Emergent Gravity: the Analogue Gravity perspective

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Emergent Time and Emergent Space in Quantum Gravity

Postdam, 15 December 2014
Emergence: process whereby larger entities, patterns and regularities arise through interactions among smaller or simpler entities that themselves do not exhibit such properties.

In the following by Emergent Gravity and/or Spacetime I mean that they are collective manifestations of very different underlying d.o.f.

And I cherish more than anything else the Analogies, my most trustworthy masters. They know all the secrets of Nature, and they ought least to be neglected in Geometry.

Johannes Kepler
Emergence.....and Gravity/Spacetime

Einstein General Relativity (GR) is (one of) the most elegant and successful theories in physics! But we want to look beyond:

- Singularities: the theory is marking its own limit of validity
- Information loss and trans-Planckian problem
- Cosmology: Dark component of the Universe
- Cosmological constant problem

There are different hints to the fact that GR could be a low energy hydrodynamics of some more fundamental constituent:

- Non-renormalizability: suggest to us that GR is a low energy EFT like hydrodynamics
- Black Hole thermodynamics and the thermodynamics of spacetime: Einstein’s equation as an equation of state [T. Jacobson, PRL (’95)]

Maybe GR and Spacetime itself are emergent phenomena, the ultimate macroscopic quantum phenomena!! [B.L. Hu, Int. Journal of Theo. Phy., 2005]

**Analogue Models for Gravity:** gives a useful tool to tackle the problem of emergence in condensed matter systems (experimentally testable)
The Analogue Gravity Program: Motivations

Analogue models: generally condensed matter (but not only) systems that share kinematical and/or dynamical features with QFT on curved spacetimes and/or theories of Gravity


A list of motivations for the interest in the program

1. Use **condensed matter physics** to gain insight into classical GR and QFTCS
2. Develop an **observational** widow on QFTCS
3. Use GR to gain insight into condensed matter physics
4. Gain insight into new and different ways of dealing with **quantum/emergent gravity**.
The Analogue Gravity Program

KINEMATICS:
Emergent spacetime $\rightarrow$ ways in which an effective pseudo-Riemannian structure is emerged

Idea: use analogies to study QFTCS effects

- Hawking radiation
- Cosmological particle production
- Superradiance

and also to test modifications of these effects due to the features of the system under study (ex. MDR)

The systems considered usually involved known physics and are typically experimentally feasible...also at home!
The Analogue Gravity Program

DYNAMICS:
Emergent Gravity $\rightarrow$ ways in which a gravitational-like dynamics emerge from the dynamics of the underlying d.o.f.

Idea: use analogue models to develop techniques useful in Emergent Gravity scenarios such as

- GFT
- Matrix models
- CST
- AdS/CFT
- ...

Here the focus is in learning lessons from simplified models
Dynamical systems where the propagation of linearised perturbations can be described via hyperbolic equations of motion possibly characterized by one single metric for all the perturbations:

- Dielectric media
- **Acoustic waves in moving fluids**
- **Gravity waves**
- High-refractive index dielectric fluids: slow light
- Optic Fibers analogues
- Quasi-particle excitations: fermionic or bosonic quasi-particles in He3
- Non-linear electrodynamics
- Solid states black holes
- **Perturbation in Bose-Einstein condensates**
- Graphene
- Many more...
The Ancestor of all analogues: Acoustic Gravity

Moving fluids drag sound waves along with them and sound waves can never escape/enter a supersonic flow region ⇒ DUMB HOLE

Sound cone:
\[
\frac{dx}{dt} = cn + v; \quad n^2 = 1
\]
\[-(c^2 - v^2)dt^2 - 2v \cdot dx dt + dx \cdot dx = 0\]
i.e. a conformal class of Lorentzian metrics. \(c\) sound velocity; \(v\) fluid velocity

