Entanglement Across a Separation in Spin Chains: Statics & Dynamics

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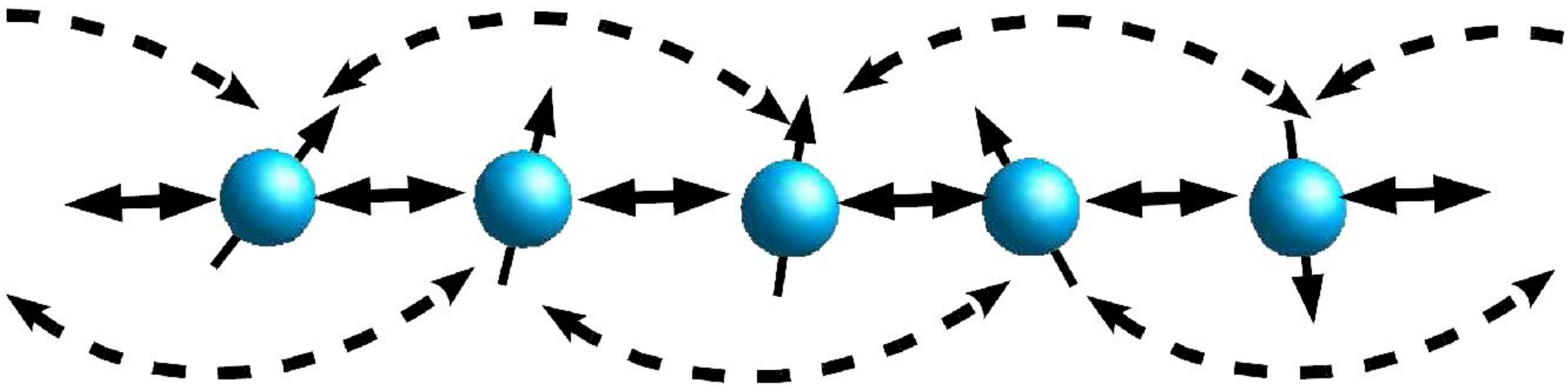
Spin chains:



Interactions:

$$\mathcal{H}_{i,j}^{\mathrm{Heis}} = \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y + \sigma_i^z \sigma_j^z = \sigma_i . \sigma_j$$

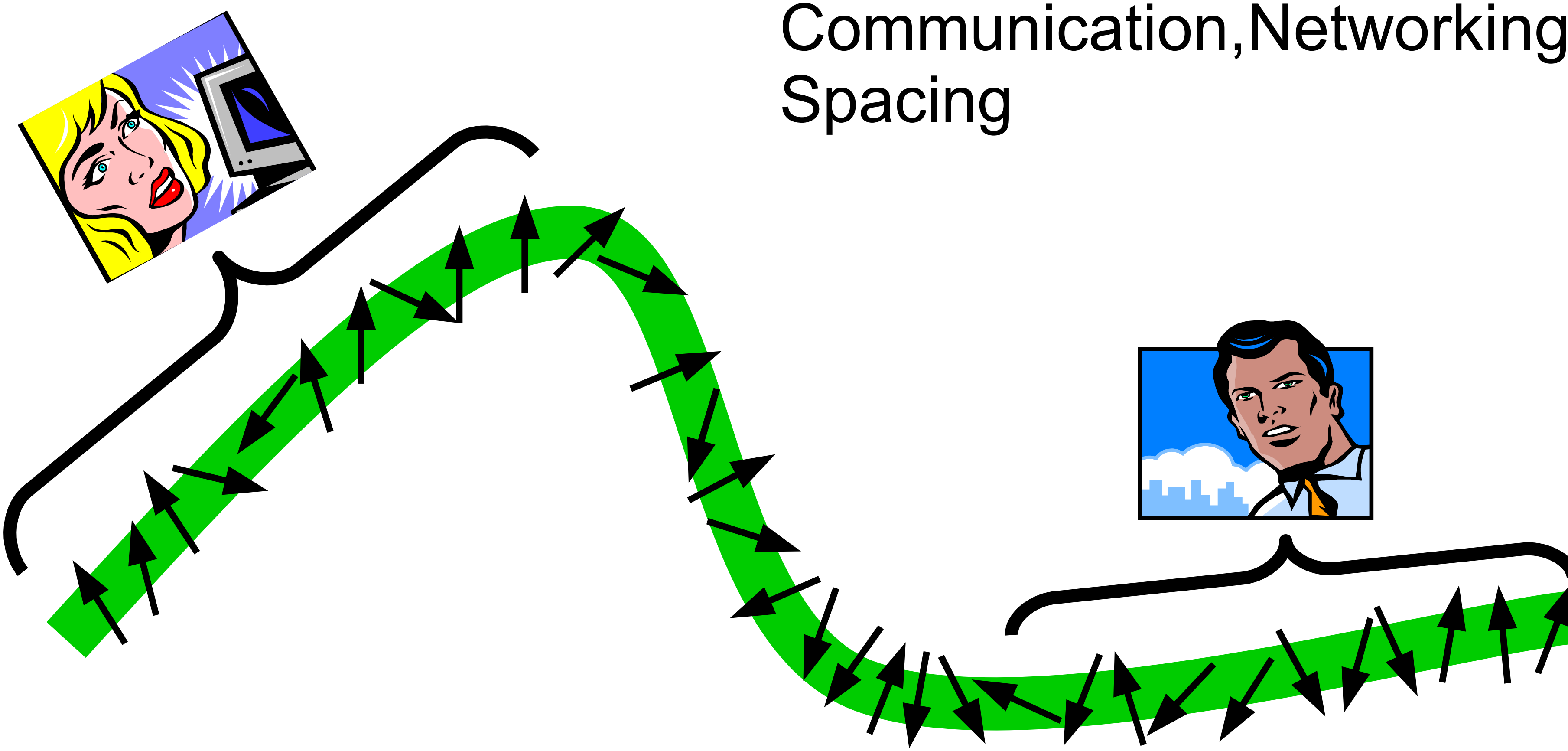
$$\mathcal{H}_{i,j}^{\mathrm{XY}} = \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y$$

$$\mathcal{H}_{i,j}^{\mathrm{Ising}} = \sigma_i^z \sigma_j^z$$

1D Magnets, JJ arrays,
Opt. Latt., ion traps,
electrons in Q.Wires
Arrays of qubits without
control of interactions!

Entanglement "between" separated systems is *in principle* a resource for QIP:

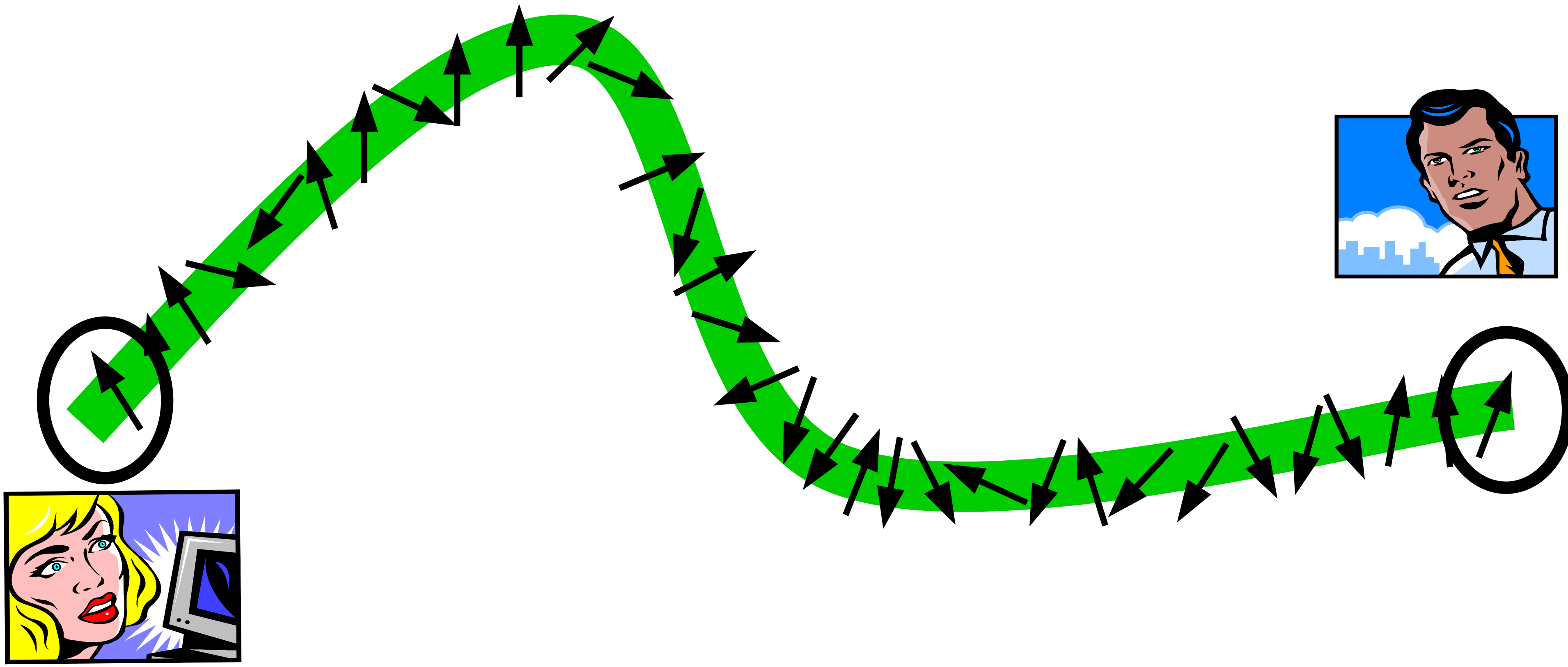
Communication,Networking, Spacing



But processing may be required to bring it to an useful form

Entanglement "between" separated systems is *in principle* a resource for QIP

Most useful form (least processing):



