Emergent gravitational dynamics from relativistic Bose-Einstein condensate

Alessio Belenchia
SISSA, International School for Advanced Studies
INFN, Sezione di Trieste

Conceptual and Technical Challenges for Quantum Gravity

Based on arXiv:1407.7896, work in collaboration with Stefano Liberati and Arif Mohd

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Different hints to the fact that GR is a low energy hydrodynamics of some more fundamental constituent:

- Non-renormalizability: said to us that GR is an EFT at low energy (long wavelength) as well as hydrodynamics
- Black Hole thermodynamics and the thermodynamics of spacetime: Einstein equation as an equation of state [T.Jacobson, PRL (’95)]

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*Particular attention is given to BEC-based analogue models since they are quantum (macroscopic) systems!*

Most analogue models offer an analogue for kinematic features of gravity theories that permit to test general features of QFT in CS

- Hawking radiation [W. G. Unruh, PRL (’81)]
- Cosmological Particle production [C.Barcelo, S.Liberati, M.Visser, PRA(2003)]
- Super-radiance [M.Richartz *et al* 2013 Class. Quantum Grav.]

- **Good:** kinematic features are independent from the particular dynamics
- **Not (so) good:** dynamical feature are essential in order to study the nature of spacetime

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Try to find some analogue model that can mimic the dynamic of (some) gravity theory ⇒ not an easy task, one model for **Newtonian-like** dynamics in BEC system [F.Girelli, S.Liberati, L.Sindoni, PRD(2008)]

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In BEC based Analogue models for gravity the order parameter gives rise to an effective metric that is felt by the linearized perturbations:

- Split the field in order parameter plus linear perturbations
- Use Madelung representation for the order parameter (condensate wave function)
- Hydrodynamic form of Gross-Pitaevskii(GP) equation $\Rightarrow$ hydrodynamics (continuity+Euler) of an irrotational fluid plus a quantum potential self-interaction term
- In the low momentum limit the linearized perturbations feels an acoustic effective metric

Quasi-particles behave as massless minimally coupled scalar field on a curved effective Lorentzian metric, i.e. the so called acoustic metric.

In general there are LIV terms in the dispersion relation of the quasi-particles that are negligible only in the low momentum limit.
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Relativistic BEC I

A non-relativistic BEC can be shown to give us:

1. A relativistic wave equation for perturbations, i.e. analogue kinematic features
2. A non-relativistic Poisson equation considering the back-reaction of perturbations on the background condensate that play the role of source of the emergent gravitational field [F.Girelli, S.Liberati, L.Sindoni, PRD(2008)]

One can try to see what happen considering relativistic BEC [S.Fagnocchi et al., New J. Phys.(2010)]

1. Good analogue system for kinematic properties: quasi-particles moves on an acoustic metric in the low momentum limit
2. The acoustic metric is the same as in a relativistic irrotational barotropic fluid [M. Visser, C. Molina-Paris, New J.Phys. 12 (2010)]
3. The dispersion relation shows LIV due to interpolation between two different relativity group (different limit speeds)
4. It is possible to obtain an analogue of Nordström gravity, that is a scalar gravity theory [AB, A.Mohd, S.Liberati, gr-qc:1407.7896]
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Relativistic BEC II

BEC (and superfluidity) can be described in a completely relativistic framework. We consider a weakly interacting $\lambda \phi^4$ complex scalar field theory in grand canonical formalism.


$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \left( \dot{\phi}_1^2 + \dot{\phi}_2^2 + (\vec{\nabla} \phi_1)^2 + (\vec{\nabla} \phi_2)^2 \right) + i \mu (\phi_2 \dot{\phi}_1 - \phi_1 \dot{\phi}_2) + V(\phi)$$

$$V(\phi) = \frac{1}{2} (m^2 - \mu^2)(\phi_1^2 + \phi_2^2) + \frac{\lambda}{4} (\phi_1^2 + \phi_2^2)^2$$

$$T_c = \frac{3}{\lambda} \left( \mu^2 - m^2 \right)$$

Note: condensation can happen at finite temperature also in the massless limit.