Theorem: Acoustic Gravity

Given a barotropic and inviscid fluid with irrotational flow then the e.o.m. for the (linear perturbation of the) velocity potential is the d’Alembert equation for a massless minimally coupled scalar field on a 3+1-Lorentzian geometry

\[
\Box_g \phi \equiv \frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu \nu} \partial_\nu \phi \right) = 0
\]

where the emergent effective metric is given by

\[
g_{\mu \nu}(t, x) \equiv \frac{\rho}{c} \begin{bmatrix}
-(c^2 - v^2) & \vdots & -v^T \\
\ldots & \vdots & \ldots \\
-v & \vdots & J
\end{bmatrix}
\]

Physical model for BH spacetimes
Sound waves in a moving fluid feel an **effective curved spacetime metric** even if the underlying physics is **non-relativistic Newtonian** physics!

- The manifold **topology** is fixed to $\mathbb{R}^4$.
- More complex topologies are possible with effectively low dimensional systems.
- The metric is **stably causal** due to the underlying Newtonian spacetime.
- **Trapped surfaces** and **Apparent horizons** can be easily defined.
- **Event (sonic) horizon** is the boundary of the region from which null geodesics (phonons) cannot escape.
- Any region of supersonic flow is an **ergoregion**

\[
    g_{\mu\nu} \left( \frac{\partial}{\partial t} \right)^\mu \left( \frac{\partial}{\partial t} \right)^\nu = g_{tt} = -(c^2 - v^2)
\]

In general the arguments shown above can be applied to different analogue systems in which an effective Lorentzian metric emerge!
Bernoulli eq. for a barotropic, inviscid fluid with irrotational flow under the action of gravity:

\[ \partial_t \phi + \frac{1}{2} (\nabla \phi)^2 = -\frac{p}{\rho} - gz - V_\parallel \]

where \( V_\parallel \) external potential to generate an horizontal flow, \( v_\parallel \), and
\( p(z = h_B) = 0, \ v_\perp (z = 0) = 0 \)

It can be shown that surface waves with long wavelength (\( \lambda \gg h_B \)) see an effective metric

\[
\frac{ds^2}{c^2} = \left[ -\left( c^2 - v_\parallel^2 \right) dt^2 - 2v_\parallel \cdot dx \cdot dt + dx \cdot dx \right]
\]

Changing the depth of the basin and the flow velocity from point to point it is possible to generalize the model and simulate different Lorentzian metrics!

- In the shallow water regime \( \omega = ck \)
- For more general waves the dispersion relation will change: useful to have analogue of MDR usually used in QG phenomenology
- Given the low velocity propagation of waves the corresponding Hawking temperature will be very low making difficult to detect analogue Hawking radiation
Let’s make a **White Hole** [G. Jannes, G. Rousseaux, arXiv:1203.6505 [gr-qc]]

Basic setup: A liquid is pumped through a nozzle and the fluid jet impacts vertically onto a horizontal plate. It will form a thin layer near the impact zone, which expands radially and at a certain distance abruptly increases in thickness.

**White Hole experiment**: try to send surface waves inside the horizon from outside, these will be not able to cross it.

**Mach Cone Experiment**: Putting an object in the three regions of interest (subsonic, supersonic and horizon). For this situation the physics is analogue to the case of a moving object in a fluid, if the velocity of the object is greater than the speed of sound this will create an observable Mach cone formed by the envelope of the wavefront when instead the cone do not form if the velocity is subsonic.
For general water waves the dispersion relation is not linear

\[ \omega = c k \sqrt{\frac{\tanh(kd)}{kd}} \]

This is useful to study the classical effect of stimulated Hawking Radiation.