Two much studied forms of bi-partite entanglement:

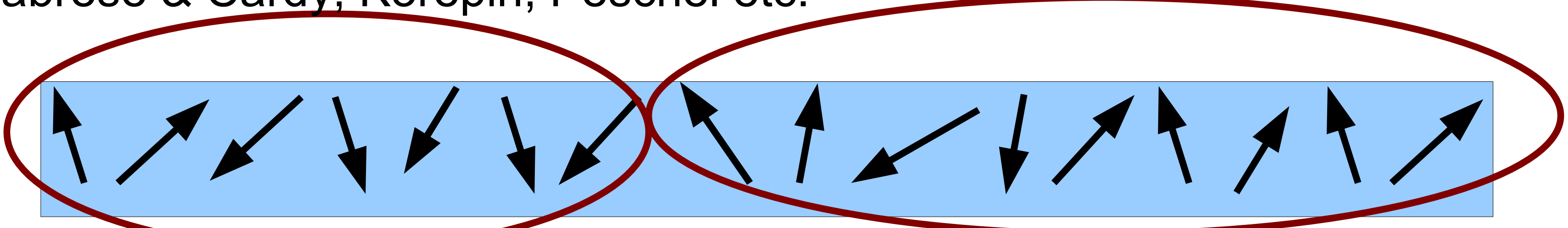
Between individual spins: e.g. Arnesen, Bose, Vedral, 2000, Osborne & Nielsen 2002, Osterloh et al 2002 etc.

Typically extremely short ranged

Mostly uncharted territory

(only exception we could find:
Audenaert, Eisert, Plenio, Werner, 2002)

Between complementary parts: e.g. Vidal et. al. 2003, Calabrese & Cardy; Korepin; Peschel etc.



Divergent at QPTs, Universal

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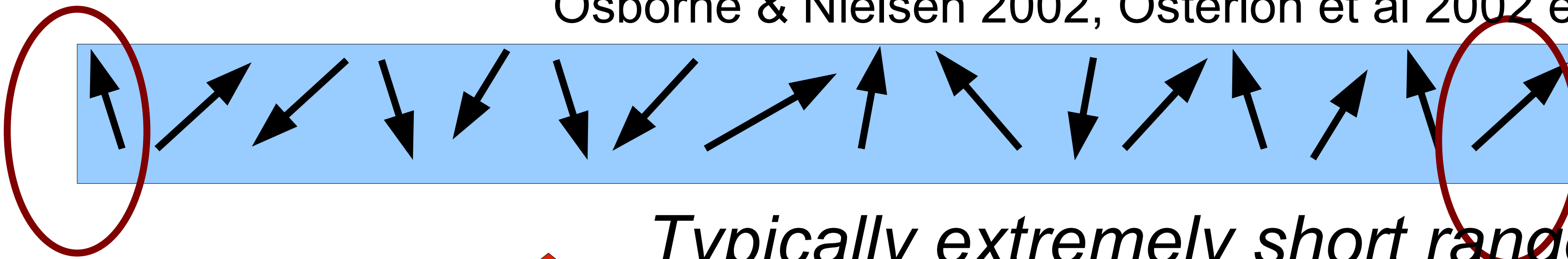
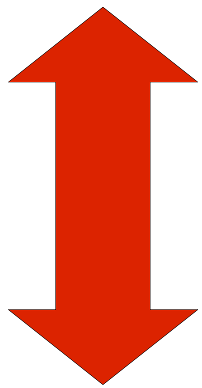


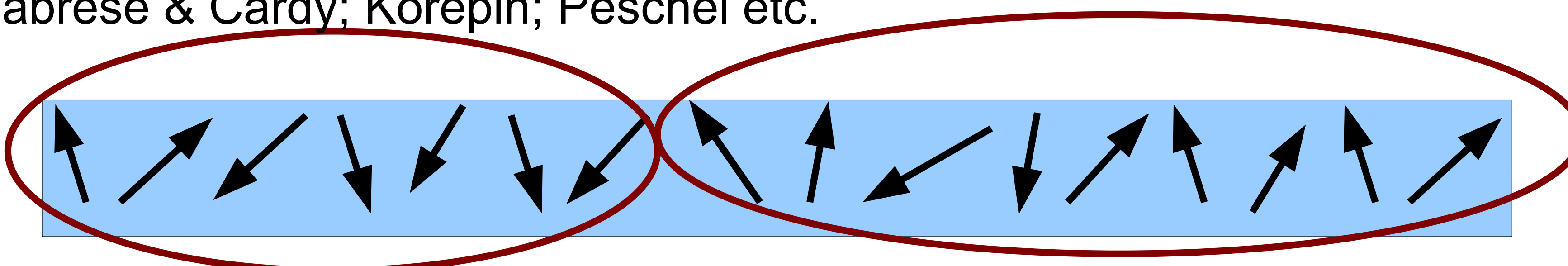
Typically extremely short ranged



Noncomplemen-tary Blocks

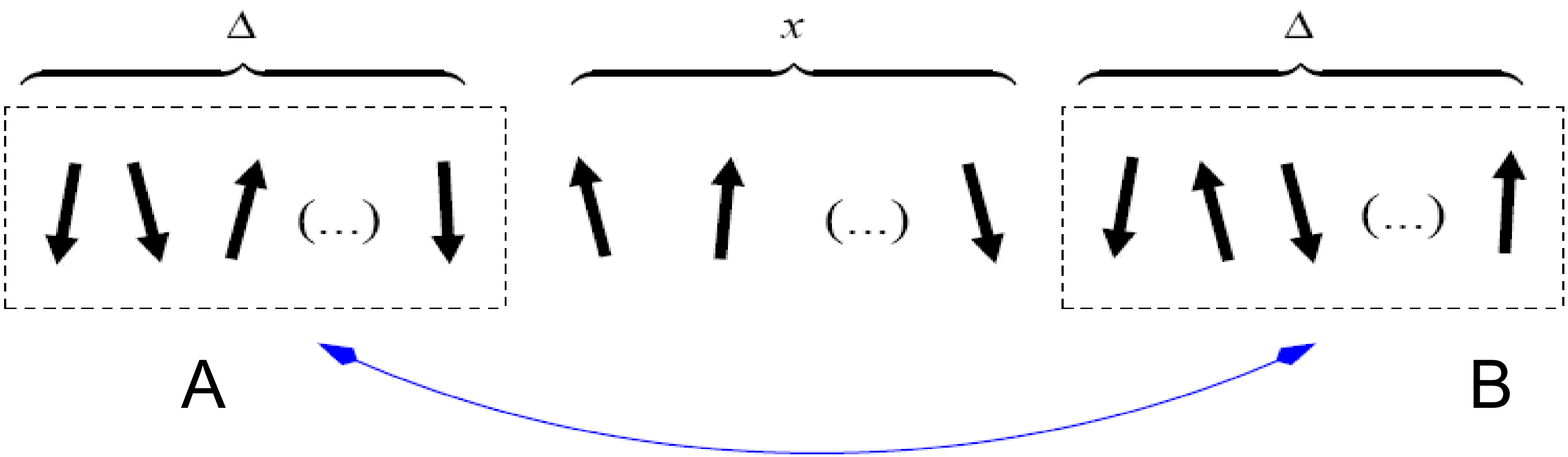
Mostly uncharted territory

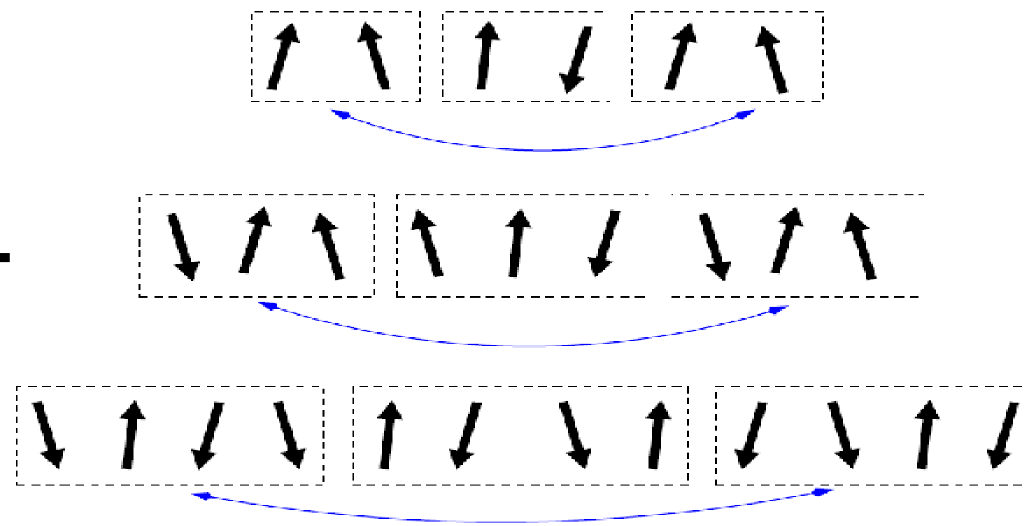
Between complementary parts: e.g. Vidal et. al. 2003, Calabrese & Cardy; Korepin; Peschel etc.



