INdAM meeting

Contemporary Trends in the Mathematics of Quantum Mechanics – Rome 4-8 July 2016

Programme

Monday 4 July 2016

9:30 ***opening***

9:40 Luca Fanelli *Frequency-dependent time-decay of Schrödinger groups*
10:50 ***coffee break***
11:20 Andrea Posilicano *Limiting absorption principle, generalised eigenfunctions and scattering matrix for Laplace operators with boundary conditions on hypersurfaces*

14:40 Costanza Benassi *Correlation inequalities for the quantum XY model*
15:50 ***coffee break***
16:10 Gian Michele Graf *Dynamical crossing of an infinitely degenerate critical point*
17:20 Alessandro Giuliani *Periodic striped ground states in Ising models with competing interactions*

Tuesday 5 July 2016

9:40 Gianluca Panati *Optimal decay of Wannier functions in Chern and Quantum Hall insulators*
10:50 ***coffee break***
11:20 Michele Correggi *Curvature effects in surface superconductivity*

14:40 Francesca Arici *Quantisation and reduction of lattice gauge fields, a non-commutative approach*
15:50 ***coffee break***
16:10 Domenico Monaco *Chern and Fu-Kane-Mele invariants as topological obstructions*
17:20 Fabio Cipriani *Transverse potential theory*

Wednesday 6 July 2016

9:20 Jan Derezinski *Almost homogeneous Schroedinger operators*
10:30 ***coffee break***
11:00 Kenji Yajima *Existence and regularity of propagators for multi-particle Schroedinger equations*
12:10 Naiara Arrizabalaga *An isoperimetric-type inequality for shell interactions for Dirac operators*
13:20 ***lunch buffet***

Thursday 7 July 2016

9:40 Serena Cenatiempo *Quantum many-body fluctuations around non-linear Schrödinger dynamics*
10:50 ***coffee break***
11:20 Nam Phan Thanh *Norm approximation for many-body quantum dynamics*

14:40 Diego Noja *The non-linear Schrödinger equation with a concentrated non-linearity: scaling limit from a regular dynamics*
15:50 ***coffee break***
16:10 Riccardo Adami *The curious occurrence of negative energy ground states for a critical NLS*
17:20 Domenico Finco *On the ground state for the NLS on a graph*

Friday 8 July 2016

9:20 Fabio Benatti *Physics of complete positivity*
10:30 ***coffee break***
11:00 Andrea Sacchetti *Quantum resonances and time decay for a double-barrier model*
12:10 Alessandro Teta *Stability problem for a quantum system with zero-range interactions*

13:20 ***closing***

Riccardo Adami ***The curious occurrence of negative energy ground states for a critical NLS***

For the L^2 critical NLS on R^n, ground states cannot have negative energy: indeed, for a ground state to exist it is necessary that the nonlinearity is focusing, and the computation of the moment of inerzia due to Glassey shows that every state with negative energy blows up. We show that if the spatial domain is a quantum graph, then the compact core of the graph can trap negative energy ground states if some topological assumption is satisfied. This is a joint result with Enrico Serra e Paolo Tilli.

Francesca Arici ***Quantisation and reduction of lattice gauge fields, a non-commutative approach***

In this talk I will review the quantisation of lattice gauge fields, describing their algebra of observables. I wille explain how reduction of gauge symmetries can be interpreted in terms of Rieffel-Induction. Finally, I will elaborate on how to use lattice refinements to study the continuum limit, and show how groupoids and their C*-algebra naturally appears in this setting. Based on joint work with R. Stienstra and W. van Suijlekom.

Naiara Arrizabalaga ***An isoperimetric-type inequality for shell interactions for Dirac operators***

In this talk we study the spectral properties of $H+V_{\lambda}$, where $H=-i\alpha\cdot\nabla+m\beta$ is the free Dirac operator in ${\mathbb R}^3$ and V_{λ} is an electrostatic shell potential, (depending on a parameter $\lambda\in \mathbb{R}$), that is located on the boundary of a smooth domain in ${\mathbb R}^3$. We also prove that there is an admissible range of λ's for which the coupling $H+V_{\lambda}$ generates pure point spectrum in (-m, m). Morerover, we give an isoperimetric-type inequality for this range of λ's, in other words, we want to determine how small can this range be under some constraint on the size of the domain and/or the boundary of the domain. We also see that the ball is the unique optimizer of this inequality.

