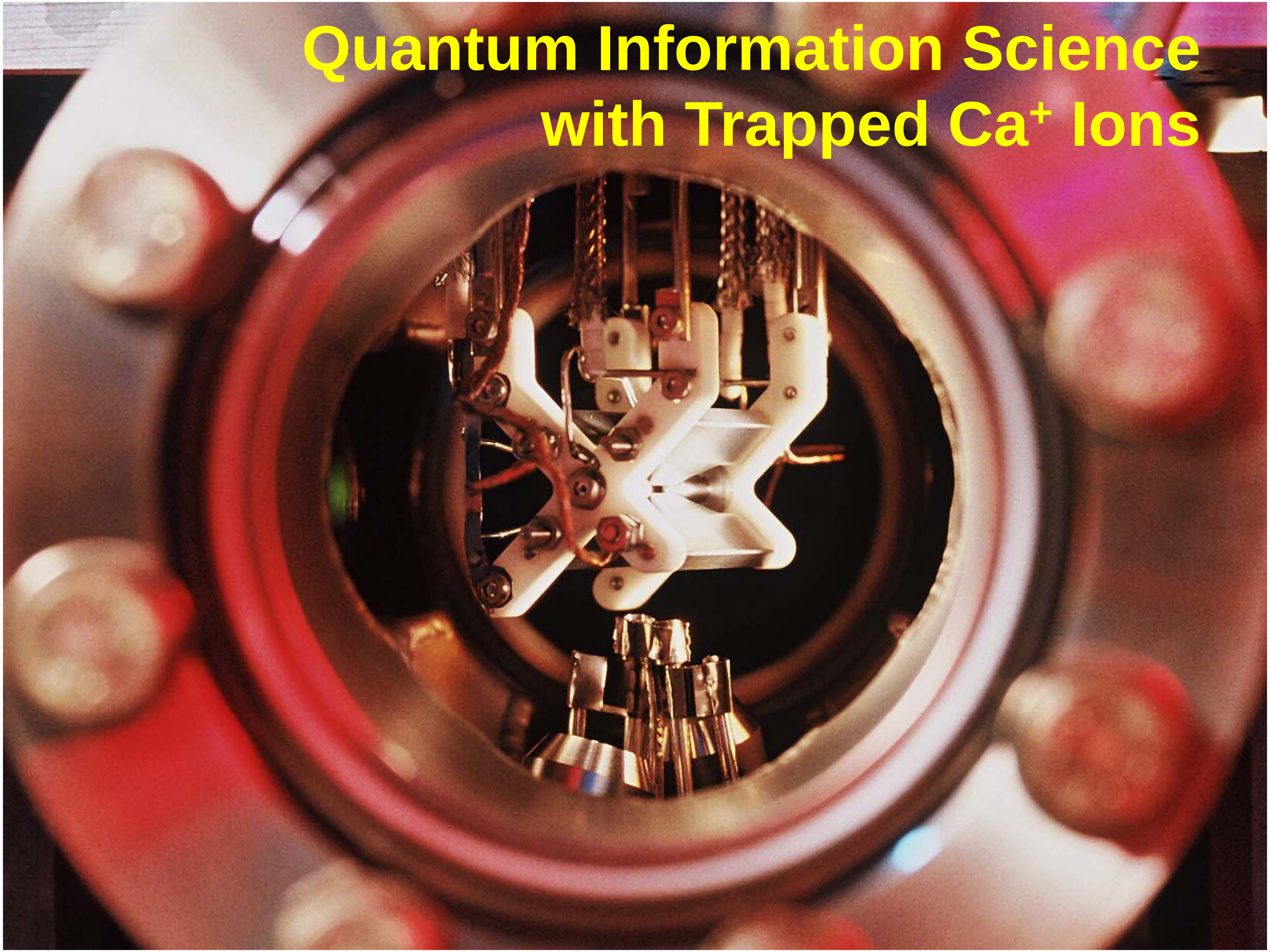
Quantum Information Science with Trapped Ca^+ Ions



Quantum Information Science with Trapped Ca^+ Ions

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Austrian Academy of Sciences

- Trapped Ca^+ for quantum information processing
- Gate operations with trapped ions
- Quantum computation with logical qubits
- Testing quantum mechanics with trapped ions
- Simulating quantum systems

€ FWF SFB
CONQUEST SCALA
Industrie Tirol
IQI GmbH
FWF bm:bwk erc
$ IARPA

universität innsbruck
AG Quantenoptik und Spektroskopie
IQI
ÖAW Österreichische Akademie der Wissenschaften

The requirements for quantum information processing

D. P. DiVincenzo, Quant. Inf. Comp. **1** (Special), 1 (2001)

I. Scalable physical system, well characterized qubits

II. Ability to initialize the state of the qubits

III. Long relevant coherence times, much longer than gate operation time

IV. “Universal” set of quantum gates

V. Measurement capability specific to implementation

VI. Ability to interconvert stationary and flying qubits

VII. Ability to faithfully transmit flying qubits between specified locations

The seven commandments for QC !!

Quantum information processing with trapped ions

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

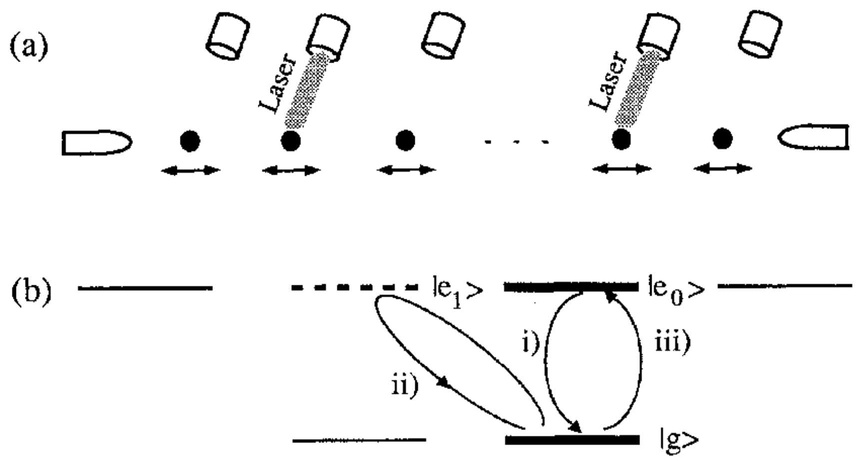


FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

controlled – NOT :

$$|\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$

$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$

$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$

$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

control bit target bit

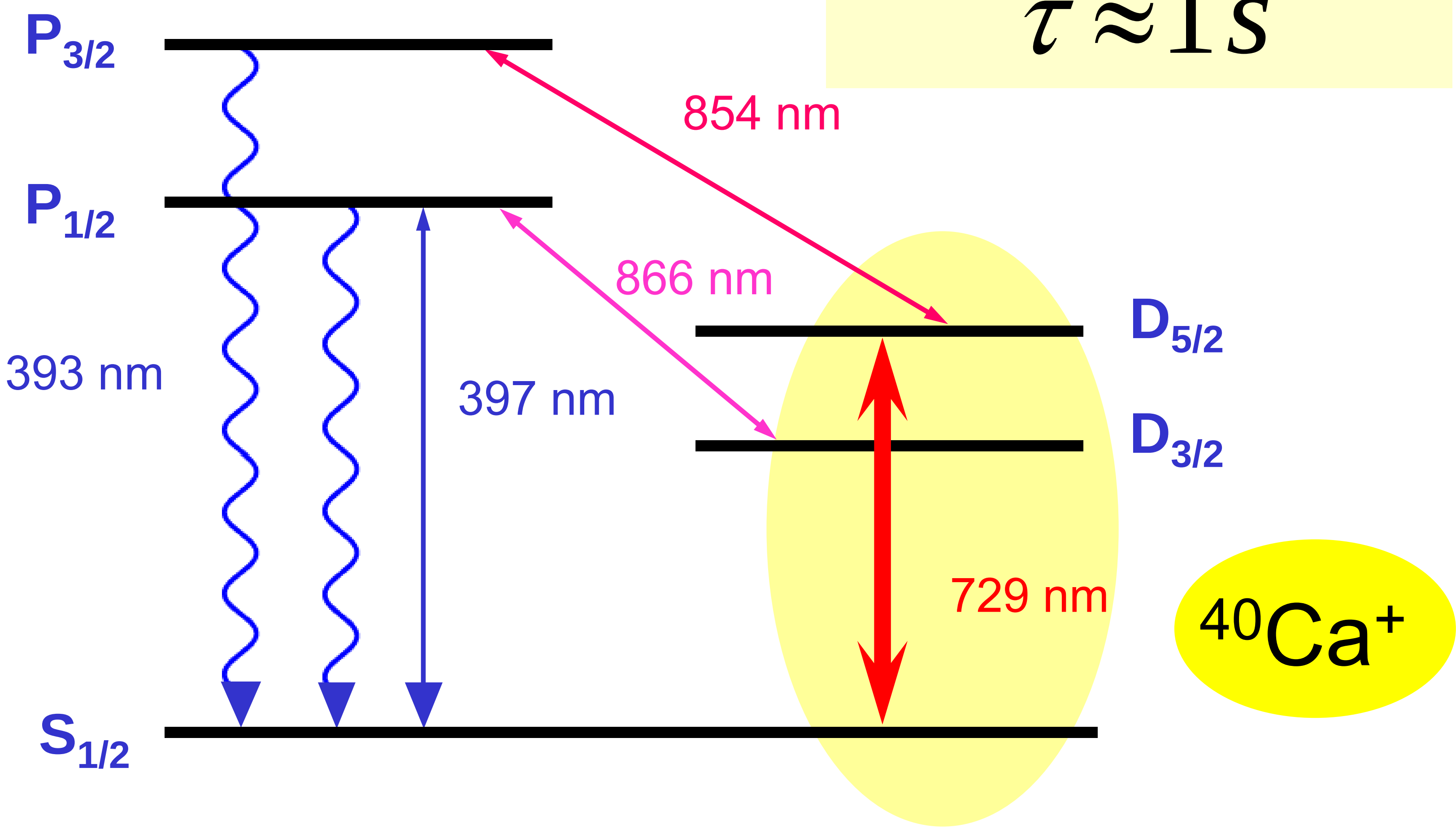
other gate proposals (and more):

- Cirac & Zoller
- Mølmer & Sørensen, Milburn
- Jonathan & Plenio & Knight
- Geometric phases
- Leibfried & Wineland

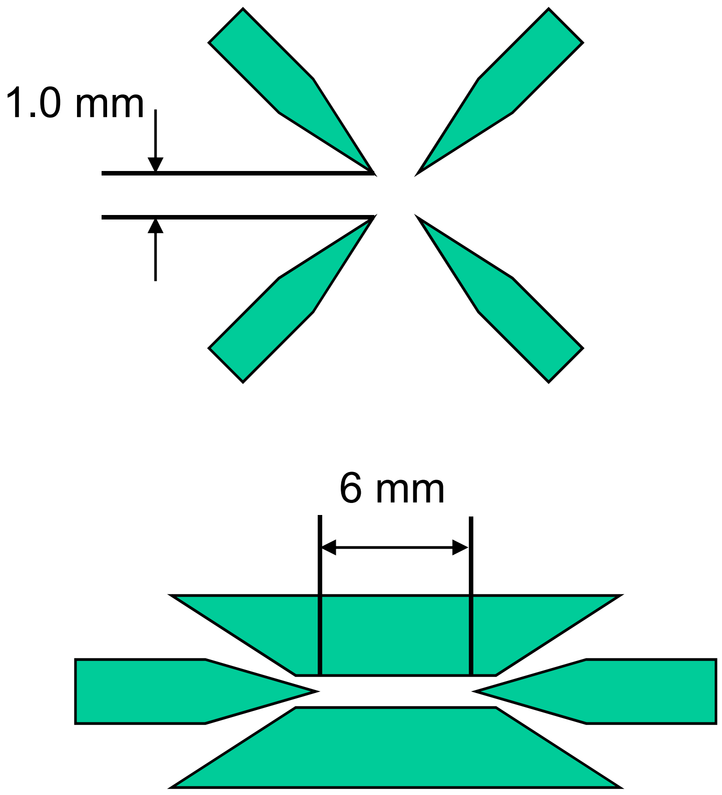
Level scheme of Ca^+

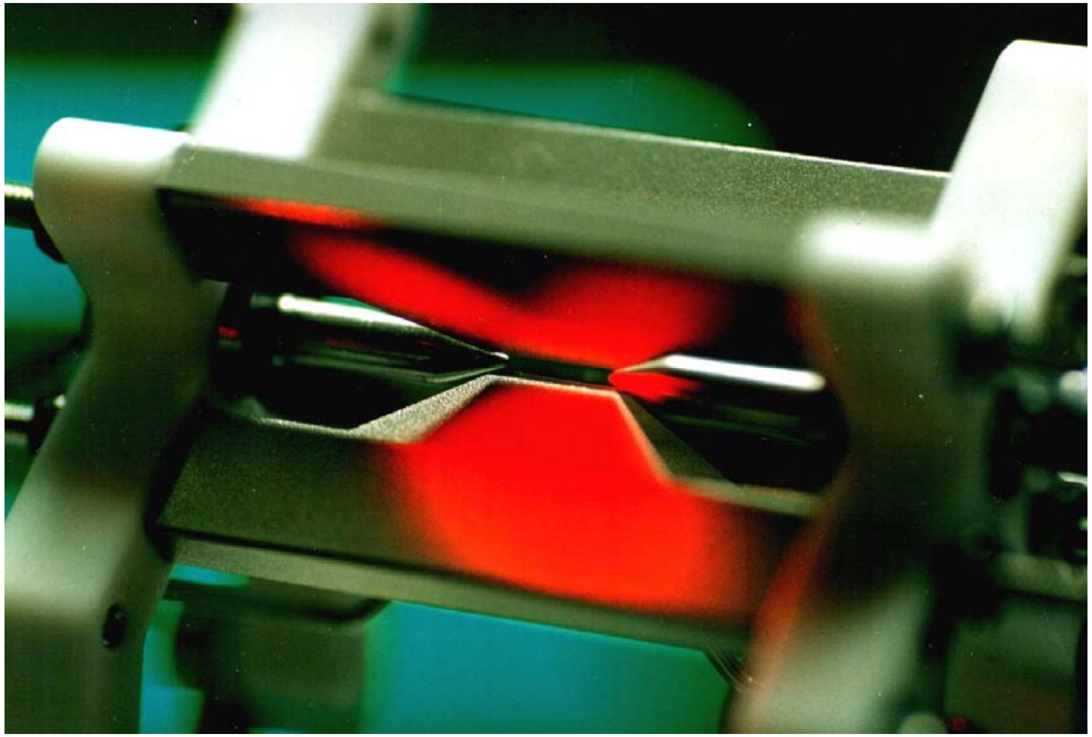
qubit on narrow S - D quadrupole transition

$$\tau \approx 1 s$$

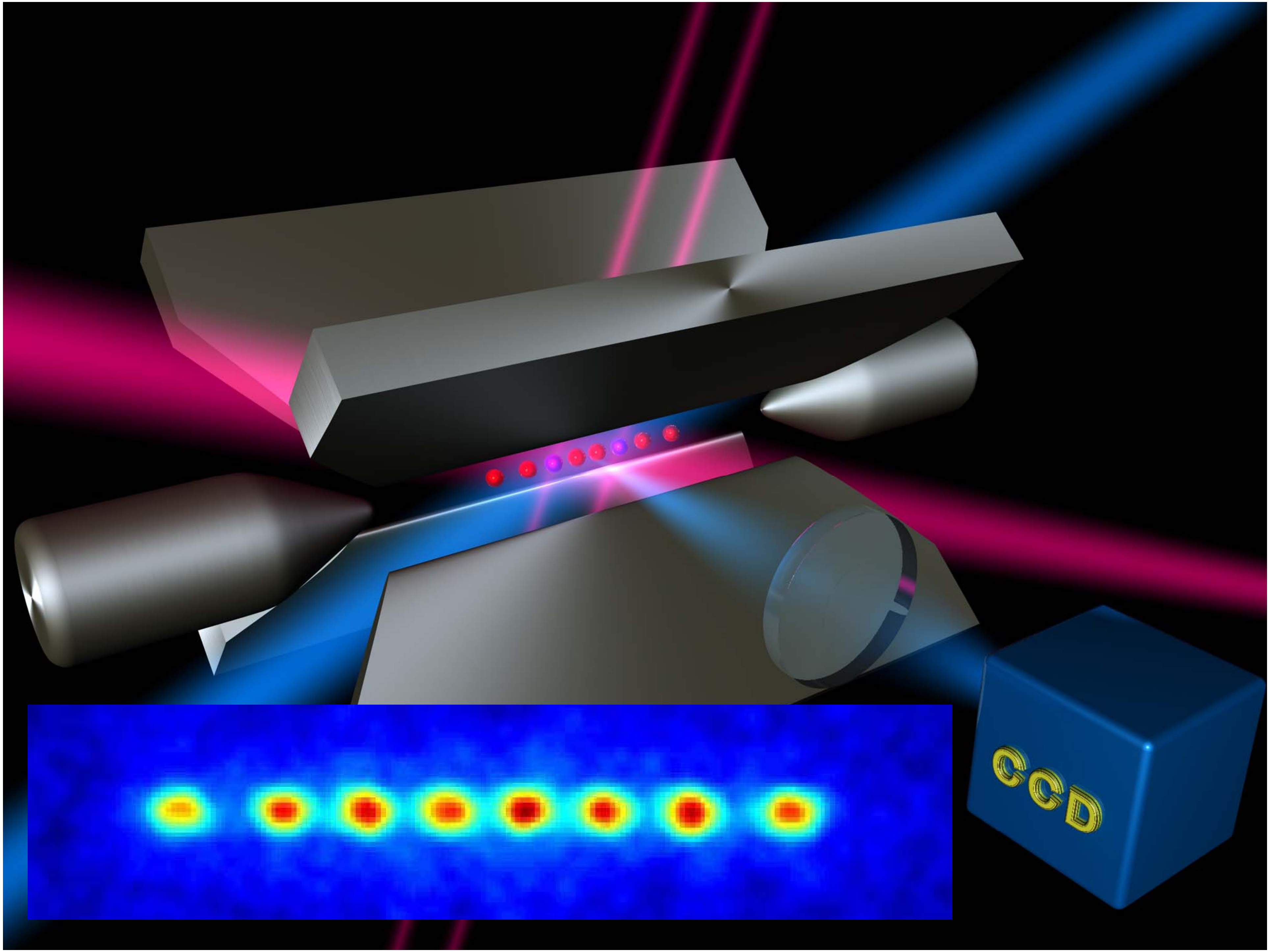


Innsbruck linear ion trap (2000)

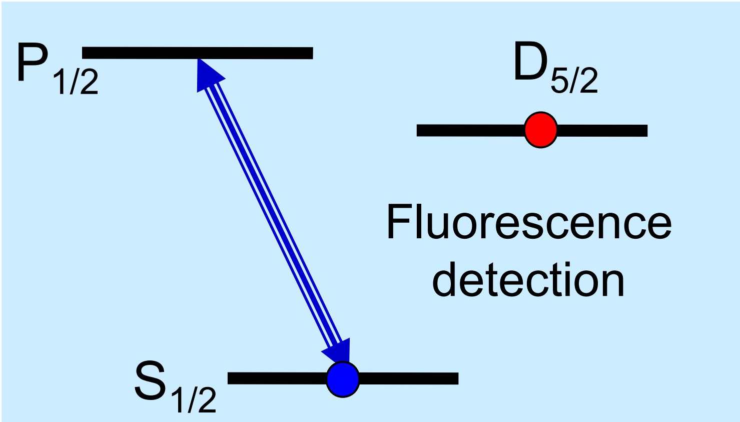




$$\omega_z \approx 0.7 - 2\,\mathrm{MHz} \qquad \omega_{x,y} \approx 1.5 - 4\,\mathrm{MHz}$$