Divergent at QPTs, Universal

Entanglement **between** separated blocks

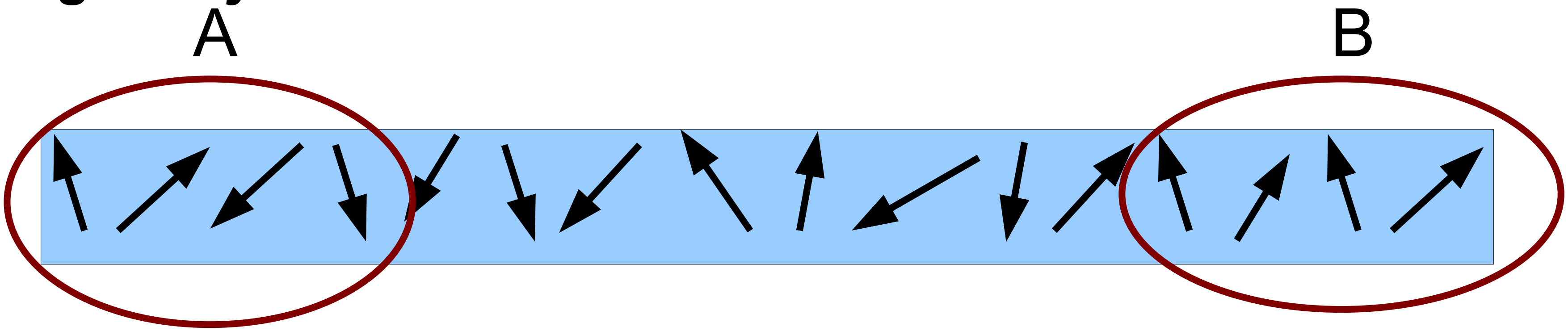




Scale invariant at QPT
(e.g. in contrast with
von Neumann entropy)

H. Wichterich, J. Molina-Vilaplana & S. Bose,
Phys. Rev. A 80, 010304(R) (2009).

Entanglement "**between**" two arbitrary dimensional systems in a mixed state – the only computable measure we know is the ***Negativity***: (Eisert 2001, Vidal and Werner 2002)



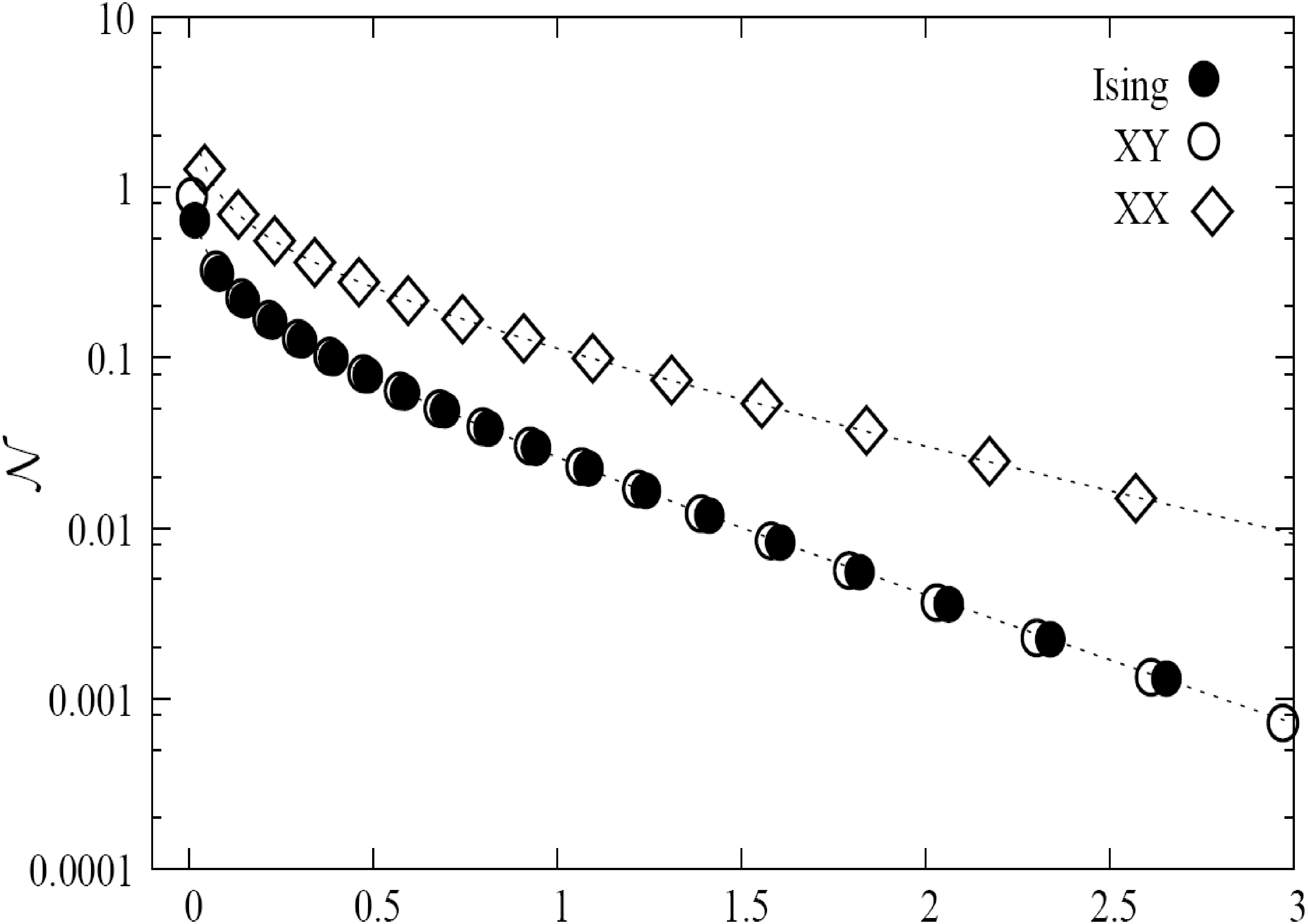
One first has to find the partial transpose ρ^{T_A} defined as

$$\langle i_A, j_B|\rho^{T_A}|k_A, l_B\rangle \equiv \langle k_A, j_B|\rho|i_A, l_B\rangle$$

then Negativity $\mathcal{N}(\rho) = \Sigma_i |a_i| - 1,$ where a_i

are the eigenvalues of ρ^{T_A}

Usually much more difficult to compute for a given many-body system than the concurrence or entropy



$\mu = \frac{x}{\Delta}$ = Separation / Block size

Scale Invariance: Entanglement is function of

$$\mu = \frac{x}{\Delta} \quad \text{only}$$

The fitted ansatz:

Polynomial decay of correlations?