Costanza Benassi ***Correlation inequalities for the quantum XY model***

Correlation inequalities have been an invaluable tool in the study of a great variety of classical models, but generalisations to the quantum case are scarse. In this talk I will focus on the Griffiths-Ginibre inequalities and show that truncated correlation functions for the quantum XY model are positive (negative). Our results hold for spin 1/2 at any temperature and for the ground state of the spin 1 system, and extend a previous result by Gallavotti. Joint work in collaboration with B.Lees and D.Ueltschi.

Fabio Benatti ***Physics of complete positivity***

That dissipative dynamical maps must be completely positive is often criticised as a mathematical request lacking a sound physical motivation. The standard argument in favour of complete positivity is indeed based on coupling the open quantum system of interest with an arbitrary, inert ancilla. We briefly review these criticisms and propose a different, more physical perspective from which it becomes apparent that complete positivity cannot be easily dispensed with.

Serena Cenatiempo Quantum many-body fluctuations around non-linear Schrödinger dynamics

We consider the many-body quantum dynamics of a system of three dimensional bosons interacting through a two-body potential $N^{3\beta -1} V(N^\beta x)$, scaling with the number of particles N. Well known results establish the convergence in trace norm of the reduced k-particle density matrices associated with the solution of the many-body Schrödinger equation towards products of solutions of a one-particle non linear Schrödinger equation. In collaboration with C. Boccato and B. Schlein we studied fluctuations around the approximate nonlinear Schrödinger dynamics and for $0<\beta<1$ we obtained a norm approximation of the evolution of an appropriate class of data on the Fock space. To this end, one need to correct the evolution of the condensate described by the nonlinear Schrödinger equation by means of a fluctuation dynamics, governed by a quadratic generator. Obstructions to the extension of this result to the Gross-Pitaevskii scaling limit $\beta=1$ will be discussed.

Fabio Cipriani ***Transverse potential theory***

We start by recalling the relation between Dirichlet forms, Markovian semigroups and associated differential calculus on operator algebras. We then illustrate the construction of the associated Dirac operator and spectral triples as well as applications to amenability of von Neumann algebras. Examples will deal initially with group algebras and group actions and then with measured groupoids, where ideas of transverse measure theory become relevant.

Michele Correggi ***Curvature effects in surface superconductivity***

We review some recent results about the behavior of extreme type-II superconductors in the surface superconductivity regime, i.e., between the second and third critical fields. Within the Ginzburg-Landau theory, we derive the effects of the (smooth) boundary curvature to the energy asymptotics and to the density of Cooper pairs. We also discuss some work in progress about the modifications induced by the presence of corners. (Joint work with N. Rougerie, E.L. Giacomelli.)

Jan Derezinski ***Almost homogeneous Schroedinger operators***

First I will describe a certain natural holomorphic family of operators with interesting spectral properties. These operators can be fully analyzed using just trigonometric functions. Then I will discuss 1-dimensional Schroedinger operators with a 1/x^2 potential, which I studied recently with S.Richard. Even though their description involves Bessel and Gamma functions, they turn out to be equivalent to the previous family.

Luca Fanelli Frequency-dependent time-decay of Schrödinger groups

After motivating the project by introducing standard uncertainty variational inequalities, we will present some recent results and further developments concerning with dispersive properties of Schrödinger propagators, at fixed frequencies.

Domenico Finco ***On the ground state for the NLS on a graph***

We discuss the existence of the ground state for the NLS on graph with a compact core and a finite number of infinite length edges, in the small mass regime.