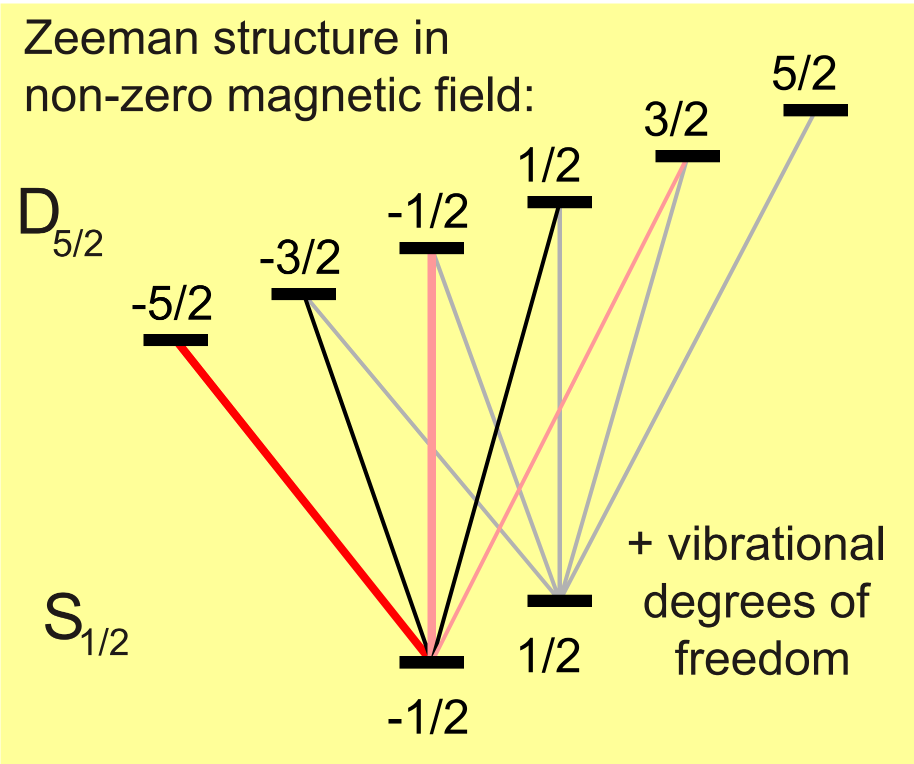

CCD

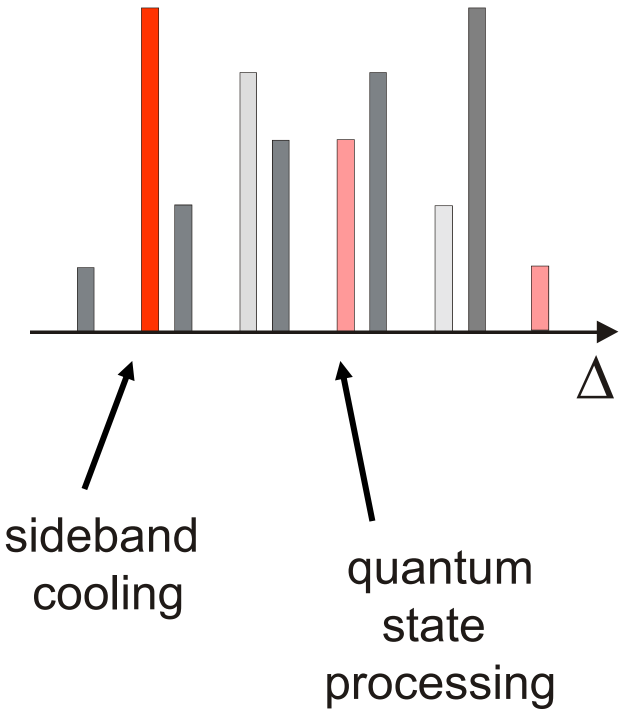
Spectroscopy of the $S_{1/2}$ – $D_{5/2}$ transition



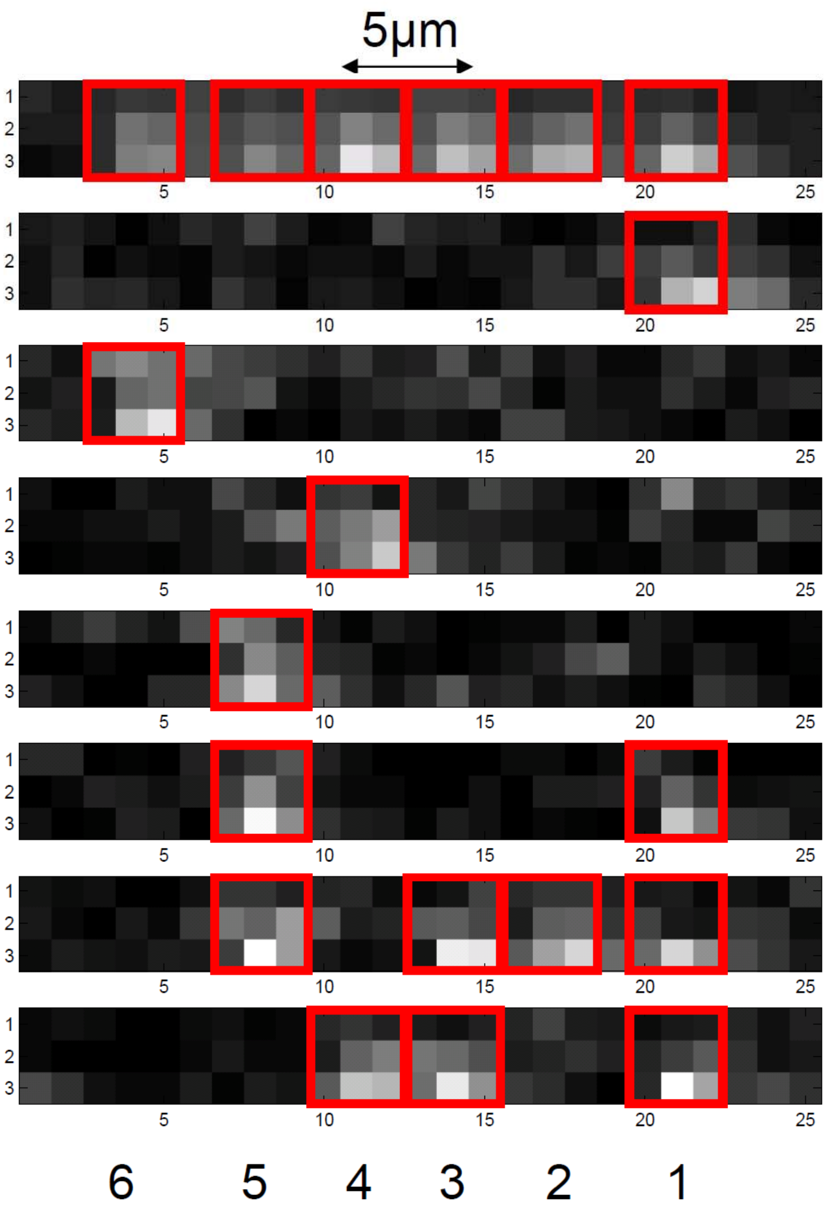
2-level-system:

$$-1/2 \longrightarrow -5/2(-1/2)$$





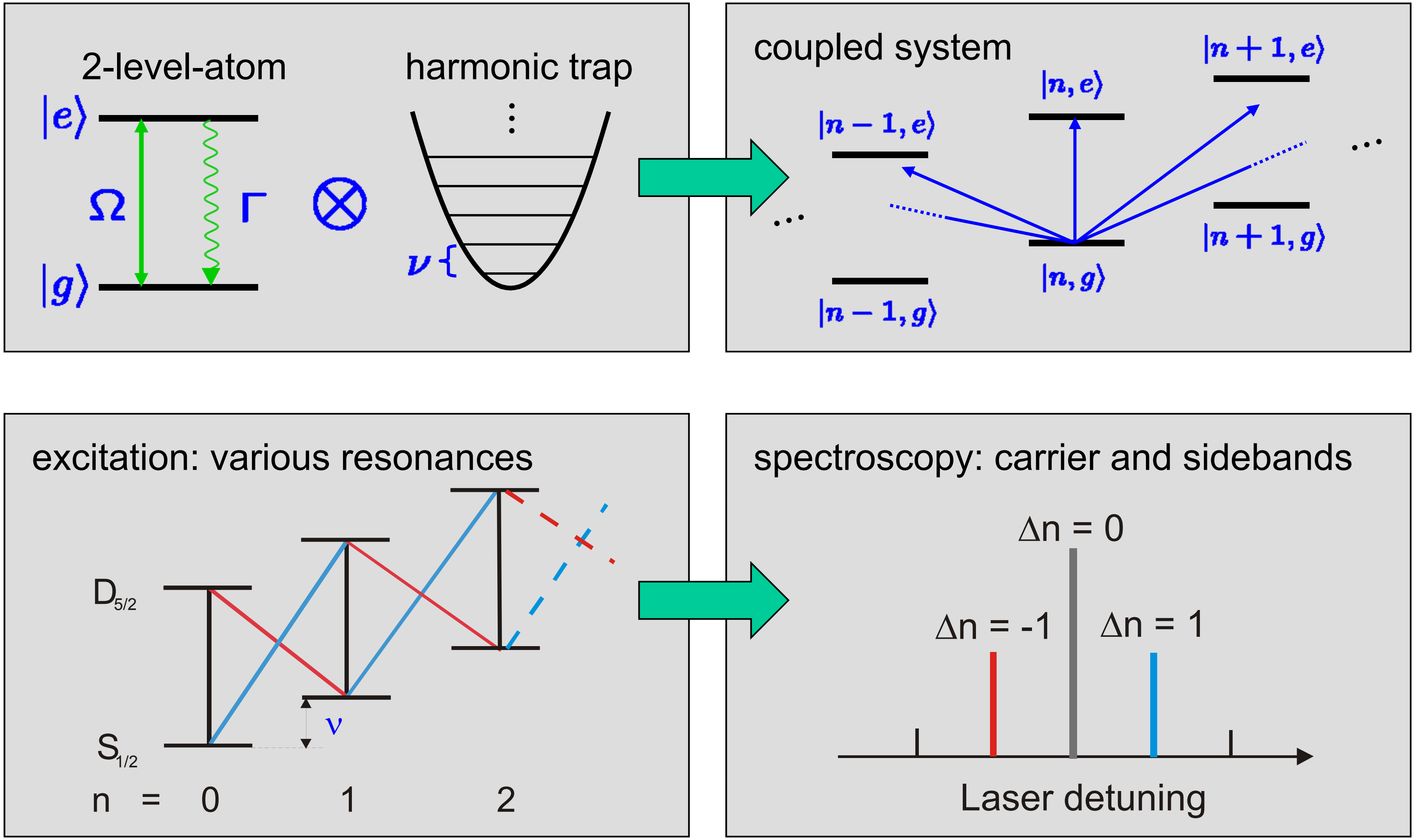
Detection of 6 individual ions



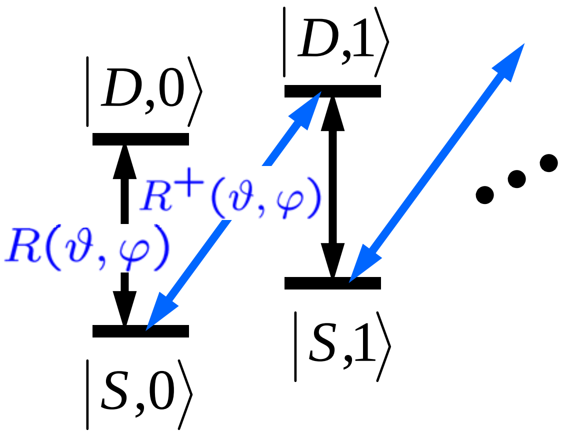
state detection on a CCD camera

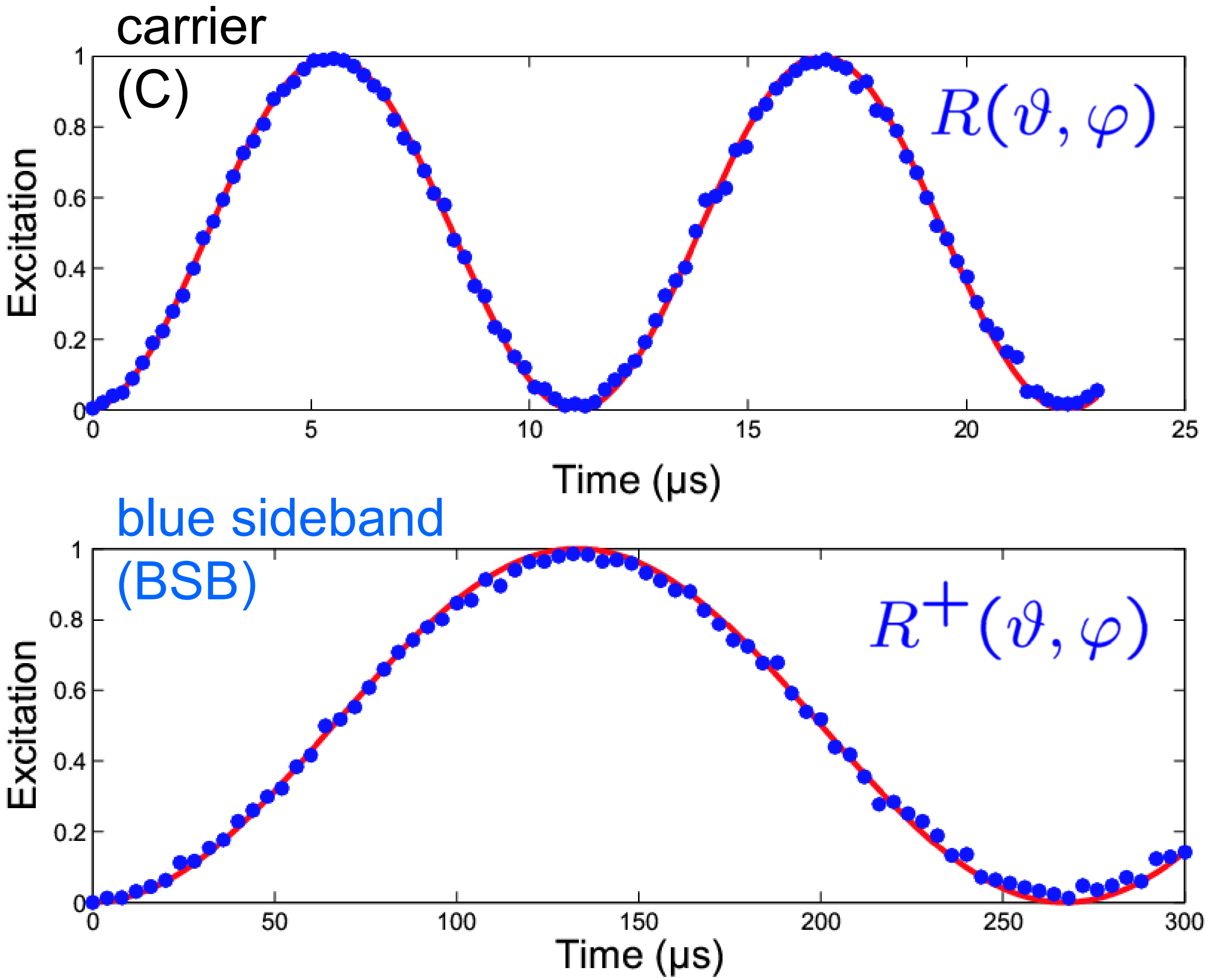
all ions in |S>	$\|SSSSSS\rangle$
ion 1 in |S>	$\|DDDDDS\rangle$
ion 6 in |S>	$\|SDDDDD\rangle$
ion 4 in |S>	$\|DDSDDD\rangle$
ion 5 in |S>	$\|DSDDDD\rangle$
ions 1 and 5 in |S>	$\|DSDDDS\rangle$
ions 1,2,3, and 5 in |S>	$\|DSSSSS\rangle$
ions 1,3 and 4 in |S>	$\|DDSSDS\rangle$

Quantized Ion Motion



Coherent state manipulation





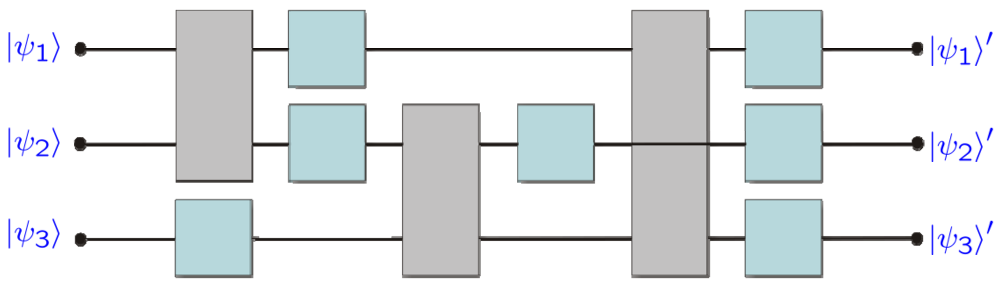
carrier and sideband
Rabi oscillations
with Rabi frequencies

$$\Omega, \quad \eta\Omega\sqrt{n+1}$$

$\eta = kx_0$ Lamb-Dicke parameter

Quantum information processing with trapped ions

- algorithms:
  sequence of single qubit and
  two-qubit gate operations



- gate operations:
  sequences of laser pulses
  (carrier and/or sideband pulses)

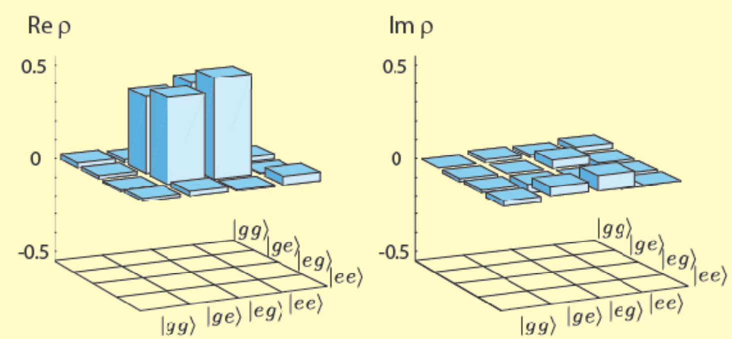
$R(\vartheta, \varphi)$, carrier

$R^{+}(\vartheta, \varphi)$, sideband

- analysis:

  measure density matrix of
  state or process (tomography)

  measure entanglement
  via parity oscillations



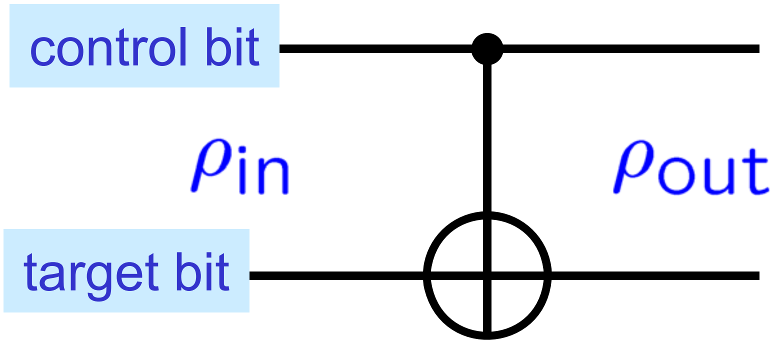
$$|\Psi^{+}\rangle = \frac{1}{\sqrt{2}}(|ge\rangle + |eg\rangle)$$

Quantum Process Tomography

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

target bit

control bit



$$\rho_{\text{out}} = \sum \chi_{ij} E_i \rho_{\text{in}} E_j^\dagger$$

$$E_i = A_i \otimes A_j$$

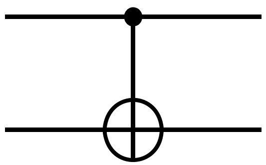
$$A_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}$$
$$\equiv \{I, X, iY, Z\}$$

$$\chi_{ij}$$

characterizes gate operation completely

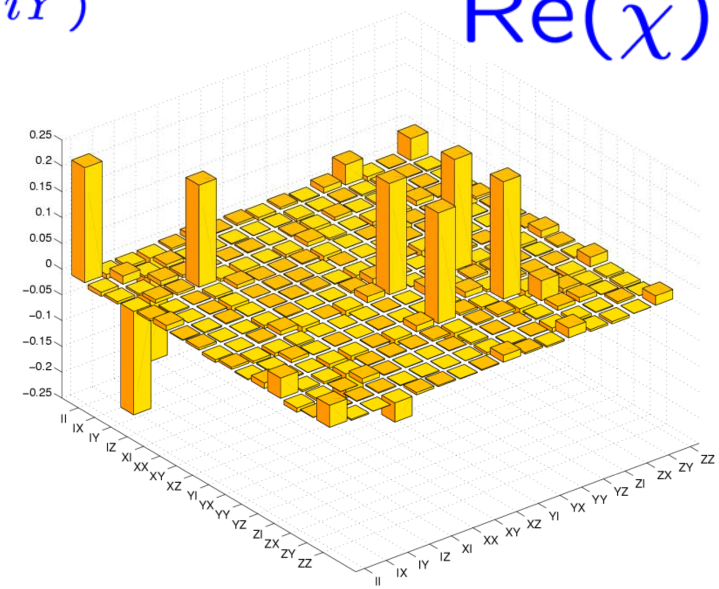
χ – matrix for observed CNOT gate operation

$$U^{12}_{\mathrm{CNOT}} = -\frac{1}{2}((I - Z)\otimes I + (I + Z)\otimes iY)$$

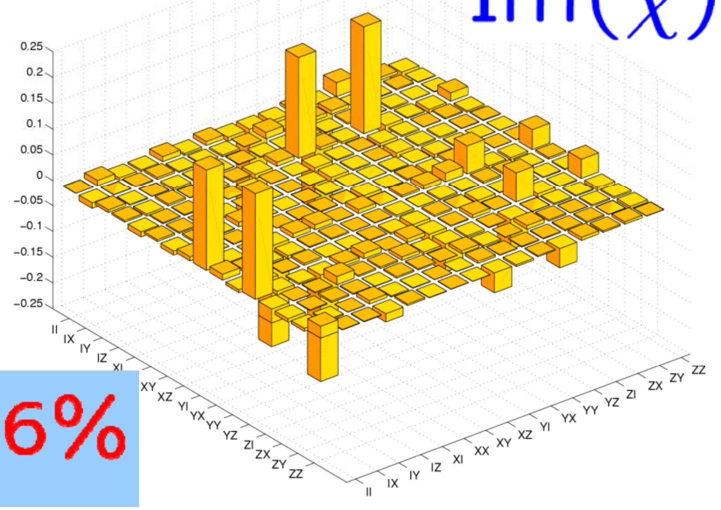


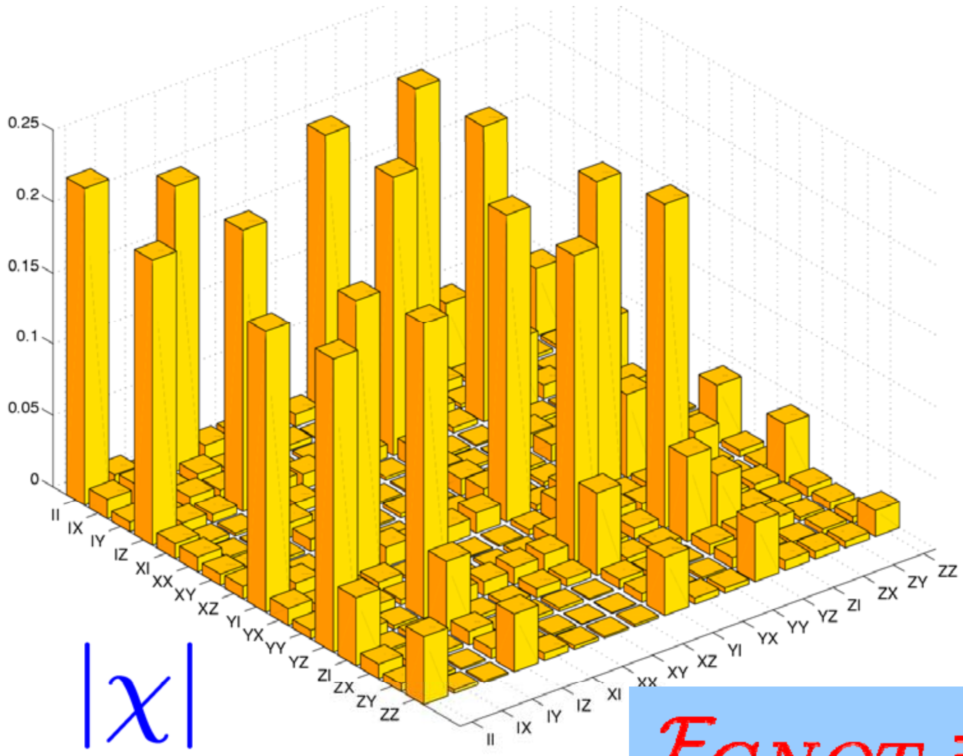
$$= \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$\mathrm{Re}(\chi)$



