Emergent gravitational dynamics from relativistic Bose-Einstein condensate

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Conceptual and Technical Challenges for Quantum Gravity

Based on **arXiv:1407.7896**, work in collaboration with *Stefano Liberati* and *Arif Mohd*

Rome, 8 September 2014

ma per seguir virtute e canoscenza
SISSA

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Different hints to the fact that **GR** is a low energy **hydrodynamics** of some more fundamental constituent:

- Non-renormalizability: said to us that GR is an EFT at low energy (long wavelength) as well as hydrodynamics
- Black Hole thermodynamics and the thermodynamics of spacetime: Einstein equation as an equation of state [T.Jacobson, PRL ('95)]

Maybe try to quantize the metric and connection forms will give us the analogous of **phonons** and not the **atoms** of the underline theory.

Analogue Models for Gravity: gives a useful tool and toy models to tackle the problem of emergence from an underline substratum in condensed matter systems (usually **experimentally testable**)

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Analogue models

Analogue Model: condensed matter systems[1] that present kinematical and/or dynamical features analogue to gravitational theories [C.Barcelo, S.Liberati, M.Visser, Living Rev.Rel.('11)]

Particular attention is given to BEC-based analogue models since they are quantum (macroscopic) systems!

Most analogue models offer an analogue for **kinematic** features of gravity theories that permit to test general features of QFT in CS

- Hawking radiation [W. G. Unruh, PRL ('81)]
- Cosmological Particle production [C.Barcelo, S.Liberati, M.Visser, PRA(2003)]
- Super-radiance [M.Richartz *et al* 2013 Class. Quantum Grav.]

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- **Not (so) good:** dynamical feature are essential in order to study the nature of spacetime

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Try to find some analogue model that can mimic the **dynamic** of (some) gravity theory ⇒ not an easy task, one model for **Newtonian-like** dynamics in BEC system [F.Girelli, S.Liberati, L.Sindoni, PRD(2008)]

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BEC Analogue Models Vademecum

In BEC based Analogue models for gravity the order parameter gives rise to an effective metric that is felt by the linearized perturbations

- Split the field in order parameter plus linear perturbations
- Use Madelung representation for the order parameter (condensate wave function)
- Hydrodynamic form of Gross-Pitaevskii(GP) equation $\Rightarrow$ hydrodynamics (continuity+Euler) of an irrotational fluid plus a *quantum potential* self-interaction term
- In the low momentum limit the linearized perturbations feels an acoustic effective metric

Quasi-particles behave as massless minimally coupled scalar field on a **curved effective Lorentzian metric**, i.e. the so called acoustic metric

In general there are LIV terms in the dispersion relation of the quasi-particles that are negligible only in the low momentum limit

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Relativistic BEC I

A **non-relativistic** BEC can be shown to give us:

1. A relativistic wave equation for perturbations, i.e. analogue **kinematic features**
2. A non-relativistic **Poisson equation** considering the back-reaction of perturbations on the background condensate that play the role of source of the emergent gravitational field [F.Girelli, S.Liberati, L.Sindoni, PRD(2008)]

One can try to see what happen considering **relativistic BEC**
[S.Fagnocchi *et al.*, New J. Phys.(2010)]

1. **Good** analogue system for kinematic properties: quasi-particles moves on an **acoustic metric** in the low momentum limit
2. The acoustic metric is the same as in a **relativistic irrotational barotropic fluid** [M. Visser, C. Molina-Paris, New J.Phys. 12 (2010)]
3. The dispersion relation shows LIV due to interpolation between two different relativity group (different limit speeds)
4. It is possible to obtain an analogue of **Nordström gravity**, that is a scalar gravity theory[AB, A.Mohd, S.Liberati, gr-qc:1407.7896]

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Relativistic BEC II

BEC (and superfluidity) can be described in a completely relativistic framework
We consider a weakly interacting $\lambda\phi^4$ complex scalar field theory in grand canonical formalism

Effective Lagrangian and Critical Temperature [J. I. Kapusta, Phys.Rev. D24, 426 (1981)]

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\left(\dot{\phi}_1^2 + \dot{\phi}_2^2 + (\vec{\nabla}\phi_1)^2 + (\vec{\nabla}\phi_2)^2\right) + i\mu(\phi_2\dot{\phi}_1 - \phi_1\dot{\phi}_2) + V(\phi)$$

$$V(\phi) = \frac{1}{2}(m^2-\mu^2)(\phi_1^2+\phi_2^2) + \frac{\lambda}{4}(\phi_1^2+\phi_2^2)^2$$

$$T_c = \frac{3}{\lambda}\left(\mu^2 - m^2\right)$$

Note: condensation can happen at finite temperature also in the massless limit.