Monogamy of entanglement?

$$\mathcal{N}(\rho) \sim \mu^{-h} e^{-\alpha\mu}$$

Interpolation formula

$h = 0.47, \alpha = 0.99$ XX

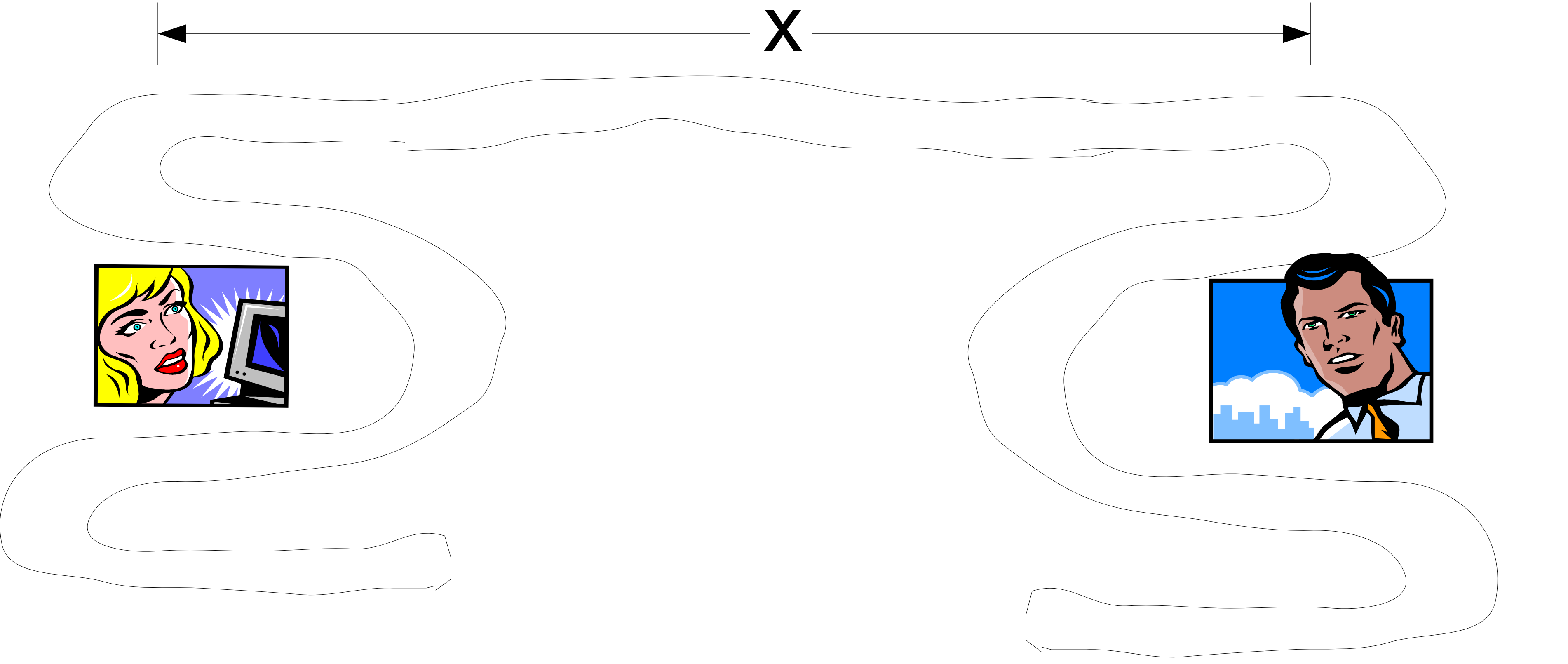
$h = 0.38, \alpha = 1.68$ Ising

Open qs: How to relate to known critical exponents?

H. Wichterich, J. Molina-Vilaplana, S. Bose, Phys. Rev. A 80, 010304(R) (2009).

See Also: **Marcovitch, Retzker, Plenio, Reznik et al PRA 2009**

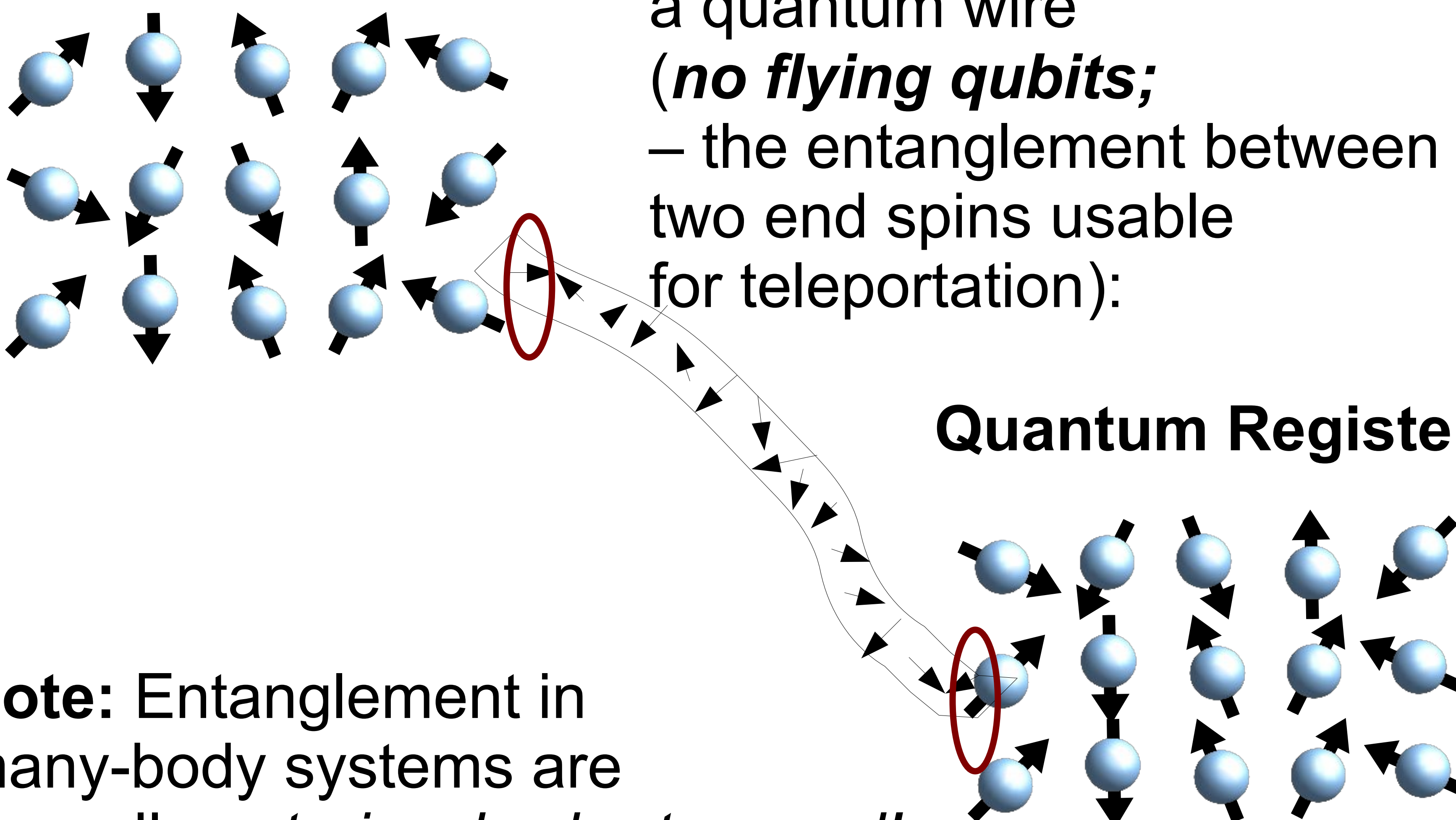
To share a good amount of entanglement across a given separation x, Alice and Bob should access regions as large as x ...



It would be more readily useful (*quantum engineering*) if two distant individual spins could be highly entangled

Quantum Register 1

Using a many-body system as a quantum wire (***no flying qubits;*** – the entanglement between two end spins usable for teleportation):



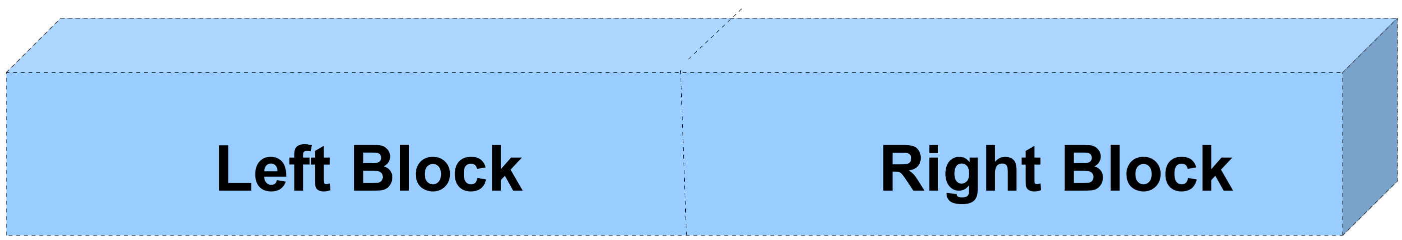
Quantum Register 2

Note: Entanglement in many-body systems are normally *notoriously short ranged!*

Can non-equilibrium dynamics induced by a quench help?

Quench is a change of the Hamiltonian. Can that be used to generate substantial entanglement between the two individual spins at the ends?