$\mathrm{Im}(\chi)$



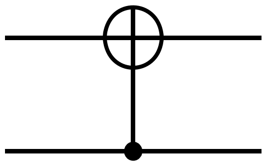


$|\chi|$

$\mathcal{F}_{CNOT} = 86\%$

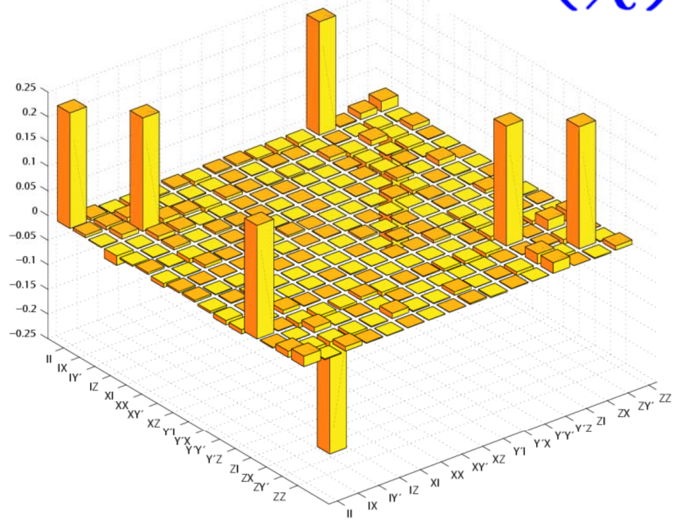
χ – matrix for observed CNOT gate operation

$$U^{21}_{\mathrm{CNOT}} = -\frac{1}{2}\left(I \otimes (I - Z) + iY \otimes (I + Z)\right)$$

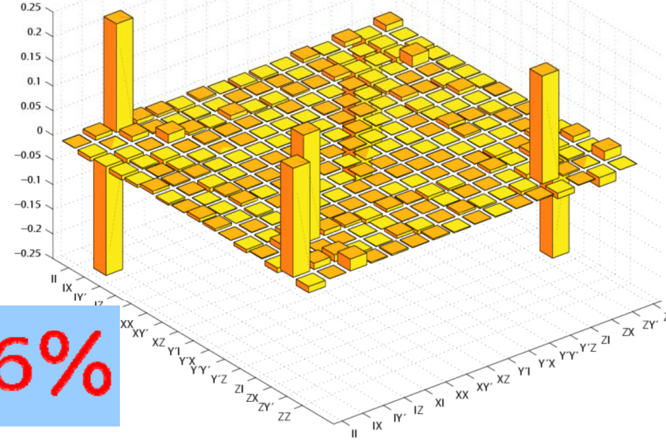


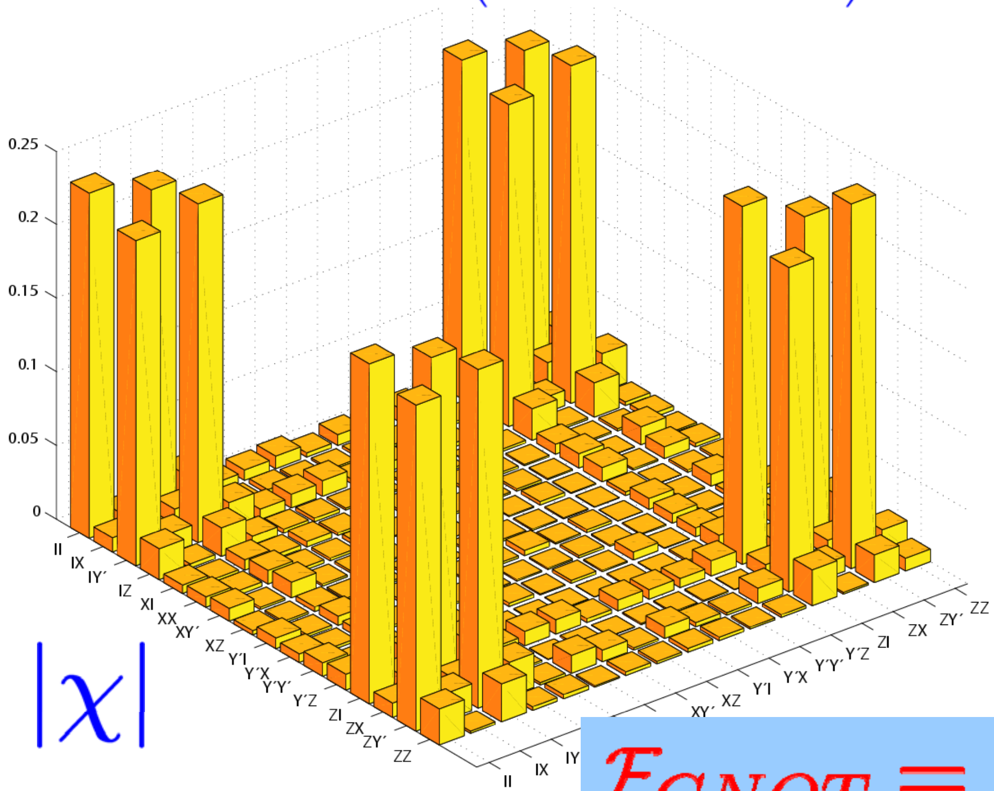
$$= \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & -1 & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Re(χ)



Im(χ)

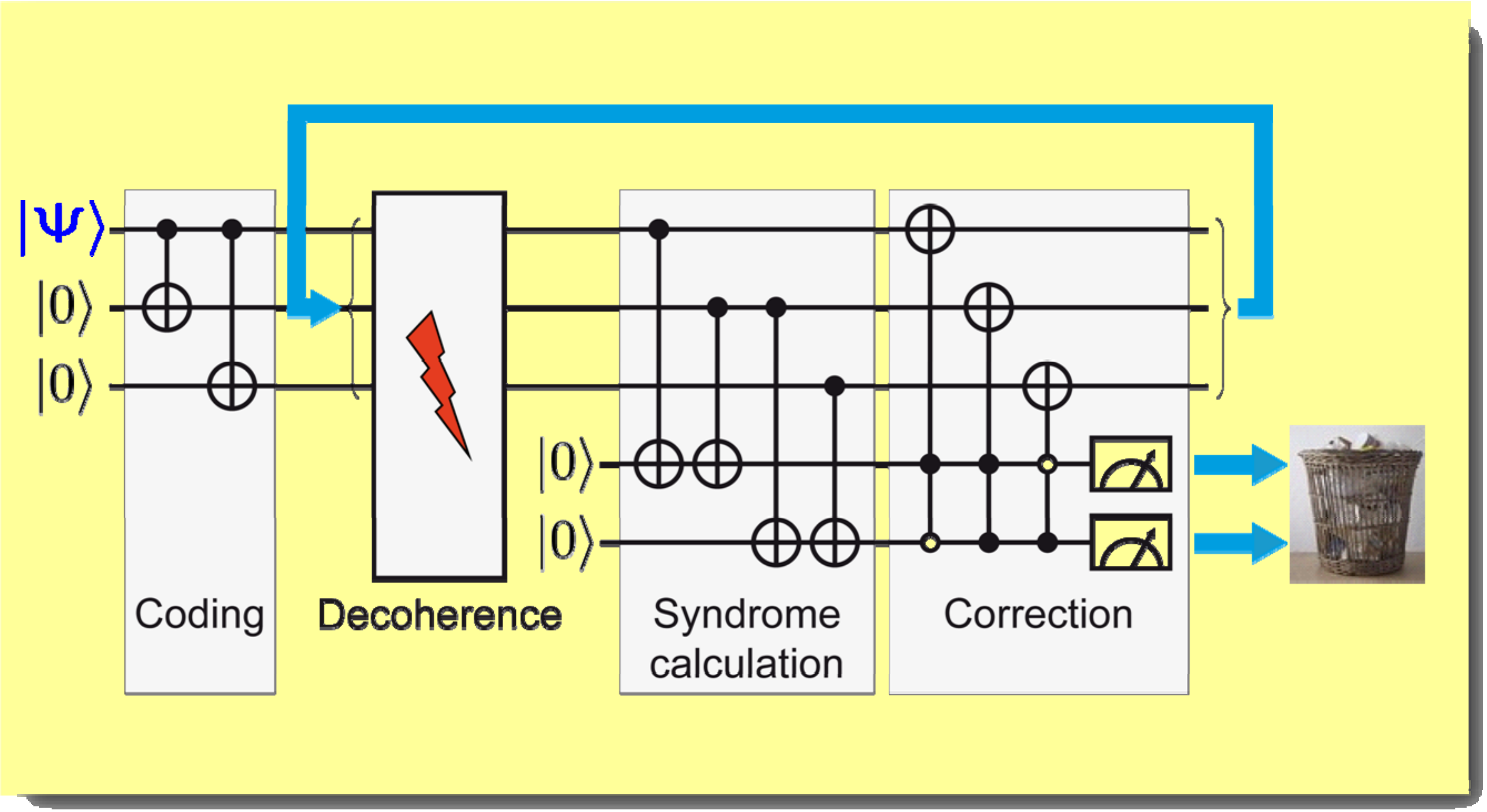




$|\chi|$

$\mathcal{F}_{CNOT} = 92.6\%$

Scalable quantum computation requires error correction

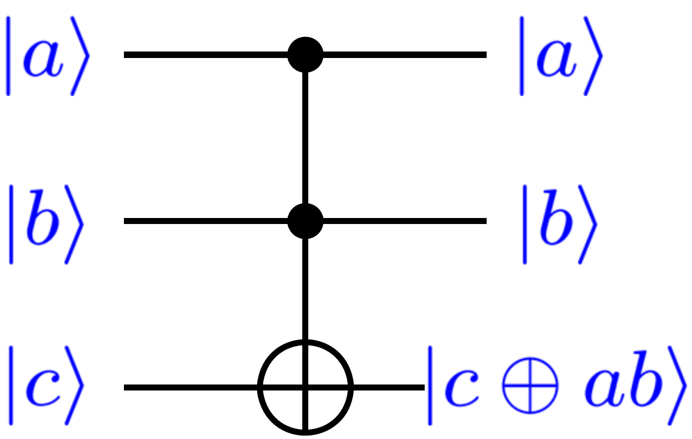


Toffoli gate: controlled-controlled NOT

Toffoli gate (Tommaso Toffoli, 1980):

…… is a universal reversible logic gate, i.e. any reversible circuit can be constructed from Toffoli gates.

also known as the controlled-controlled-NOT or CCNOT-gate operation



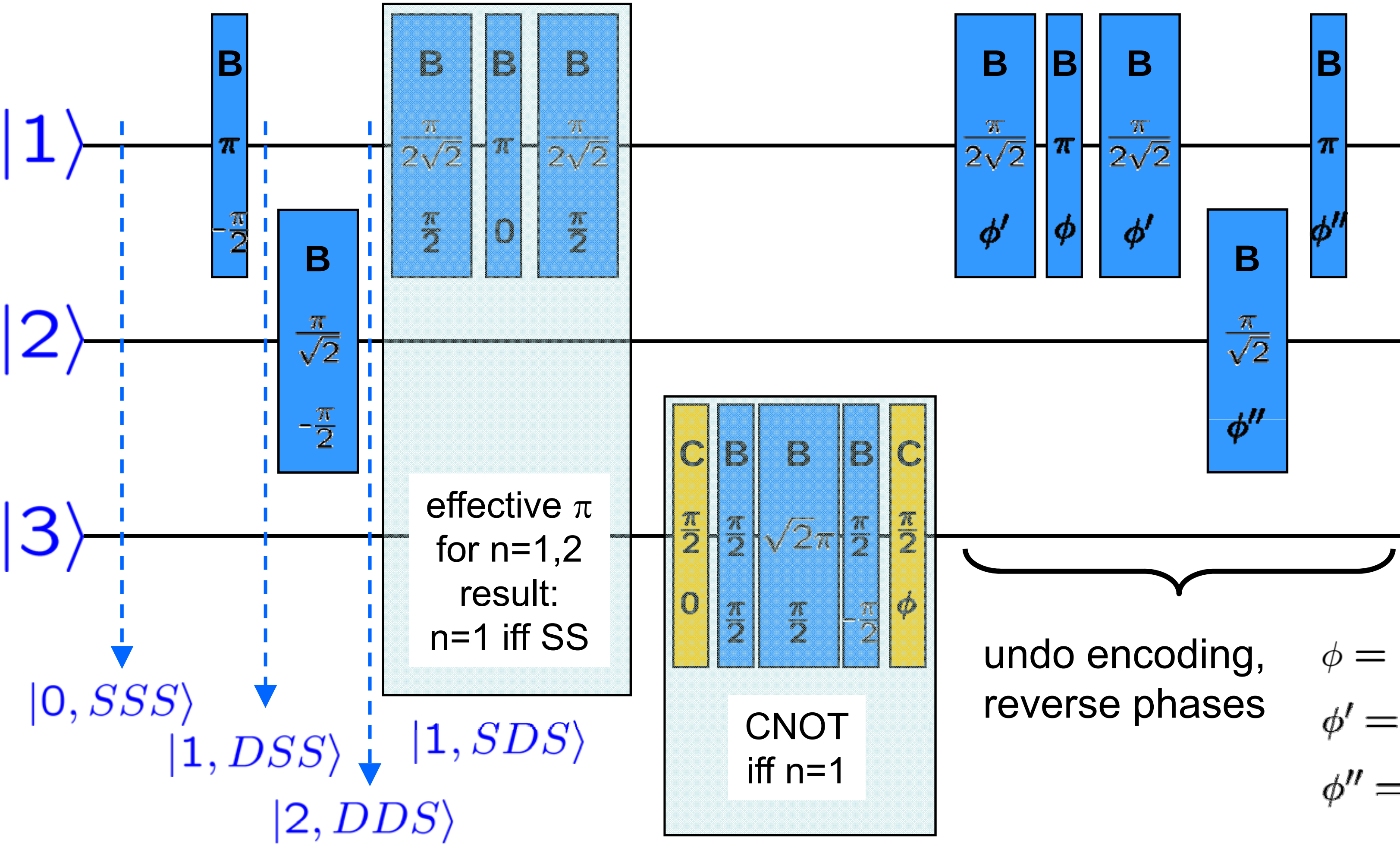
$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \end{pmatrix}$$

useful, e.g. for error correction

Toffoli gate: pulse sequence

use 2-phonon excitation

Th. Monz et al.,
Phys. Rev. Lett. **102**, 040501 (2009)



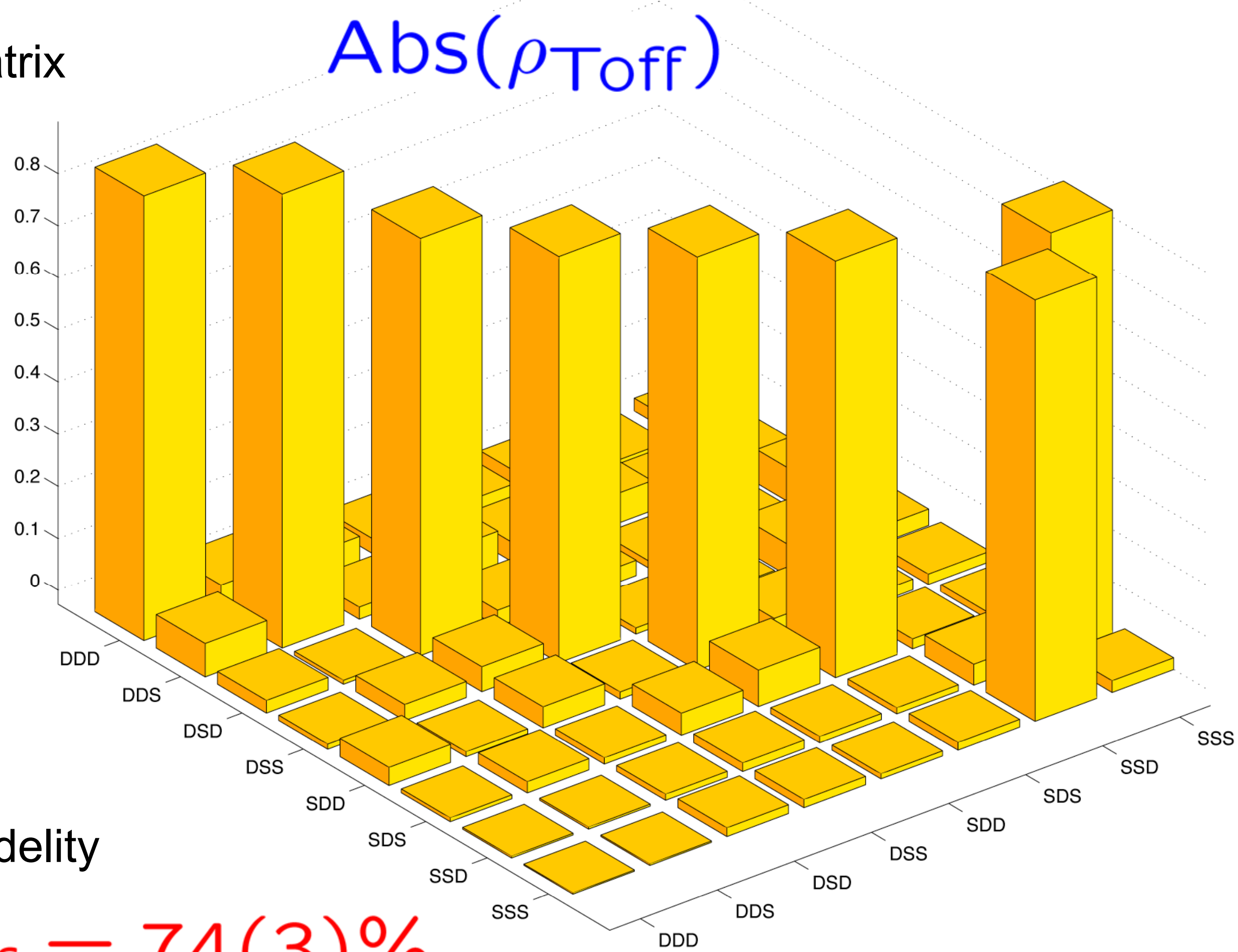
$\phi = (\frac{1}{\sqrt{2}} - 1)\pi$

$\phi' = (-\frac{1}{2} + \frac{1}{\sqrt{2}})\pi$

$\phi'' = (\frac{1}{2} + \frac{1}{\sqrt{2}})\pi$

Toffoli gate: experimental truth table

density matrix



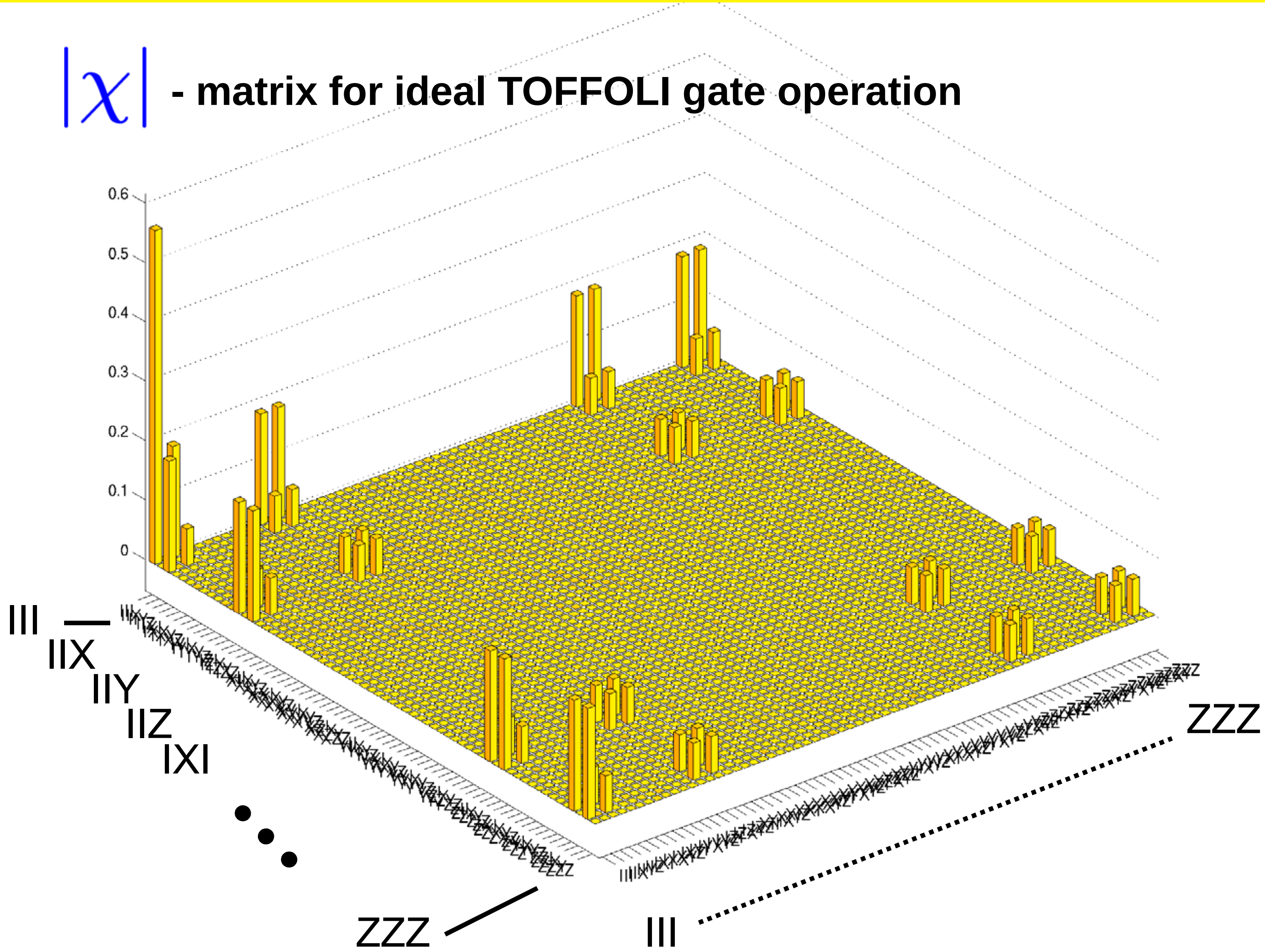
State Fidelity

$$F_{\mathrm{Toff}} = 74(3)\%$$

Th. Monz et al., Phys. Rev. Lett. **102**, 040501 (2009)

