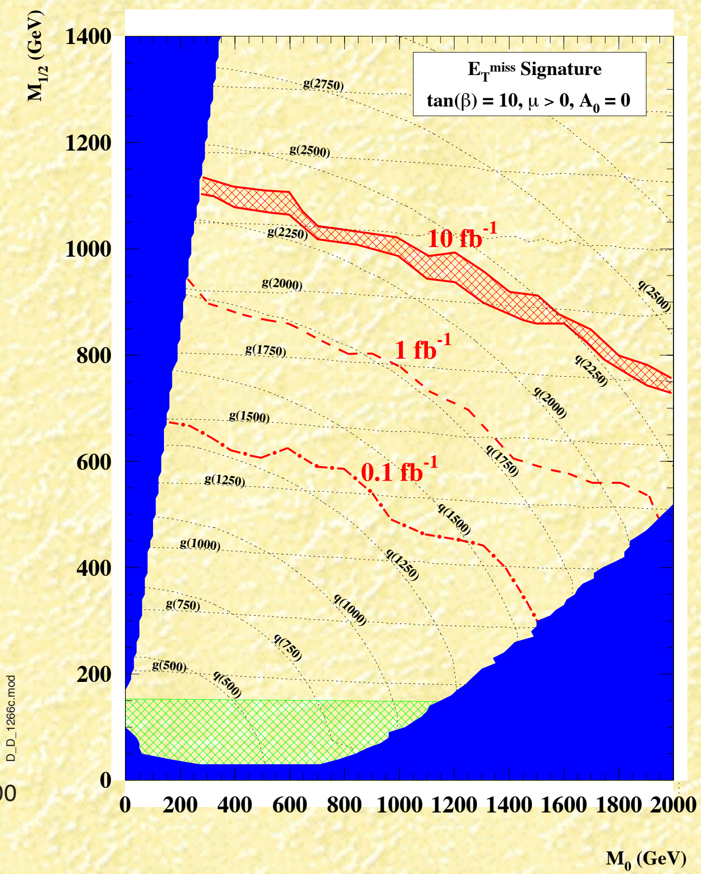
* Trade $|\mu|$ for M_Z, m^2_{ud} for $\tan\beta$ (see below):

  $-\infty < m^2_0 < \infty$, $-\infty < A_0 < \infty$, $M_{1/2} > 0$, $0 \le \beta \le \pi/2$, $\mathrm{sign}(\mu) = \pm 1$

* Plots often in m_0-$M_{1/2}$ plane for fixed β, A_0, $\mathrm{sign}(\mu)$

Example:
CMS reach





Electroweak symmetry breaking (EWSB) in the MSSM

Electroweak symmetry breaking (EWSB) in the MSSM

$$V = V_{\rm susy} + V_{\rm soft} = V(h_u, h_d, \tilde{q}_i, \tilde{u}^c_i, \tilde{d}^c_i, \tilde{l}_i, \tilde{e}^c_i)$$

* Issues:

  1. V bounded from below? ("UFB" directions)
  2. $\langle\tilde{q}_i\rangle = \langle\tilde{u}^c_i\rangle = \langle\tilde{d}^c_i\rangle = \langle\tilde{l}_i\rangle = \langle\tilde{e}^c_i\rangle = 0$? ("CCB" (and L breaking) minima)
  3. $\langle h_u\rangle$, $\langle h_d\rangle$ preserve $U(1)_{em}$?

$$\boxed{\begin{aligned} m_u^2 &\equiv m_{h_u}^2 + |\mu|^2 \\ m_d^2 &\equiv m_{h_d}^2 + |\mu|^2 \end{aligned}}$$

1. Not guaranteed. E.g. along $\langle h_u \rangle = \begin{pmatrix} 0 \\ w \end{pmatrix}$, $\langle h_d \rangle = \begin{pmatrix} w \\ 0 \end{pmatrix}$, $\left\langle \tilde{f} \right\rangle = 0$

   $V = (m_u^2 + m_d^2 - m_{ud}^2)\, w^2$ is unbounded from below unless $m_u^2 + m_d^2 > m_{ud}^2$

2. Not guaranteed. E.g. along $\langle h_d \rangle = \begin{pmatrix} w \\ 0 \end{pmatrix}$, $\left\langle \tilde{l}_i \right\rangle = \begin{pmatrix} 0 \\ w \end{pmatrix}$, $\langle \tilde{e}^c_i \rangle = -we^{-\phi(A^E_{ii})}$, $\langle \text{else} \rangle = 0$

   V(w) has a (deep) $\cancel{U(1)}_{em}$ minimum unless $|A^E_{ii}|^2 < 3\lambda^2_{e_i}\left[(\tilde{m}^2_l)_{ii} + (\tilde{m}^2_{e^c})_{ii} + m^2_d\right]$

   Analogously: $|A^D_{ii}|^2 < 3\lambda^2_{d_i}\left[(\tilde{m}^2_q)_{ii} + (\tilde{m}^2_{d^c})_{ii} + m^2_d\right]$

   Also: check positivity of mass eigenvalues $|A^U_{ii}|^2 < 3\lambda^2_{u_i}\left[(\tilde{m}^2_q)_{ii} + (\tilde{m}^2_{u^c})_{ii} + m^2_u\right]$

   Note: $|A| \lesssim \lambda\, \tilde{m}$, $A \equiv \lambda\, \hat{A}$

3. Guaranteed (provided that 1. and 2. are fine)

* Assume $\langle\tilde{q}_i\rangle = \langle\tilde{u}^c_i\rangle = \langle\tilde{d}^c_i\rangle = \langle\tilde{l}_i\rangle = \langle\tilde{e}^c_i\rangle = 0$. Then

$$V = \frac{g^2 + g'^2}{8}\left(h_u^\dagger h_u - h_d^\dagger h_d\right)^2 + \frac{g^2}{2}\left|h_u^\dagger h_d\right|^2 + |\mu|^2\left(h_u^\dagger h_u + h_d^\dagger h_d\right) \quad \text{from } \mathcal{L}_{\rm susy}$$

$$+ m_{h_u}^2 h_u^\dagger h_u + m_{h_d}^2 h_d^\dagger h_d + m_{ud}^2\left(h_u h_d + \text{h.c.}\right) \quad \text{from } \mathcal{L}_{\rm soft}$$

* Up to a gauge transformation: $h_u = v_u \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \; h_d = v_d e^{i\phi} \begin{pmatrix} \cos\chi \\ \sin\chi \end{pmatrix}$ $\quad v_{u,d} > 0$, $\; 0 \le \chi \le \pi/2$

* $\chi \neq 0 \Leftrightarrow U(1)_{em}$ spontaneously broken

  $e^{i\phi} \neq \pm 1 \Leftrightarrow$ CP spontaneously broken

* V minimum at $\chi = 0$, $e^{i\phi} = 1$ (for given $v_{u,d}$)

* $$\boxed{h_u = v_u \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \; h_d = v_d \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{array}{ll} v_u = v\sin\beta & v \simeq 174\,\text{GeV} \\ v_d = v\cos\beta & 0 \le \beta \le \pi/2 \end{array}}$$

* $V(v_u,v_d) = \frac{g^2+g'^2}{8}\left(v_u^2-v_d^2\right)^2 + m_u^2 v_u^2 + m_d^2 v_d^2 - 2m_{ud}^2 v_u v_d$ $\boxed{\begin{array}{l} m_u^2 \equiv m_{h_u}^2 + |\mu|^2 \\ m_d^2 \equiv m_{h_d}^2 + |\mu|^2 \end{array}}$
* Quartic term dominates at large v, except for tanβ = 1 ($v_u = v_d = v/\surd 2$), in which case: $V(v/\sqrt{2}, v/\sqrt{2}) = (m_u^2 + m_d^2 - 2m_{ud}^2)\, v^2/2$. V bounded from below iff

$$\boxed{m_u^2 + m_d^2 \geq 2m_{ud}^2 (\geq 0)}$$

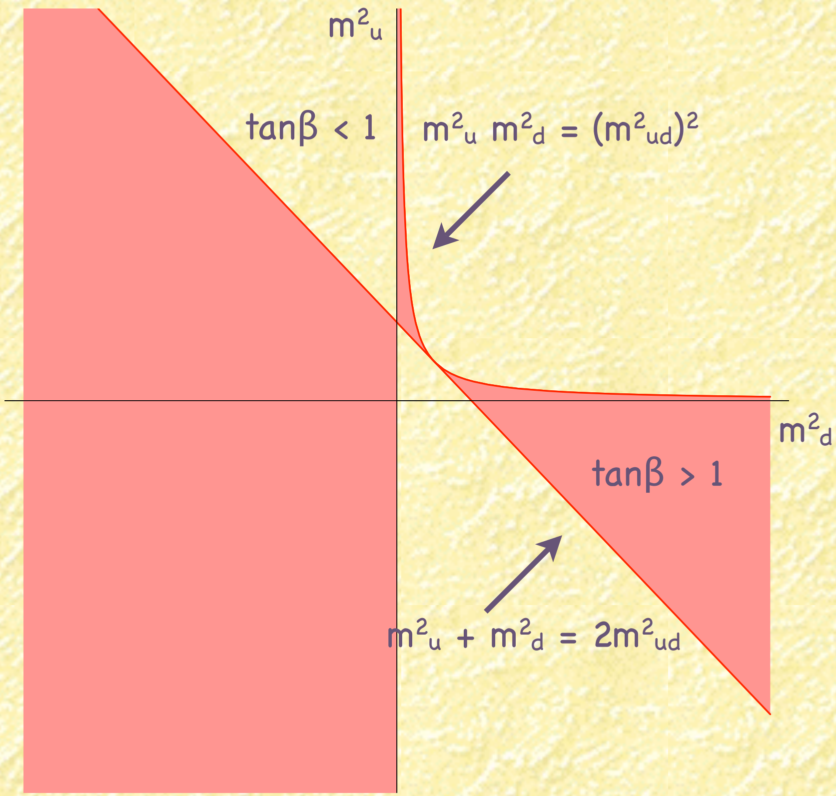
* Local extrema:
  - v = 0, V = 0
  - v ≠ 0: iff $\boxed{m_u^2 m_d^2 \leq (m_{ud}^2)^2}$ from $\begin{array}{l} v_d\partial_d V - v_u \partial_u V \\ v_d \partial_u V + v_u \partial_d V \end{array}$: V = $-\frac{4}{g^2+g'^2}\left(m_u^2 s_\beta^2 - m_d^2 c_\beta^2\right)^2$

$$\boxed{\frac{g^2+g'^2}{4} v^2 = -\frac{m_u^2 \tan^2\beta - m_d^2}{\tan\beta^2 - 1} = \frac{M_Z^2}{2}} \qquad \boxed{\sin 2\beta = \frac{2m_{ud}^2}{m_u^2 + m_d^2}}$$

β is given by the solution with tanβ ≷ 1 if $m^2_d \gtrless m^2_u$

* Bounds on β:
  - λ_t Landau pole beyond M_{Pl}: tanβ ≳ 1 (see below)
  - Higgs mass bound: tanβ ≳ 2 (see below)
  - B-physics: tanβ ≲ 60

Radiative corrections lower m^2_u more than m^2_d



We typically need $m^2{}_{hu} < 0$

while $m^2{}_{hd}$, $\tilde{m}^2{}_f > 0$:

an accident?