Alessandro Giuliani ***Periodic striped ground states in Ising models with competing interactions***

We consider Ising models in two and three dimensions, with short range ferromagnetic and long range, power-law decaying, anti-ferromagnetic interactions. We let *J* be the ratio between the strengths of the ferromagnetic and of the anti-ferromagnetic interaction. The competition between these two kinds of interactions induces the system to form domains of minus spins in a background of plus spins, or vice versa. If the decay exponent *p* of the long range interaction is larger than *d*+1, with *d* the space dimension, this happens for all values of *J* smaller than a critical value $J_c(p)$, beyond which the ground state is homogeneous. In this talk, we give a characterisation of the infinite volume ground states of the system, for *p*>2*d* and *J* in a left neighborhood of $J_c(p)$. In particular, we report a proof that the quasi-one-dimensional states consisting of infinite stripes (*d*=2) or slabs (*d*=3), all of the same optimal width and orientation, and alternating magnetization, are infinite volume ground states. We shall explain the key aspects of the proof, which is based on localization bounds combined with reflection positivity. Joint work with Robert Seiringer.

Gian Michele Graf Dynamical crossing of an infinitely degenerate critical point

A quantum evolution can be qualified as adiabatic or sudden in relation to the time dependence of the Hamiltonian. The canonical crossover behavior between these two cases is expressed for two-level systems by a formula about nearly avoided crossings found independently by Landau, Majorana, and Zener. We discuss the behavior in two related but very different situations: (a) In presence of decoherence (dephasing) and, more extensively, (b) for a model of a dynamical phase transition. The signature of the transition is that the discrete energy spectrum of the system becomes ever denser as the critical point is approached, where it then turns to a continuum (or nearly so), just to return being discrete past that point. The model is solvable and exhibits the Kibble-Zurek mechanism, according to which excitations (or defects) are generated by the transition "out of nothing". Concretely, the model is a time-dependent harmonic oscillator and the solution is expressed in terms of squeezed states. (Joint work with S. Bachmann and M. Fraas.)

Domenico Monaco ***Chern and Fu-Kane-Mele invariants as topological obstructions***

The use of topological invariants to describe different geometric phases of quantum matter has become an essential tool in modern solid state physics. The first instance of this paradigmatic trend can be traced to the study of the quantum Hall effect, in which the Chern number (or TKNN invariant) underlies the quantization of the transverse Hall conductivity. This integer-valued topological invariant can be related to the obstruction to the existence of a frame of (quasi-)Bloch states for the crystal which is both continuous and periodic with respect to the crystal momentum. More recently, in the framework of time-reversal symmetric topological insulators and quantum spin Hall systems, a new topological classification has been proposed by Fu, Kane and Mele, where the invariants take value in integers modulo 2. We show that obstruction theory provides a geometric interpretation for these invariants as well, measuring the possibility to impose a further time-reversal symmetry constraint on the Bloch frame. This is based on joint work with D. Fiorenza and G. Panati.

Nam Phan Thanh ***Norm approximation for many-body quantum dynamics***

Starting from the many-body Schroedinger equation for bosons, I will discuss the rigorous derivation of the Hartree equation for the condensate and the Bogoliubov equation for the excited particles. The effective equations allows us to construct an approximation for the many-body wavefunction in norm. This talk is based on joint works with M. Lewin, B. Schlein and M. Napiórkowski.

Diego Noja ***The non-linear Schrödinger equation with a concentrated non-linearity: scaling limit from a regular dynamics***

In this talk a nonlinear Schrödinger equation with a space dependent nonlinearity in dimension three will be considered and the limit in which the range of the nonlinear term shrinks to a point will be studied. Some previous results in dimension one will be briefly reviewed and discussed. Then the more difficult three dimensional case will be presented, considering a regularized and scaled "mean field" nonlinearity and performing the removal of the regularization. It is shown that that a limit dynamics exists and it is non trivial (different form a free dynamics) if the nonlinearity and the scaling are fine tuned and suitably renormalized. The limit dynamics is a nonlinear version of point interaction (delta potential) in dimension three and it has been previously studied as regards the well posedness, blow-up and asymptotic properties of solutions, but this is the first justification of the model as the point limit of a regularized dynamics. This is a work in collaboration with Claudio Cacciapuoti, Domenico Finco and Alessandro Teta.