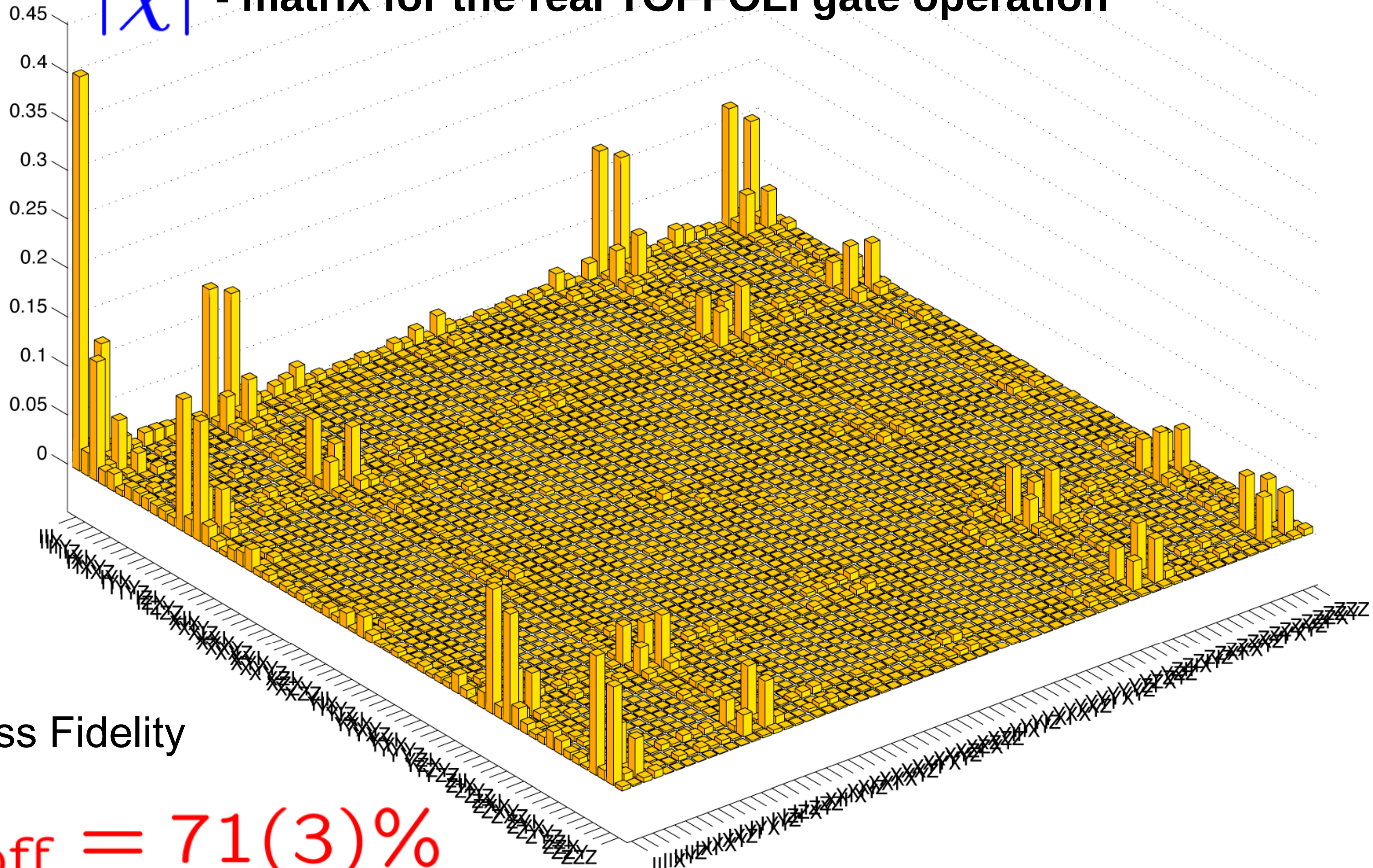
Toffoli gate: process tomography

$|\chi|$ **- matrix for ideal TOFFOLI gate operation**



Toffoli gate: process tomography

$|\chi|$ **- matrix for the real TOFFOLI gate operation**



Mean
Process Fidelity

$F_{\text{Toff}} = 71(3)\%$

Th. Monz et al., Phys. Rev. Lett. **102**, 040501 (2009)

Quantum procedures and fidelities

► single qubit operations	$|\psi\rangle = \alpha|g\rangle + \beta|e\rangle$	> 99 %
► 2-qubit CNOT gate	$|\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$	~ 93 %
► Bell states	$\Psi_- = \frac{1}{\sqrt{2}}(|SD\rangle - |DS\rangle)$	93-95 %
► W and GHZ states	$|\psi\rangle_{GHZ} = |SSS + DDD\rangle$	85-90 %
► Quantum teleportation	$|\Psi\rangle_A \longrightarrow |\Psi\rangle_B$	83 %
► Entanglement swapping	$|Bell\rangle_{ab}, |Bell\rangle_{cd} \longrightarrow |Bell\rangle_{ad,bc}$	~ 80 %
► Toffoli gate operation	$|a\rangle|b\rangle|c\rangle \rightarrow |a\rangle|b\rangle|c \oplus ab\rangle$	71 %

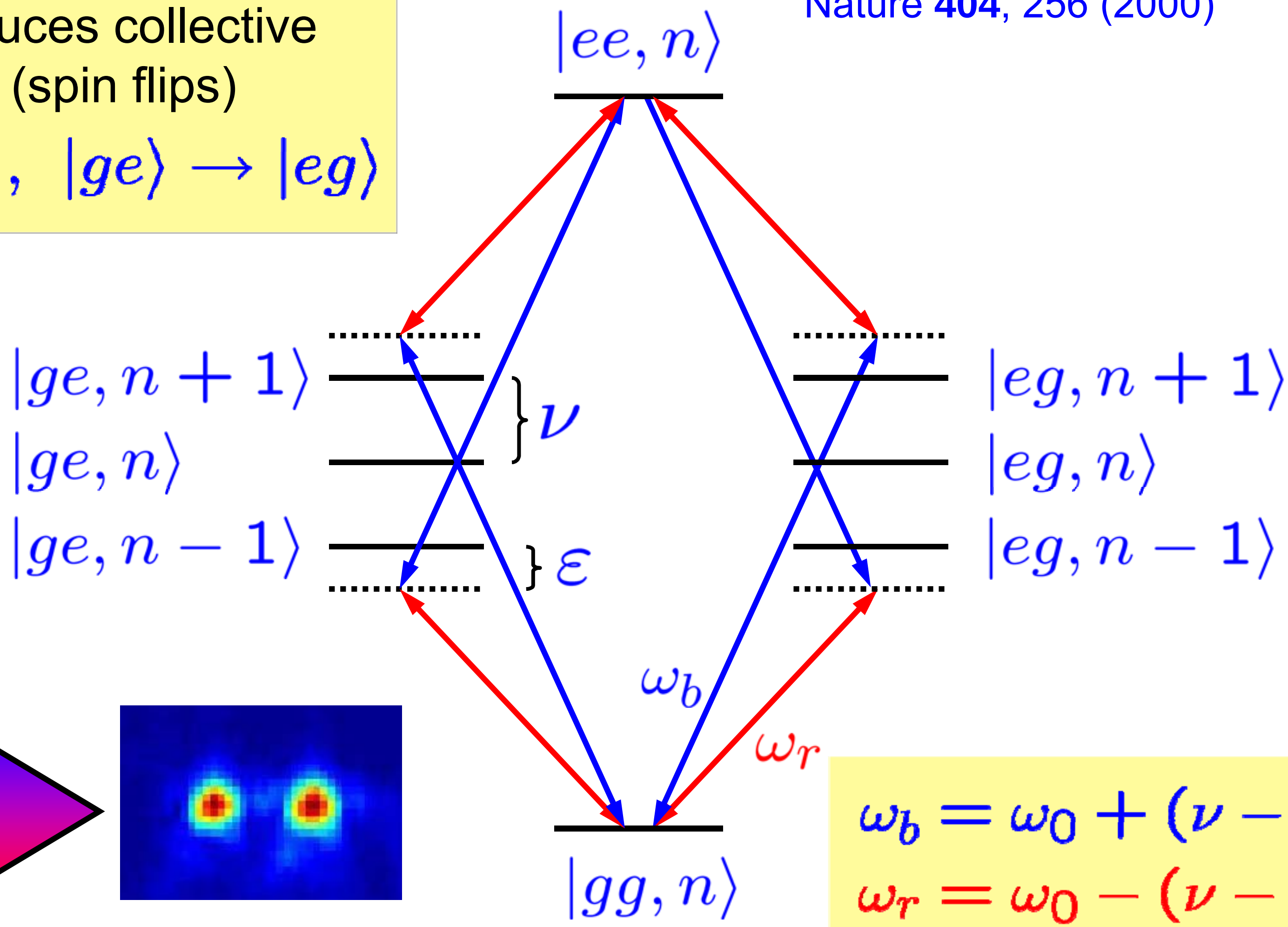
BUT: for fault-tolerant operation needed > 99 % !

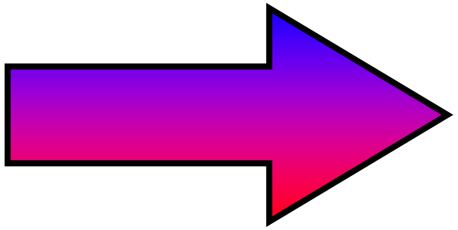
Mølmer - Sørensen gate - operation

bichromatic laser excitation close to upper and lower sidebands induces collective state changes (spin flips)

$|gg\rangle \rightarrow |ee\rangle, \quad |ge\rangle \rightarrow |eg\rangle$

K. Mølmer, A. Sørensen,
Phys. Rev. Lett. **82**, 1971 (1999)
C. A. Sackett et al.,
Nature **404**, 256 (2000)



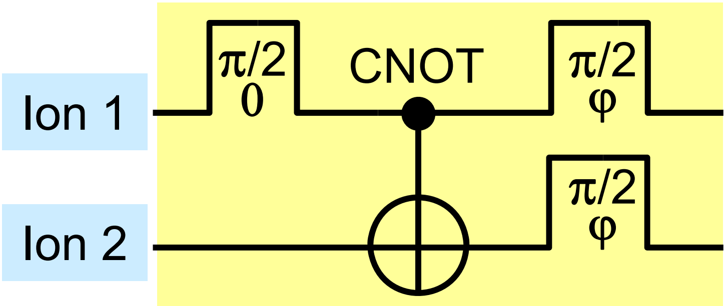


$$\omega_b = \omega_0 + (\nu - \varepsilon)$$
$$\omega_r = \omega_0 - (\nu - \varepsilon)$$

Measuring entanglement

C. A. Sackett et al., Nature **404**, 256 (2000)

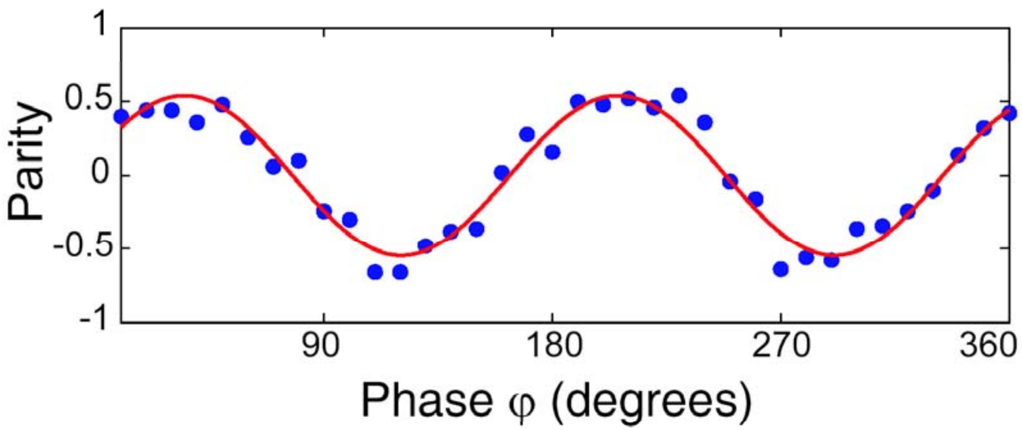
measure the parity Π:



$$\Pi = \sum_{j=0}^{2} (-1)^j P_j$$

$P_0 \equiv P_{DD}, P_1 \equiv P_{SD,DS}, P_2 \equiv P_{SS}$

Π oscillates with 2φ !
54% visibility



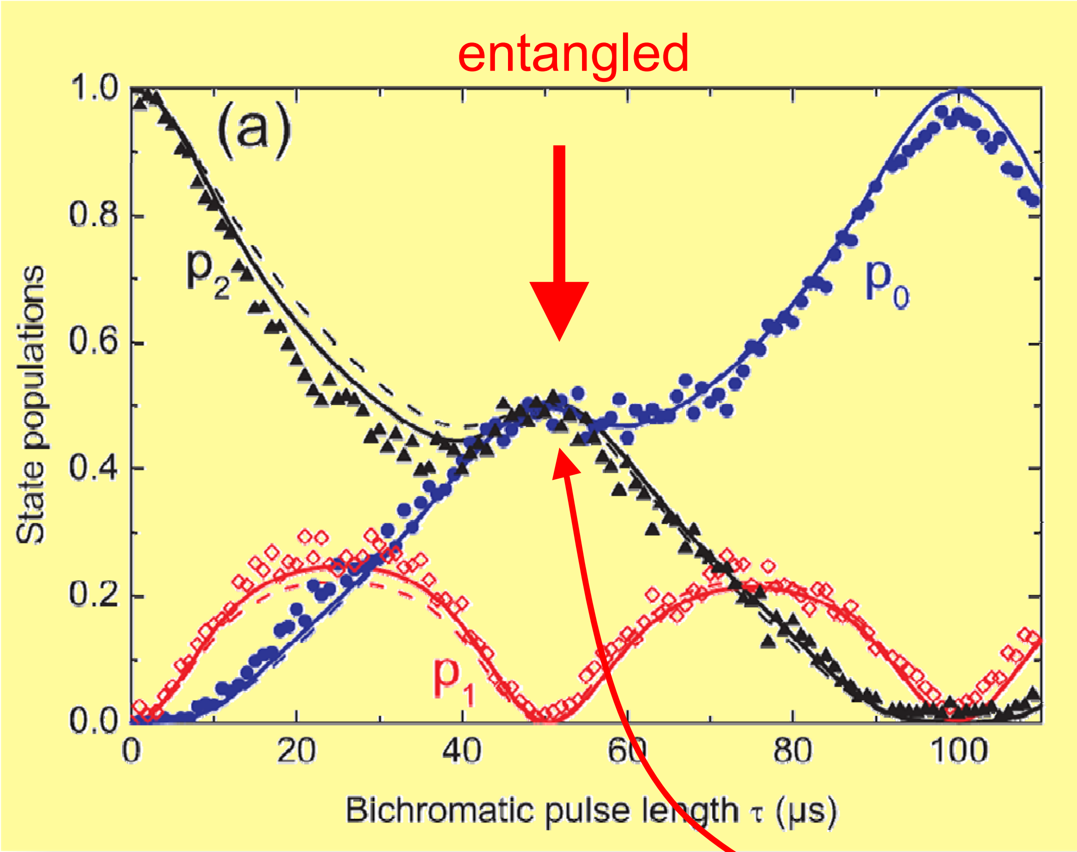
Fidelity =

$0.5(P_{SS}+P_{DD}+\text{visibility}) =$

71(3) %

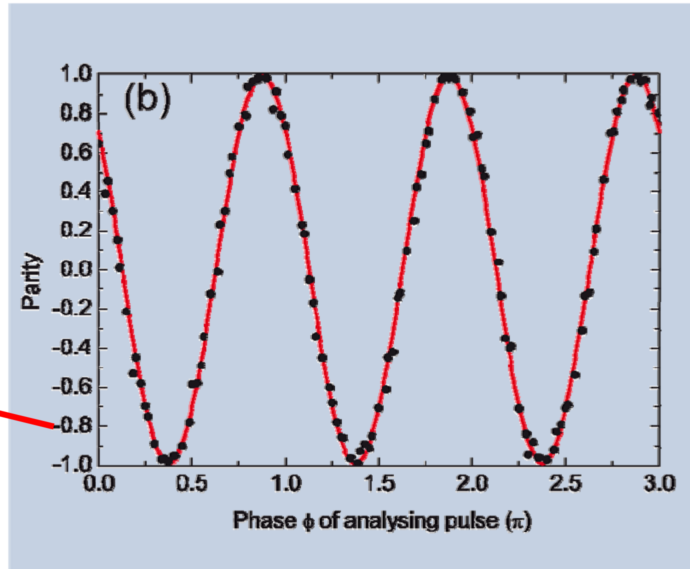
F. Schmidt-Kaler et al., Nature **422**, 408 (2003)