Radiative EWSB

* Soft terms generated at $M_0 \gg$ TeV   e.g. in sugra $M_0 = M_{Pl}$
* Rad corrs to soft terms enhanced by large logs: $t = \frac{1}{(4\pi)^2} \log \frac{M^2_{\rm GUT}}{Q^2} \simeq 0.4$
* RGEs: $$\begin{aligned} \frac{d}{dt}\tilde{m}^2_{q_3} &= \frac{16}{3} g_3^2 M_3^2 + 3 g_2^2 M_2^2 + \frac{1}{15} g_1^2 M_1^2 - \lambda_t^2 \left(\tilde{m}^2_{q_3} + \tilde{m}^2_{t^c} + \tilde{m}^2_{h_u} + |\hat{A}_t|^2 \right) \\ \frac{d}{dt}\tilde{m}^2_{t^c} &= \frac{16}{3} g_3^2 M_3^2 + \frac{4}{15} g_1^2 M_1^2 - 2\lambda_t^2 \left(\tilde{m}^2_{q_3} + \tilde{m}^2_{t^c} + \tilde{m}^2_{h_u} + |\hat{A}_t|^2 \right) \\ \frac{d}{dt} m^2_{h_u} &= \times \quad 3 g_2^2 M_2^2 + \frac{3}{5} g_1^2 M_1^2 - \boxed{3}\lambda_t^2 \left(\tilde{m}^2_{q_3} + \tilde{m}^2_{t^c} + \tilde{m}^2_{h_u} + |\hat{A}_t|^2 \right) \end{aligned}$$

$$\frac{d}{dt} m^2_{\rm others} = \text{only gauge terms}$$

[Martin Vaughn, PRD50 (1994)
Barger Berger Ohmann, PRD49 (1994)]

* BTW: $\frac{d}{dt} g_i^2 = -b_i g_i^4, \; \frac{d}{dt} M_i = -b_i g_i^2 M_i \Rightarrow \frac{M_i(Q_1)}{M_i(Q_2)} = \frac{g_i^2(Q_1)}{g_i^2(Q_2)}$

$M_1 = M_2 = M_3$, $g_1 = g_2 = g_3$ @ M_{GUT} $\Rightarrow$ $M_1 : M_2 : M_3 = g^2{}_1 = g^2{}_2 = g^2{}_3$

$M_1 : M_2 : M_3 \approx 1 : 2 : 7$

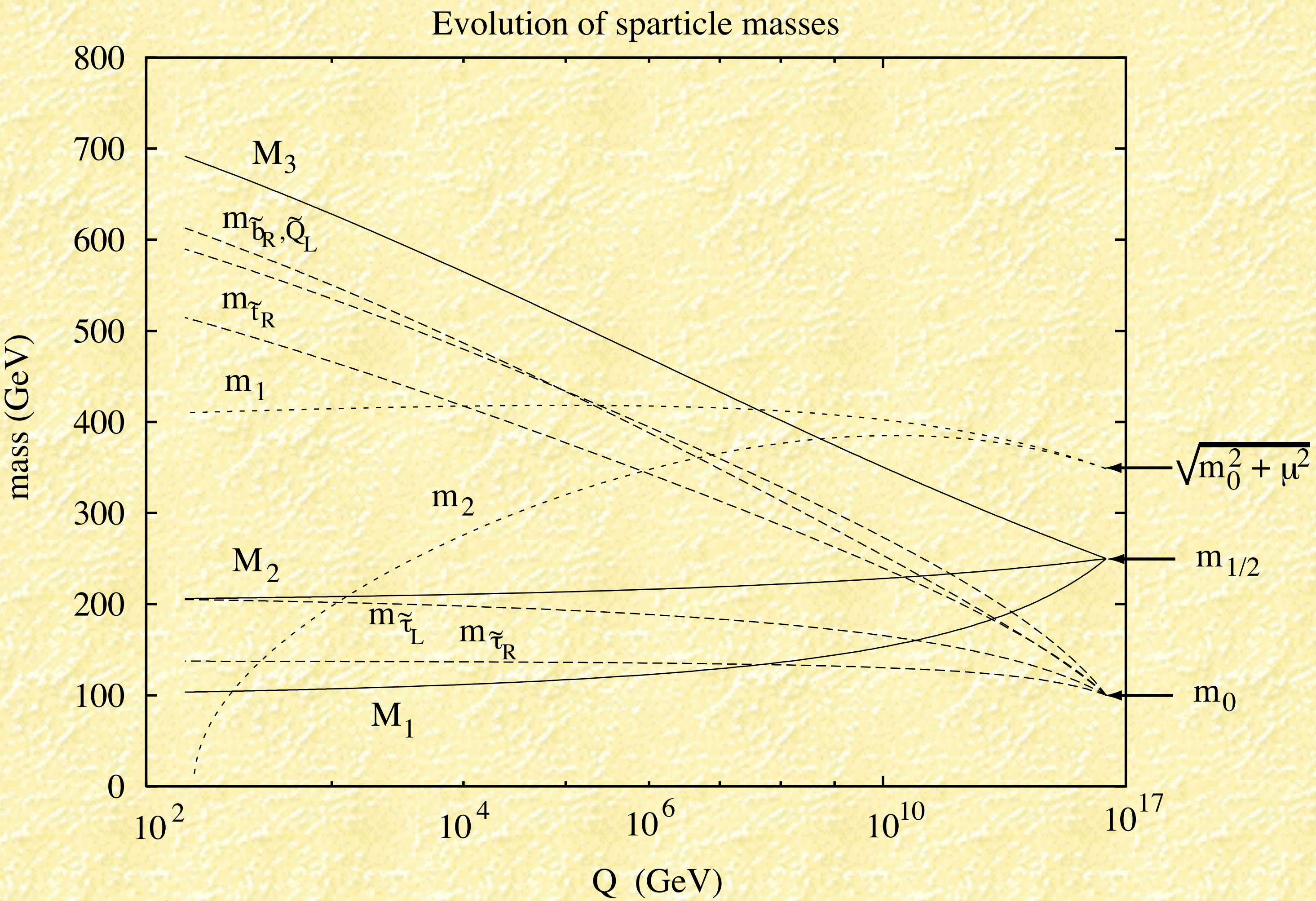
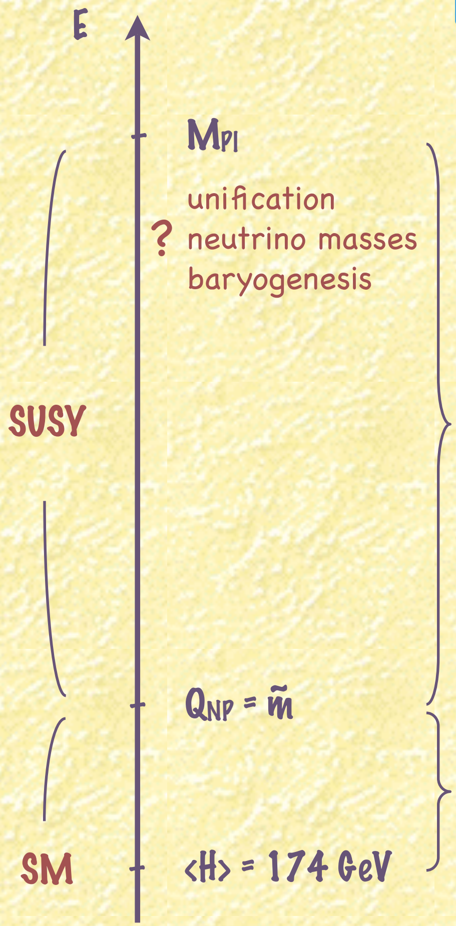


Fig. 1. An example of the running of the soft-supersymmetry breaking parameters for $\alpha_s(M_Z) = 0.120$, $m_t(m_t) = 150$ GeV, $\tan\beta = 10$, $m_{\frac{1}{2}} = 250$ GeV, $m_0 = 100$ GeV, and $A^G = 0$, where the superscript G denotes the GUT scale.

Barger Berger Ohmann, hep-ph/9311269

Fine-tuning in the MSSM



$$\delta m_h^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 Q_{\mathrm{NP}}^2 \log \frac{M_{\mathrm{Pl}}^2}{Q_{\mathrm{NP}}^2}$$

$$\delta m_h^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 Q_{\mathrm{NP}}^2 = \begin{cases} m_h^2 \left(\dfrac{Q_{\mathrm{NP}}}{0.5\,\mathrm{TeV}}\right)^2 & \text{if } m_h = 115\,\mathrm{GeV} \\ m_h^2 \left(\dfrac{Q_{\mathrm{NP}}}{2\,\mathrm{TeV}}\right)^2 & \text{if } m_h = 250\,\mathrm{GeV} \end{cases}$$

* $M_Z^2 = -2\dfrac{m_{h_u}^2 \tan^2\beta - m_{h_d}^2}{\tan^2\beta - 1} - 2|\mu|^2 \approx -2m_{h_u}^2 - 2|\mu|^2 \quad (\text{large } \tan\beta)$

  $$\approx -2\left(m_{h_u}^2(M_0) + |\mu|^2\right) + 2\,\delta m_{h_u}^2$$

* Large logs + color factors + lower bounds on gluinos and squarks: $\delta m_{h_u}^2 \gg M_Z^2$
  A moderate (up to %) fine-tuning is required to obtain M_Z = 91 GeV

$\tilde{m}_Q = \tilde{m}_H$: "focus point"

$$M_Z^2 \approx (91\,\mathrm{GeV})^2 \left[\frac{\tilde{m}_Q^2}{(70\,\mathrm{GeV})^2} - \frac{\tilde{m}_H^2}{(80\,\mathrm{GeV})^2} + \frac{M_{1/2}^2}{(40\,\mathrm{GeV})^2} - \frac{\mu^2}{(70\,\mathrm{GeV})^2} \right]$$

* FT ≈ maximum contribution in [...] (+ possibly in tanβ and m_t)
* Benchmark points:

$$M_{1/2} = (250 \div 1840)\,\mathrm{GeV}: \quad \mathrm{FT} \simeq 40 \div 2000$$

[De Roeck, Ellis, Gianotti, Moortgat, Olive, Pape]

$$\tilde{m}_Q = (1500 \div 4300)\,\mathrm{GeV}: \quad \mathrm{FT} \simeq 430 \div 3700 \quad \text{or} \quad M_{1/2} = 500\,\mathrm{GeV}: \quad \mathrm{FT} \simeq 150$$

[Kane, Lykken, Mrenna, Nelson, Wang, Wang]