First direct observation of negative-frequency waves converted from positive-frequency ones in a moving medium with analogue WH. [Rousseaux et al., New J.Phys., 2008]

HR mode conversion at white hole horizon has been detected in a shallow water basin! [S.Weinfurtner et al., Phys.Rev.Lett.106, 2011]

Important test of the dependence of HR on high frequency behaviour of the theory. Relevant Bogoliubov coefficients have been experimentally measured and are observed to satisfy the expected Boltzmann relation

\[ \frac{|\beta|^2}{|\alpha|^2} = \exp\left\{ \frac{-2\pi \omega}{g_H/c_H} \right\} \]

Superradiance, the process of amplification of a wave incident on a rotating black hole. Draining bathtub geometry can permit to simulate this process. SISSA–Nottingham experiment: an analogue of superradiant scattering (PI: S. Weinfurtner)
Quantum Model: BEC

Hawking Radiation and Cosmological Particle production are QFTCS phenomena. Search for systems in which the linearized perturbations are quantized ⇒ phonons

Bose Einstein Condensate (BEC), particular phase of a system of identical bosons in which a single energy level acquires a macroscopic occupation number

Some good features of BEC based analogue models:

- Excitations are quantize hence mimicking QFT on CS
- Very low temperature: in order to have $T_H \approx T_{environment}$
- Low sound speed, makes it easier to produce horizons!
BEC Analogue model

The system is described by the Hamiltonian in second quantization formalism

$$H = \int dx \hat{\Psi}^\dagger(t, x) \left[ -\frac{\hbar^2}{2m} \nabla^2 - \mu + \frac{\kappa}{2} |\hat{\Psi}|^2 \right] \hat{\Psi}(t, x)$$

- $\kappa$ represent the strength of the interaction. Proportional to the scattering length
- Mean field approximation: $\hat{\Psi}(t, x) = \psi(t, x) + \hat{\phi}(t, x)$
- $\psi(t, x) \equiv \langle \Omega | \hat{\psi} | \Omega \rangle$, **condensate wave function**, classical complex scalar field that plays the role of **order parameter** of the phase transition
- Fock vacuum of quasi-particles is inequivalent to the Fock vacuum of atoms: $\hat{\Psi} |\Omega\rangle \neq 0$

(Time dependent) Gross-Pitaevskii(GP) equation and equation for the excitations:

$$i\hbar \frac{\partial}{\partial t} \psi(t, x) = \left( -\frac{\hbar^2}{2m} \nabla^2 - \mu + \kappa|\psi(t, x)|^2 \right) \psi(t, x)$$

$$i\hbar \frac{\partial}{\partial t} \hat{\phi} = -\frac{\hbar^2}{2m} \nabla^2 \hat{\phi} + \mu \hat{\phi} + \mu \hat{\phi}^\dagger$$
**BEC Analogue model**

Madelung representation for the order parameter:

$$\psi(t, x) \equiv \rho(t, x)e^{-i\theta(t,x)/\hbar}$$

Continuity+Euler equation

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0; \quad m\partial_t \mathbf{v} + \nabla \left( \frac{m\mathbf{v}^2}{2} - \mu + \kappa \rho - \nabla \cdot \frac{\hbar^2}{2m} \frac{\nabla \sqrt{\rho}}{\sqrt{\rho}} \right) = 0$$

$$\mathbf{v} \equiv \frac{\nabla \theta}{m}$$ is the (locally) irrotational velocity field.

Hydrodynamical form of GP equation ⇒ hydrodynamics of an irrotational, inviscid fluid plus a quantum potential self-interaction term

The hydrodynamical limit is the low momentum limit ($\lambda >$healing length) in which the quantum potential is negligible. In this regime the equation for the perturbations is

$$\partial_{\mu} \left( f^{\mu\nu} \partial_{\nu} \delta \hat{\theta} \right) = 0$$

$$g_{\mu\nu}(t, x) \equiv \frac{\rho}{mc_s} \begin{bmatrix} -(c_s^2 - v^2) & \vdots & -v_j \\ \cdots & \ddots & \cdots \\ -v_i & \vdots & \delta_{ij} \end{bmatrix}$$

$$c_s^2 \equiv \frac{\lambda \rho}{m}$$ is the sound velocity, i.e. the speed of the phonons in the condensate.
BEC Analogue model: Summary

**Emergent Lorentz** invariance of the (IR) spectrum from an underlying **Galielian** system; Accidental symmetry of the **low energy** theory

\[ \omega^2 = c^2 k^2 + \frac{\hbar^2 k^4}{(2m)^2} \]

The separation scale between IR and UV regimes is given by the **healing length**, \( \xi = \frac{1}{8\pi \rho} \) characterizing the typical scale of the BEC dynamic.