Deterministic Bell states using the Mølmer-Sørensen gate



J. Benhelm, G. Kirchmair, C. Roos
Theory: C. Roos,
New Journ. of Physics **10,** 013002 (2008)

measure entanglement via parity oscillations



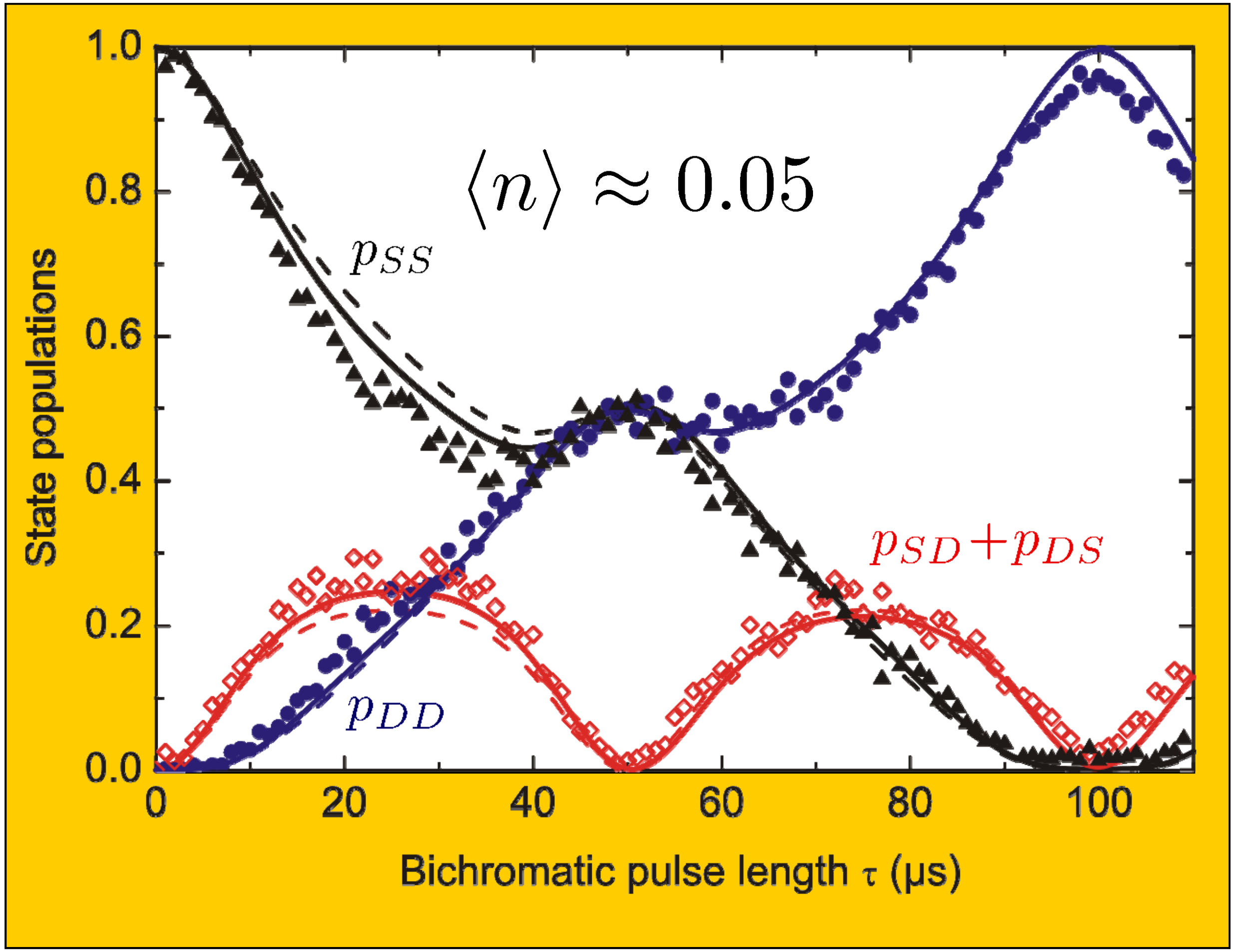
gate duration $51\mu s$

average fidelity

$$F_{\mathsf{MS}} = 99.3(0.2)\%$$

Mølmer-Sørensen gate: thermal states

Gate operation after ground state cooling



Bell state:

$$\Psi = |SS\rangle + i|DD\rangle$$

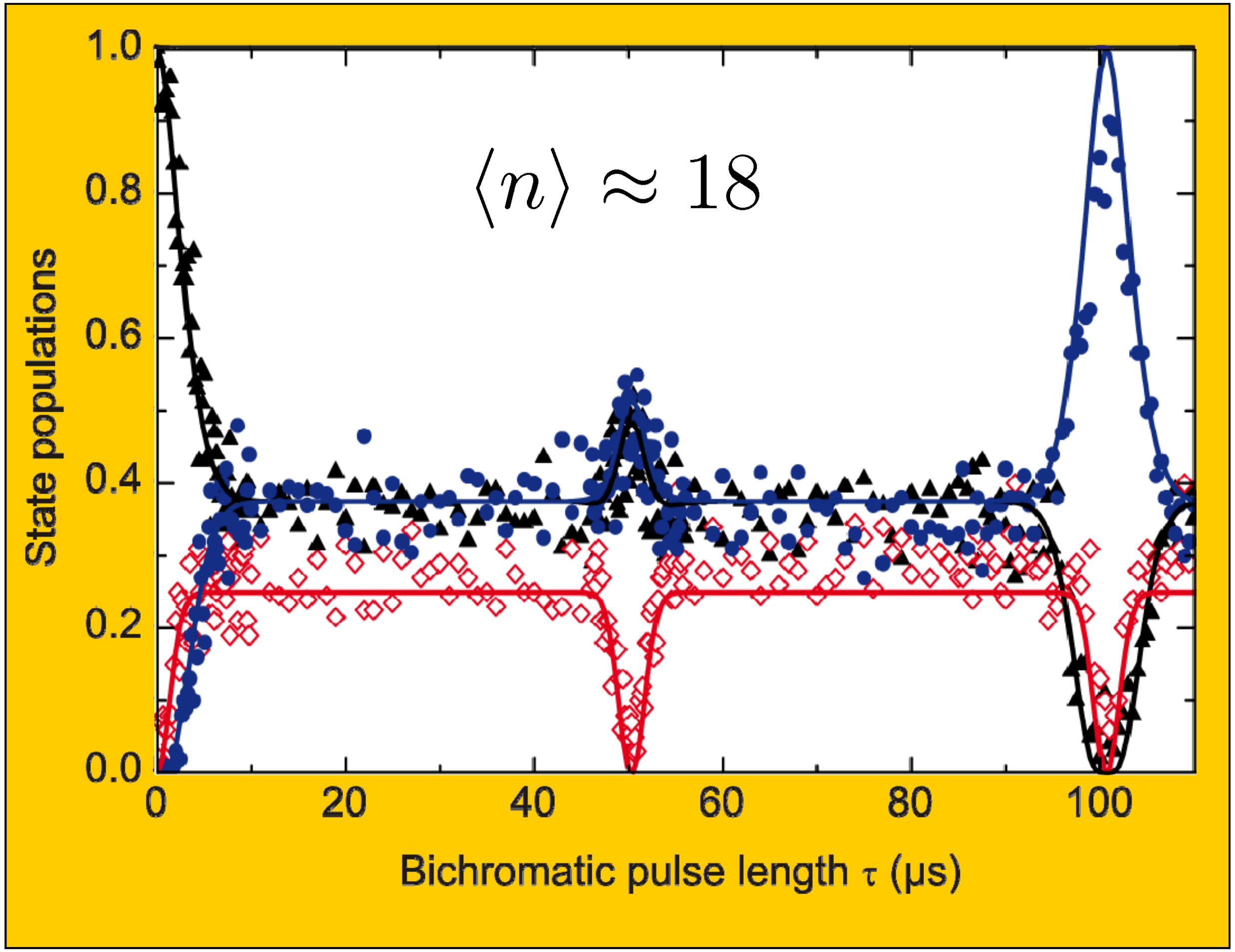
Fidelity :

F = 99.3(1) %

G. Kirchmair et al., New. J. Phys. **11**, 023002 (2009)

Mølmer-Sørensen gate: thermal states

Gate operation after Doppler cooling



Bell state:

$\Psi = |SS\rangle + i|DD\rangle$

Fidelity :

F = 98.0(1) %

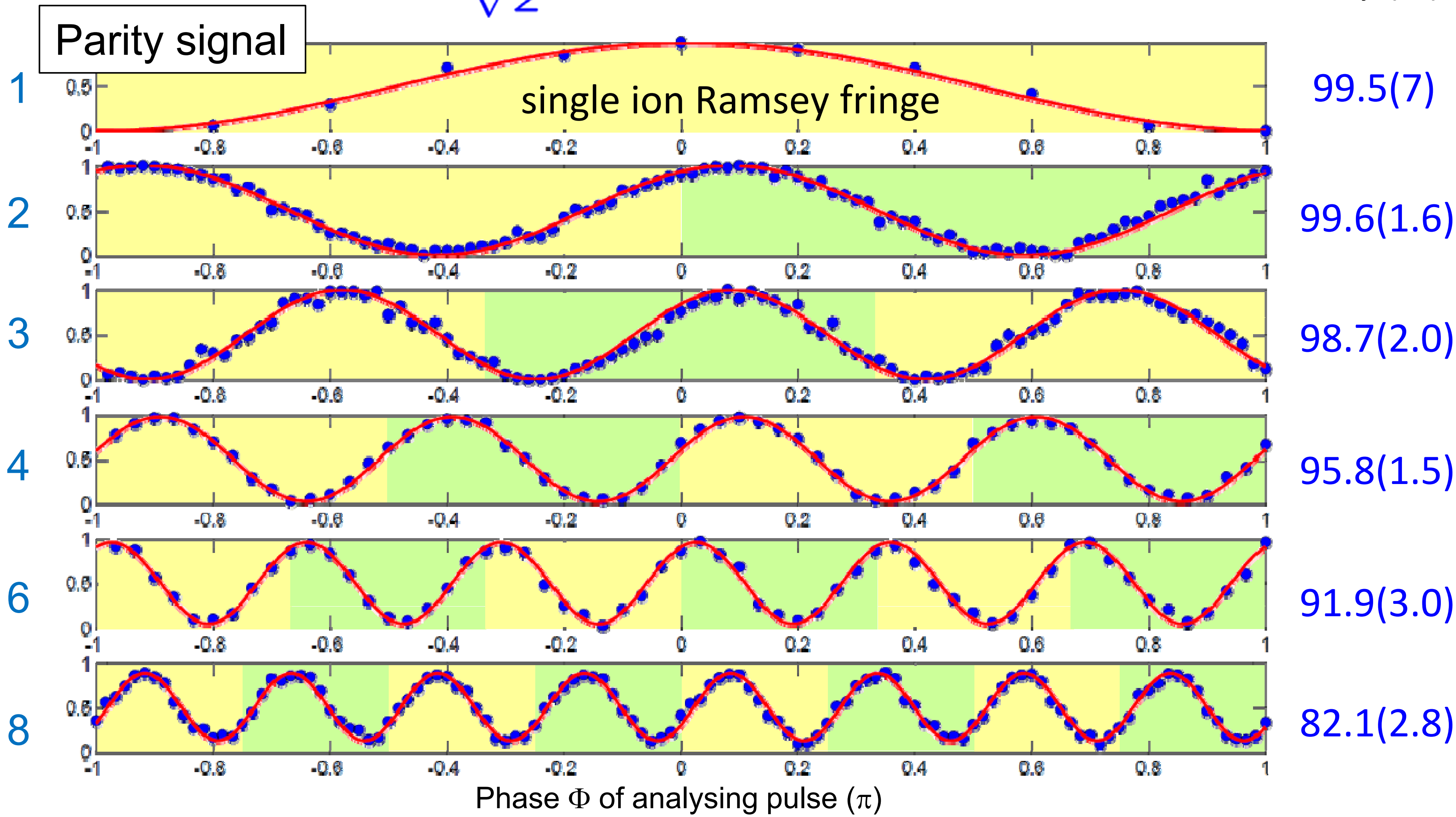
Gate operation ≈ independent of motional state !

G. Kirchmair et al., New. J. Phys. **11**, 023002 (2009)

n - qubit GHZ state generation with global MS gates

$$n = \qquad \Psi = \frac{1}{\sqrt{2}}(|gg\ldots g\rangle + |ee\ldots e\rangle)$$

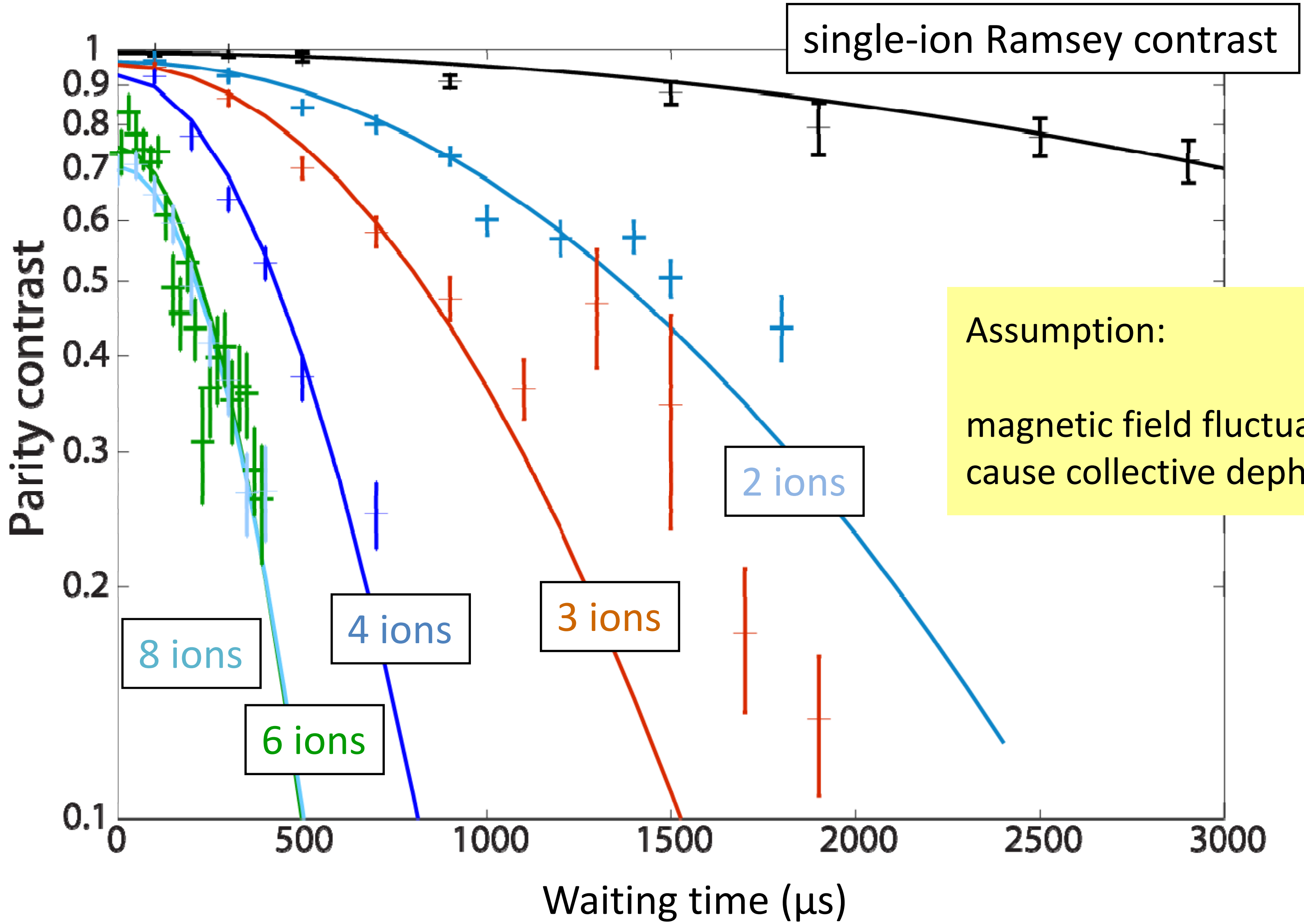
Fidelity (%)



T. Monz, P. Schindler, J. Barreiro, M. Chwalla, M. Hennrich, Innsbruck (2009)

Fidelity decay of n - qubit GHZ states (preliminary)

T. Monz, P. Schindler, J. Barreiro, M. Chwalla, M. Hennrich, Innsbruck (2009)



Assumption:

magnetic field fluctuations cause collective dephasing

Collective entangling gates + individual light shifts

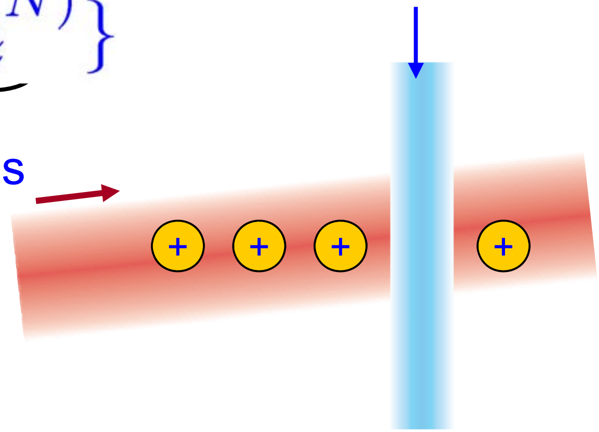
N ions Basic set of operations:

$$H_i \in \{ {S_y}^2, S_y, \underbrace{\sigma_z^{(1)}, \sigma_z^{(2)} \ldots \sigma_z^{(N)}} \}$$

Mølmer-Sørensen gate

collective spin flips

individual light shift gates



- favorable ion addressing by light shifts (~Ω^2)
- no interferometric stability between beams required

$H_i, H_j \rightarrow [H_i, H_j]$ generate Lie algebra $\mathcal{L}$ with dim $\mathcal{L} = 4^N$

→ Arbitrary unitary operations can be achieved !

...but how ?

Optimal control for arbitrary quantum gates

V. Nebendahl et al., Phys. Rev. A **79**, 012312 (2009)

Quantum optimal control: $$H(t) = \sum_{k=1}^{n} \alpha_k(t) H_k$$

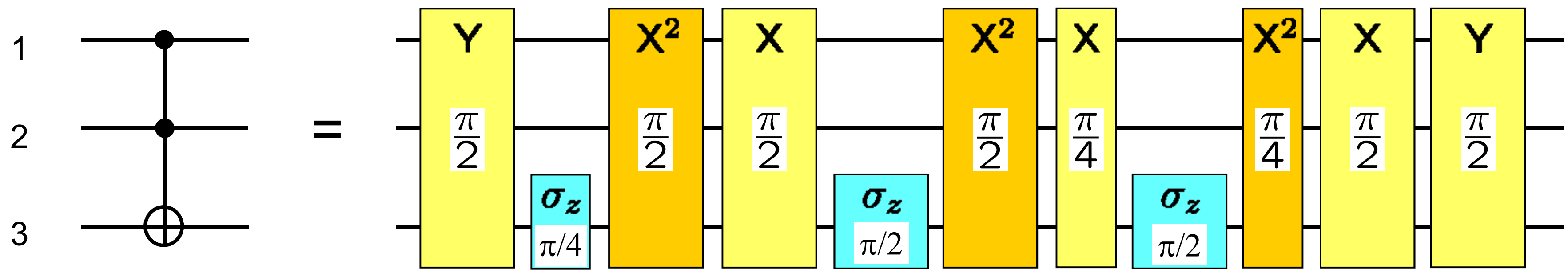
Find $\{\alpha_k(t), k = 1 \ldots n\}$ such that $$U_{gate} \stackrel{!}{=} \mathcal{T} \int_0^{\tau} dt \, e^{-\frac{i}{\hbar} \sum_k \alpha_k(t) H_k}$$

Gradient ascent algorithm: N. Khaneja *et al*., J. Magn. Res. **172**, 296 (2005)

Modification of search algorithm:

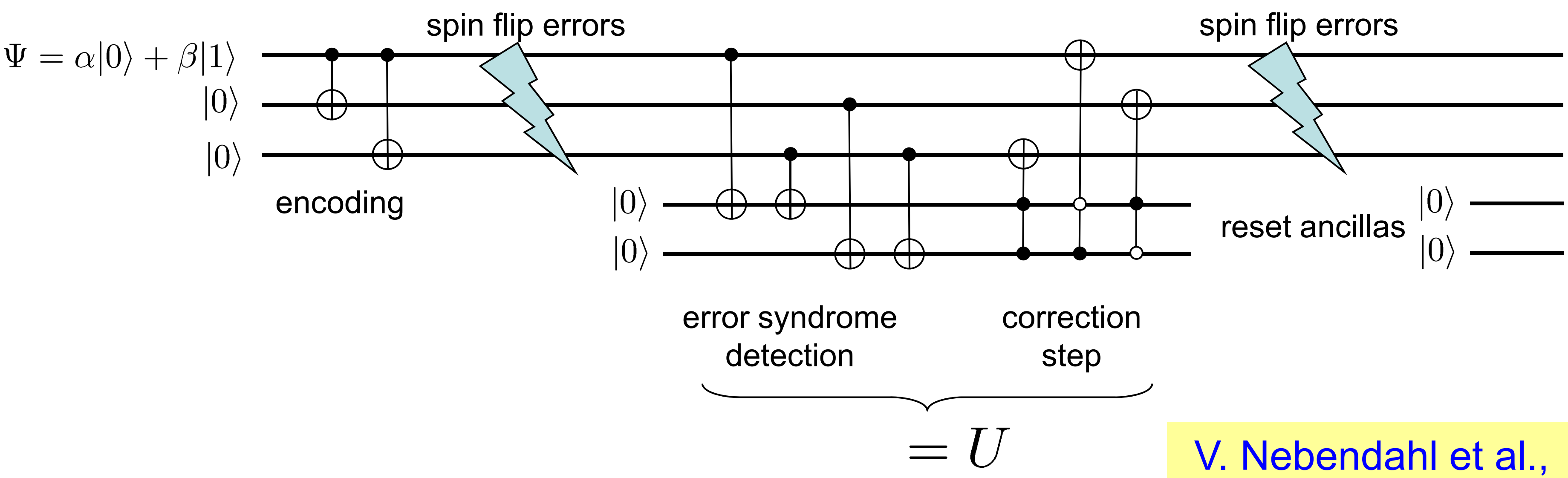
- no simultaneous application of several Hamiltonians
- sequence of pulses with variable length

