Non-equilibrium transport through double quantum dot devices: A non-Fermi liquid critical point

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OUTLINE

- Critical point in 2 impurity Kondo model
- Experimental realizations
- Exact critical behaviour
- Non-equilibrium transport and noise

Critical Point in 2-Impurity Kondo Model

Single Impurity Case:

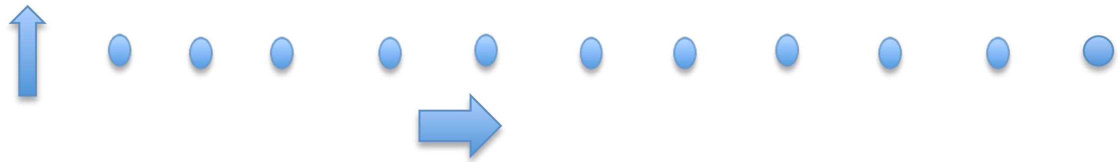
- A single S=1/2 impurity (at r=0) has an exchange interaction with a single channel of conduction electrons: $H_{\text{int}} = J\vec{S}\cdot\psi(0)^{+}\vec{\sigma}\psi(0)/2$
- J renormalizes to infinity at low energies
- Low energy ($E<T_K$) Fermi liquid fixed point corresponds to one electron from the Fermi sea forming an entangled singlet state with impurity
- Other electrons suffer π/2 phase shift to stay orthogonal to entangled electron

When 2 impurities couple independently to 2 channels and also have exchange coupling with each other there is frustration:

$$H_{\text{int}} = J\left[\vec{S}_1 \cdot \psi_1(0)^+ \vec{\sigma}\psi_1(0)/2 + \vec{S}_2 \cdot \psi_2(0)^+ \vec{\sigma}\psi_2(0)/2\right] + K\vec{S}_1 \cdot \vec{S}_2$$

- When $K<K_c$, each spin forms singlet with a conduction electron, with a π/2 phase shift in each channel
- When $K>K_c$, 2 impurities form singlet together and there are no phase shifts
- K_c is of order T_K

•Phase shifts are a sort of trivial conformally invariant boundary condition (BC)
•At quantum critical point, $K=K_c$, a non-trivial
•conformally invariant BC occurs
•Singlet and triplet states of 2 impurities are
•degenerate at this point
•Behaviour near Quantum Critical Point (QCP) at $K=K_c$, is equivalent to that of critical quantum Ising chain with boundary field (IA, A.W.W. Ludwig, 1992)