* Direct lower limits on squark and gluinos

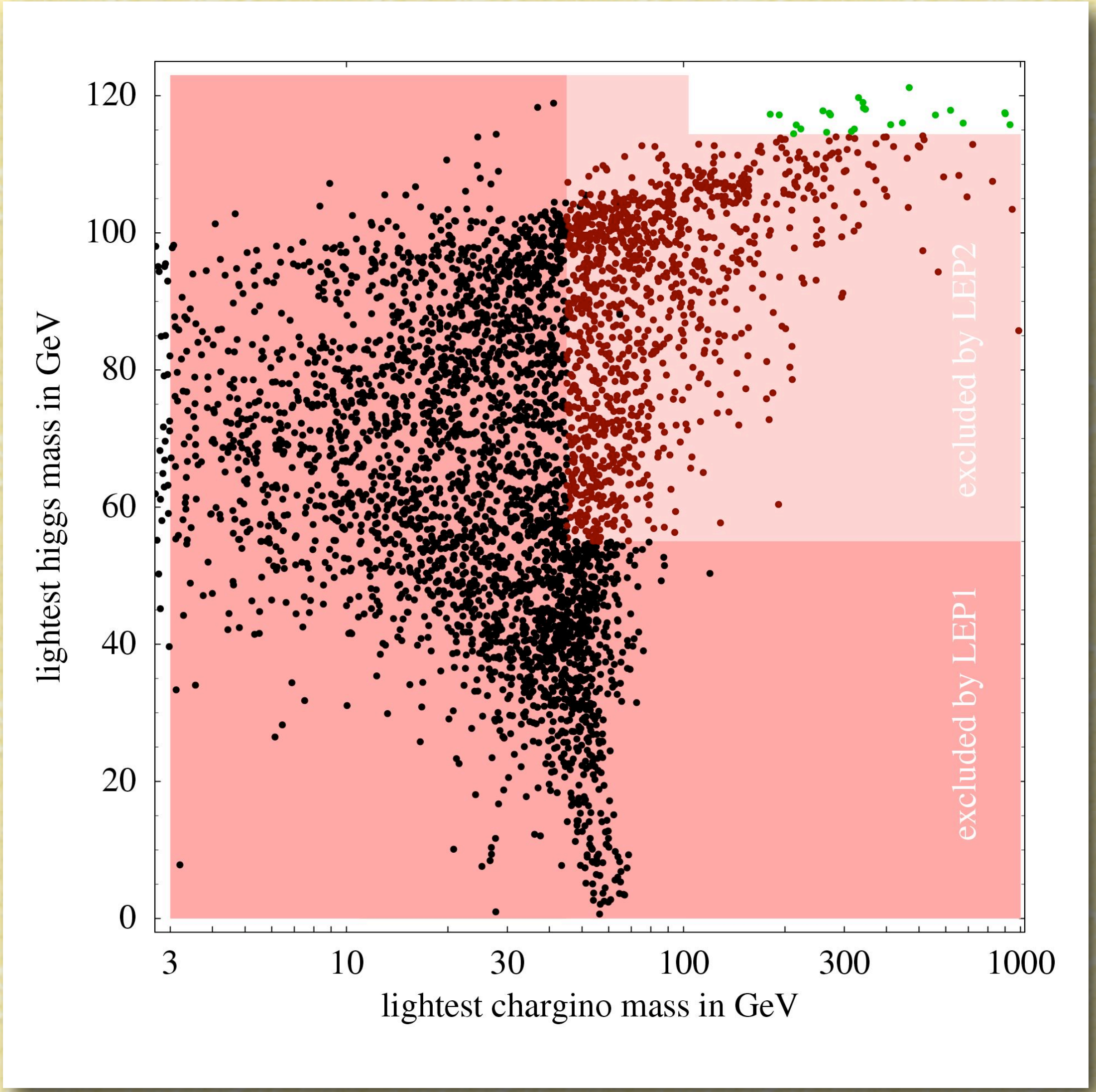
$$M_{\tilde{g}} \gtrsim \begin{cases} 195\,\mathrm{GeV} \\ 260\,\mathrm{GeV} \\ 500\,\mathrm{GeV} \end{cases} \Rightarrow \mathrm{FT} \gtrsim \begin{cases} 3 \\ 6 \\ 20 \end{cases} \qquad m_{\tilde{t}} \gtrsim \begin{cases} 300\,\mathrm{GeV} \\ 260\,\mathrm{GeV} \\ 100\,\mathrm{GeV} \end{cases} \Rightarrow \begin{cases} 25 \\ 10 \\ 50 \end{cases}$$

* Indirect lower limit on the stop mass

$$(114\,\mathrm{GeV})^2 < m_h^2 < M_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} h_t^2 m_t^2 \log \frac{\tilde{m}_t^2}{m_t^2} \Rightarrow \mathrm{FT} \sim 50 \div 100$$

What is left?

* Quantitative measure of naturalness nicely taking into account and combining all the considerations above
    - Scan the relative sizes of SUSY parameters and the SM parameters in their ranges
    - Set the overall scale of SUSY parameters from <H> = 174 GeV
    - Calculate SUSY spectrum and compare with experiment
* Few O(1%) of points satisfy all experimental constraints



[Giusti R Strumia]

A comment on numerical scanning procedures

- The FT problem typically introduces a bias in numerical scans of the MSSM parameter space
- Physical parameters (besides gauge, fermion masses and mixings)

  $-\infty < \mathbf{m^2_0} < \infty$, $-\infty < \mathbf{A_0} < \infty$, $\mathbf{|\mu|} > 0$, $\mathbf{M_{1/2}} > 0$, $\mathbf{m^2_{ud}} > 0$, $\mathbf{sign(\mu)} = \pm 1$
- $|\mu|$ is traded for M_Z, which means that the (necessary) cancellation is forced to take place between μ^2 and all the rest in

$$M_Z^2 = -2\frac{m^2_{h_u}\tan^2\beta - m^2_{h_d}}{\tan^2\beta - 1} - 2|\mu|^2 \approx -2m^2_{h_u} - 2|\mu|^2 \quad (\text{large} \tan\beta)$$

$$\approx -2\left(m^2_{h_u}(M_0) + |\mu|^2\right) + 2\,\delta m^2_{h_u}$$

- Example: LSP is rarely an Higgsino

Addressing the FT problem

* Low M_0
* NMSSM
* Supersymmetric Little Higgs
* Sliding overall soft mass scale
* Environment
* Who cares?

The particle spectrum of the MSSM

MSSM fields:

$$g_\mu \; W_\mu \; B_\mu \quad \tilde{g} \; \tilde{W} \; \tilde{B} \quad q_i \; u_i^c \; d_i^c \; l_i \; e_i^c \; \tilde{h}_u \; \tilde{h}_d \quad \tilde{q}_i \; \tilde{u}_i^c \; \tilde{d}_i^c \; \tilde{l}_i \; \tilde{e}_i^c \; h_u \; h_d$$

Mass matrices → masses + expressions in terms of mass eigenstates

Selection rules (after EWSB): spin, color, charge, R_P

Gauge bosons

$g^A_\mu \quad W^a_\mu \quad B_\mu$

$$\mathcal{L} \supseteq \left|\left(gW^z_\mu T_a + g' B_\mu Y_u\right)\langle h_u\rangle\right|^2 + \left|\left(gW^z_\mu T_a + g' B_\mu Y_d\right)\langle h_d\rangle\right|^2$$

$$\boxed{M_W^2 = \frac{g^2}{2}v^2} \qquad \boxed{M_Z^2 = \frac{g^2+g'^2}{2}v^2}$$

$$\boxed{\begin{aligned} & g_s g^A_\mu T_A + g W^a_\mu T_a + g' B_\mu Y \\ & \qquad = g_s g^A_\mu T_A + \frac{g}{\sqrt{2}}(W^+_\mu T_+ + W^-_\mu T_-) + \frac{g}{c_W} Z_\mu (T_3 - s^2_W Q) + e A_\mu Q \end{aligned}}$$

Same as in the SM, with $v^2 = v^2_u + v^2_d$

R_P = 1 (SM) fermions

* q_i u^c_i d^c_i l_i e^c_i

* $-\mathcal{L} \supseteq \lambda^U_{ij} u^c_i q_j h_u + \lambda^D_{ij} d^c_i q_j h_d + \lambda^E_{ij} e^c_i l_j h_d \quad \rightarrow \quad \begin{aligned} m_U &= \lambda_U v \sin\beta \\ m_D &= \lambda_D v \cos\beta \\ m_E &= \lambda_E v \cos\beta \end{aligned}$

* $\dfrac{m_t}{m_b} = \dfrac{\lambda_t}{\lambda_b} \tan\beta$: $m_b \ll m_t$ either because $\lambda_b \ll \lambda_t$ (as in the SM)
or because $\tan\beta \gg 1$ (allows $\lambda_b \sim \lambda_t$, relevant for rad corrs, Yukawa unification (PQ symmetry?))

* $\lambda_t = \dfrac{m_t}{v \sin\beta}$: $\lambda_t(M_{GUT}) < \infty \Rightarrow \tan\beta \gtrsim 1$ (depending on what goes on from M_Z to M_{GUT})

* $$m_U = U^T_{u^c} \, m^{\text{diag}}_U \, U_u \qquad m_D = U^T_{d^c} \, m^{\text{diag}}_D \, U_d \qquad m_E = U^T_{e^c} \, m^{\text{diag}}_E \, U_e$$

$$\boxed{\begin{aligned} q_i &= \begin{pmatrix} (U^\dagger_u)_{ij} u'_j \\ (U^\dagger_d)_{ij} d'_j \end{pmatrix} & u^c_i &= (U^\dagger_{u^c})_{ij} u^{c\prime}_i \\ & & d^c_i &= (U^\dagger_{d^c})_{ij} d^{c\prime}_i \\ l_i &= \begin{pmatrix} (U^\dagger_\nu)_{ij} \nu'_j \\ (U^\dagger_e)_{ij} e'_j \end{pmatrix} & e^c_i &= (U^\dagger_{e^c})_{ij} e^{c\prime}_i \end{aligned}}$$

* $\boxed{V = U_u U^\dagger_d}$ appears in SM CC interactions, fermion-sfermion relative orientation appears in supersymmetric interactions

$R_P = -1$ fermions (gauginos and Higgsinos)

* $\tilde{g}_A$ $\tilde{W}_a$ $\tilde{B}$ $\tilde{h}_u$ $\tilde{h}_d$
* $\tilde{h}_u = \begin{pmatrix} \tilde{h}_u^+ \\ \tilde{h}_u^0 \end{pmatrix} \quad \tilde{h}_d = \begin{pmatrix} \tilde{h}_d^0 \\ \tilde{h}_d^- \end{pmatrix} \quad \tilde{W}^{\pm} = \frac{\tilde{W}_1 \mp i\tilde{W}_2}{\sqrt{2}} \quad \tilde{W}^0 = \tilde{W}^3$
* $\tilde{g}_A$ have mass M_3
* $\tilde{h}^+_u$ $\tilde{W}^+$ / $\tilde{h}^-_d$ $\tilde{W}^-$ can mix ("charginos")
* $\tilde{h}^0_u$ $\tilde{h}^0_d$ $\tilde{W}^0$ $\tilde{B}$ can mix ("neutralinos")

* **Charginos:** $-\mathcal{L} \supseteq \left(\tilde{W}^- \; \tilde{h}_d^-\right) M_C \begin{pmatrix} \tilde{W}^+ \\ \tilde{h}_u^+ \end{pmatrix} + \text{h.c.}$

$$\boxed{M_C = \begin{pmatrix} M_2 & \sqrt{2} M_Z c_W s_\beta \\ \sqrt{2} M_Z c_W c_\beta & |\mu| e^{i\phi_\mu} \end{pmatrix}}$$

e.g. $\sqrt{2} M_Z c_W c_\beta$ from $\sqrt{2} h_u^\dagger (g \frac{\sigma_a}{2} \tilde{W}_a + g' \frac{1}{2} \tilde{B}) \tilde{h}_u + \sqrt{2} h_d^\dagger (g \frac{\sigma_a}{2} \tilde{W}_a - g' \frac{1}{2} \tilde{B}) \tilde{h}_d$

* **Neutralinos:** $-\mathcal{L} \supseteq \frac{1}{2} \left(\tilde{B} \; \tilde{W}^3 \; \tilde{h}_d^0 \; \tilde{h}_u^0\right) M_N \begin{pmatrix} \tilde{B} \\ \tilde{W}^3 \\ \tilde{h}_d^0 \\ \tilde{h}_u^0 \end{pmatrix} + \text{h.c.}$

$$\boxed{M_N = \begin{pmatrix} M_1 & 0 & -\sqrt{2} M_Z s_W c_\beta & \sqrt{2} M_Z s_W s_\beta \\ 0 & M_2 & \sqrt{2} M_Z c_W c_\beta & -\sqrt{2} M_Z c_W s_\beta \\ -\sqrt{2} M_Z s_W c_\beta & \sqrt{2} M_Z c_W c_\beta & 0 & -|\mu| e^{i\phi_\mu} \\ \sqrt{2} M_Z s_W s_\beta & -\sqrt{2} M_Z c_W s_\beta & -|\mu| e^{i\phi_\mu} & 0 \end{pmatrix}}$$