Example: quantum Toffoli gate



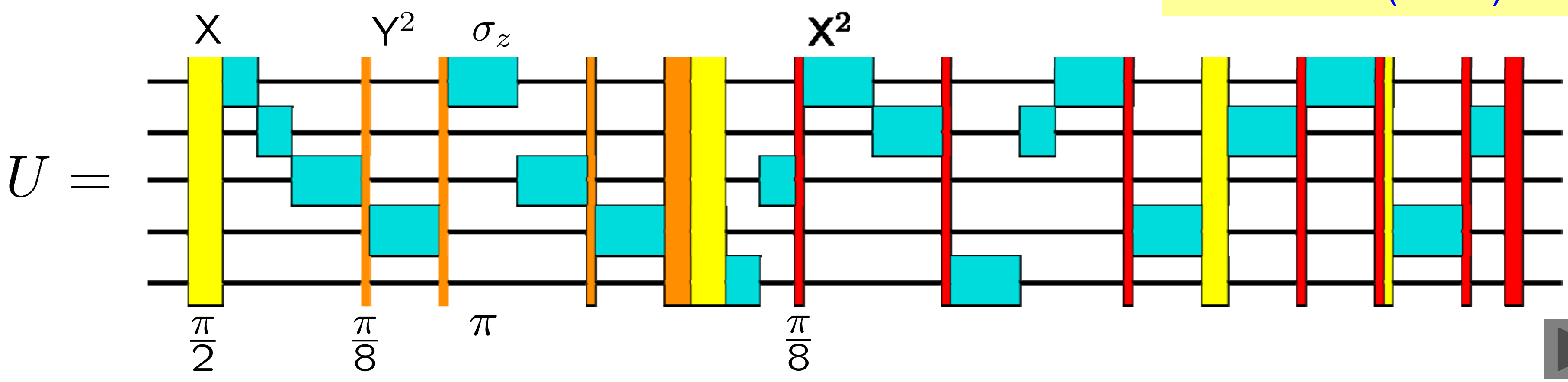
Optimal control : Quantum Error Correction

Quantum Error Correction: 3 qubits encode logical qubit (protection against spin flips)



V. Nebendahl et al., Phys. Rev. A **79**, 012312 (2009)

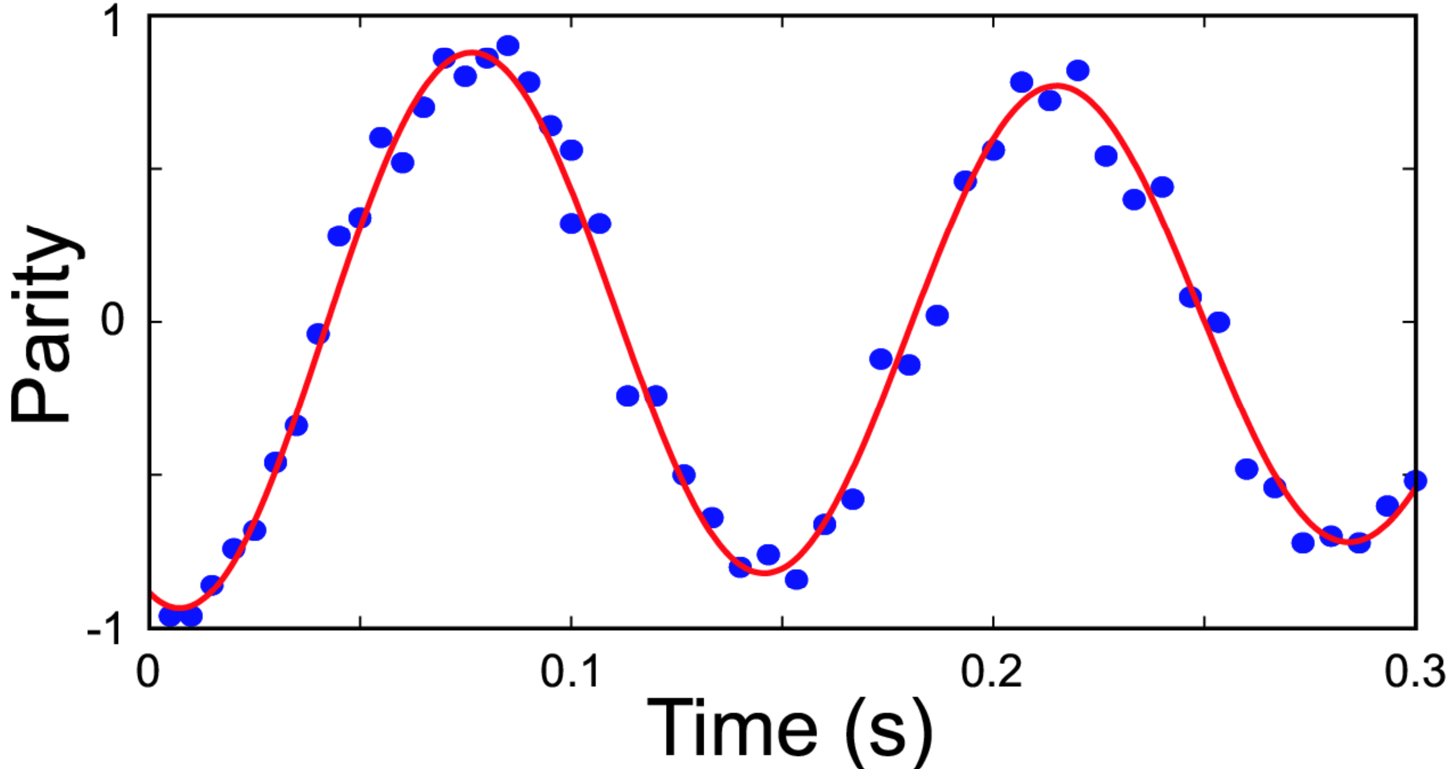
Implementation : 34 laser pulses (11 entangling pulses)



Scalability is facilitated by error avoidance

Decoherence-free Bell states:

$$\Psi_- = \frac{1}{\sqrt{2}}(|SD\rangle - |DS\rangle)$$



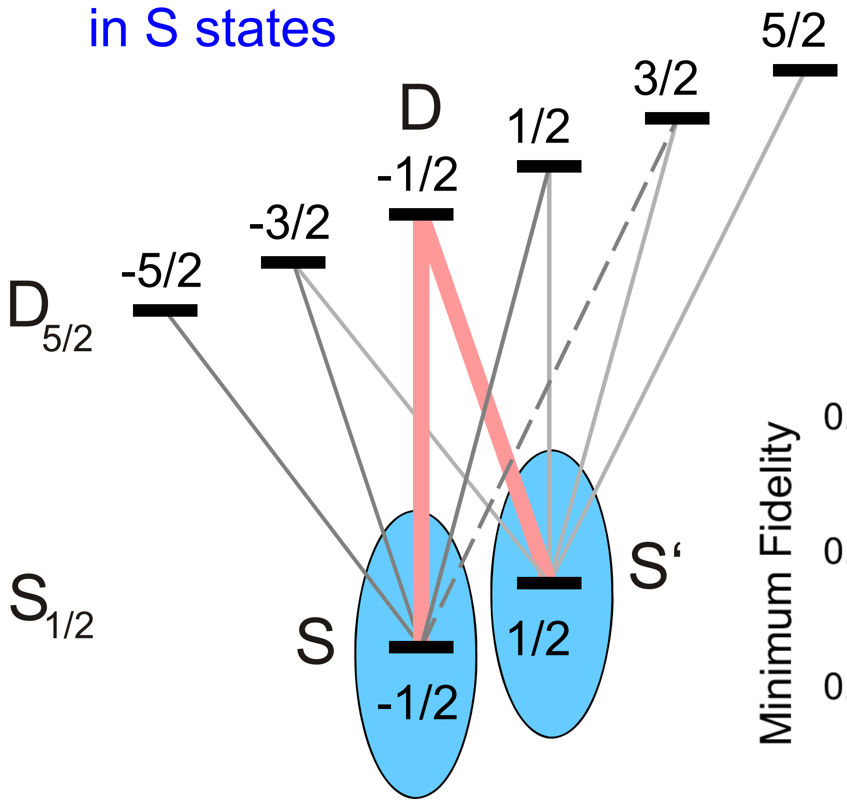
decoherence-time:
0.5 x 1.05(15) s

C. Roos et al., Phys. Rev. Lett. **92**, 220402 (2004)

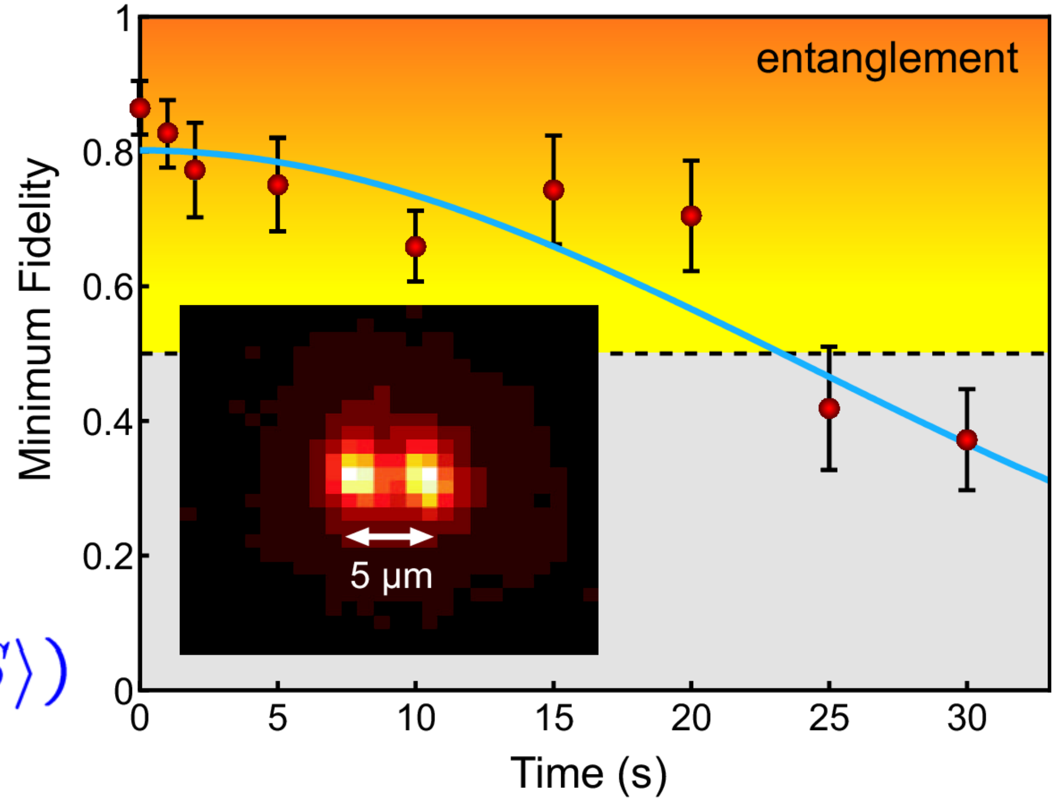
Robust entanglement

H. Häffner et al., Appl. Phys. B **81**, 151 (2005)

prepare qubits in S states



Hiding states in S, S' states avoids decoherence from spontaneous emission

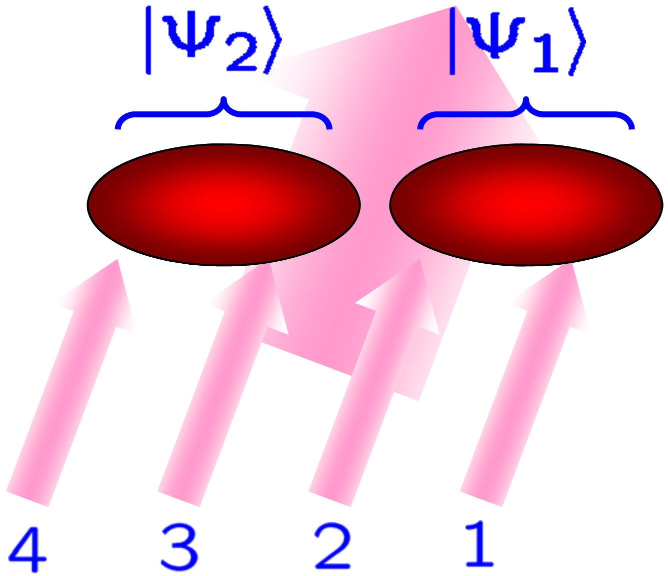


$$\psi = \frac{1}{\sqrt{2}}(|SS'\rangle + e^{i\phi}|S'S\rangle)$$

Quantum gates within a decoherence free subspace

K. Kim et al., Innsbruck, 2006

Idea: use long-lived Bell states as logical qubits



$$|0\rangle_L \longrightarrow |SS'\rangle$$

$$|1\rangle_L \longrightarrow |S'S\rangle$$

Gate operations by

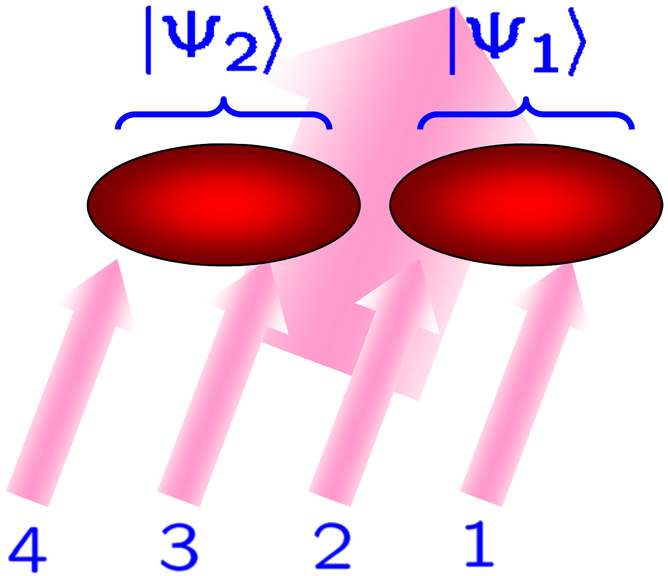
- addressing individual ions
- simultaneous addressing of innermost ions

Trade off more ions (2x) for much extended coherence times (100-1000x)

Universal quantum computation with logical qubits

Th. Monz, K. Kim,
Ph. Schindler (2008)

Idea: use long-lived Bell states as logical qubits



$$|0\rangle_L \longrightarrow |SS'\rangle$$

$$|1\rangle_L \longrightarrow |S'S\rangle$$

Gate operations by
- addressing individual ions and MS-gates
- simultaneous addressing of innermost ions

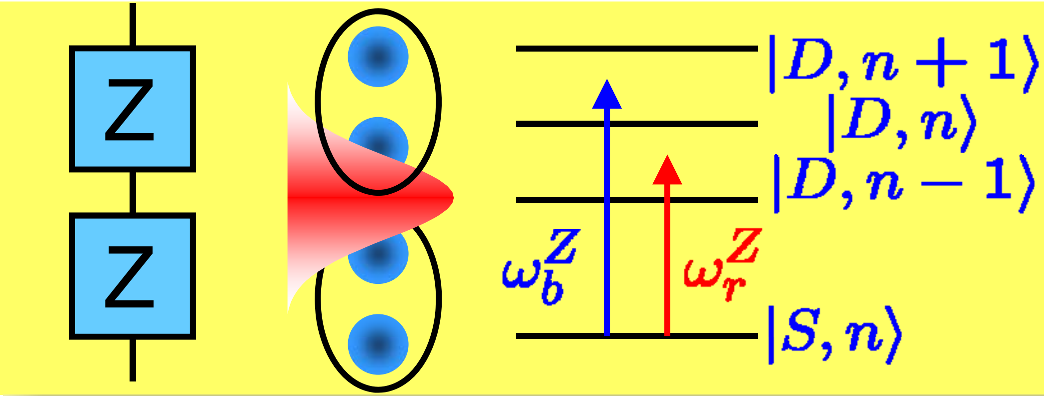
Analysis by state and process tomography

1st experiment:

$$|0\rangle_L \longrightarrow |SD\rangle$$

$$|1\rangle_L \longrightarrow |DS\rangle$$

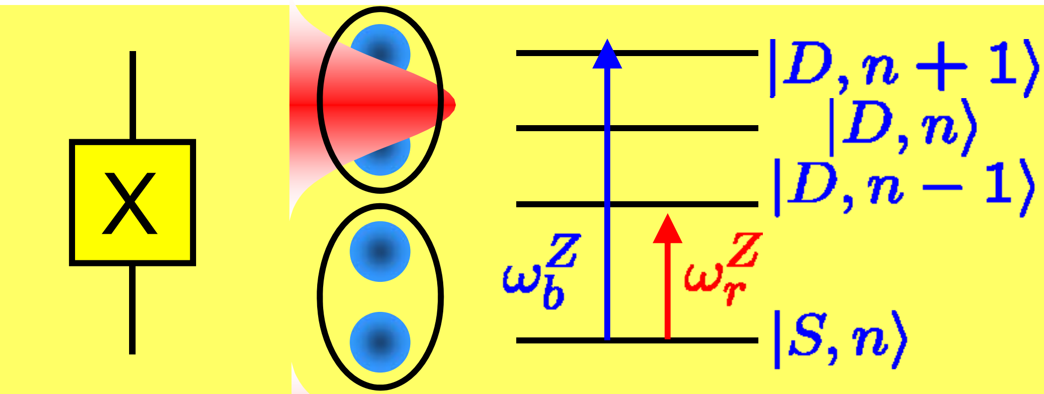
Gate operations with logical qubits



$$\omega_b^Z = \omega_c + \nu/2 + \delta_z$$

$$\omega_r^Z = \omega_c - \nu/2 - \delta_z$$

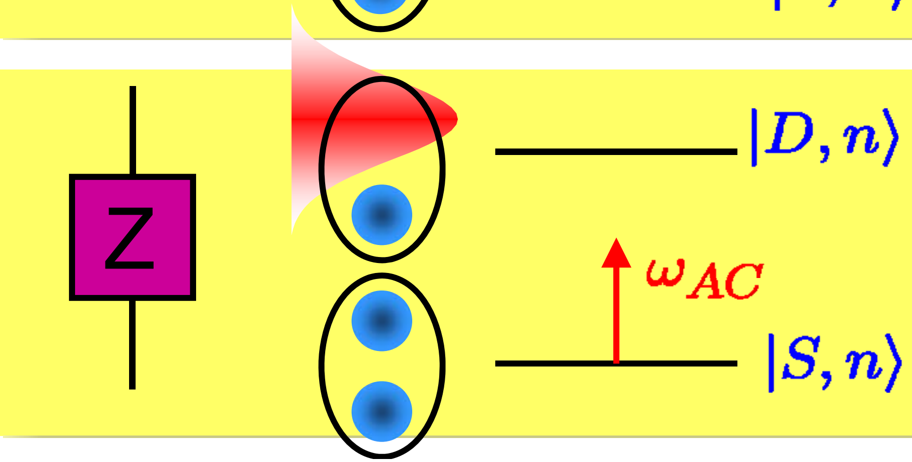
K. Kim et. al.
Phys. Rev. **A77**, 050303 (2008)



$$\omega_b^{MS} = \omega_c + \nu + \delta_{MS}$$

$$\omega_r^{MS} = \omega_c - \nu - \delta_{MS}$$

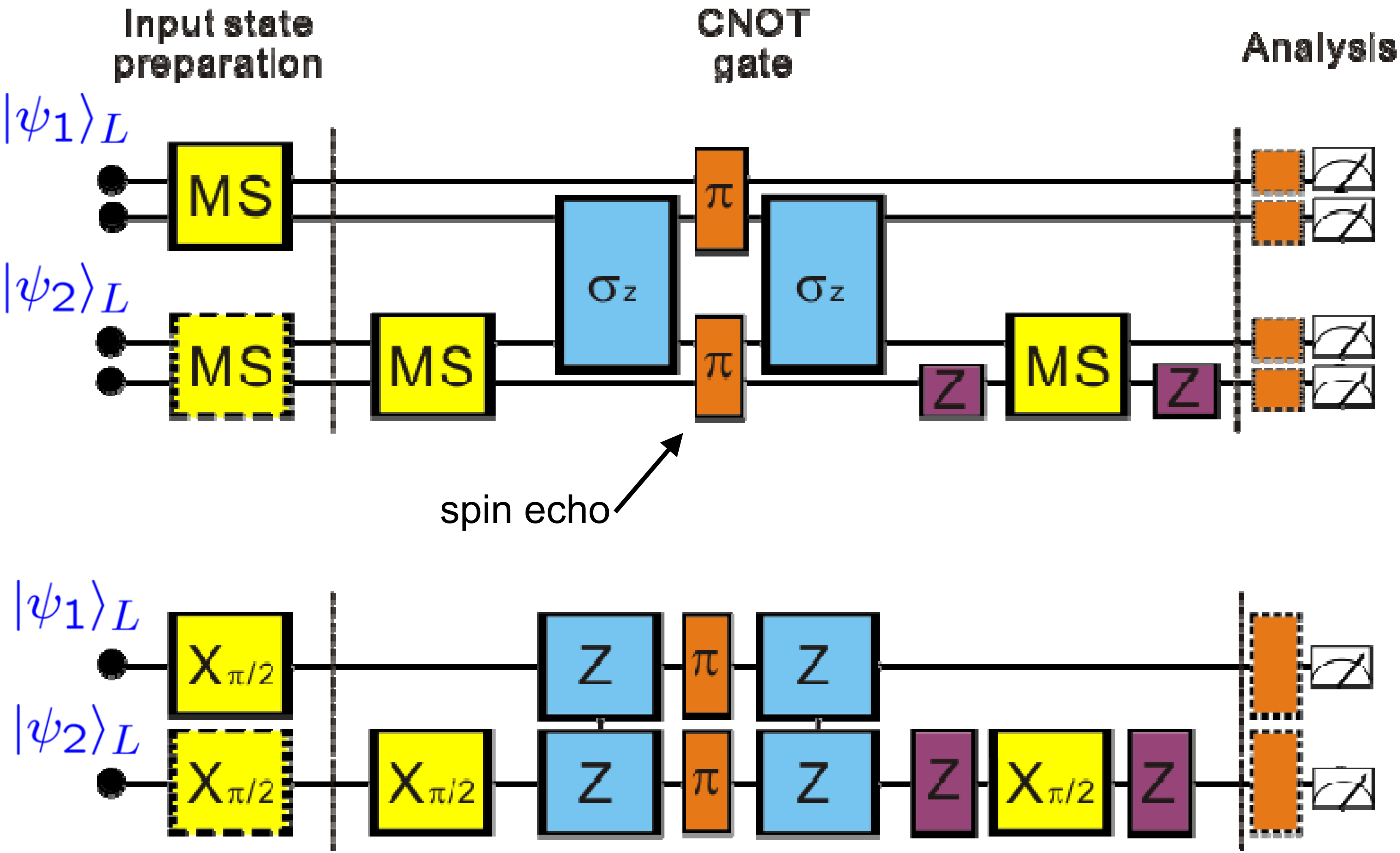
C. Roos,
New Journ. Phys. **10** (2008)