Studied already in the context of block entanglement:



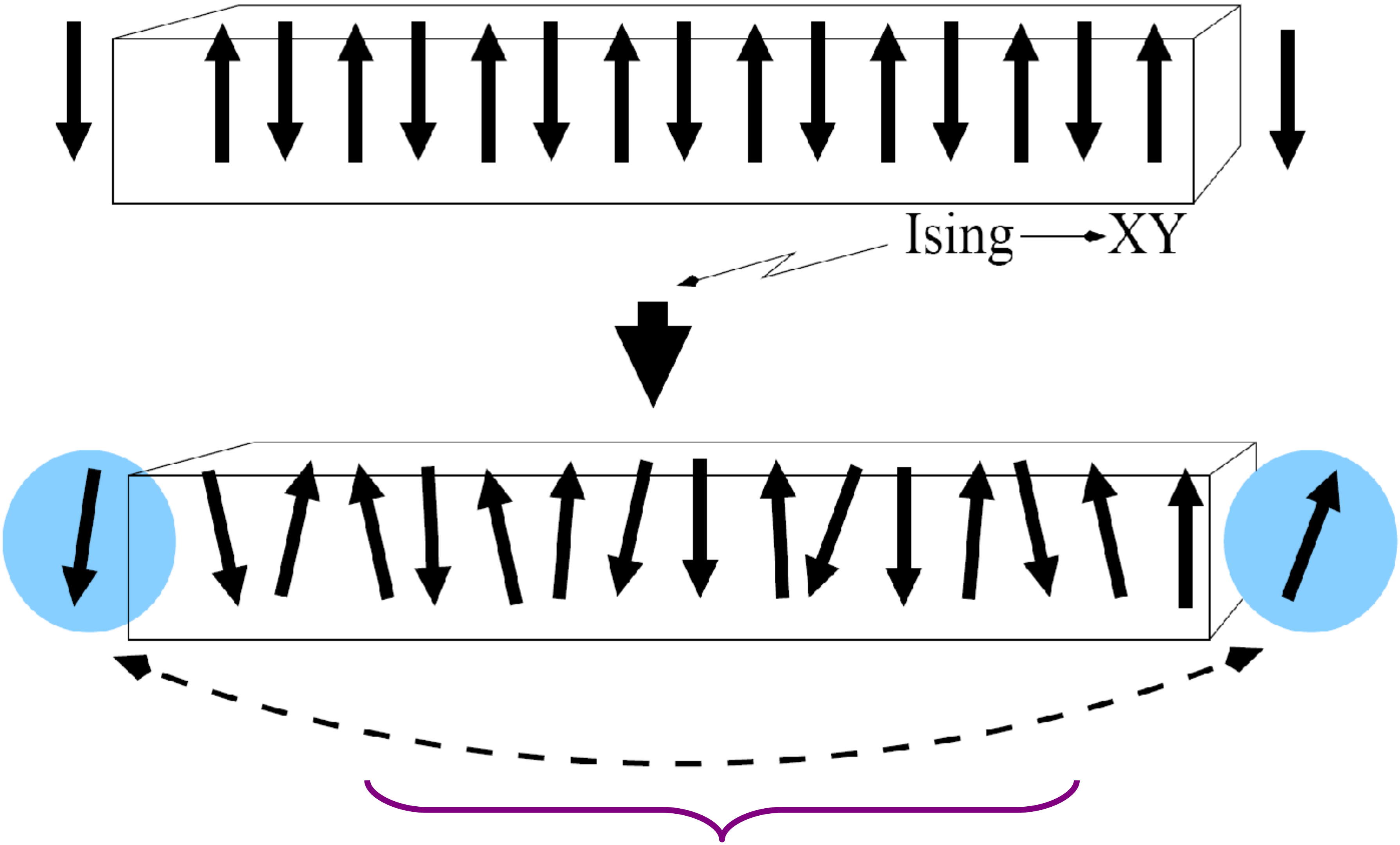
P. Calabrese and J. Cardy, J. Stat. Mech. P04010 (2005).

G. De Chiara, S. Montangero, P. Calabrese, and R. Fazio, J. Stat. Mech. P03001 (2006).

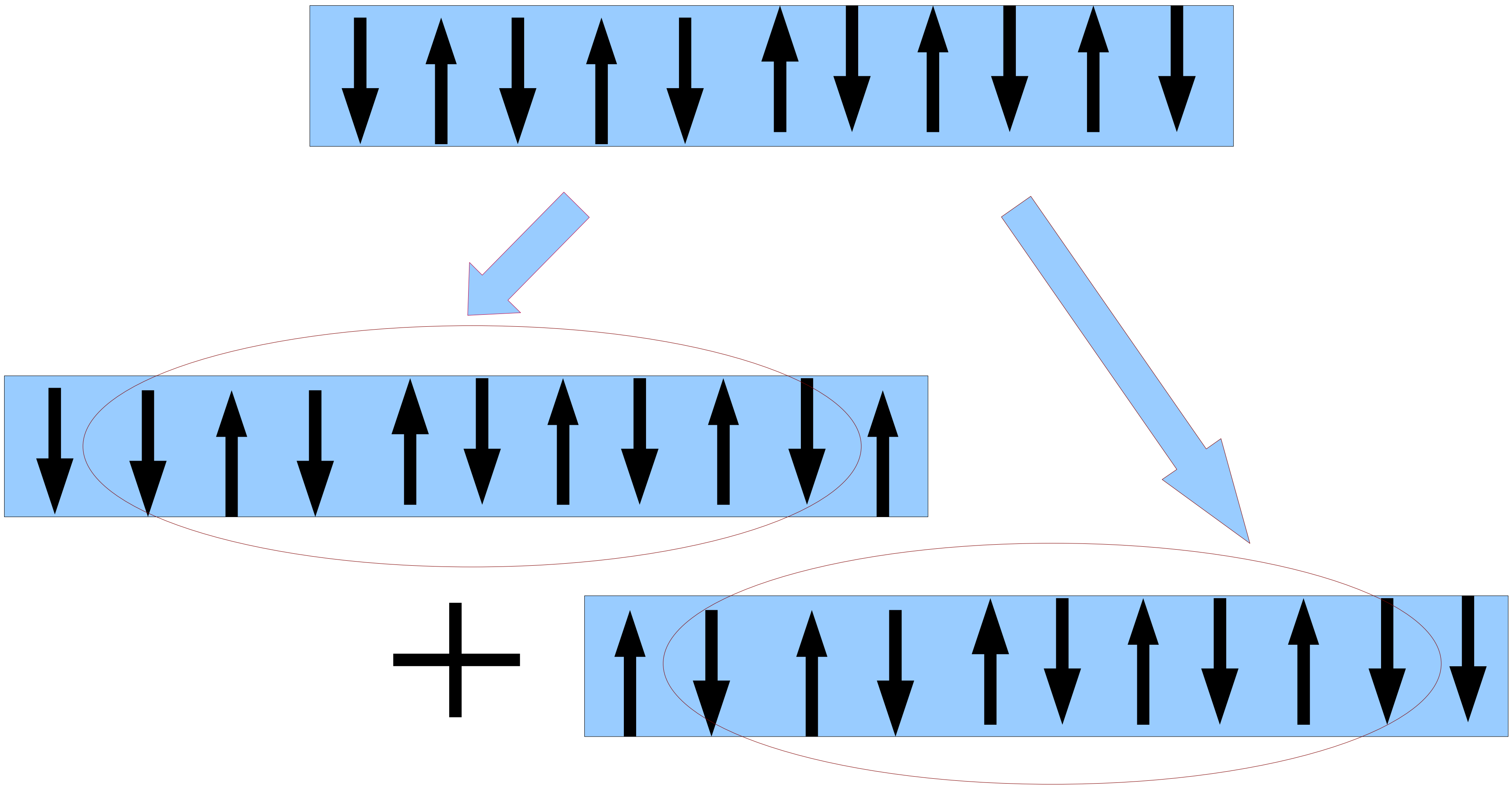
Does not guarantee that individual spins at the ends will be significantly entangled.

Quantum Communication Resource from Quench

Hannu Wichterich & Sougato Bose, Phys. Rev. A 79, 060302(R) (2009)



Required configurations for entanglement:



$$H = \sum_{k=1}^{N-1} \frac{J}{2} \left(\sigma_k^{\mathrm{x}} \sigma_{k+1}^{\mathrm{x}} + \sigma_k^{\mathrm{y}} \sigma_{k+1}^{\mathrm{y}} + \Delta\, \sigma_k^{\mathrm{z}} \sigma_{k+1}^{\mathrm{z}} \right)$$

Quench: $\Delta_1 \rightarrow \infty$ to $\Delta_2 = 0$

Ising

XY

$$\rho_0 = \frac{1}{2} \Big(|\mathcal{N}_1\rangle\langle\mathcal{N}_1| + |\mathcal{N}_2\rangle\langle\mathcal{N}_2| \Big) \longrightarrow \quad ?$$

where

$$|\mathcal{N}_1\rangle \equiv |\downarrow_1, \uparrow_2, \downarrow_3, \cdots\rangle$$

$$|\mathcal{N}_2\rangle \equiv |\uparrow_1, \downarrow_2, \uparrow_3, \cdots\rangle$$

Starting from a mixed state

Mapped to free fermions: $c_k^\dagger \equiv \left(\prod_{l=1}^{k-1} -\sigma_l^z\right)\sigma_k^+$

Components of the density matrix at any time:

$$\langle\downarrow\uparrow|\rho_{1,N}|\uparrow\downarrow\rangle = \frac{1}{2}\left((-1)^{M+1}\langle c_N^\dagger c_1\rangle_1 + c.c.\right)$$