Gianluca Panati ***Optimal decay of Wannier functions in Chern and Quantum Hall insulators***

We investigate the localization properties of independent electrons in a periodic background, possibly including a periodic magnetic field, as, e.g., in Chern insulators and in Quantum Hall systems. Since, generically, the spectrum of the Hamiltonian is absolutely continuous, localization is characterized by the decay, as |x| goes to infinity, of the composite Wannier functions associated to the Bloch bands below the Fermi energy, which is supposed to be in a spectral gap. We prove the validity of a *localization dichotomy*, in the following sense: either there exist exponentially localized composite Wannier functions, and correspondingly the system is in a trivial topological phase with vanishing Hall conductivity, *or* the decay of *any* composite Wannier function is such that the expectation value of the squared position operator, or equivalently of the Marzari-Vanderbilt localization functional, is infinite. In the latter case, the Bloch bundle is topologically non-trivial, and one expects a non-zero Hall conductivity. The talk is based on a joint work with Domenico Monaco, Adriano Pisante, and Stefan Teufel.

Andrea Posilicano ***Limiting absorption principle, generalised eigenfunctions and scattering matrix for Laplace operators with boundary conditions on hypersurfaces***

We provide a limiting absorption principle for the self-adjoint realizations of Laplace operators corresponding to boundary conditions on (relatively open parts Σ of) compact hypersurfaces $\Gamma=\partial\Omega$, $\Omega\subset\mathbb{R}^n$. For any of such self-adjoint operators we also provide the generalized eigenfunctions and the scattering matrix; both these objects are written in terms of operator-valued Weyl functions. We make use of a Krein-type formula which provides the resolvent difference between the operator corresponding to self-adjoint boundary conditions on the hypersurface and the free Laplacian on the whole space $\mathbb{R}^n$. Our results apply to all standard examples of boundary conditions, like Dirichlet, Neumann, Robin, δ and δ'-type, either assigned on Γ or on $\Sigma\subset\Gamma$.

Andrea Sacchetti ***Quantum resonances and time decay for a double-barrier model***

In this talk we consider the time evolution of a one-dimensional quantum system with a double barrier given by a couple of two repulsive Dirac's deltas. In such a *pedagogical* model we give, by means of the theory of quantum resonances, the explicit expression of the dominant terms of < \psi , e^{-it H} \phi >, where *H* is the double-barrier Hamiltonian operator and where ψ and ϕ are two test functions. From: A.S., J. Phys. A 49 175301 (2016.

Alessandro Teta ***Stability problem for a quantum system with zero-range interactions***

We discuss some mathematical aspects of the dynamics of a system of quantum particles in dimension three with two-body zero-range interactions. Such systems are relevant in the physics of the so-called ultra-cold atoms, in connection with Efimov and Thomas effects. In particular, we review the state of the art concerning the rigorous construction of the corresponding Hamiltonian as a self-adjoint operator and the conditions for its lower boundedness. Exploiting a quadratic form method, we show that self-adjointness and lower boundedness hold for three bosons excluding the "s-wave" subspace and for N identical fermions plus a different particle for suitable values of the mass ratio.

Kenji Yajima ***Existence and regularity of propagators for multi-particle Schroedinger equations***

We consider Schrödinger equations for N quantum particles in the electro-magnetic fields described by (A(t,x),φ(t,x). We assume that fields are smooth, magnetic fields B(t,x)=rot A(t,x) are long range perturbation of a constant fied and for |α|≥2. We prove that equation generate a unique propagator with the invariant subspace , the domain of the harmonic oscillator . The inter-particle interaction V(t,x) is the sum of D-body interactions , D⊂{1,…,N} which are functions on the space of relative motions of particles inside D and, can be spatially locally as singular as and if and similar for lower dimensions. The vector valued Strichartz estimates for N independent particles in the field and integration by parts for integrals of operator valued functions will be used in the proof.