single qubit phase shift by
AC Stark shifts

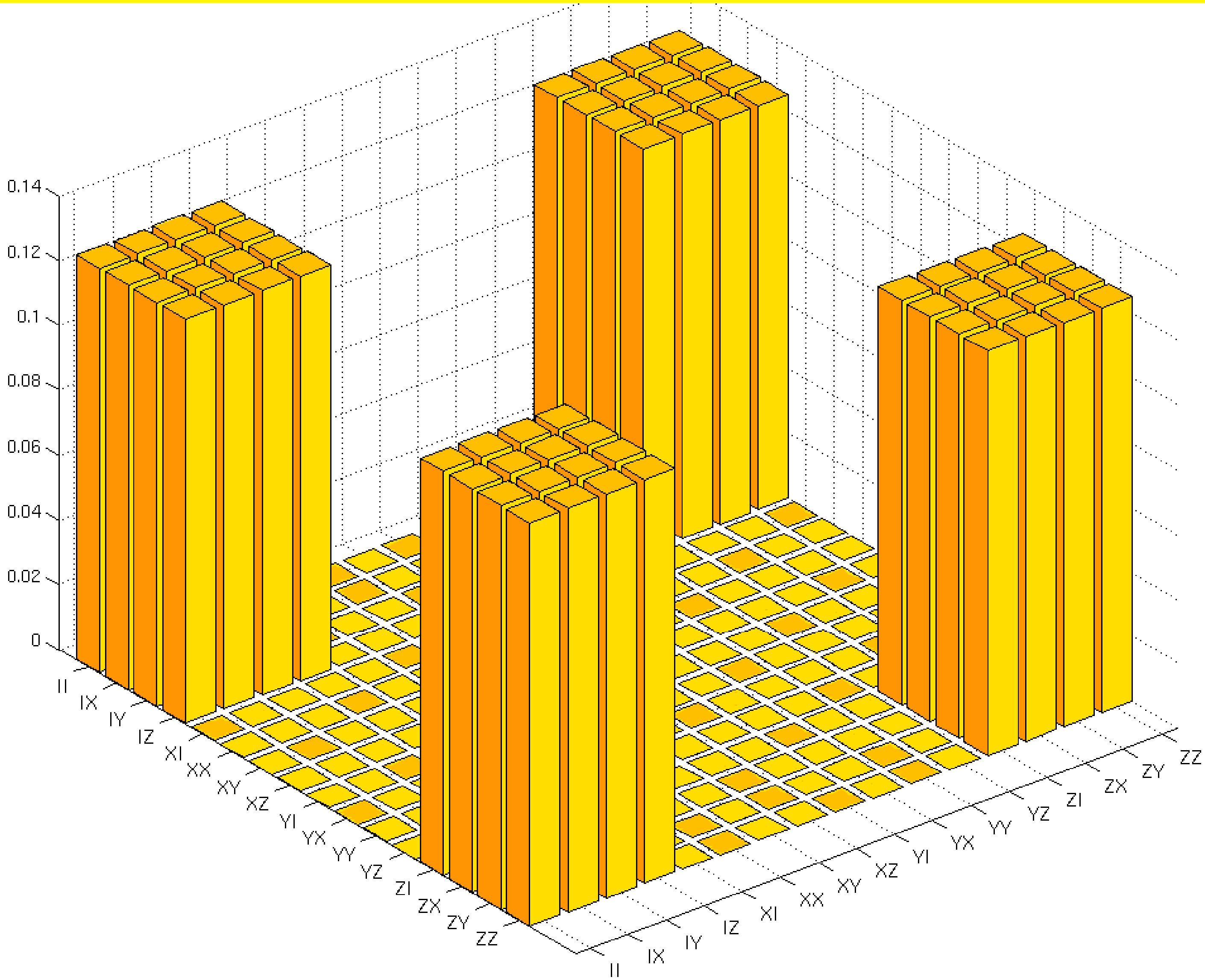
F. Schmidt-Kaler et. al.
Europhys. Lett. **65**, 587 (2004)

CNOT gate operation with a logical qubit



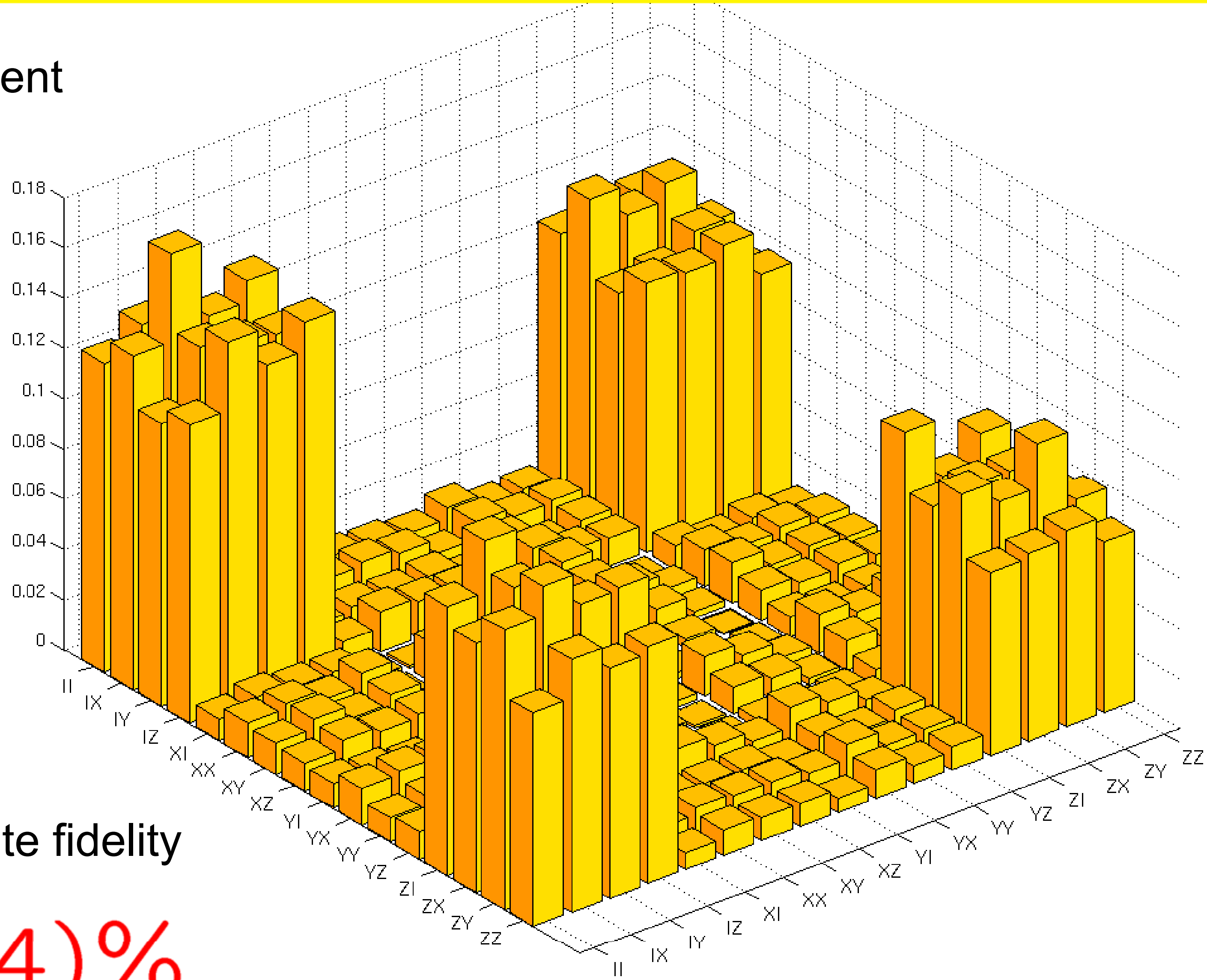
Process tomography of CNOT with logical qubits

theory



Process tomography of CNOT with logical qubits

experiment



mean gate fidelity

89(4)%

T. Monz, K. Kim et al., Phys. Rev. Lett. **103**, 200503 (2009)

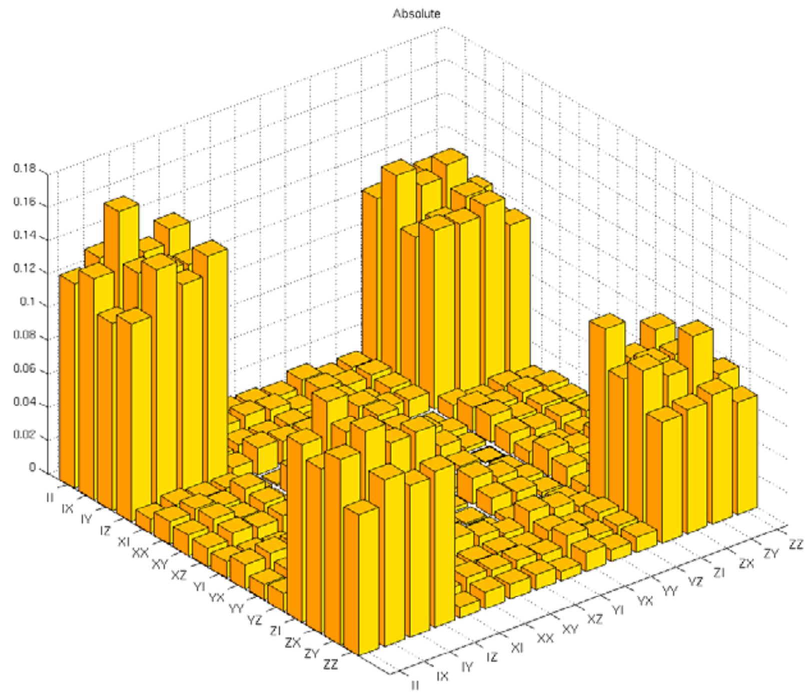
Quantum state manipulation with logical qubits

results and limitations:

- mean gate fidelity: 89(4)%
- limited by addressing errors, spurious laser noise, position stability, equal illumination of neighbouring ions

advantages:

- insensitive to laser linewidth,
- insensitive to AC-Stark shifts (work in progress)
- lifetime limited coherence time (work in progress)



T. Monz, K. Kim et al., Phys. Rev. Lett. **103**, 200503 (2009)

Testing quantum mechanics with trapped ions

- Do observables have predetermined values, fixed by hidden variables?

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

J. S. Bell

"No physical theory of local hidden variables can reproduce all of the predictions of quantum mechanics"

S. Kochen

E. Specker

"Non-contextual hidden variable theory cannot reproduce the predictions of quantum mechanics"

Kochen - Specker theorem

"Non-contextual hidden variable theory cannot reproduce the predictions of quantum mechanics"

Non-Contextuality:

Any measurement A should give a value independent of other **compatible measurements** B :

$$\nu(AB) = \nu(A) \cdot \nu(B)$$

Experimental tests:
Huang, Y.F., Phys. Rev. Lett. **90**, 250401 (2003)
Hasegawa, Y., Phys. Rev. Lett. **97**, 230401 (2006)

Specker, R., Dialectica **14**, 239-246 (1960)
Bell, J.S., Rev. Mod. Phys. **38**, 447-452 (1966)
Kochen, S., & Specker, E.P., J. Math. Mech. (1967)

The Peres – Mermin square

Measure observables A_{ij} with $v(A_{ij}) = \pm 1$, where the observables in each row and column are mutually compatible

A. Peres, Phys. Lett. **A 151**, 107–108 (1990)
N. D. Mermin, Phys. Rev. Lett. **65**, 3373–3376 (1990)

$$R_k = v(A_{k1})v(A_{k2})v(A_{k3})$$
$$C_k = v(A_{1k})v(A_{2k})v(A_{3k})$$

$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(1)} \otimes \sigma_z^{(2)}$	$\langle R_1 \rangle = 1$
$\sigma_x^{(2)}$	$\sigma_x^{(1)}$	$\sigma_x^{(1)} \otimes \sigma_x^{(2)}$	$\langle R_2 \rangle = 1$
$\sigma_z^{(1)} \otimes \sigma_x^{(2)}$	$\sigma_x^{(1)} \otimes \sigma_z^{(2)}$	$\sigma_y^{(1)} \otimes \sigma_y^{(2)}$	$\langle R_3 \rangle = 1$
$\langle C_1 \rangle = 1$	$\langle C_2 \rangle = 1$	$\langle C_3 \rangle = -1$	

Testing the Kochen-Specker theorem

- $\langle \chi_{KS} \rangle = \langle R_1 \rangle + \langle R_2 \rangle + \langle R_3 \rangle + \langle C_1 \rangle + \langle C_2 \rangle - \langle C_3 \rangle$
- Non-contextual hidden variable theory
  $\langle \chi_{KS} \rangle \leq 4$
- quantum mechanics
  $\langle \chi_{KS} \rangle = 6$
  for any input state

$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(1)} \otimes \sigma_z^{(2)}$	$\langle R_1 \rangle = 1$
$\sigma_x^{(2)}$	$\sigma_x^{(1)}$	$\sigma_x^{(1)} \otimes \sigma_x^{(2)}$	$\langle R_2 \rangle = 1$
$\sigma_z^{(1)} \otimes \sigma_x^{2}$	$\sigma_x^{(1)} \otimes \sigma_z^{(2)}$	$\sigma_y^{(1)} \otimes \sigma_y^{(2)}$	$\langle R_3 \rangle = 1$
$\langle C_1 \rangle = 1$	$\langle C_2 \rangle = 1$	$\langle C_3 \rangle = -1$	

A. Cabello, Phys. Rev. Lett. **101**, 210401 (2008)

Measuring operators like $\sigma_x \otimes \sigma_x$

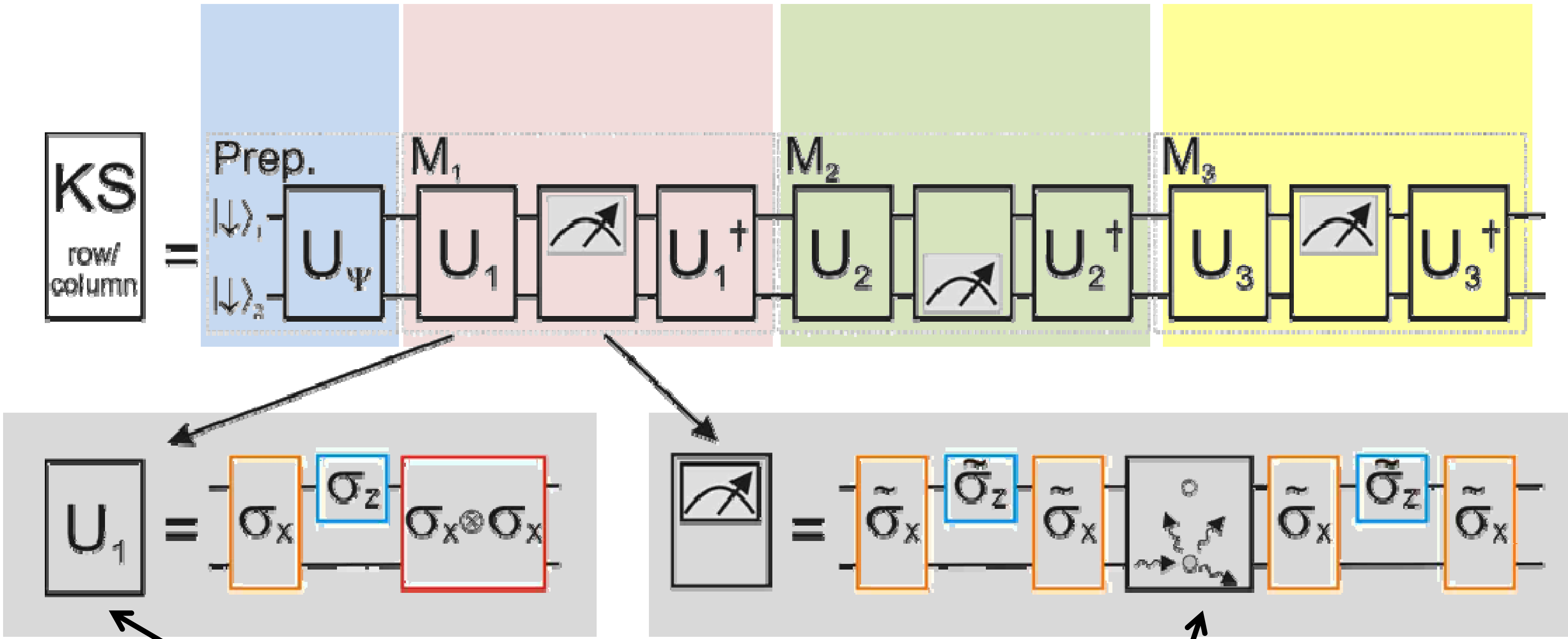
$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(1)} \otimes \sigma_z^{(2)}$
$\sigma_x^{(2)}$	$\sigma_x^{(1)}$	$\sigma_x^{(1)} \otimes \sigma_x^{(2)}$
$\sigma_z^{(1)} \otimes \sigma_x^{(2)}$	$\sigma_x^{(1)} \otimes \sigma_z^{(2)}$	$\sigma_y^{(1)} \otimes \sigma_y^{(2)}$

We have to measure all observables with the same setup
whether measured in a row or a column → map to single ion

Experimental procedure

G. Kirchmair et al., Nature **460**, 494 (2009)

Measurement scheme for rows and columns of PM square:



unitary for respective element of PM square

actual measurement

Results

Input state:

$\Psi = |SS\rangle - |DD\rangle$

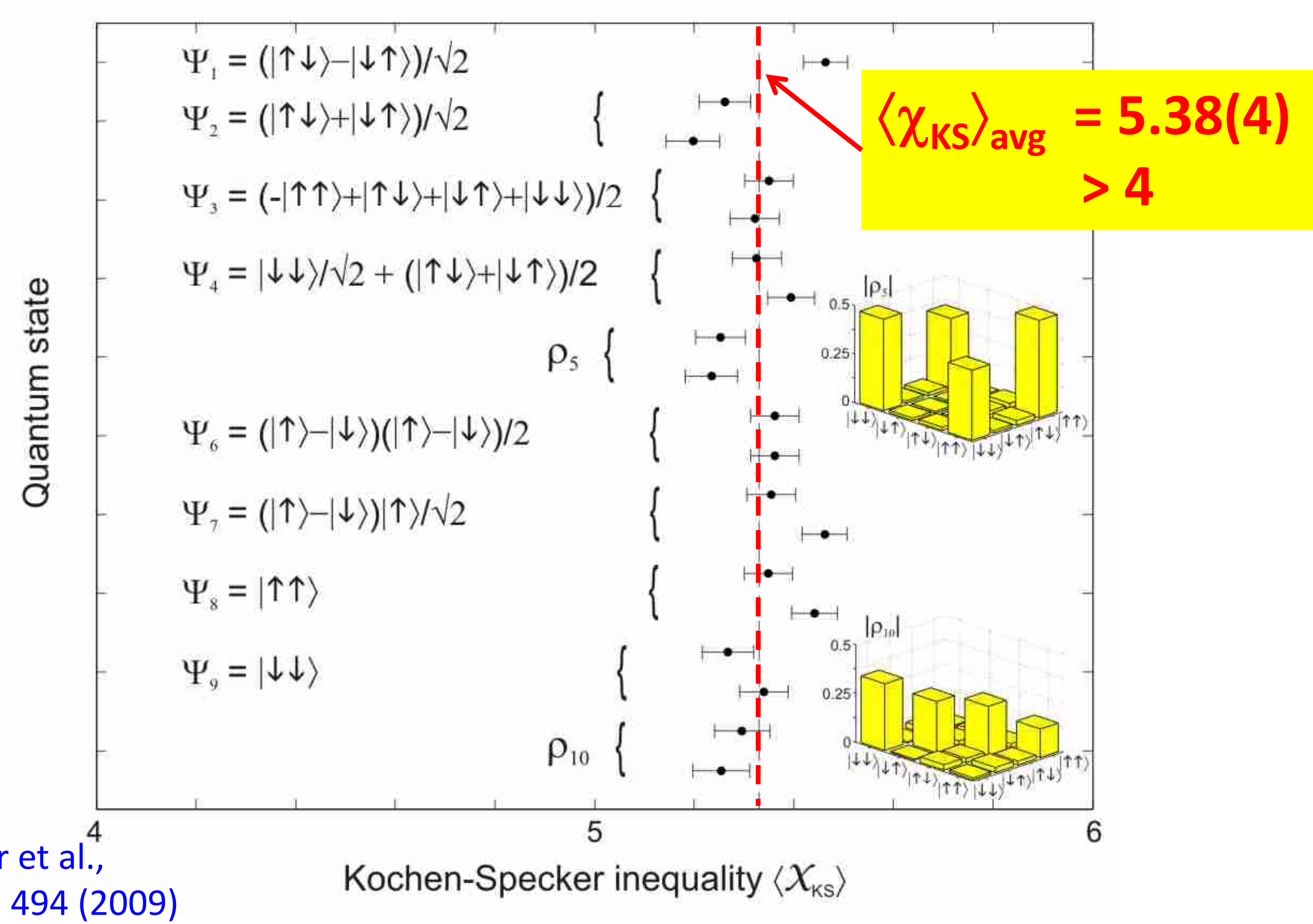
$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(1)} \otimes \sigma_z^{(2)}$	$\langle R_1 \rangle = 0.92(1)$
$\sigma_x^{(2)}$	$\sigma_x^{(1)}$	$\sigma_x^{(1)} \otimes \sigma_x^{(2)}$	$\langle R_2 \rangle = 0.93(1)$
$\sigma_z^{(1)} \otimes \sigma_x^{2}$	$\sigma_x^{(1)} \otimes \sigma_z^{(2)}$	$\sigma_y^{(1)} \otimes \sigma_y^{(2)}$	$\langle R_3 \rangle = 0.90(1)$
$\langle C_1 \rangle = 0.90(1)$	$\langle C_2 \rangle = 0.89(1)$	$\langle C_3 \rangle = -0.91(1)$	

$\langle \chi_{KS} \rangle$ = 5.46(4) > 4

G. Kirchmair et al., Nature **460**, 494 (2009)

State independence

States are prepared with 97(2) % fidelity (quantum state tomography)



G. Kirchmair et al.,
Nature **460**, 494 (2009)

Simulating the Dirac equation

… with a single trapped ion

PRL **98,** 253005 (2007) PHYSICAL REVIEW LETTERS week ending 22 JUNE 2007

Dirac Equation and Quantum Relativistic Effects in a Single Trapped Ion

L. Lamata,[1] J. León,[1] T. Schätz,[2] and E. Solano[3,4]

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[3]*Physics Department, ASC, and CeNS, Ludwig-Maximilians-Universität, Theresienstrasse 37, 80333 Munich, Germany*
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(Received 27 March 2007; published 22 June 2007)

We present a method of simulating the Dirac equation in $3 + 1$ dimensions for a free spin-$1/2$ particle in a single trapped ion. The Dirac bispinor is represented by four ionic internal states, and position and momentum of the Dirac particle are associated with the respective ionic variables. We show also how to simulate the simplified $1 + 1$ case, requiring the manipulation of only two internal levels and one motional degree of freedom. Moreover, we study relevant quantum-relativistic effects, like the *Zitterbewegung* and Klein's paradox, the transition from massless to massive fermions, and the relativistic and nonrelativistic limits, via the tuning of controllable experimental parameters.