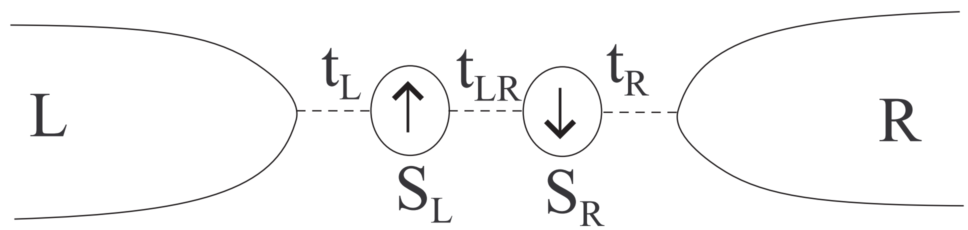


$$H = -hS_0^z - \sum_{i=0}^{\infty}\left[S_i^z S_{i+1}^z + S_i^x\right]$$

NB: bulk chain is tuned to critical transverse field

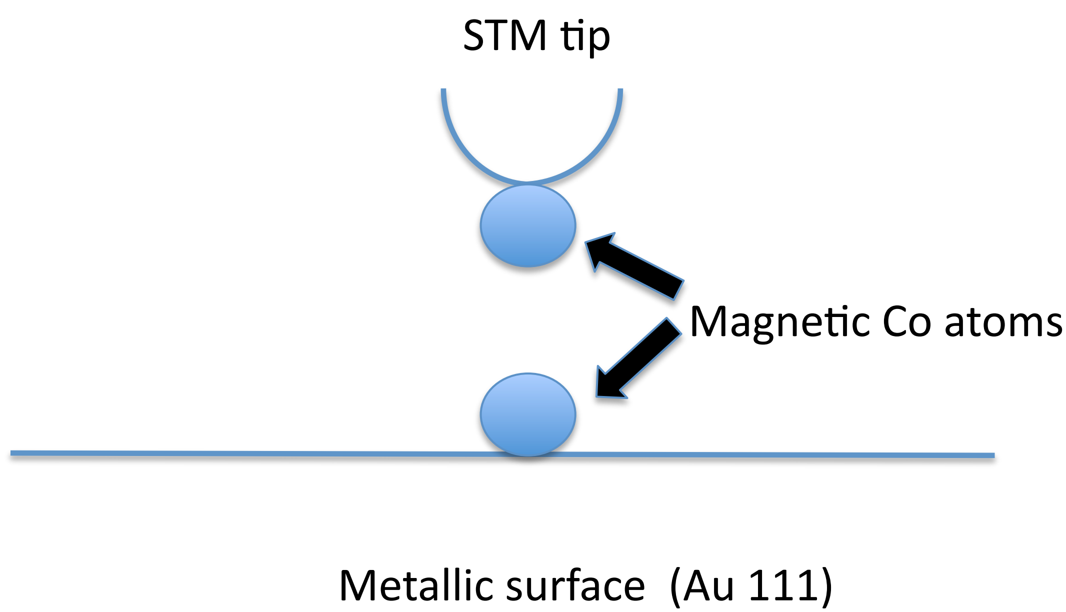
- At h=0, 2 states of "impurity" at i=0 are degenerate
- h is a relevant boundary perturbation that leads to renormalization group (RG) flow to fixed spin boundary condition at i=0
- Spin up or down state of boundary spin correspond to singlet or triplet in 2 impurity Kondo model
- An exact correspondence near QCP

Experimental Realizations



•2 gated semi-conductor quantum dots in series between 2 leads
$K \sim t_{LR}^2/U$ where U is dot charging energy
•System has been studied experimentally as a 2-qubit realization

System was recently realized in STM experiment (K. Kern, private communication) with one magnetic atom picked up by STM tip and one on surface



Exact Critical Behaviour

- Connection with Ising model can be used by mapping to 8 Majorana fermions
- First bosonize, then refermionize introducing charge, spin, flavour and "spin-flavour" fermions
- Each of these 4 Dirac fermions can then be split into 2 Majorana fermions

- Various fixed points correspond to various simple BC's on Majorana fermions
- At zero Kondo coupling: $\chi_{out}^{i} = \chi_{in}^{i}$
- QCP corresponds to a sign change in BC for 1 Majorana component of spin-charge fermion: $\chi_{out}^{1} = -\chi_{in}^{1}$, other BC's unchanged
- Relevant $(K-K_c)$ interaction at QCP can be written in terms of χ_1 at origin after transforming to scattering basis: $H_{int}=\lambda_1\chi_1(0)a$
- Here $\lambda_1 \prec (K-K_c)$
- a is an extra Majorana fermion that lives at x=0 only and flips impurities between singlet and triplet

•In series double dot set-up there is generally another relevant operator which transmits charge between 2 leads ("channels")

$\delta H = V_{LR}[\psi_1^{+}(0)\psi_2(0) + h.c.]$, $V_{LR} \prec t_L t_{LR} t_R / U^2$

•At QCP this becomes linear in a <u>flavour</u> Majorana fermion, $\delta H = \lambda_2 \chi_2(0) a$ with $\lambda_2 \prec V_{LR}$

•Both relevant interactions have RG scaling dimension ½

•Related to each other by SO(8) rotation

Non-equilibrium transport and noise

This series double dot system is amenable to exact calculations in critical region since:

✓Perturbations are harmonic in χ^i and a-fermions

✓Current operator is quadratic in χ^i's

•H_{eff} maps onto one studied earlier as a special (and highly unrealistic) limit of single dot model by Schiller-Hershfeld and Komnik-Gogolin, as does current operator

•We may simply borrow their results

•Current and noise can be calculated close to critical point

•Distance from critical point determined by 2 parameters: $K-K_c$ and V (L-R tunneling)