This DR is of the type used in QG phenomenology for describe some **LIV EFT!!**

**Why BEC analogue are so interesting?**

1. **Emergence** of QFTCS in the IR from a non relativistic system
2. Possibility to simulate exotic processes as **signature changing events**
3. Testing the robustness of **HR** and **Inflationary particle production** to different **UV modifications**
4. Possibility to detect HR via **correlation measurement** between inside and outside the BH!! **We can since it is a tabletop BH!!**
Analogue gravity program is successful in providing a specific and precise example of $discrete \rightarrow continuum \rightarrow differential\ geometry$ chain of development

**BUT**

What about the **dynamics** of the effective spacetime?? Emerging Einstein Equation from condensed matter systems is not an easy task

**WW theorem** is not a big deal in analogue models:
- It requires **strict Lorentz Invariance at all scales**
- There are condensed matter systems that show the emergence of helicity $\pm 2$ particles
- There are also other way out since it is **just** a no-go theorem!

Let’s see now a couple of systems in which we have **emergent gravity-like dynamics**!
Emergent Newtonian-like Gravity from BEC:


Non-relativistic equation ⇒ at most **Newtonian gravity**!!

For this we need to give a mass to the phonon ⇒ Introduce a soft $U(1)$ **breaking term**!

From **Goldstone** to **Pseudo-Goldstone** bosons!!

- Physically reasonable for a BEC in contact with a reservoir that preserve in average the number of atoms
- Realized in magnons system

For the purpose of analyzing **dynamical aspect** we have to resort to a refinement of the mean field method to take into account **backreaction** of the fluctuations on the condensate i.e. Bogoliubov-de Gennes equation:

$$i\hbar \frac{\partial}{\partial t} \psi(t, x) = \left(-\frac{\hbar^2}{2m} \nabla^2 - \mu + \kappa |\psi|^2\right) + 2\kappa (\tilde{n}\psi + \tilde{m}\psi^*) - \lambda \psi^*$$

Dispersion relation for quasi-particles:

$$\mathcal{E}^2(k) = \frac{k^4}{4m^2} + c_s^2k^2 + \mathcal{M}^2c_s^4$$

\[\mathcal{M} \ll m; \quad \text{if } \lambda \ll \mu\]

Time independent background solution:

$$\rho \equiv |\psi|^2 = \frac{\mu + \lambda}{\kappa}$$
Emergent Newtonian-like Gravity from BEC

Consider a nearly homogeneous BEC...the BEC is allowed to be non-homogeneous only in the bulk: asymptotically flat $\iff$ asymptotically homogeneous

Condensate nearly homogeneous:

$$\psi = \sqrt{\frac{\mu + \lambda}{\kappa}} (1 + u(x)); \quad u(x) \ll 1$$

The velocity perturbations can be neglected

The gravitational potential is encoded in the density perturbations! To find the actual form it is necessary to inspect the Hamiltonian of quasi-particles in the non-relativistic limit. The result is

$$\Phi_{grav}(x) = \frac{(\mu + 4\lambda)(\mu + 2\lambda)}{2\lambda m} u(x)$$

Finally we want to see if this satisfies some sort of modified Poisson equation

For the case of one localized quasi-particle as source the BdG equation becomes:

$$\left( \frac{\hbar^2}{2m} \nabla^2 - 2(\mu + \lambda) \right) u(x) = 2\kappa \left( \tilde{n}(x) + \frac{1}{2} \tilde{m}(x) \right) + 2\kappa \left( n_\Omega + \frac{1}{2} m_\Omega \right)$$

$$\tilde{n}(x) = \tilde{n}(x) - n_\Omega$$

$$\tilde{m}(x) = \tilde{m}(x) - m_\Omega$$

These are the quasi-particle vacuum backreaction terms
The equation can be cast in the form of a generalized Poisson equation:

$$\left( \nabla^2 - \frac{1}{L^2} \right) \Phi_{grav} = 4\pi G_N \rho_{matter} + \Lambda$$