Simulating the Dirac equation

L. Lamata, J. León, T. Schätz, E. Solano, PRL **98**, 253005 (2007)

The Dirac equation

$$i\hbar\frac{\partial\psi}{\partial t} = \mathcal{H}_D\psi = (c\,\vec{\alpha}\cdot\vec{p} + \beta mc^2)\psi$$

can be cast in a 1+1 dimensional form (1 spatial + 1 spinor degree of freedom)

$$\mathcal{H}_D^{(1)} = 2\eta\Delta\tilde{\Omega}\sigma_x p_x + \hbar\Omega\sigma_z$$

resonant bichromatic excitation   Stark shift

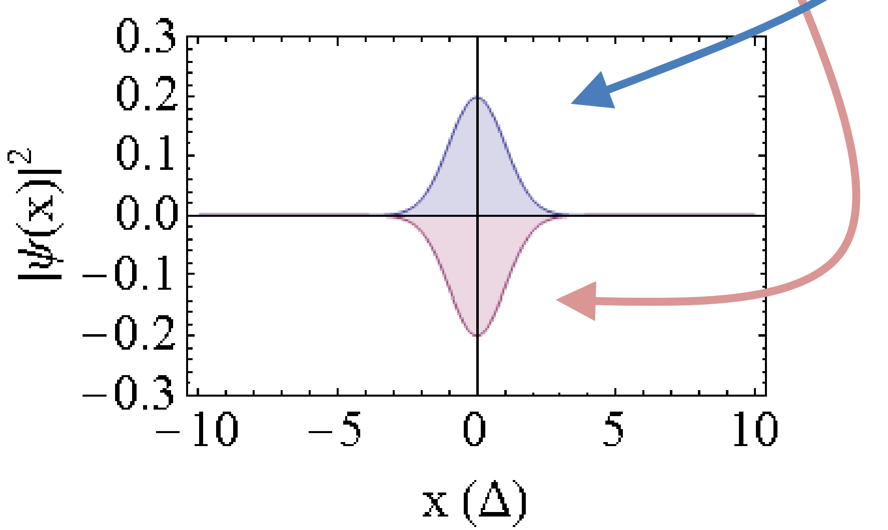
with the replacements $c := 2\eta\Delta\tilde{\Omega}, \quad mc^2 := \hbar\Omega$

Dynamics of a relativistic wave packet

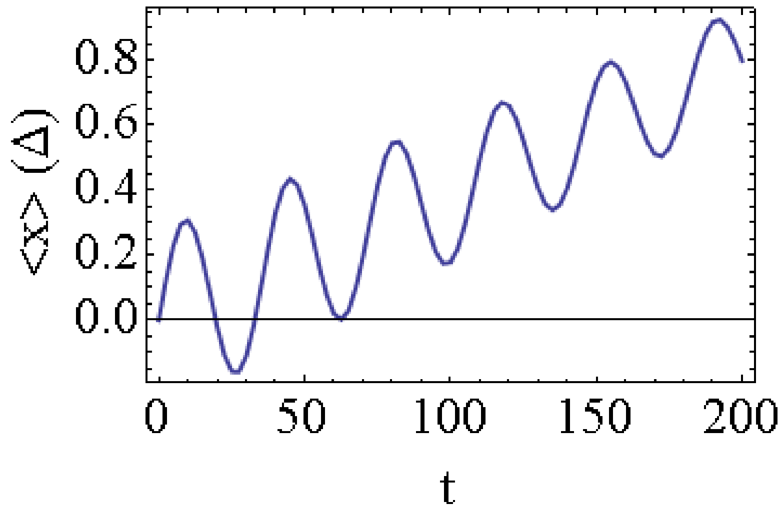
$$i\hbar\frac{\partial\psi}{\partial t} = \mathcal{H}_D\psi = (c\,\vec{\alpha}\cdot\vec{p} + \beta mc^2)\psi$$

For the 1d Dirac equation we trace over the spin degrees of freedom and the remaining spinor represents the matter and anti-matter components

$$\psi(x;t=0) = \frac{e^{-\frac{x^2}{4\Delta^2}}}{(\sqrt{2\pi}2\Delta)^{\frac{1}{2}}}\begin{pmatrix}1\\1\end{pmatrix}$$



„Zitterbewegung"

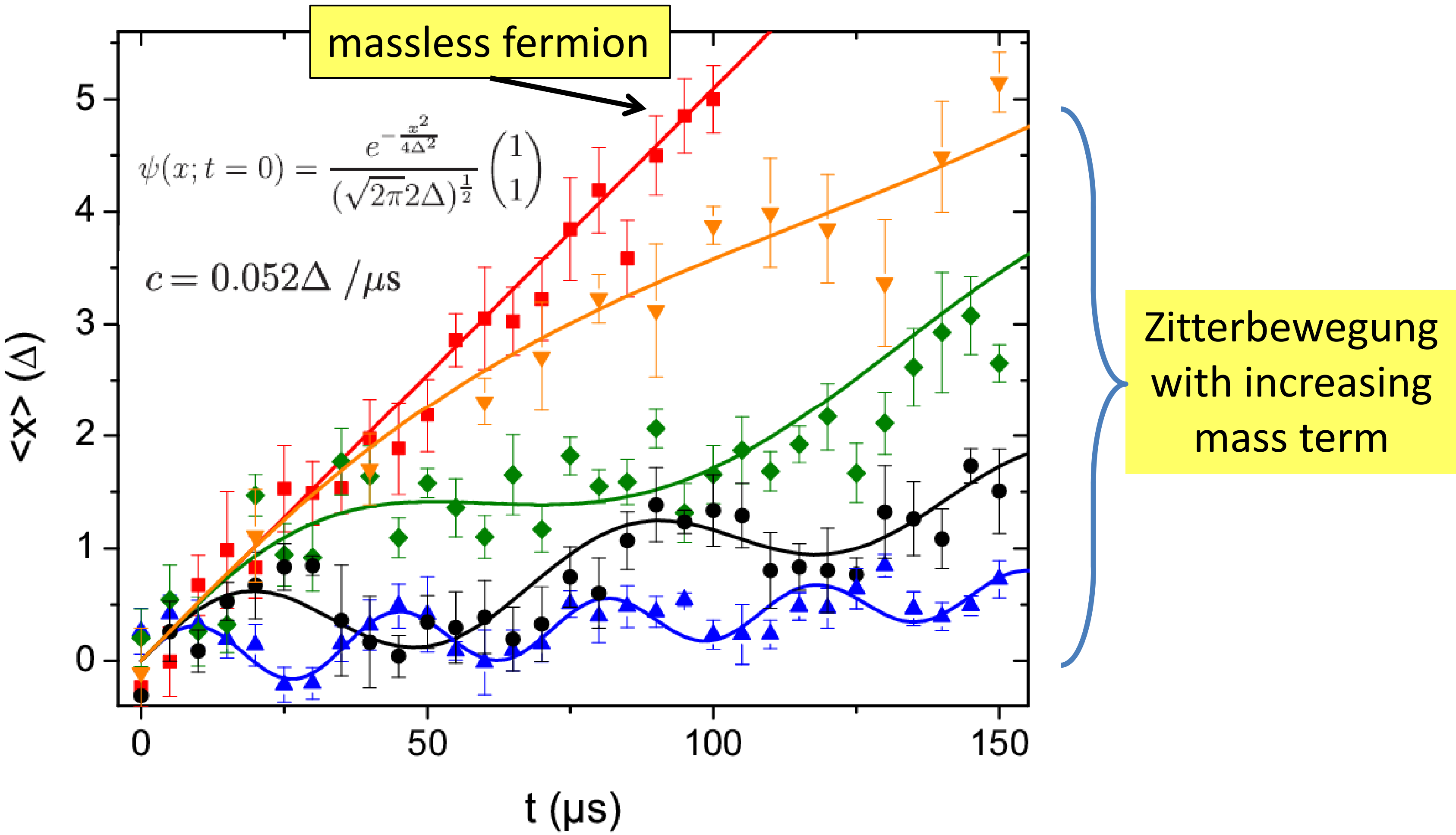


due to interference between positive and negative energy parts of spinor

Experiment: Simulating the Dirac equation

R. Gerritsma, G. Kirchmair, F. Zähringer, E. Solano, R. Blatt, C. Roos, Nature **463**, 58 (2010)

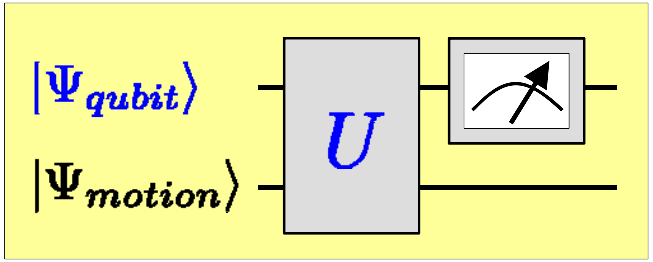
Apply the Hamiltonian and measure the expectation value of the position $\langle \hat{x} \rangle$



Measurements in position space

How to measure $|\Psi_{motion}(x)|^2$?

Entangle motion with qubit and measure σ_z !



For $U = e^{-i\theta\sigma_x(a^\dagger + a)}$

$$\sigma_z \longrightarrow A = U^\dagger \sigma_z U = \cos(2\theta\underbrace{(a^\dagger + a)}_{\propto \hat{x}})\,\sigma_z + \sin(2\theta(a^\dagger + a))\,\sigma_y$$

For $|\Psi_{qubit}\rangle = |+\rangle_z$: $\langle A\rangle = \langle\cos(k\hat{x})\rangle$

For $|\Psi_{qubit}\rangle = |+\rangle_y$: $\langle A\rangle = \langle\sin(k\hat{x})\rangle$

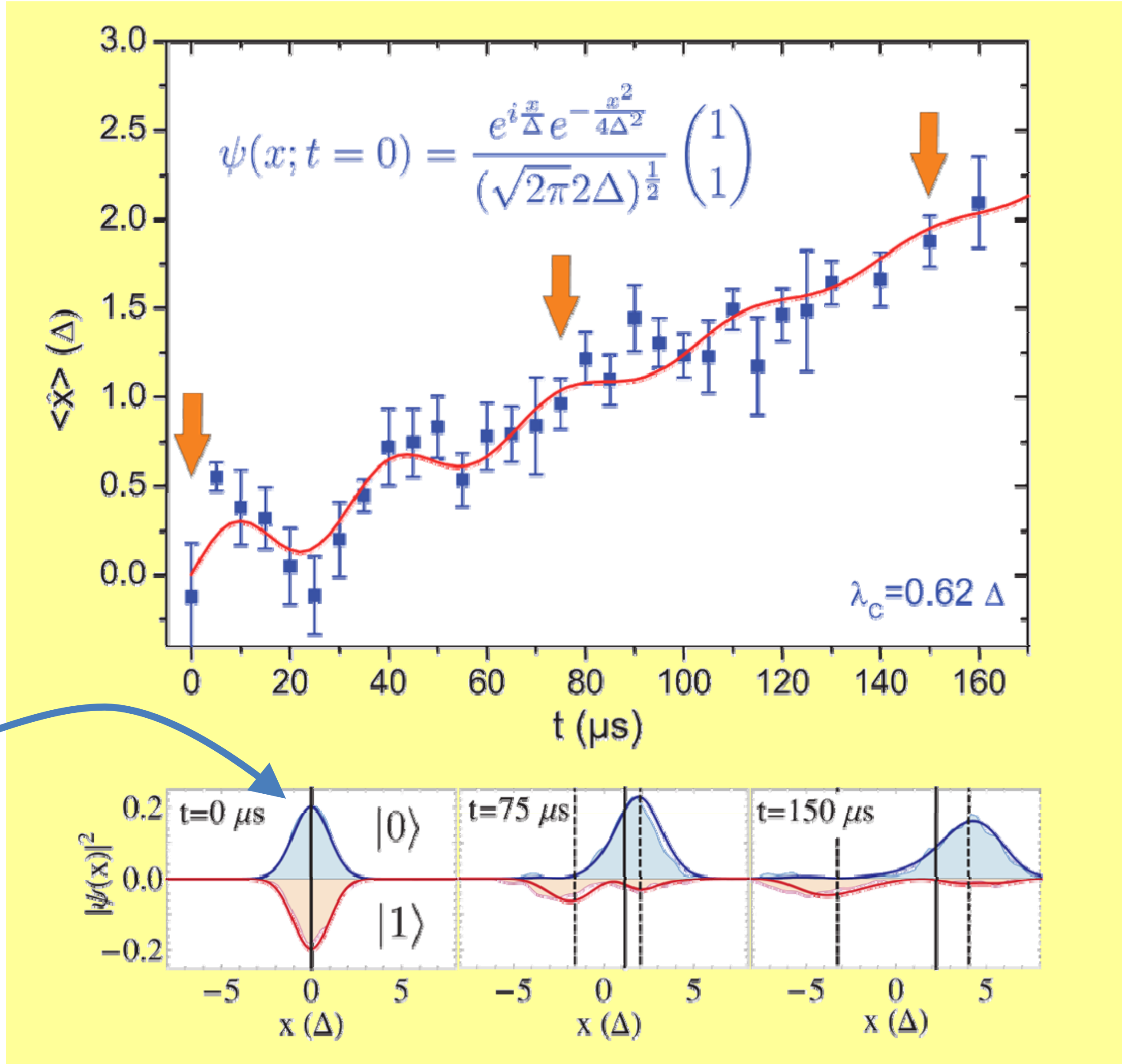
Fourier transformation of $\langle\cos(k\hat{x})\rangle + i\langle\sin(k\hat{x})\rangle \longrightarrow |\Psi_{motion}(x)|^2$

Experiment: Simulating the Dirac equation

Zitterbewegung

- due to interference between positive and negative energy parts of spinor
- Present when components have overlap in position and momentum space

initial state prepared with non-zero average momentum



R. Gerritsma, G. Kirchmair, F. Zähringer, E. Solano, R. Blatt, C. Roos, Nature **463**, 68 (2010)

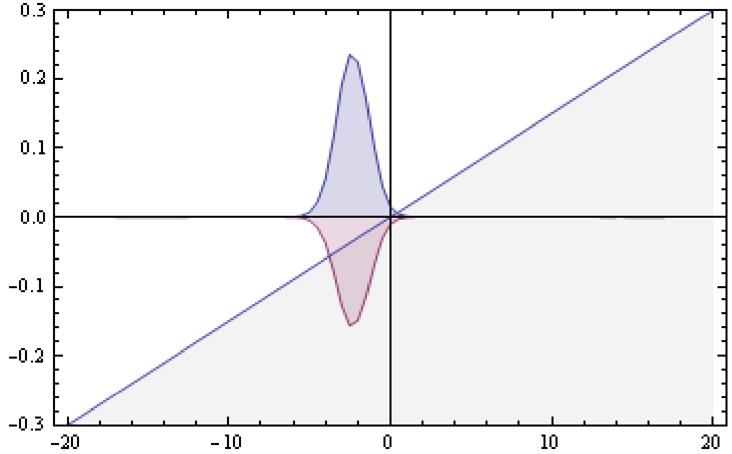
Dirac particle in external potentials

J. Casanova, J. Garcia-Ripoll, R. Gerritsma, C. Roos, E. Solano, arXiv: 1004.5400

$$H_D = c\hat{p}\sigma_x + mc^2\sigma_z + V(\hat{x})$$

For $V(\hat{x}) = a\hat{x}$:

Klein tunneling



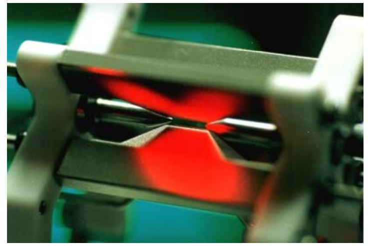
Realization of a linear potential with trapped ions: $V(\hat{x}) = a\hat{x}\sigma_x^{(2)}$

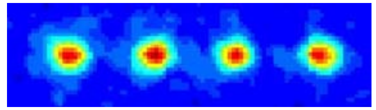
A second ion driven by bichromatic light field and prepared in an eigenstate of σ_x

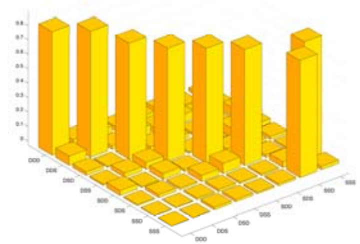
$$H_D = c\hat{p}\sigma_x^{(1)} + mc^2\sigma_z^{(1)} + a\hat{x}\sigma_x^{(2)}$$

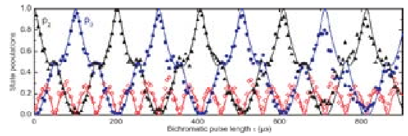
Quantum Information Science with Trapped Ions

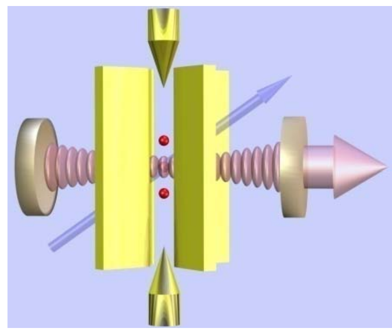
- Ca^{+} for quantum information processing
- Quantum process tomography, gate operations
- Gate operations with trapped ions
- Quantum computation with logical qubits
- Testing quantum mechanics with trapped ions
- Simulating the Dirac equation with a trapped ion
- Quantum (random) walk







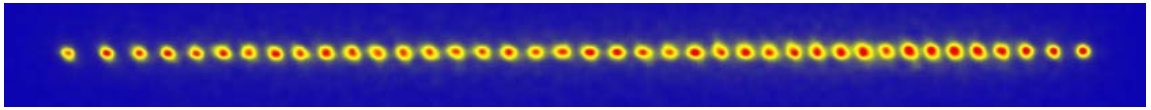




Future:

- further optimization of logic operations
- error correction protocols with three and five qubits
- implementation with $^{43}Ca^{+}$, logical qubits + scalability
- miniaturize traps, interface quantum information

Future goals and developments



- more qubits (~20 – 50)
- better fidelities
- faster gate operations
- faster detection

} cryogenic trap, micro-structured traps

- development of 2-d trap arrays, onboard addressing, electronics etc.
- entangling of large(r) systems: characterization ?
- implementation of error correction, keep „**qubit alive**“
- applications
  - small scale QIP (e.g. repeaters)
  - quantum metrology, enhanced S/N, tailored atoms and states
  - quantum simulations
  - quantum computation

AG Quantenoptik
und Spektroskopie

The international Team 2009

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