* Small v/M_i, $v/|\mu|$:
  - $W^+ W^-$ → 1 Dirac spinor, mass M_2
  - $h^+_u\ h^-_d$ → 1 Dirac spinor, mass $|\mu|$
  - $h^0_u\ h^0_d$ → 1 Dirac spinor, mass $|\mu|$
  - B, W^0 → 2 Majorana spinors, mass M_1, M_2
* In general:

$$\begin{pmatrix}\chi_1^-\\ \chi_2^-\end{pmatrix} = U\begin{pmatrix}\tilde{W}^-\\ \tilde{h}_d^-\end{pmatrix} \quad \begin{pmatrix}\chi_1^+\\ \chi_2^+\end{pmatrix} = V\begin{pmatrix}\tilde{W}^+\\ \tilde{h}_d^+\end{pmatrix} \quad \begin{pmatrix}\chi_1^0\\ \chi_2^0\\ \chi_3^0\\ \chi_4^0\end{pmatrix} = N\begin{pmatrix}\tilde{B}\\ \tilde{W}^0\\ \tilde{h}_d^0\\ \tilde{h}_u^0\end{pmatrix}$$

$$\boxed{\begin{aligned} M_C &= V^T\ M_C^{\mathrm{diag}}\ U \\ M_N &= N^T\ M_N^{\mathrm{diag}}\ N \end{aligned}}$$

$$\text{mass terms} = M_{\chi_i^+}\chi_i^+\chi_i^- + \frac{1}{2}M_{\chi_j^0}\chi_j^0\chi_j^0 + \text{h.c.} = M_{\chi_i^\pm}\bar{C}_i C_i + \frac{1}{2}M_{\chi_j^-}\bar{N}_j N_j$$

$$C_i = \begin{pmatrix}\epsilon\chi_i^{-\,*}\\ \chi_i^+\end{pmatrix} \quad N_i = \begin{pmatrix}\epsilon\chi_i^{0\,*}\\ \chi_i^0\end{pmatrix}$$

(ordered by mass: $M_i \le M_j$ if $i < j$)

$$\tilde{h}_u = \begin{pmatrix} V^\dagger_{2n}\chi^+_n \\ N^\dagger_{4m}\chi^0_m \end{pmatrix} \quad \tilde{h}_d = \begin{pmatrix} N^\dagger_{3m}\chi^0_m \\ U^\dagger_{2n}\chi^+_n \end{pmatrix}$$

$$g_s\tilde{g}^A T_A + g\tilde{W}^a T_a + g'\tilde{B}Y = g_s\tilde{g}^A T_A + \frac{g}{\sqrt{2}}U^\dagger_{1n}\chi^-_n T_- + \frac{g}{\sqrt{2}}V^\dagger_{1n}\chi^+_n T_+ + \left(gN^\dagger_{W_3m}T_3 + g'N^\dagger_{Bm}Y\right)\chi^0_m$$

- $M_{\chi^\pm} \gtrsim (90 - 105)$ GeV (depending on scenarios)
- $M_{\chi^\pm} > Q \Rightarrow M^2_2 + \mu^2 > 2Q^2 + 2QM_W$
- The LSP can easily be in the neutralino/chargino sector
- Composition of the lightest neutralino/chargino:
  - In the limit of small EWSB effects and assuming gaugino unification: χ^0_1 mainly Bino if $M_1 \lesssim \mu$, mainly Higgsino if $M_1 \gtrsim \mu$
  - If $M_1 \gtrsim \mu$, EWSB and loop effects guarantee $M^\pm_{\chi_1} \geq M^0_{\chi_2}$

R_P = 1 scalars (Higgs sector)

* h_u h_d 8 real dofs: 2x(Q=1) + 2x(Q=-1) + 2x(Q=0,CP+) + 2x(Q=0,CP-)

V(h_u, h_d) breaks $SU(2)_W$x$U(1)_Y$, preserves $U(1)_{em}$, CP

(barring $\phi_{\mu,A}$ effects through loop corrections, neglecting δ_{CKM})

* 3 massless Goldstones G^+ G^- G^0 (CP-)

* 5 physical dofs: H^+ H^- A (CP-) ϕ_u ϕ_d (CP+)

$$\boxed{h_u = \begin{pmatrix} c_\beta H^+ + i s_\beta G^+ \\ v s_\beta + \dfrac{\phi_u - i(s_\beta G^0 + c_\beta A)}{\sqrt{2}} \end{pmatrix} \qquad h_d = \begin{pmatrix} v c_\beta + \dfrac{\phi_d + i(c_\beta G^0 - s_\beta A)}{\sqrt{2}} \\ s_\beta H^- + i c_\beta G^- \end{pmatrix}}$$

* Masses: the 8x8 mass matrix decomposes into
  - a vanishing 3x3 block corresponding to the Goldstones G^+ G^- G^0
  - a mass term for H^+H^-: $m^2_{H^\pm} = \left.\frac{\partial^2 V_\pm}{\partial H^+ \partial H^-}\right|_{H^\pm=0}$ $\quad V_\pm = V\left(\begin{pmatrix} c_\beta H^+ \\ v s_\beta \end{pmatrix}, \begin{pmatrix} v c_\beta \\ s_\beta H^- \end{pmatrix}\right)$
  - a mass term for A: $m^2_A = \left.\frac{\partial^2 V_A}{\partial A^2}\right|_{A=0}$
  - a 2x2 mass matrix for ϕ_u ϕ_d: $-\mathcal{L} \supseteq -\frac{1}{2}\,(\phi_u\ \phi_d)\, M^2_\phi \begin{pmatrix} \phi_u \\ \phi_d \end{pmatrix}$

$$M^2_\phi = R(\alpha) \begin{pmatrix} m^2_H & \\ & m^2_h \end{pmatrix} R(\alpha)^{-1} \quad m^2_h < m^2_H \quad R(\alpha) = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}$$

$$\boxed{\begin{aligned} \phi_d &= c_\alpha H - s_\alpha h \\ \phi_u &= c_\alpha h + s_\alpha H \end{aligned}}$$

* Decoupling limit: $m_A \gg v \Leftrightarrow m_{H\pm} \gg v \Leftrightarrow m_H \gg v$ ($m_h \sim v$) $\alpha \approx \beta - \pi/2$

In the MSSM

* m^2_h m^2_H $m^2_{H\pm}$ m^2_A α β ↔ MSSM parameters

$$\boxed{\begin{aligned} m_A^2 &= m_u^2 + m_d^2 = m_{h_u}^2 + m_{h_d}^2 + 2|\mu|^2 \\ m_{H^\pm}^2 &= m_A^2 + M_W^2 \end{aligned}}$$

$$M_\phi^2 = \begin{pmatrix} m_A^2 s_\beta^2 + M_Z^2 c_\beta^2 & -s_\beta c_\beta (m_A^2 + M_Z^2) \\ -s_\beta c_\beta (m_A^2 + M_Z^2) & m_A^2 c_\beta^2 + M_Z^2 s_\beta^2 \end{pmatrix}$$

* Decoupling limit: $m^2_h \approx M^2_Z \cos^2 2\beta$
* In general: $\boxed{m_{h,H}^2 = \frac{1}{2}\left[M_Z^2 + m_A^2 \pm \sqrt{(M_Z^2+m_A^2)^2 - 4M_Z^2 m_A^2 \cos^2 2\beta}\right]}$

$$\boxed{\tan 2\alpha = \frac{m_A^2 + M_Z^2}{m_A^2 - M_Z^2}\tan 2\beta}$$

$$\left(\begin{aligned} \cos 2\alpha &= \frac{M_Z^2 - m_A^2}{m_H^2 - m_h^2}\cos 2\beta \\ \sin 2\alpha &= -\frac{M_Z^2 + m_A^2}{m_H^2 - m_h^2}\sin 2\beta \end{aligned}\right)$$

* $m_h^2 \leq M_Z^2 \cos^2 2\beta \quad \text{(tree level)}$

[Ellis Ridolfi Zwirner]

* 1-loop corrections (very basic approx): $m_h^2 \lesssim M_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} h_t^2 m_t^2 \log\frac{\tilde{m}_t^2}{m_t^2} \lesssim 130\,\text{GeV}$
  - Lower limit on m^2_h → lower limit on $\tilde{m}_t$ → lower limit on FT for $\tilde{m}_t \lesssim 1\text{-}2\,\text{TeV}$
  - lower tanβ requires a larger correction (upper limit on m_t → lower limit on tanβ)
  - m^2_h > 115 GeV can be evaded in the MSSM but requires even more FT

Radiative corrections to m_h

- Full 1-loop computation: Coleman-Weinberg potential + self-energy
- Moderate tanβ: corrections dominated by top-stop sector
- The stop mixing ($A_t + \mu\cot\beta$) has a significant impact on the results
- $\log(\tilde{m}_t^2/m_t^2)$-enhanced contributions:
  - consider the limit $\tilde{m}_t^2 \gg m_t^2$
  - match the MSSM at Q > m̃ with the SM at Q < m̃: $\begin{cases} \lambda_h(\tilde{m}_t) = \dfrac{g^2 + g'^2}{4}\cos^2 2\beta \\ h_t = \lambda_t \sin\beta = m_t/v \end{cases}$
  - compute leading-log corrections to the SM Higgs coupling

    $$\lambda_h(m_t) = \lambda_h(\tilde{m}_t) + 6\frac{h_t^2}{(4\pi)^2}\log\frac{\tilde{m}_t^2}{m_t^2}$$
  - $m_h^2 = 2\lambda_h(m_t)v^2 = M_Z^2\cos^2 2\beta + 12\dfrac{h_t^2}{(4\pi)^2}m_t^2\log\dfrac{\tilde{m}_t^2}{m_t^2}$

R_P = -1 scalars (squarks and sleptons)

* $\tilde{q}_i = \begin{pmatrix} \tilde{u}_i \\ \tilde{d}_i \end{pmatrix}$ $\quad \begin{matrix} \tilde{u}_i^c \\ \tilde{d}_i^c \end{matrix}$ $\quad \tilde{l}_i = \begin{pmatrix} \tilde{\nu}_i \\ \tilde{e}_i \end{pmatrix}$ $\quad \tilde{e}_i^c$ $\quad \tilde{q}_i^* = \begin{pmatrix} \tilde{u}_i^* \\ \tilde{d}_i^* \end{pmatrix}$ $\quad \begin{matrix} \tilde{u}_i^{c*} \\ \tilde{d}_i^{c*} \end{matrix}$ $\quad \tilde{l}_i^* = \begin{pmatrix} \tilde{\nu}_i^* \\ \tilde{e}_i^* \end{pmatrix}$ $\quad \tilde{e}_i^{c*}$