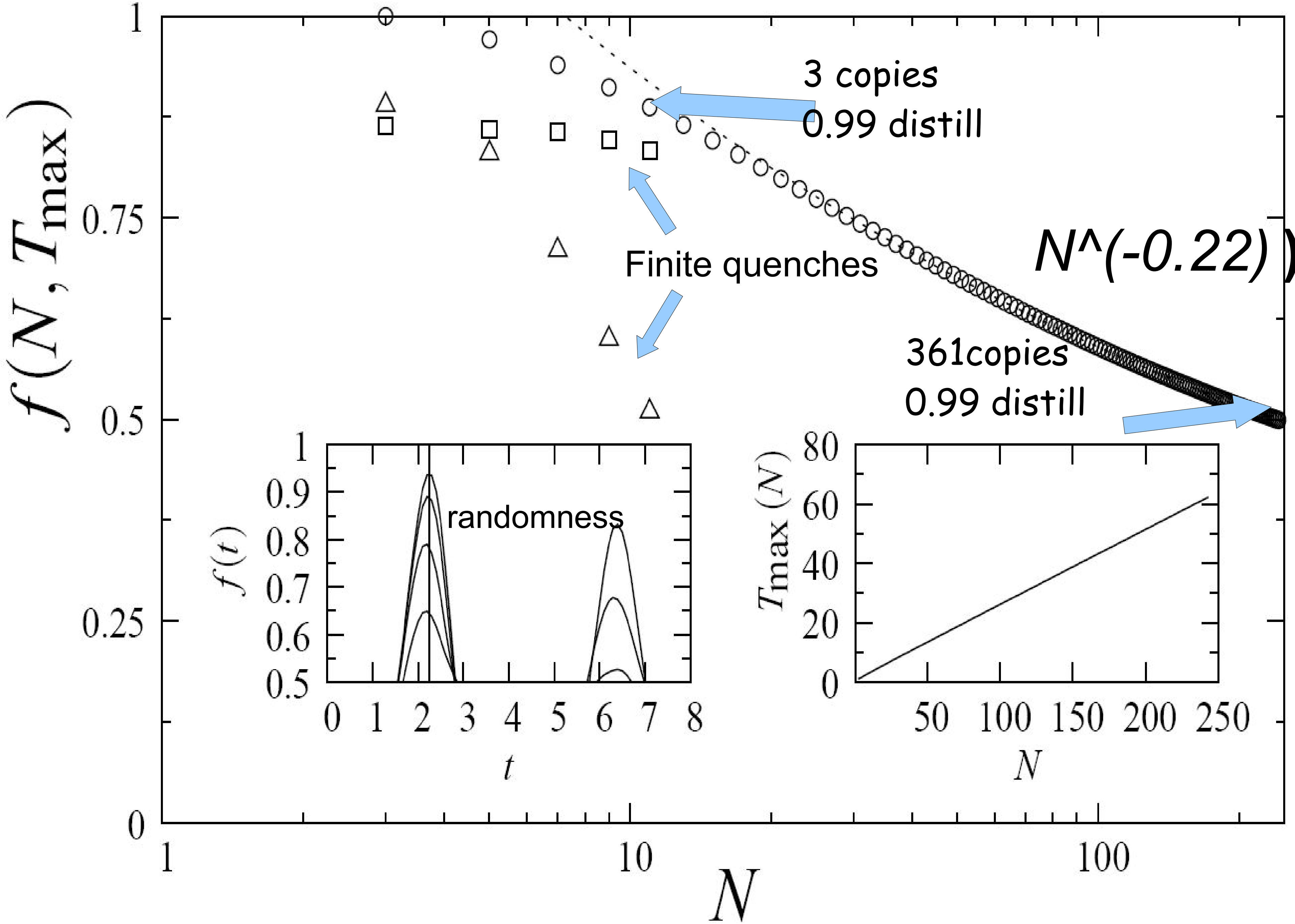
Free fermion amplitudes

$$\langle c_i^\dagger(t)c_j(t)\rangle = \sum_{k,l=1}^{N} f_{i,k}(t)f_{j,l}^*(t)\ \langle c_k^\dagger(0)c_l(0)\rangle$$

$$\langle c_k^\dagger(0)c_l(0)\rangle_1 = \delta_{k,l}\delta_{k,2m},\ (m = 1, 2, \ldots, M),$$

Odd chain only:

$$\rho_{1,N} \simeq f|\psi^+\rangle\langle\psi^+| + \frac{(1-f)}{2}(|\uparrow,\uparrow\rangle\langle\uparrow,\uparrow| + |\downarrow,\downarrow\rangle\langle\downarrow,\downarrow|)$$

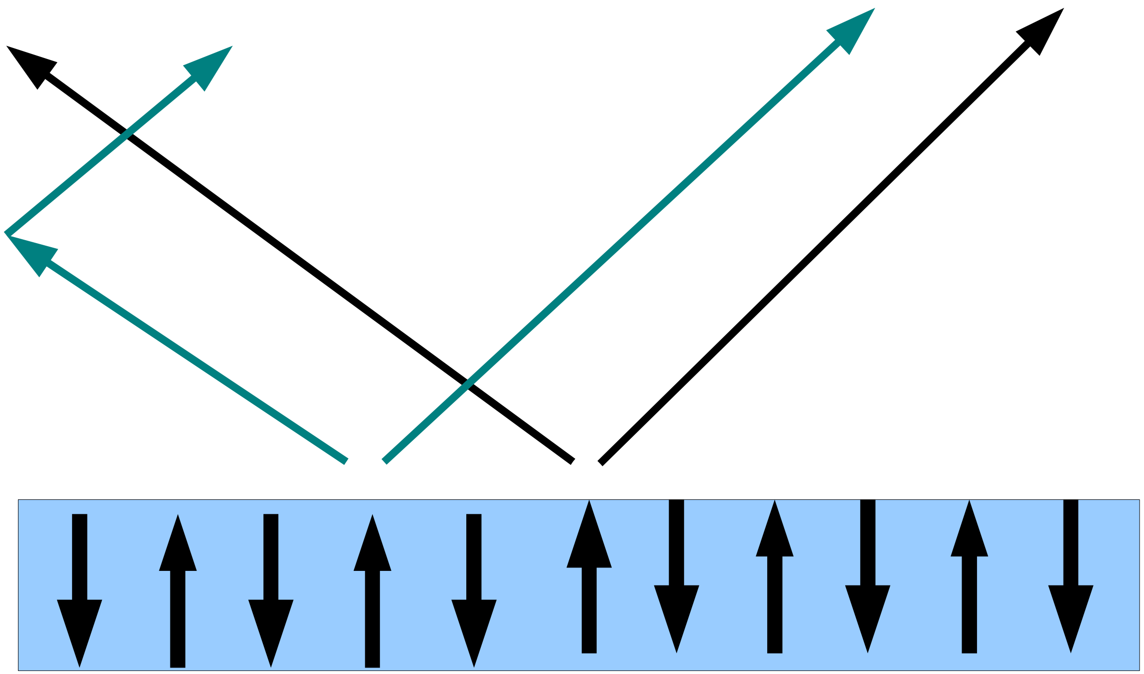


3 copies
0.99 distill
Finite quenches
N^(-0.22)
361copies
0.99 distill
randomness
f(N, Tmax)
N
f(t)
t
Tmax (N)
N

Explanation: A number of sources cooperatively give the same entangled state at ~ T/2

Left & right components become equidistant from respective ends

Each fermion evolves independently – does not see each other except for statistics --- whose effects are also cancelled here by the special initial state



Hannu Wichterich & Sougato Bose, Phys. Rev. A 79, 060302(R) (2009)

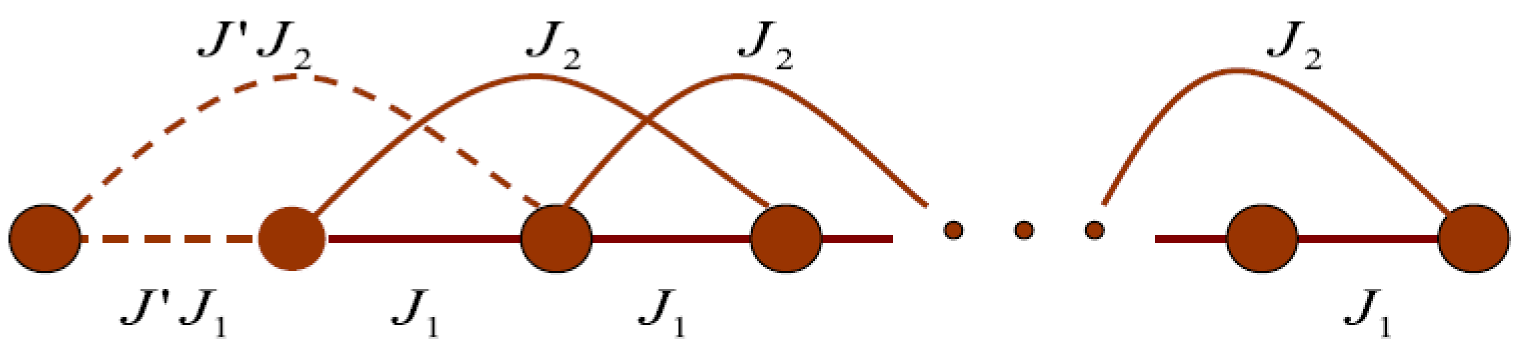
Related Work: **Galve, Zueco et. al. Phys. Rev. A 79, 032332 (2009).**