After a field redefinition: $\phi \to \varphi e^{i\mu t}$ we ended with the starting equation of [S.Fagnocchi *et al.*, New J. Phys.(2010)]

$$\boxed{\left(\Box - m^2\right)\varphi - 2\lambda|\varphi|^2\varphi = 0}$$

In the following we will assume that a condensate is actually present and we have a non-zero chemical potential

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rBEC Analogue Model: general case

In the general case we can follow the *vademecum*:

1. Splitting in order parameter plus *fractional* perturbations: $\varphi = \varphi_0(1+\psi)$
2. Using Madelung representation to obtain the background equation plus back-reaction as in non-relativistic BEC
3. Equation for perturbations:
$$\Box\psi + 2\eta^{\mu\nu}(\partial_\mu \ln \varphi_0)\partial_\nu\psi - |\varphi_0|^2(\psi+\psi^\dagger) = 0$$
4. Low momentum limit:
$$\Box_g \psi = 0$$

The effective metric that the perturbations feel is the so called acoustic metric:

Acoustic metric

$$g_{\mu\nu} = \rho\frac{c}{c_s}\left[\eta_{\mu\nu} + \left(1-\frac{c_s^2}{c^2}\right)\frac{v_\mu v_\nu}{c^2}\right]$$

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Nordström (Analogue) Gravity from RBEC I

$$\boxed{\left(\Box - m^2\right)\varphi - 2\lambda|\varphi|^2\varphi = 0}$$

- Mean field plus (fractional) fluctuations splitting: $\varphi = \varphi_0(1 + \psi_1 + i\psi_2)$
- Assume the order parameter to be **real**: $\varphi_0 \in \mathbb{R}$

Relativistic GP equation+back-reaction

$$(\Box - m^2)\varphi_0 - 2\lambda\varphi_0^3 - 2\lambda\varphi_0^3\left[3\left\langle\psi_1^2\right\rangle + \left\langle\psi_2^2\right\rangle\right] = 0$$

- Equation of motion of the (linear, fractional) perturbations

$$\boxed{\Box\psi_1 + 2\eta^{\mu\nu}\partial_\mu(\ln\varphi_0)\partial_\nu\psi_1 - 4\lambda\varphi_0^2\psi_1 = 0}$$

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Nordström (Analogue) Gravity from RBEC II

- For $g_{\mu\nu} = \varphi_0^2 \eta_{\mu\nu}$

$$\Box_g f = \frac{1}{\varphi_0^2} \Box + \frac{2}{\varphi_0^2} \eta^{\mu\nu} \partial_\mu (\ln \varphi_0) \partial_\nu f$$

- The perturbations feel an effective curved (conformally flat) metric

$$\Box_g \psi_1 - 4\lambda\psi_1 = 0 \quad \Box_g \psi_2 = 0$$

- Ricci scalar for the conformal metric:

$$R_g \equiv -6 \frac{\Box \varphi_0}{\varphi_0^3}$$

- The GP equation can be rewritten in a (almost) geometrical form

$$R_g + 6\frac{m^2}{\varphi_0^2} + 12\lambda = -12\lambda \left[3\left\langle \psi_1^2 \right\rangle + \left\langle \psi_2^2 \right\rangle \right]$$

- Taking the **massless** limit one recover something similar to the Einstein-Fokker equation describing Nordström gravity $R + \Lambda = 24\pi \frac{G_N}{c^4} T$

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Nordström (Analogue) Gravity from RBEC III

- The action of the system can be recast in a geometrical form
- From that we can obtain the trace of the Stress Energy tensor of the perturbations

$$T_{\mu\nu} \equiv -\frac{1}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\mathcal{L}_2\right)}{\delta g^{\mu\nu}}$$

- Matching it with the RHS of the geometrical equation we can determine the expression for the emergent Newton constant

Analogue Nordström gravity

$$\boxed{R + \Lambda = 24\pi \frac{G_{\rm N}^{eff}}{c^4} \langle T \rangle}$$

where

$$\Lambda = 12\lambda \frac{\mu^2}{c\hbar},$$

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Summary

- rBEC as Analogue Model also for the **dynamics** of Gravity Theories as in the non-relativistic case
- It can be obtain an analogue of **Nordström (scalar) theory of gravity** from a peculiar corner of the general case
- Emergence of Nordström gravity expected due to **d.o.f.** of the model
- More general Lagrangian are needed for more **realistic models**
- The $\lambda\phi^4$ interaction gives rise to a cosmological constant such that

$$\frac{\epsilon_\Lambda}{\epsilon_P} \simeq \frac{3\lambda\hbar c}{4\pi^2}$$

- Emergence of a (more as possible) general covariant dynamics for the background given the back-reaction of the perturbations in the limit in which **LIV are tuned to vanish**
- **Proof of concept** that is possible to have an emergent fully LI system [2]
- Possible interesting **toy model** for geometrogenesis and/or nature of spacetime singularities in emergent gravity scenarios

[2] At least at the level of linear perturbations