* Possible mixing between

  - $SU(3)_c$ triplets, Q=2/3 (up squarks): u_i $u^c{}_i{}^*$
  - $SU(3)_c$ triplets, Q=-1/3 (down squarks): d_i $d^c{}_i{}^*$
  - $SU(3)_c$ singlets, Q=-1 (charged sleptons): e_i $e^c{}_i{}^*$
  - $SU(3)_c$ singlets, Q=0 (sneutrinos): ν_i

$$-\mathcal{L} = (\tilde{u}^* \; \tilde{u}^c)\mathcal{M}_U^2 \begin{pmatrix} \tilde{u} \\ \tilde{u}^{c*} \end{pmatrix} + (\tilde{d}^* \; \tilde{d}^c)\mathcal{M}_D^2 \begin{pmatrix} \tilde{d}_i \\ \tilde{d}_i^{c*} \end{pmatrix} + (\tilde{e}^* \; \tilde{e}^c)\mathcal{M}_E^2 \begin{pmatrix} \tilde{e} \\ \tilde{e}^{c*} \end{pmatrix} + \tilde{\nu}^* M_\nu^2 \tilde{\nu}$$

$$\mathcal{M}_U^2 = \begin{pmatrix} \tilde{m}_q^2 + M_U^\dagger M_U + M_Z^2 z_u c_{2\beta}\mathbf{1} & -(\hat{A}_U^\dagger + \mu \cot\beta) M_U^\dagger \\ -M_U(\hat{A}_U + \mu^* \cot\beta) & \tilde{m}_{u_R}^2 + M_U M_U^\dagger + M_Z^2 z_{u_c} c_{2\beta}\mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathrm{LL} & \mathrm{LR} \\ \mathrm{RL} & \mathrm{RR} \end{pmatrix}$$

$$\mathcal{M}_D^2 = \begin{pmatrix} \tilde{m}_q^2 + M_D^\dagger M_D + M_Z^2 z_d c_{2\beta}\mathbf{1} & -(\hat{A}_D^\dagger + \mu \tan\beta) M_D^\dagger \\ -M_D(\hat{A}_D + \mu^* \tan\beta) & \tilde{m}_{d_R}^2 + M_D M_D^\dagger + M_Z^2 z_{d_c} c_{2\beta}\mathbf{1} \end{pmatrix}$$

$$\mathcal{M}_E^2 = \begin{pmatrix} \tilde{m}_l^2 + M_E^\dagger M_E + M_Z^2 z_e c_{2\beta}\mathbf{1} & -(\hat{A}_E^\dagger + \mu \tan\beta) M_E^\dagger \\ -M_E(\hat{A}_E + \mu^* \tan\beta) & \tilde{m}_{e_R}^2 + M_E M_E^\dagger + M_Z^2 z_{e_c} c_{2\beta}\mathbf{1} \end{pmatrix}$$

$$M_\nu^2 = \tilde{m}_l^2 + M_Z^2 z_\nu c_{2\beta}\mathbf{1} \qquad A_{U,D,E} \equiv \lambda_{U,D,E} \hat{A}_{U,D,E} \quad m_R^2 \equiv (m_c^2)^*$$

$$z_A \equiv t_3(A) - \sin^2\theta_W q(A)$$

- Super-CKM basis: write the scalar mass matrices in the basis in flavour space in which the corresponding fermions are diagonal (U **or** D)

* FCNC/sugra-inspired ansatz for colliders: (neglecting small off-diagonal entries, $V_{cb,ub}$)

$$(\tilde{m}^2_{ij}) = \begin{pmatrix} \tilde{m}^2 & & \\ & \tilde{m}^2 & \\ & & \tilde{m}^2_3 \end{pmatrix}$$

* I and II families up squarks:

$$\tilde{m}^2_{u_{1,2}} = \tilde{m}^2_q + z_u c_{2\beta} M_Z^2$$
$$\tilde{m}^2_{u^c_{1,2}} = \tilde{m}^2_{u^c} + z_{u^c} c_{2\beta} M_Z^2$$

* III family (stops):

$$\begin{pmatrix} \tilde{m}^2_{q_3} + m_t^2 + z_u c_{2\beta} M_Z^2 & -m_t(A_t + \mu \cot\beta) \\ -m_t(A_t + \mu\cot\beta) & \tilde{m}^2_{u^c_3} + m_t^2 + z_{u^c} c_{2\beta} M_Z^2 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} \qquad 0 \leq \theta \leq \pi, \quad \tilde{m}_{t_1} < \tilde{m}_{t_2}$$

* Analogously in the D, E sectors. Relevant LR mixing in the third family only for large $\tan\beta$

* In general:

$$\mathcal{M}_U^2 = \mathcal{U}_U^\dagger \mathcal{M}_U^{2\text{diag}} \mathcal{U}_U \qquad \mathcal{M}_U^{2\text{diag}} = (\tilde{m}^2_{U_I})_{I=1}^6$$
$$\mathcal{M}_D^2 = \mathcal{U}_D^\dagger \mathcal{M}_D^{2\text{diag}} \mathcal{U}_D \qquad \mathcal{M}_D^{2\text{diag}} = (\tilde{m}^2_{D_I})_{I=1}^6$$
$$\mathcal{M}_E^2 = \mathcal{U}_E^\dagger \mathcal{M}_E^{2\text{diag}} \mathcal{U}_E \qquad \mathcal{M}_E^{2\text{diag}} = (\tilde{m}^2_{E_I})_{I=1}^6$$
$$M_\nu^2 = u_\nu^\dagger\, M_\nu^{2\text{diag}}\, u_\nu \qquad M_\nu^{2\text{diag}} = (\tilde{m}^2_{\nu_i})_{i=1}^3$$

$$\boxed{\tilde{q}_i = \begin{pmatrix} (\mathcal{U}_U^\dagger)_{iJ}\tilde{U}_J \\ (\mathcal{U}_D^\dagger)_{iJ}\tilde{D}_J \end{pmatrix} \quad \begin{matrix} \tilde{u}_i^{c*} = (\mathcal{U}_U^\dagger)_{(i+3)J}\tilde{U}_J \\ \tilde{d}_i^{c*} = (\mathcal{U}_D^\dagger)_{(i+3)J}\tilde{D}_J \end{matrix} \quad \tilde{l}_i = \begin{pmatrix} (u_\nu^\dagger)_{ij}\tilde{\nu}_j \\ (\mathcal{U}_E^\dagger)_{iJ}\tilde{E}_J \end{pmatrix} \quad \tilde{e}_i^{c*} = (\mathcal{U}_E^\dagger)_{(i+3)J}\tilde{E}_J}$$

* $\mathcal{W}_U \equiv \begin{pmatrix} U_u & \\ & U_{u_c}^* \end{pmatrix} \mathcal{U}_U^\dagger$ = relative rotation between up quarks and squarks enters supersymmetric gauge interactions and extra Yukawa interactions (analogously in D, E, ν sectors)

Interactions and phenomenology

$$\mathcal{L}_{\rm susy} = \text{Kinetic} + \text{gauge for } A_i,\, \psi_i,\, v^\mu_A,\, \lambda_A$$
$$- \left(\frac{1}{2}\partial_i\partial_j W(A)\psi_i\psi_j + \sqrt{2} g_A A_i^\dagger T_A^{ij} \lambda^A \psi_j + \text{h.c.} \right) - V(A)$$
$$V(A) = F_i^\dagger F_i + \frac{1}{2} D_A^2 \geq 0$$

$$\mathcal{L} = \mathcal{L}_{\rm susy} + \mathcal{L}_{\rm soft}$$
$$-\mathcal{L}_{\rm soft} = m^2_{ij} A_i^\dagger A_j + \left(\frac{M_{AB}}{2} \lambda_A \lambda_B + w(A) + \text{h.c.} \right)$$

$$W = \lambda^U_{ij} \hat{u}^c_i \hat{q}_j \hat{h}_u + \lambda^D_{ij} \hat{d}^c_i \hat{q}_j \hat{h}_d + \lambda^E_{ij} \hat{e}^c_i \hat{l}_j \hat{h}_d + \mu\, \hat{h}_u \hat{h}_d$$
$$\begin{aligned}
-\mathcal{L}_{\rm soft} = {} & A^U_{ij} \tilde{u}^c_i \tilde{q}^j h_u + A^D_{ij} \tilde{d}^c_i \tilde{q}^j h_d + A^E_{ij} \tilde{e}^c_i \tilde{l}^j h_d + m^2_{ud} h_u h_d + \text{h.c.} \\
& + (\tilde{m}^2_q)_{ij} \tilde{q}_i^\dagger \tilde{q}_j + (\tilde{m}^2_{u^c})_{ij} (\tilde{u}^c_i)^\dagger \tilde{u}^c_j + (\tilde{m}^2_{d^c})_{ij} (\tilde{d}^c_i)^\dagger \tilde{d}^c_j + (\tilde{m}^2_l)_{ij} \tilde{l}_i^\dagger \tilde{l}_j \\
& + (\tilde{m}^2_{e^c})_{ij} (\tilde{e}^c_i)^\dagger \tilde{e}^c_j + m^2_{h_u} h_u^\dagger h_u + m^2_{h_d} h_d^\dagger h_d \\
& + \frac{M_3}{2} \tilde{g}_A \tilde{g}_A + \frac{M_2}{2} \tilde{W}_a \tilde{W}_a + \frac{M_1}{2} \tilde{B}\tilde{B} + \text{h.c.}
\end{aligned}$$

+ express fields in terms of mass eigenstate

Example: dark matter detection

- Assume the LSP is the lightest neutralino
- The detection process proceeds through h (spin independent) or Z_μ (spin dependent) exchange
- $h\,\chi_1^0\chi_1^0$ from $h_u^\dagger\left(g\tilde{W}^a\frac{\sigma_a}{2}+g'\tilde{B}\frac{1}{2}\right)\tilde{h}_u+h_d^\dagger\left(g\tilde{W}^a\frac{\sigma_a}{2}-g'\tilde{B}\frac{1}{2}\right)\tilde{h}_d+\text{h.c.}$

$$\supseteq h\chi_1^0\chi_1^0\left[c_\alpha\left(g\,t_3(h_u^0)N^*_{1\tilde{W}^0}+g'y(h_u^0)N^*_{1\tilde{B}}\right)\tilde{N}^*_{1\tilde{h}_u^0}-s_\alpha\left(g\,t_3(h_d^0)N^*_{1\tilde{W}^0}+g'y(h_d^0)N^*_{1\tilde{B}}\right)\tilde{N}^*_{1\tilde{h}_d^0}\right]$$

note: the coupling vanishes in the small v/M limit

- ${\chi_1^0}^\dagger\sigma^\mu\chi_1^0Z_\mu$ from $\tilde{h}_u^\dagger i\sigma^\mu D_\mu\tilde{h}_u+\tilde{h}_d^\dagger i\sigma^\mu D_\mu\tilde{h}_d+\tilde{W}^\dagger i\sigma^\mu D_\mu\tilde{W}+\tilde{B}^\dagger i\sigma^\mu D_\mu\tilde{B}$

$$\left[D_\mu\supseteq\frac{g}{c_W}\left(T_3-s_W^2Y\right)Z_\mu\right]$$

$$\supseteq\frac{g}{c_W}z_{h_u^0}\left(N_{1h_u^0}N^*_{1h_u^0}-N_{1h_d^0}N^*_{1h_d^0}\right){\chi_1^0}^\dagger\sigma^\mu\chi_1^0Z_\mu$$
$$=\frac{g}{c_W}z_{h_u^0}\left(N_{1h_u^0}N^*_{1h_u^0}-N_{1h_d^0}N^*_{1h_d^0}\right)\overline{N_{1L}}\gamma^\mu N_{1L}Z_\mu$$

note: the coupling vanishes in the small v/M limit

$$\overline{\Psi_1}\Psi_2=\psi_1^c\psi_2+(\psi_1\psi_2^c)^*\quad\overline{\Psi_1}\gamma^\mu\Psi_2=\psi_1^\dagger\sigma^\mu\psi_2-(\psi_2^c)^\dagger\sigma^\mu\psi_1^c$$