Where the emergent quantities are:

$$\rho_{matter} = \left( \bar{n}(x) + \frac{1}{2} \bar{m}(x) \right)$$

$$\Lambda = \frac{2\kappa(\mu + 4\lambda)(\mu + 2\lambda)}{\lambda \hbar^2} \left( n_{\Omega} + \frac{1}{2} m_{\Omega} \right)$$

Lessons:

- **Emergent Newtonian-like** gravity form a BEC analogue model
- The emergent gravity is short range, related to the nature of the condensate and the healing length
- **Cosmological constant** naturally arise; the vacuum gravitate
- Cosmological constant naturally small

$$\mathcal{E}_P = \frac{c_s^7}{\hbar G_N^2}, \quad \mathcal{E}_\Lambda = \frac{\Lambda c_s^4}{4\pi G_N}; \quad \frac{\mathcal{E}_\Lambda}{\mathcal{E}_P} \propto \rho a^3 \left( \frac{\lambda}{\kappa \rho} \right)^{-5/2}$$
The treatment can be generalized to the Relativistic BEC case

[S.Fagnocchi et al., New J. Phys., 2010]

A Relativistic BEC has proven to be a good analogue model for **kinematics**. Moreover the DR shows LIV due to interpolation between two different relativity groups.

In a suitable regime one can obtain an analogue of **scalar gravity**!

- Mean field: $\varphi = \varphi_0 (1 + \psi_1 + i\psi_2)$
- Assume the order parameter to be **real**: particular choice of the vev

Equation of motion for the linear fractional perturbations…..

\[ \Box \psi_1 + 2\eta^{\mu\nu} \partial_\mu (\ln \varphi_0) \partial_\nu \psi_1 - 4\lambda \varphi_0^2 \psi_1 = 0 \]

\[ \Box \psi_2 + 2\eta^{\mu\nu} \partial_\mu (\ln \varphi_0) \partial_\nu \psi_2 = 0 \]

The emergent metric is **conformally flat**, we have effectively only one background d.o.f. to play with!!
The Ricci scalar for the conformal metric:

\[ R_g \equiv -6 \frac{\Box \varphi_0}{\varphi_0^3} \]

The relativistic BdG equation can be rewritten in a geometrical form

\[ R_g + 12 \lambda = -12 \lambda \left[ 3 \langle \psi_1^2 \rangle + \langle \psi_2^2 \rangle \right] \]

c.f. the Einstein-Fokker equation describing Nordström gravity \( R + \Lambda = 24\pi \frac{G_N}{c^4} T \)

**Analogue Nordström gravity**

\[ R + \Lambda = 24\pi \frac{G_{\text{eff}}^N}{c^4} \langle T \rangle \]

where

\[ \Lambda = 12 \lambda \frac{\mu^2}{\hbar c}, \]

\[ G_{\text{eff}}^N = \hbar c^5 / (4\pi \mu^2) \]
Nordström (Analogue) Gravity from RBEC: Summary

- rBEC as Analogue Model for the dynamics of Gravity Theories, as in the non-relativistic case
- One can obtain an analogue of Nordström (scalar) theory of gravity from a peculiar corner of the general case
- Emergence of Nordström gravity expected due to the d.o.f. of the model
- More general Lagrangians are needed for realistic models
- The $\lambda \phi^4$ interaction gives rise to a cosmological constant small compared with the emergent Planck scale
  \[ \frac{\mathcal{E}_A}{\mathcal{E}_P} \ll 1 \]
- Proof of concept that is possible to have an emergent fully LI system
Conclusions:

Analogue Models for Gravity:

I  Possibility of testing QFTCS predictions in lab experiments

II New ways to tackle problems in semiclassical-gravity like the Trans-Planckian problem of BH and inflation

III Physical systems that exhibit the emergence QFT in CS, LI and a Lorentzian metric at low energies

IV Ideas for QG phenomenology and emergent spacetime/gravity scenarios