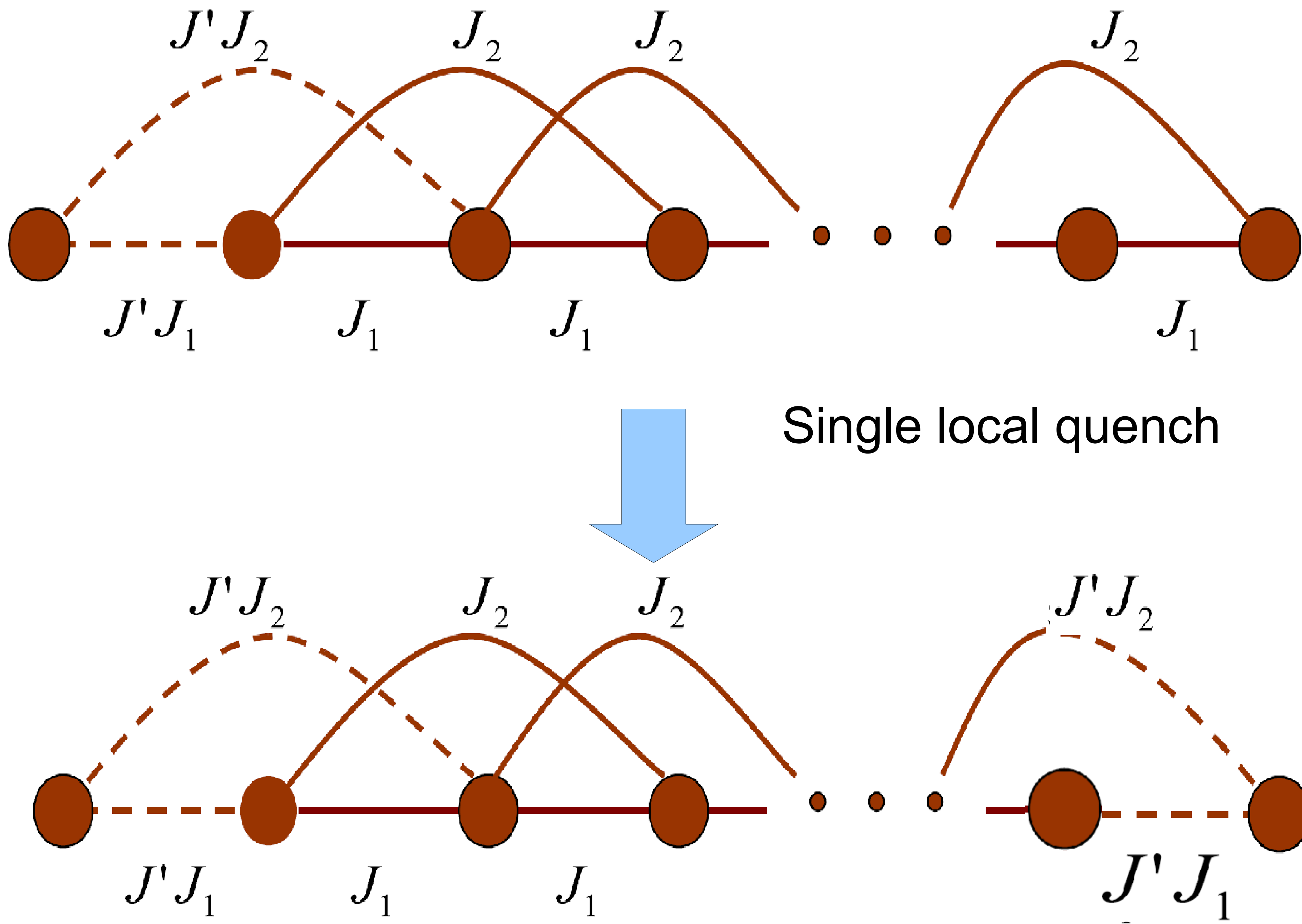
Can one get a distance *independent* entanglement of a high amount between the ends of a spin chain?

One possibility is to have weakly coupled end spins (Campos-Venuti *et al* 2006) --- robustness to temperature unclear

Another possibility with stronger, but nonuniform couplings: To exploit the Kondo cloud in a ***Kondo spin chain*** (*Affleck and co-workers*)

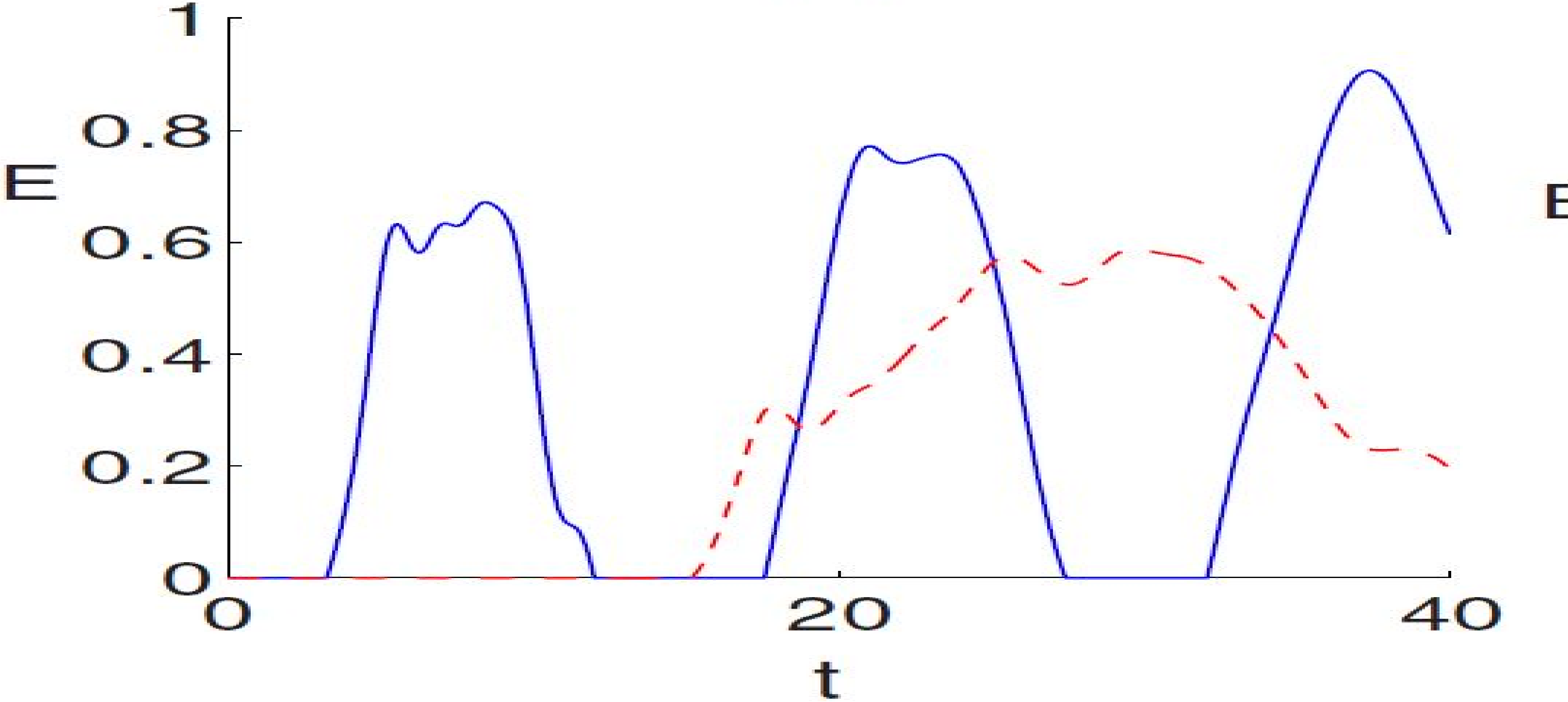
(a)




J' J_2
J_2
J_2
J_2
J' J_1
J_1
J_1
J_1
Single local quench
J' J_2
J_2
J_2
J' J_2
J' J_1
J_1
J_1
J' J_1

$$|\psi(t)\rangle = e^{-iH_F t}|GS_I\rangle$$

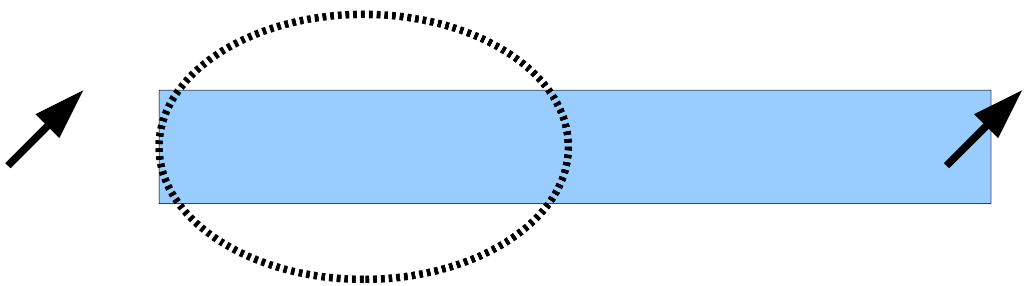
(a)