Dark matter abundance

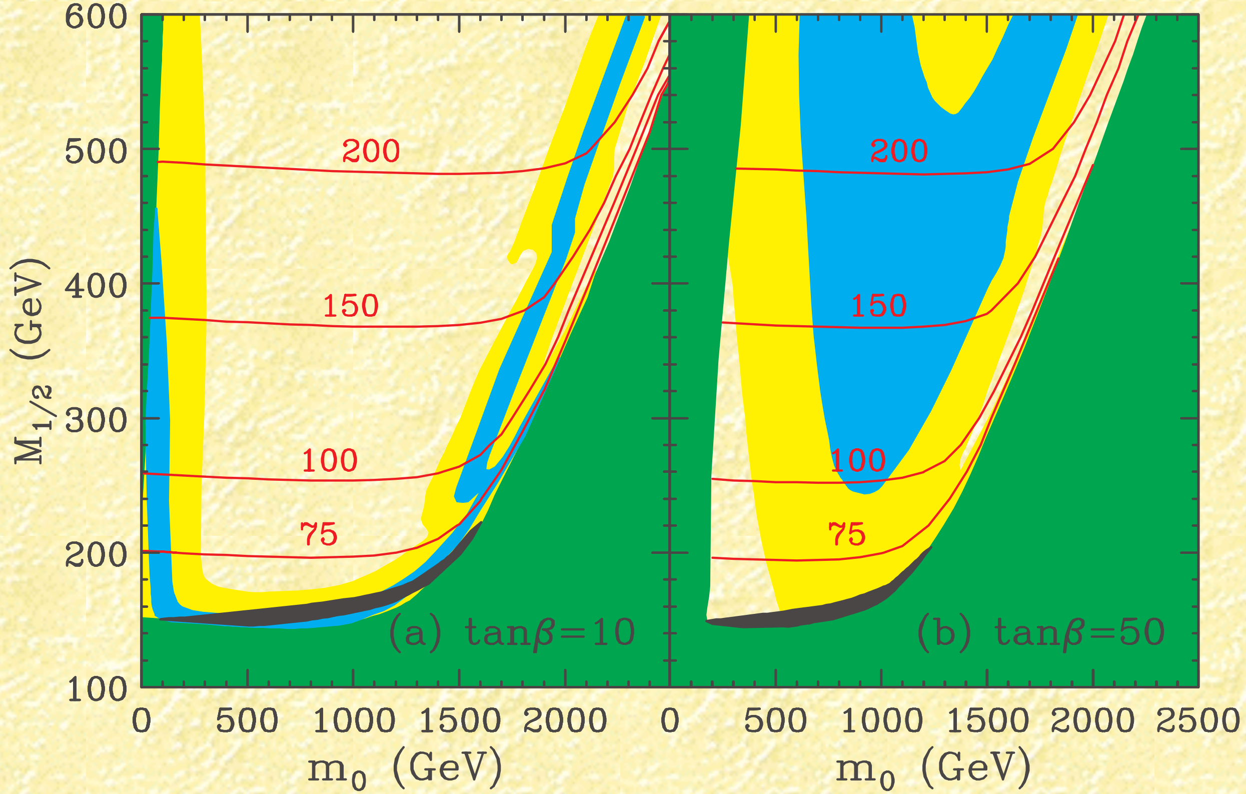


FIG. 1. Contours of constant LSP mass m_χ in GeV in the $(m_0, M_{1/2})$ plane for $A_0 = 0$, $\mu > 0$, $m_t = 174$ GeV, and two representative values of $\tan\beta$. The green shaded regions are excluded by the requirement that the LSP be neutral (left) and by the chargino mass limit of 95 GeV (bottom and right). We have also delineated the regions with potentially interesting values of the LSP relic abundance: $0.025 \le \Omega_\chi h^2 \le 1$ (yellow) and $0.1 \le \Omega_\chi h^2 \le 0.3$ (light blue). In the black region, $|2m_\chi - m_h| < 5$ GeV, and neutralino annihilation is enhanced by a Higgs resonance.

[Feng Matchev Wilczek, hep-ph/0008115]

Example: supersymmetric contributions to ϵ_K

- ϵ_K: CP-violation in $K^0(d\bar{s})$ - $\bar{K}^0(\bar{d}s)$ oscillations
- Induced by $\mathcal{H} \supseteq C_i Q_i \quad Q_i \sim (s^\dagger d)^2$

$$Q_1 = (\bar{s}_\alpha \gamma^\mu P_L d_\alpha)(\bar{s}_\beta \gamma_\mu P_L d_\beta) \qquad Q_6 = \tilde{Q}_1 = Q_1|_{L\leftrightarrow R}$$

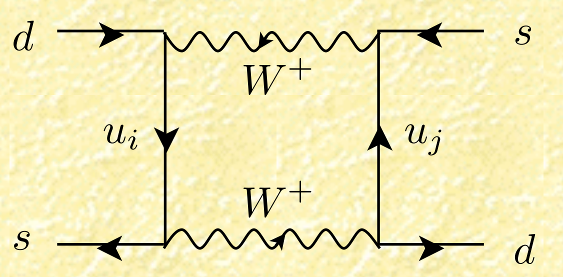
$$Q_2 = (\bar{s}_\alpha P_L d_\alpha)(\bar{s}_\beta P_L d_\beta) \qquad Q_7 = \tilde{Q}_2 = Q_2|_{L\leftrightarrow R}$$

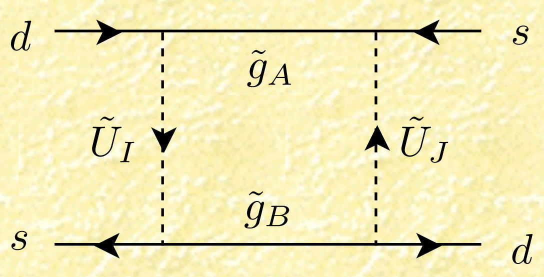
$$Q_3 = (\bar{s}_\alpha P_L d_\beta)(\bar{s}_\beta P_L d_\alpha) \qquad Q_8 = \tilde{Q}_3 = Q_3|_{L\leftrightarrow R}$$

$$Q_4 = (\bar{s}_\alpha P_L d_\alpha)(\bar{s}_\beta P_R d_\beta)$$

$$Q_5 = (\bar{s}_\alpha P_L d_\beta)(\bar{s}_\beta P_R d_\alpha)$$

- SM interactions only contribute to C_1Q_1, supersymmetry to all





+ another diagram
+ chargino and neutralino exchange

$$-\mathcal{L} \supseteq \sqrt{2} g_s \tilde{q}_i^\dagger \tilde{g}^A \frac{\lambda_A}{2} q_i - \sqrt{2} g_s (\tilde{d}_i^c)^\dagger \tilde{g}^A \frac{\lambda_A^T}{2} d_i^c + \mathrm{h.c.}$$

$$= \sqrt{2} g_s \mathcal{W}^\dagger_{D_J d_i^L} \tilde{D}_J^\dagger \frac{\lambda_A}{2} d_i \tilde{g}^A - \sqrt{2} g_s \mathcal{W}^\dagger_{D_J d_i^R} \tilde{D}_J^\dagger \frac{\lambda_A}{2} \left(d_i^c \tilde{g}^A\right)^* + \mathrm{h.c.}$$

$$= \sqrt{2} g_s \tilde{D}_J^\dagger \frac{\lambda_A}{2} \overline{G}^A \left(\mathcal{W}^\dagger_{D_J d_i^L} P_L + \mathcal{W}^\dagger_{D_J d_i^R} P_R\right) \psi_d$$

$$\overline{\Psi_1} \Psi_2 = \psi_1^c \psi_2 + (\psi_1 \psi_2^c)^* \quad \overline{\Psi_1} \gamma^\mu \Psi_2 = \psi_1^\dagger \sigma^\mu \psi_2 - (\psi_2^c)^\dagger \sigma^\mu \psi_1^c$$

Gauge coupling unification

	SU(3)	SU(2)	U(1)
L_i	1	2	-1/2
e^c_i	1	1	1
Q_i	3	2	1/6
u^c_i	3^*	1	1/3
d^c_i	3^*	1	-2/3
			Y

Gauge coupling unification

	SU(3)	SU(2)	U(1)	SO(10)
L_i	1	2	-1/2	
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Q_i	3	2	1/6	16
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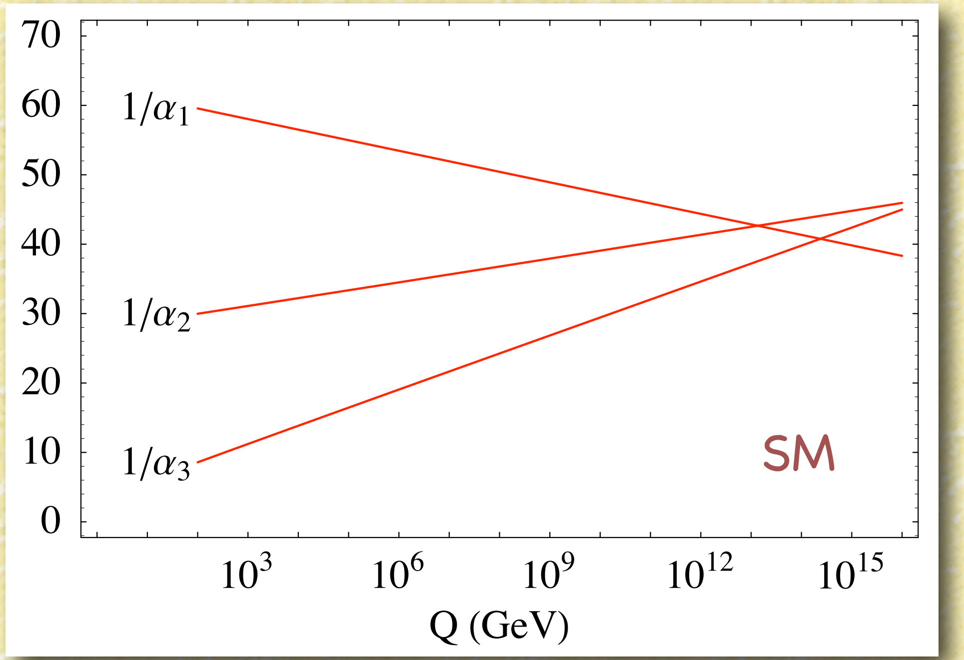

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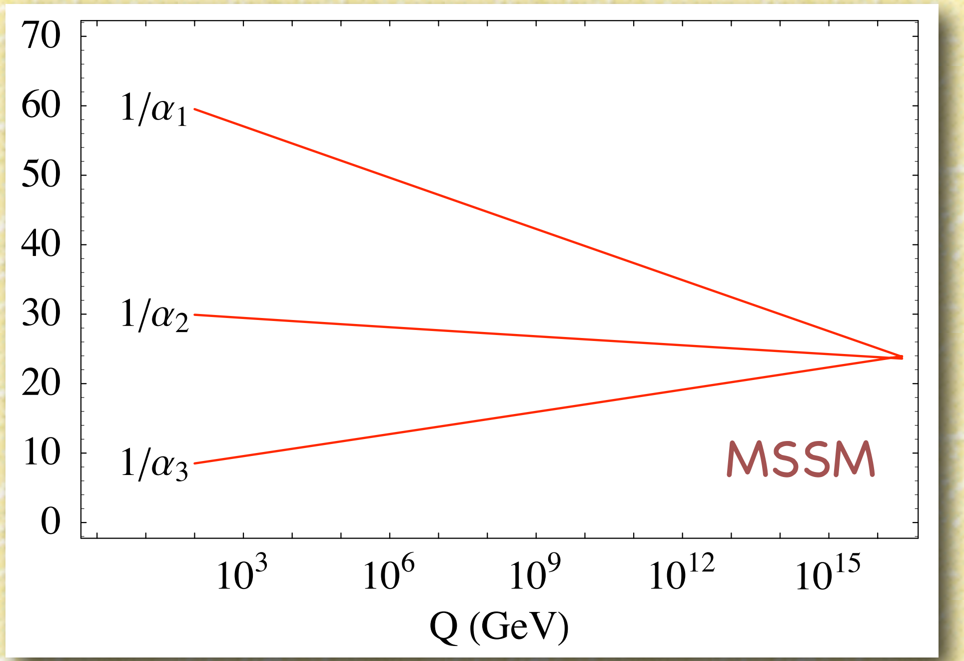
SO(10)

16



$$4\pi \frac{d\alpha_i^{-1}}{d\log(M^2/Q^2)} = b_i$$

$$b_3 = -7 + \frac{N_3}{3} \qquad \mathrm{Tr}(\mathbf{T}_A\mathbf{T}_B) = \frac{N_3}{2}\delta_{AB} \quad N_3 \geq 0 \text{ integer}$$

$$b_2 = -\frac{19}{6} + \frac{N_2}{3} \qquad \mathrm{Tr}(\mathbf{t}_a\mathbf{t}_b) = \frac{N_2}{2}\delta_{ab} \quad N_2 \geq 0 \text{ integer}$$

$$b_1 = \frac{41}{10} + \frac{N_1}{15} \quad \text{(SU(5) norm.)} \qquad \mathrm{Tr}(\mathbf{Y^2}) = \frac{N_1}{6} \quad N_1 \geq 0 \text{ integer (from SU(5) multiplets)}$$

