Casual–set Non-Locality

Alessio Belenchia

SISSA, International School for Advanced Studies
INFN, Sezione di Trieste

Astro@TS, 3 June 2015

Discreteness, Lorentz Symmetry and all of that

Discreteness of Spacetime: Arise in different ways in many approaches to QG (LQG, CST, Spinfoam, ecc.). Hints from general arguments involving QT+GR of the existence of a **minimal length scale**.

QG phenomenology aim: find some observational signature of such a discreteness

$\Rightarrow$ Look for a fundamental **discreteness** of (quantum) spacetime

WARNING: Discreteness and Lorentz Invariance, [J. Collins *et al.*, PRL('93)]

A general result in QFT show that unless unnaturally tuned at the Planck scale UV Lorentz violation will percolate in the IR not suppressed by any mass scale.

- There is an **extreme fine-tuning**. Another fine tuning problem to add to the list
- Most of the theories with Lorentz violation are already experimentally **ruled out**

Try to look for Lorentz invariant preserving candidates and to their phenomenology!

Causal set theory (CST) is an example of a discrete and Lorentz invariant proposal for a theory of QG [L.Bombelli *et al.*, Modern Phys. Lett. A(2009)]

Discreteness, Lorentz Symmetry and all of that

Discreteness of Spacetime: Arise in different ways in many approaches to QG (LQG, CST, Spinfoam, ecc.). Hints from general arguments involving QT+GR of the existence of a **minimal length scale**.

QG phenomenology aim: find some observational signature of such a discreteness

$$E^2 = p^2 + m^2 + \sum_{n=0}^{\infty} \xi_n \frac{p^n}{M^{n-2}}$$

Order	photon	e^-/e^+	Protrons	Neutrinos[a]
n=2	N.A.	$O(10^{-16})$	$O(10^{-20})$ (CR)	$O(10^{-8} \div 10^{-10})$
n=3	$O(10^{-16})$ (GRB)	$O(10^{-16})$ (CR)	$O(10^{-14})$ (CR)	$O(40)$
n=4	$O(10^{-8})$ (CR)	$O(10^{-8})$ (CR)	$O(10^{-6})$ (CR)	$O(10^{-7})^*$ (CR)

[S.Liberati, Class. Quantum Grav.(2013)]

Try to look for Lorentz invariance preserving candidates and to their phenomenology!

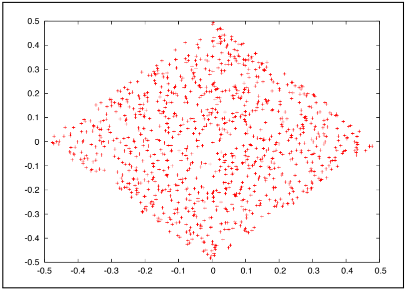
Causal set theory (CST) is an example of a discrete and Lorentz invariant proposal for a theory of QG [R.Sorkin, arXiv:gr-qc/0309009]

A discrete theory of spacetime

Causal Set: is a partial order with a discreteness axiom, $\mathcal{C}$.

VOLUME+ORDER$\Rightarrow$GEOMETRY $\longrightarrow$ NUMBER+ORDER$\Rightarrow$GEOMETRY

Sprinkling process:Poisson random process of choosing countable subset of spacetime points for which the expected number of points chosen from any given region of spacetime is equal to its volume in fundamental unit (in Planck volume)

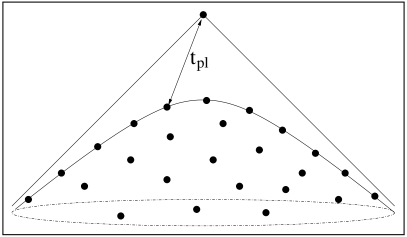


Sprinkling process is **(Locally) Lorentz Invariant** so that the causal set produced this way is also Lorentz Invariant [L.Bombelli *et al.*, Mod. Phys. Lett. A(2009)]

A spacetime M is a good approximation of a causal set $\mathcal{C}$, $M \approx \mathcal{C}$, if $\mathcal{C}$ can be obtained by the sprinkling process with relatively high probability.

Discreteness, Lorentz Invariance and Non-Locality

The price to pay in order to maintain Lorentz Invariance and having a fundamentally discrete spacetime is **non-locality**
but this non-locality is **causal**!