•Crossover scale is:

$T^*=\lambda^2=\lambda_1^2+\lambda_2^2$ where $\lambda_1 = c_1(K-K_c)/\sqrt{T_K}$, $\lambda_2 = c_2 \nu V_{LR}\sqrt{T_K}$

•Here ν is density of states; c_i are constants of order 1

Must have $T^* \ll T_K$ for theory to apply

Conductance at source-drain voltage, V
and temperature T:

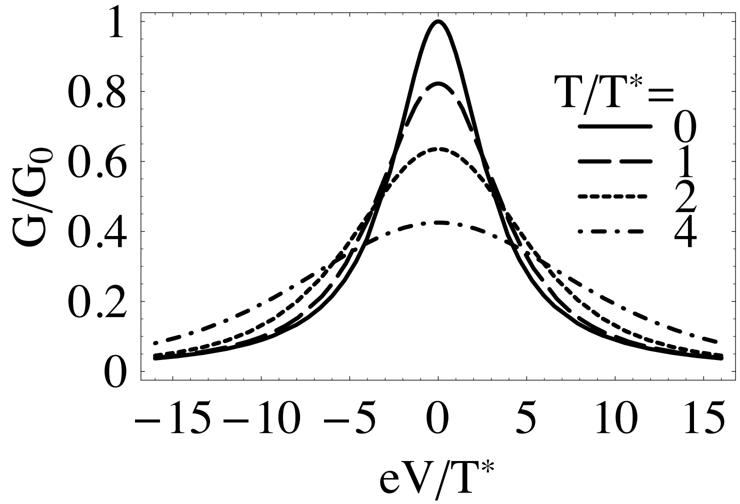
$G=(2e^2/h)(T^*_{LR}/T^*)^2F(T/T^*, eV/T^*)$

Here T^*_{LR} is part of crossover scale due to V_{LR}:

$T^*_{LR}=\lambda_2{}^2=(c_2\nu V_{LR})^2T_K$ and:

$F(t,v)=\text{Re}\ \psi_1(1/2+1/4\pi t+iv/2\pi t)/4\pi t$

where ψ_1 is the trigamma function



- At T=0, a very narrow resonance at Fermi energy of width $T^* << T_K$
- Peak height is $2e^2/h$ at $K=K_c$
- NB: at larger V~K, presumably get split peaks as predicted earlier and seen in experiments

"full counting statistics", as defined by Levitov and Lesovik also determined exactly: Letting P(Q) be the probability of transmitting charge Q in waiting time T, the generating function defined by

$$\sum_{Q=-\infty}^{\infty} P(Q)e^{i\mu Q} \equiv \chi(\mu)$$ is given by $$\frac{\ln \chi(\mu)}{T} = \int_{-\infty}^{\infty} \frac{d\varepsilon}{4\pi} \ln\left[1 + \sum_{n=-2}^{2} A_n(\varepsilon)(e^{i\mu n} - 1)\right]$$

$$A_1(\varepsilon) = \frac{2\lambda_1^2\lambda_2^2}{4\varepsilon^2 + \lambda^4}\left[n_F(\varepsilon)(1 - n_L(\varepsilon)) + n_R(\varepsilon)(1 - n_F(\varepsilon))\right]$$

with

$$A_2(\varepsilon) = \frac{\lambda_2^4}{4\varepsilon^2 + \lambda^4} n_L(\varepsilon)(1 - n_R(\varepsilon))$$

With A_{-n} obtained from A_n by switching L and R; $n_F(\varepsilon)=1/(e^{\varepsilon/T}+1)$, $n_{R/L}(\varepsilon)=n_F(\varepsilon \pm eV)$

From χ(μ) the current, I, noise, S and higher cumulants can be extracted:

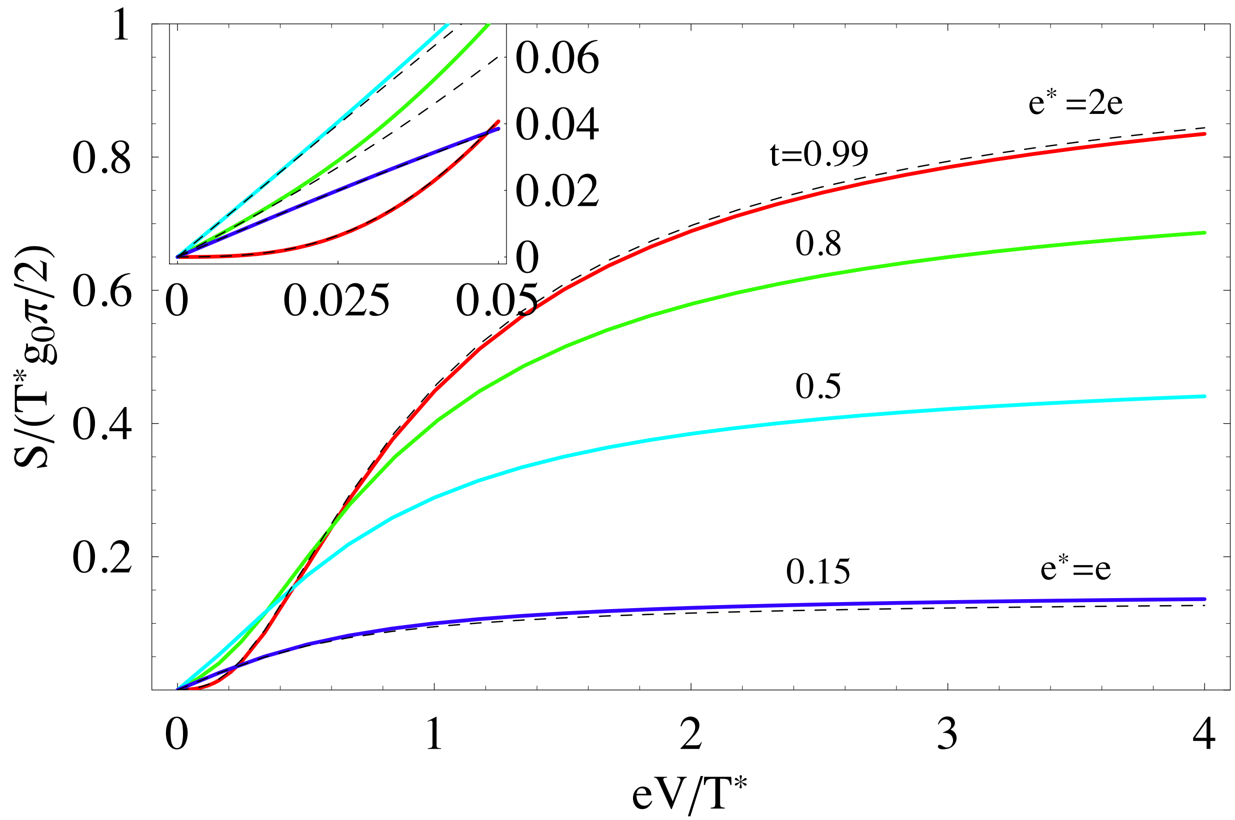
$$I = -ie \frac{\partial \ln \chi}{\partial \mu}\bigg|_{\mu=0} / T \qquad S = -2e \frac{\partial^2 \ln \chi}{\partial \mu^2}\bigg|_{\mu=0} / T$$

Effective charge, e*, is often defined by:

$$S_{fit} = 2e^* \frac{2e^2}{h} \int_0^V t(V')[1 - t(V')]dV'$$

where $t(V)=(h/2e^2)dI/dV$.

•This definition is motivated by noise for non-interacting electrons



•Close to critical point ($K \approx K_c$, $t \approx 1$) $e^* = 2e$ gives good fit except at very small V

•This reflects cross-over from NFL fixed point at higher energies to FL at low energies

Collaborators

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CONCLUSIONS

- Competition between Kondo screening and inter-impurity singlet formation leads to a non-Fermi liquid quantum critical point
- In series double quantum dot device, induced tunnelling amplitude between leads destabilizes this QCP but it determines higher E physics
- Exact crossover from NFL to FL critical points has been calculated by mapping to free Majorana fermions