(only fermions; with scalars: N → $N_f+N_s/4$)

The μ-problem

$$W = \lambda^U_{ij}\hat{u}^c_i \hat{q}_j \hat{h}_u + \lambda^D_{ij}\hat{d}^c_i \hat{q}_j \hat{h}_d + \lambda^E_{ij}\hat{e}^c_i \hat{l}_j \hat{h}_d + \mu\, \hat{h}_u \hat{h}_d$$

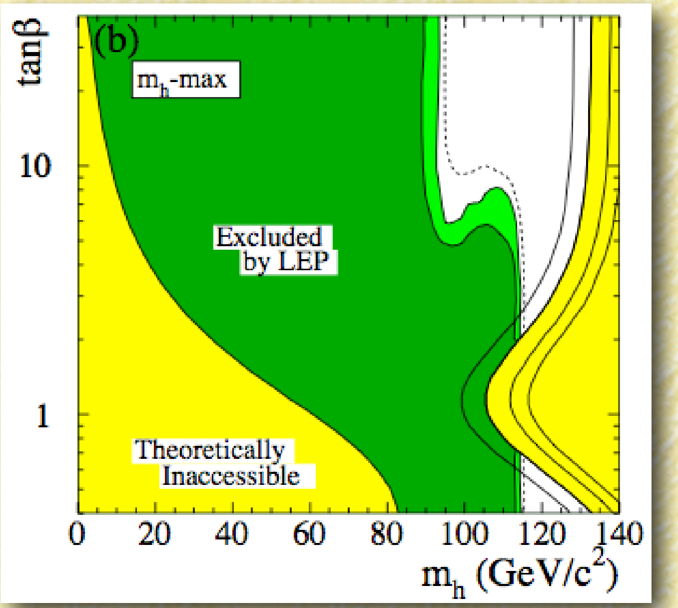
$$\begin{aligned} -\mathcal{L}_{\rm soft} =\; & A^U_{ij}\tilde{u}^c_i \tilde{q}^j h_u + A^D_{ij}\tilde{d}^c_i \tilde{q}^j h_d + A^E_{ij}\tilde{e}^c_i \tilde{l}^j h_d + m^2_{ud} h_u h_d + \text{h.c.} \\ & + (\tilde{m}^2_q)_{ij}\tilde{q}^\dagger_i \tilde{q}_j + (\tilde{m}^2_{u^c})_{ij}(\tilde{u}^c_i)^\dagger \tilde{u}^c_j + (\tilde{m}^2_{d^c})_{ij}(\tilde{d}^c_i)^\dagger \tilde{d}^c_j + (\tilde{m}^2_l)_{ij}\tilde{l}^\dagger_i \tilde{l}_j \\ & + (\tilde{m}^2_{e^c})_{ij}(\tilde{e}^c_i)^\dagger \tilde{e}^c_j + m^2_{h_u} h^\dagger_u h_u + m^2_{h_d} h^\dagger_d h_d \\ & + \frac{M_3}{2}\tilde{g}_A \tilde{g}_A + \frac{M_2}{2}\tilde{W}_a \tilde{W}_a + \frac{M_1}{2}\tilde{B}\tilde{B} + \text{h.c.} \end{aligned}$$

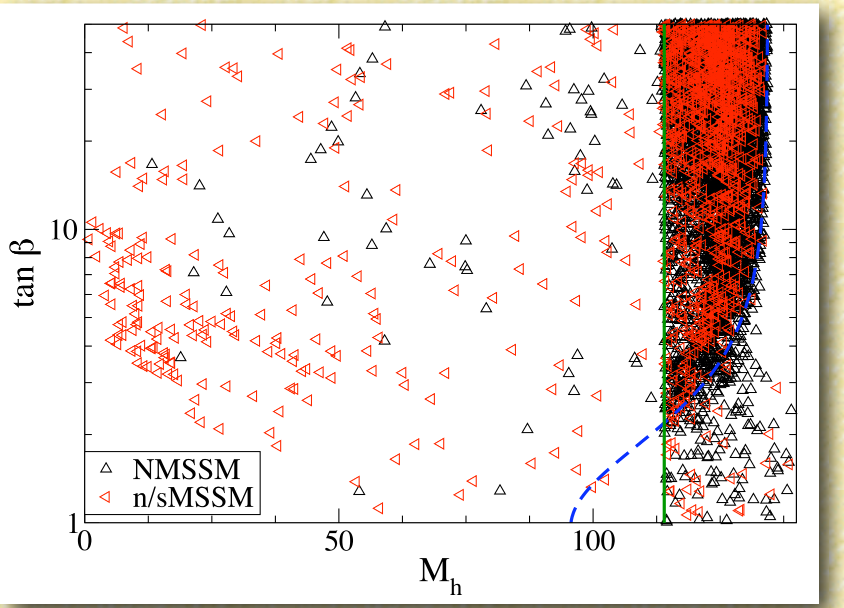
- 100 GeV $\lesssim \mu \lesssim$ TeV

- As the soft supersymmetry breaking parameters: why?
  - μ is actually a supersymmetry-breaking parameter (Giudice-Masiero) Reminder
  - μ = <S>, <S> induced by supersymmetry breaking (NMSSM)

Beyond MSSM: xMSSM

* Minimal extension: $\lambda S H_u H_d$ (with no $\mu H_u H_d$ because of symmetries)
  * harmless (unification OK)
  * welcome ($\mu = \lambda\langle S\rangle \approx$ susy scale)
* Spectrum: h H → h_1 h_2 h_3, A → a_1 a_2, $N_1 \ldots N_4$ → N_0 $N_1 \ldots N_4$
* Help with FT from $(114\,\mathrm{GeV})^2 < m_h^2 < M_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} h_t^2 m_t^2 \log \frac{\tilde{m}_t^2}{m_t^2}$:
  * $\lambda_H = \frac{g^2 + g'^2}{4} \cos^2 2\beta + \frac{\lambda^2}{2} \sin^2 2\beta + \mathrm{loops}$ (λ bound by Landau poles)
  * $m_h^2 < (114\,\mathrm{GeV})^2$ through invisible decays h → aa (m_a protected by PQ, R)
* Persistent FT from
  * direct bounds on SUSY partners
  * arranging the invisible decay [Shuster Toro hep-ph/0512189]
* Signatures:





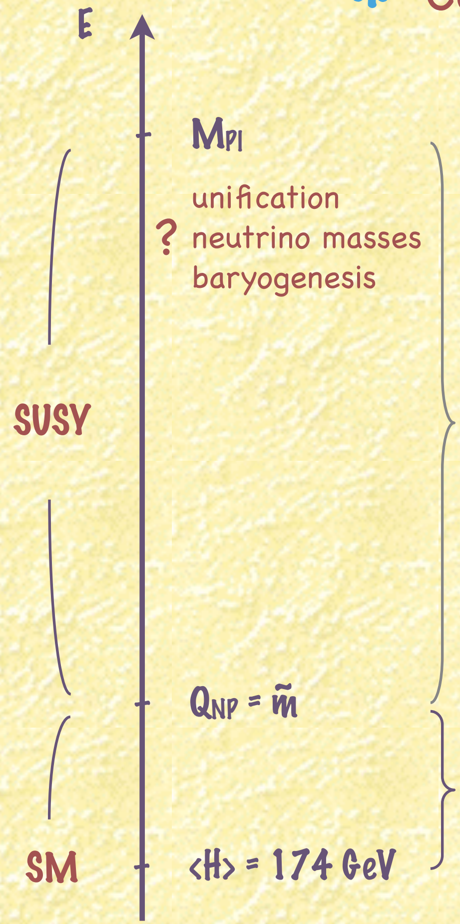
[Barger Langacker Lee Shaughnessy hep-ph/0603247]

* Invisible Higgs decays: $h \to aa \to 4X$ [No loose theorem? Ellwanger Gunion Hugonie Moretti hep-ph/0401228, ...]
* 3leptons → multileptons from additional steps in chargino/neutralino decays
  * C_1+N_2 and then
  * $N_2 \to N_1+2l \to N_0+4l$ (if N_0 is lightest and mainly singlino)
  * $C_1 \to N_0+l+\nu$ (5l overall) or even $C_1 \to N_1+l+\nu \to N_0+3l+\nu$ (7l overall)
* Deviation from MSSM coupling relations: $VVh = VHA = \sin^2(\alpha-\beta)$, $VVH = VhA = \cos^2(\alpha-\beta)$ (optimistic)
* Z′ if μ is protected by a gauge symmetry

Other variations on the MSSM

* Combine MSSM with extra-dimensions not far from TeV

[Pomarol Quiros hep-ph/9806263
Barbieri Hall Nomura hep-ph/0011311]



$$\delta m_h^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 Q_{\rm NP}^2 \log \frac{M_{\rm Pl}^2}{Q_{\rm NP}^2}$$

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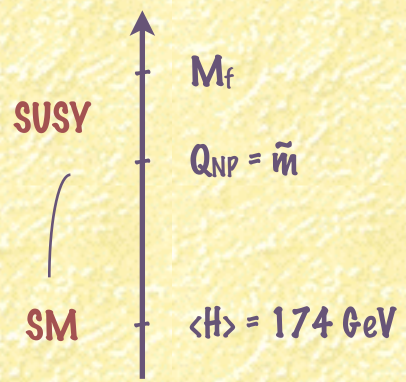

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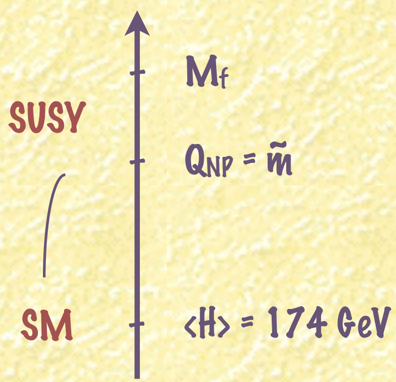

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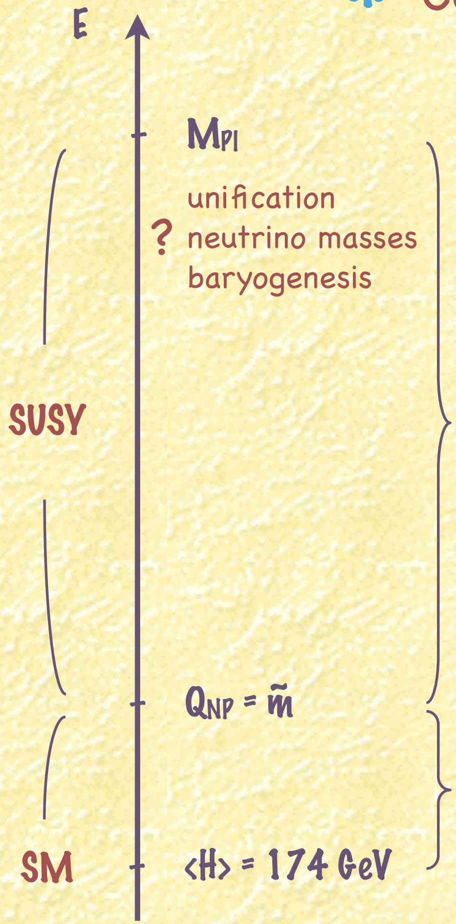
Other variations on the MSSM

* Combine MSSM with (Simplest) Little Higgs

[Berezhiani Chankowski Falkovski Pokorski hep-ph/0509311
Roy Schmaltz hep-ph/0509357]