The nearest neighbors points to a fixed one are one Planck unit proper time from this

⇓

they will almost stay on a hyperboloid and there will be an infinite number of them (in Minkowski)

QG phenomenological goal: studying the effect of such a LI non-locality

Non-local wave operators I: [[R.Sorkin, arXiv:gr-qc/0703099; D.Benincasa, F.Dowker, PRL(2010)....many others]]

Idea: build a CS discrete version of the wave operator such as its mean over sprinkling of Minkowski spacetime converge in the continuum approximation to the usual d'Alembert's operator

$$\lim_{\rho\to\infty}\left[\mathbb{E}\left(B^{(d)}\phi(x)\right)\right] = \Box\phi(x)$$

Wild **fluctuations** require the introduction of a **non-locality** scale much bigger then the discreteness one

$$\rho = \frac{1}{l_{nl}^D}, \quad l_{nl} \gg l_P$$

Example: 4D *minimal* operator

$$\Box_\rho^{(4)}\phi(x) = -\frac{4}{\sqrt{6}}\left[\rho^{1/2}\phi(x) - \rho^{3/2}\int_{J^-(x)} d^4y\left(1 - 9\rho V + 8\rho^2V^2 - \frac{4}{3}\rho^3V^3\right)\phi(y)e^{-\rho V}\right]$$

Non-local wave operators II

What can be extracted from these operators?

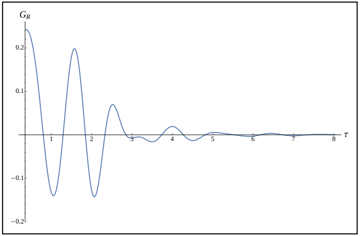
First correction to the local operator due to non-locality: [AB, Benincasa, Liberati, *JHEP*('15), S. Johnston, *arXiv:1411.2614*]

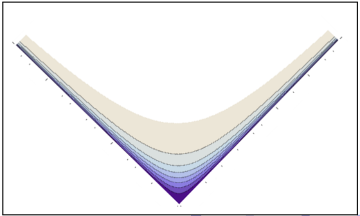
$$\Box_\rho^{(4)} \approx \Box - \frac{3}{2\pi\sqrt{6}}\left[3\gamma - 2 + \ln\left(\frac{3\Box^2}{2\pi\rho}\right)\right]\frac{\Box^2}{\sqrt{\rho}} + \ldots .$$

WORK IN PROGRESS

Spectral dimension: peculiar behavior of the spectral dimension that approach from above the Hausdorff dimension in the IR but run to 2 in the UV in any dimension. Connected to the good UV properties of the non-local operators (possible **UV LI regulators**!?![S. Aslanbeigi, M. Saravani, R. Sorkin,*JHEP('14)*]) [AB, Benincasa, Marcianó, Modesto, *in preparation*]

Huygens Principle violations: there are tails inside the lightcone for massless fields in flat spacetime. Violation of the Huygens principle that usually happens only in odd dimensions (or in presence of curvature) [AB, Benincasa, Liberati, *JHEP*('15)]





Non-local wave operators II

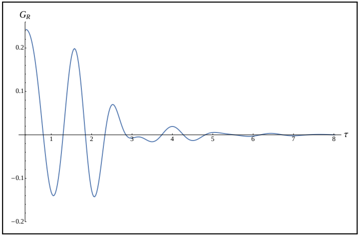
What can be extracted from these operators?

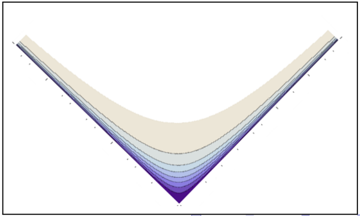
First correction to the local operator due to non-locality: [AB, Benincasa, Liberati, *JHEP*('15), S. Johnston, *arXiv:1411.2614*]

$$\square_{\rho}^{(4)} \approx \square - \frac{3}{2\pi\sqrt{6}}\left[3\gamma - 2 + \ln\left(\frac{3\square^2}{2\pi\rho}\right)\right]\frac{\square^2}{\sqrt{\rho}} + \dots$$

WORK IN PROGRESS

Spectral dimension: peculiar behavior of the spectral dimension that approach from above the Hausdorff dimension in the IR but run to 2 in the UV in any dimension. Connected to the good UV properties of the non-local operators (possible **UV LI regulators**!?![S. Aslanbeigi, M. Saravani, R. Sorkin,*JHEP('14)*]) [AB, Benincasa, Marcianó, Modesto, *in preparation*]
Huygens Principle violations: there are tails inside the lightcone for massless fields in flat spacetime. Violation of the Huygens principle that usually happens only in odd dimensions (or in presence of curvature) [AB, Benincasa, Liberati, *JHEP*('15)]





Summary and Outlook

- A quantization scheme for free fields was proposed recently [AB, Benincasa, Liberati, *JHEP*('15)]
- There are issues with extensions to an interacting theory
- Propagation of massless fields in 4D is modified: effect that is grasped by the spectral dimension
- Possible interesting phenomenology of **violation of Huygens principle**: Astrophysics, laboratory q.o. experiment?
- Non-relativistic phenomenology: *work in progress* with **microscopic quantum oscillators experiments**