After a field redefinition: $\phi \rightarrow \varphi e^{i\mu t}$ we ended with the starting equation of [S.Fagnocchi et al., New J. Phys.(2010)]

$$\left( \Box - m^2 \right) \varphi - 2\lambda |\varphi|^2 \varphi = 0$$

In the following we will assume that a condensate is actually present and we have a non-zero chemical potential.
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**Effective Lagrangian and Critical Temperature** [J. I. Kapusta, Phys.Rev. D24, 426 (1981)]

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In the general case we can follow the *vademecum*:

1. Splitting in order parameter plus *fractional* perturbations: \( \varphi = \varphi_0 (1 + \psi) \)
2. Using Madelung representation to obtain the background equation plus back-reaction as in non-relativistic BEC
3. Equation for perturbations:

\[
\Box \psi + 2 \eta^{\mu\nu} (\partial_\mu \ln \varphi_0) \partial_\nu \psi - |\varphi_0|^2 (\psi + \psi^\dagger) = 0
\]

4. Low momentum limit:

\[
\Box g \psi = 0
\]

The effective metric that the perturbations feel is the so called acoustic metric:

\[
g_{\mu\nu} = \rho \frac{c}{c_s} \left[ \eta_{\mu\nu} + \left( 1 - \frac{c_s^2}{c^2} \right) \frac{\nu_\mu \nu_\nu}{c^2} \right]
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rBEC Analogue Model: general case

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Nordström (Analogue) Gravity from RBEC I

\[
\left(\Box - m^2\right) \varphi - 2\lambda |\varphi|^2 \varphi = 0
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- Mean field plus (fractional) fluctuations splitting: \( \varphi = \varphi_0 (1 + \psi_1 + i\psi_2) \)
- Assume the order parameter to be real: \( \varphi_0 \in \mathbb{R} \)

Relativistic GP equation + back-reaction

\[
(\Box - m^2) \varphi_0 - 2\lambda \varphi_0^3 - 2\lambda \varphi_0^3 \left[ 3 \langle \psi_1^2 \rangle + \langle \psi_2^2 \rangle \right] = 0
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- Equation of motion of the (linear, fractional) perturbations

\[
\Box \psi_1 + 2\eta^{\mu \nu} \partial_\mu (\ln \varphi_0) \partial_\nu \psi_1 - 4\lambda \varphi_0^2 \psi_1 = 0
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Nordström (Analogue) Gravity from RBEC II

- For $g_{\mu\nu} = \varphi_0^2 \eta_{\mu\nu}$
  \[
  \Box_g f = \frac{1}{\varphi_0^2} \Box + \frac{2}{\varphi_0^2} \eta^{\mu\nu} \partial_\mu (\ln \varphi_0) \partial_\nu f
  \]

- The perturbations feel an effective curved (conformally flat) metric
  \[
  \Box_g \psi_1 - 4\lambda \psi_1 = 0 \quad \Box_g \psi_2 = 0
  \]

- Ricci scalar for the conformal metric:
  \[
  R_g \equiv -6 \frac{\Box \varphi_0}{\varphi_0^3}
  \]

- The GP equation can be rewritten in a (almost) geometrical form
  \[
  R_g + 6 \frac{m^2}{\varphi_0^2} + 12\lambda = -12\lambda \left[ 3 \langle \psi_1^2 \rangle + \langle \psi_2^2 \rangle \right]
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- Taking the massless limit one recover something similar to the Einstein-Fokker equation describing Nordström gravity $R + \Lambda = 24\pi \frac{G_N}{c^4} T$
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Nordström (Analogue) Gravity from RBEC III

- The action of the system can be recast in a geometrical form.
- From that we can obtain the trace of the Stress Energy tensor of the perturbations

\[ T_{\mu\nu} \equiv -\frac{1}{\sqrt{-g}} \frac{\delta (\sqrt{-g}L_2)}{\delta g^{\mu\nu}} \]

- Matching it with the RHS of the geometrical equation we can determine the expression for the emergent Newton constant.

\[ R + \Lambda = 24\pi \frac{G_N^{\text{eff}}}{c^4} \langle T \rangle \]

where

\[ \Lambda = 12\lambda \frac{\mu^2}{\hbar c}, \]

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**Analogue Nordström gravity**

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Summary

- rBEC as Analogue Model also for the **dynamics** of Gravity Theories as in the non-relativistic case
- It can be obtain an analogue of **Nordström (scalar) theory of gravity** from a peculiar corner of the general case
- Emergence of Nordström gravity expected due to d.o.f. of the model
- More general Lagrangian are needed for more **realistic models**
- The $\lambda\phi^4$ interaction gives rise to a cosmological constant such that
  \[
  \frac{\epsilon_\Lambda}{\epsilon_p} \approx \frac{3\lambda\hbar c}{4\pi^2}
  \]
- Emergence of a (more as possible) general covariant dynamics for the background given the back-reaction of the perturbations in the limit in which LIV are tuned to vanish
- **Proof of concept** that is possible to have an emergent fully LI system \(^2\)
- Possible interesting **toy model** for geometrogenesis and/or nature of spacetime singularities in emergent gravity scenarios

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\(^2\) At least at the level of linear perturbations