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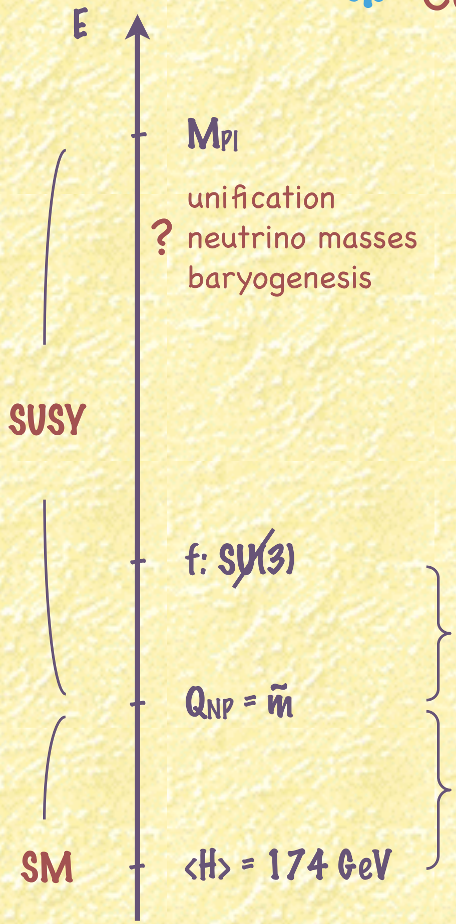


$$\delta m_h^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 Q_{\rm NP}^2 \log \frac{f^2}{Q_{\rm NP}^2}$$

$$\delta m_h^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 Q_{\rm NP}^2$$

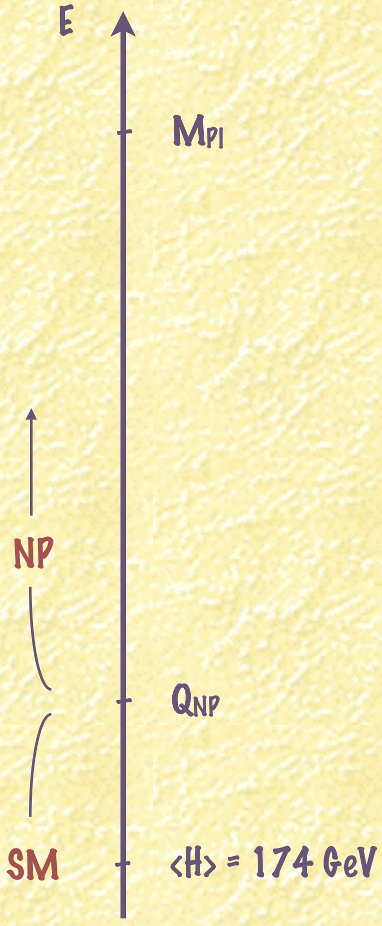
* Issues
  * Potentially > 100 parameters (CMSSM)
  * FCNCs and CP-violation in particular EDMs (SUSY breaking mechanism, symmetries)
  * Proton decay from dimension 5 operators (non minimal models)
  * Gravitino and moduli problem (low reheating T)
  * Fine-tuning (NMSSM)

scalars

* Successes of the MSSM
  * Gauge coupling unification
  * Natural dark matter candidate (with R-parity)

fermions

Is a natural m_H unavoidable?



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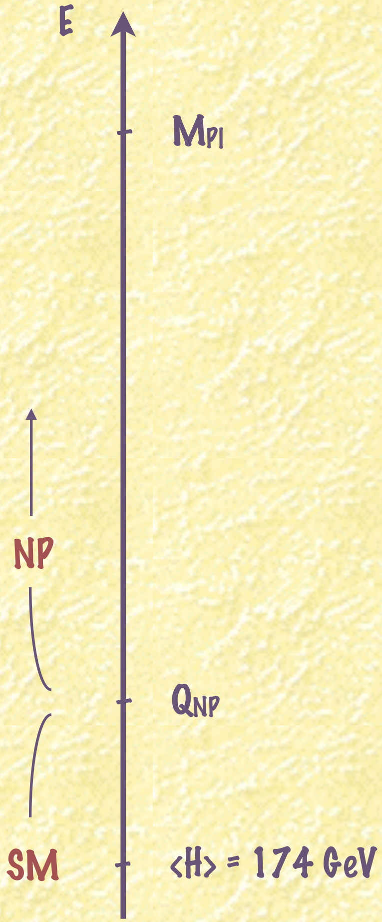
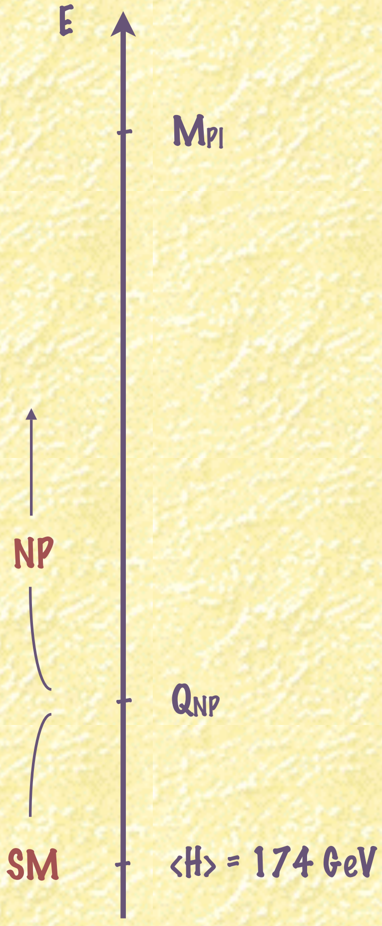


* What about the cosmological constant?

Is a natural m_H unavoidable?



* What about the cosmological constant?
* If the m_h naturalness criterium is irrelevant, what are the observable consequences?

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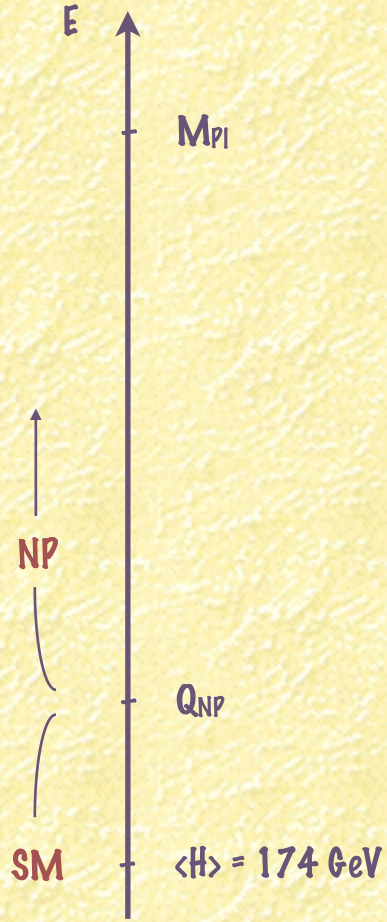

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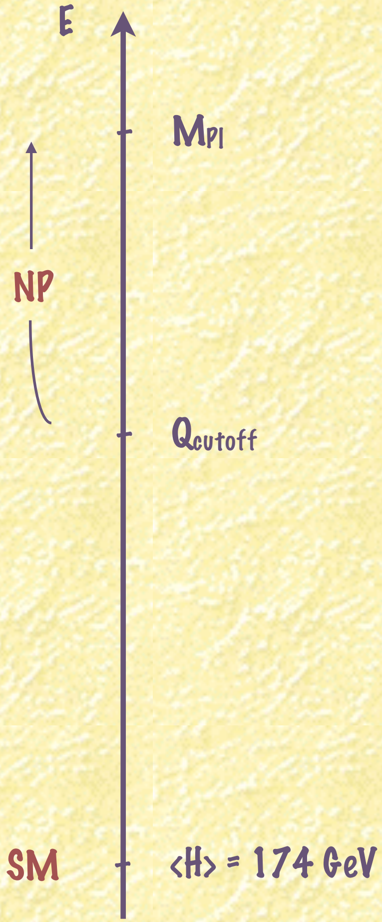

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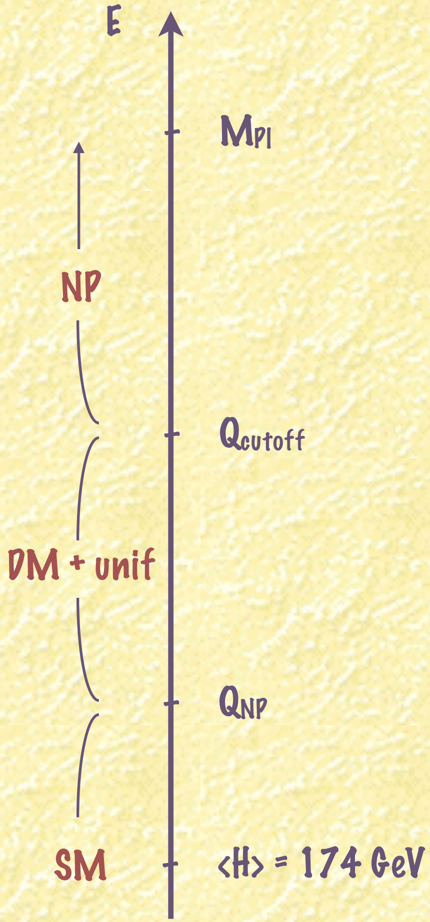
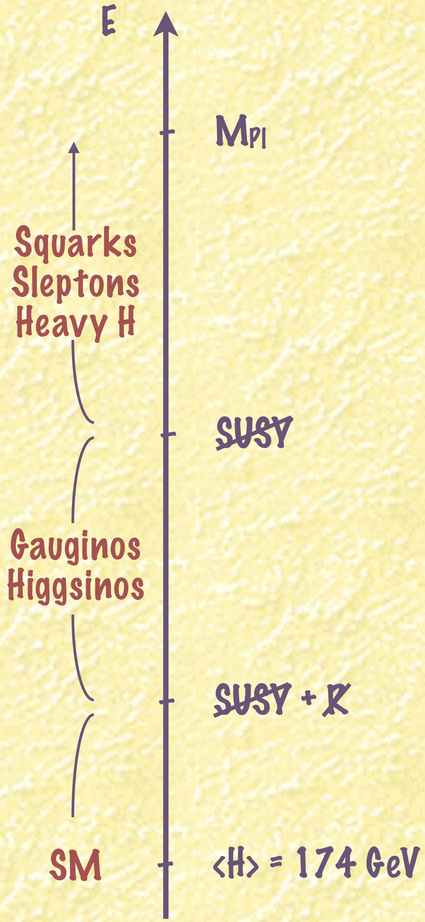


* What about the cosmological constant?
* If the m_h naturalness criterium is irrelevant, what are the observable consequences?
* LHC..?
* Dark matter still motivates NP at the TeV scale

[Arkani-Hamed Dimopoulos 04,
Giudice R 04,
Arkani-Hamed Dimopoulos Giudice R 04]

Split Supersymmetry



* DM: μ < 1.2 TeV ($M_1 < M_2$), mostly Bino favourable for LHC
* No bounds from EWPTs
* m_H < 170 GeV, in terms of of $\tilde{m}$, $\tan\beta$
* Long-lived gluino R-hadrons (charged: slow, highly ionizing track; neutral: missing energy, mild hadronic activity; actually: Energy, charge, Baryon-number exchange) LHC sensitivity up to (1-2.5) TeV [Kilian Plehn Richardson Schmidt hep-ph/0408088, Hewett Lillie Masip Rizzo hep-ph/0408248, Kraan Hansen Nevski hep-ex/0511014]
* (quasi-stable coloured particles also e.g stop in some 5D SUSY models or in MSSM with fine-tuned $\tilde{m}_t \approx M_{N1}$)
* Wilder: stopping gluinos (1-2 jets in any direction from denser parts of the detector + m.e.), displaced vertexes (low m), charge flips