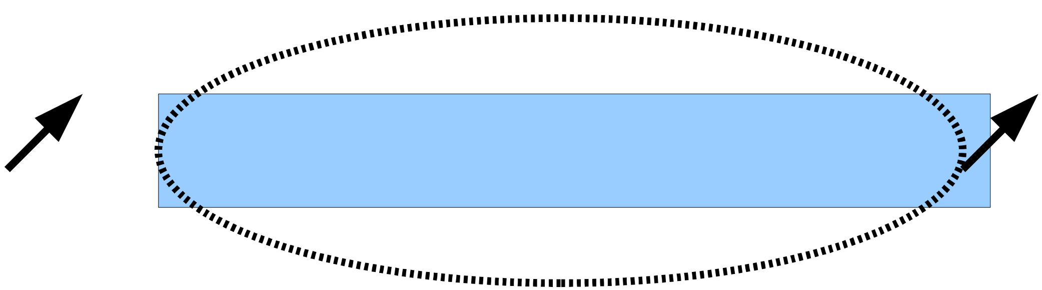
(c)

Interpretation in terms of Kondo Cloud

No entanglement
on quench



Finite entanglement
on quench

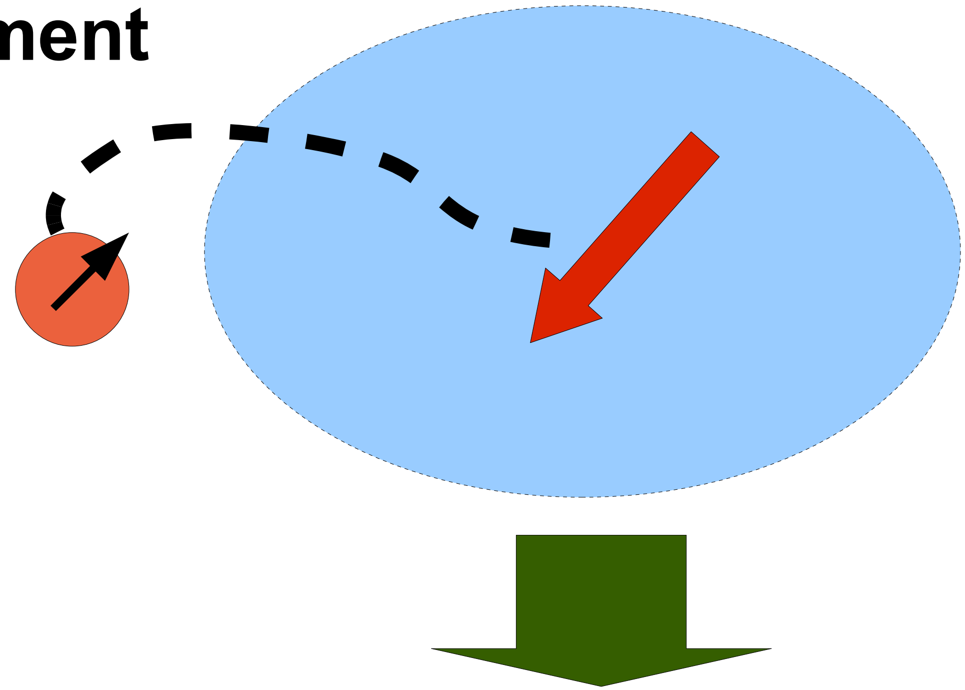


Optimal entanglement
on quench

Conversion of useless natural entanglement to an useful form

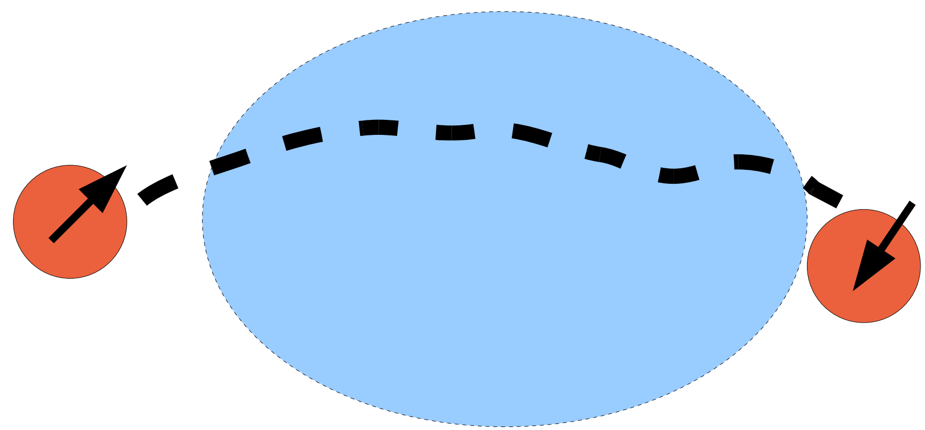
Kondo entanglement

Useless
Entanglement



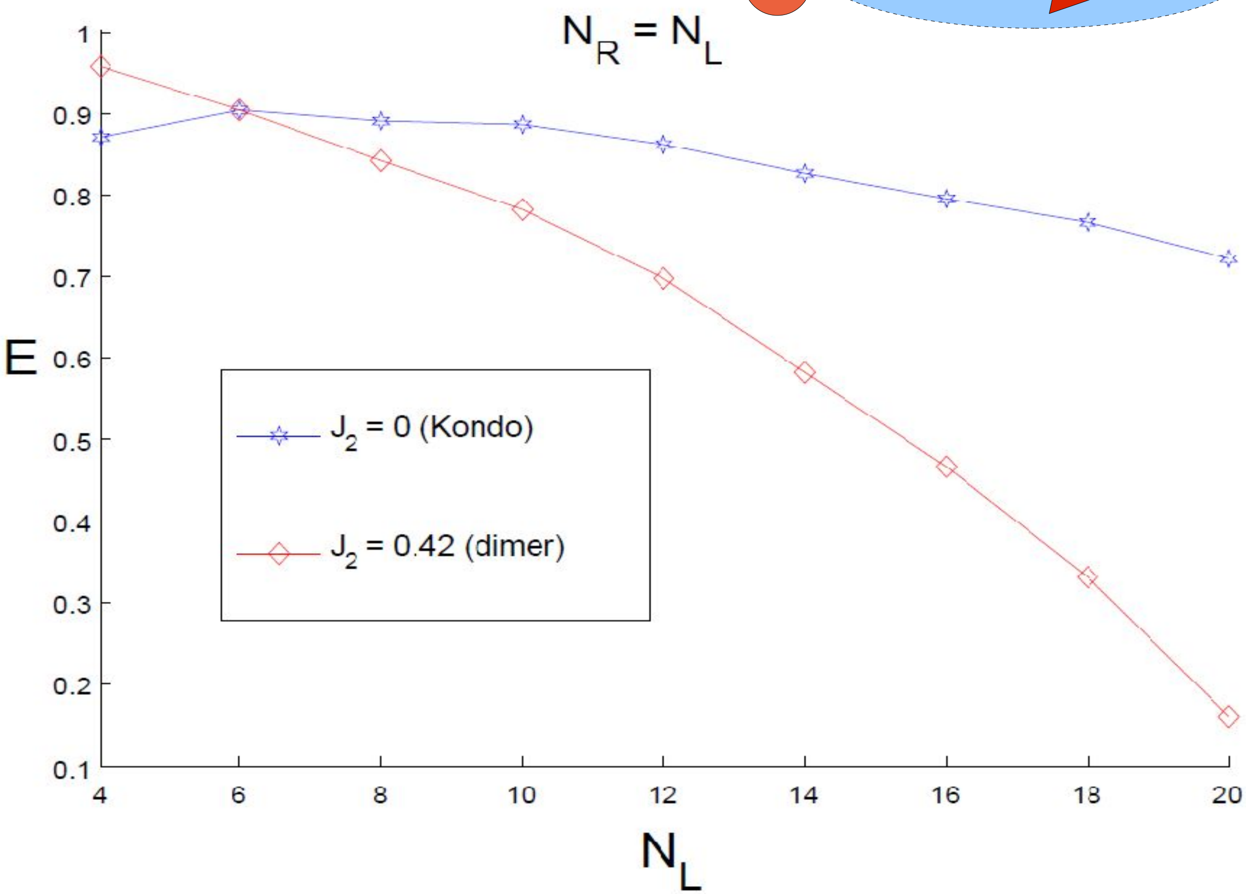
After Quench

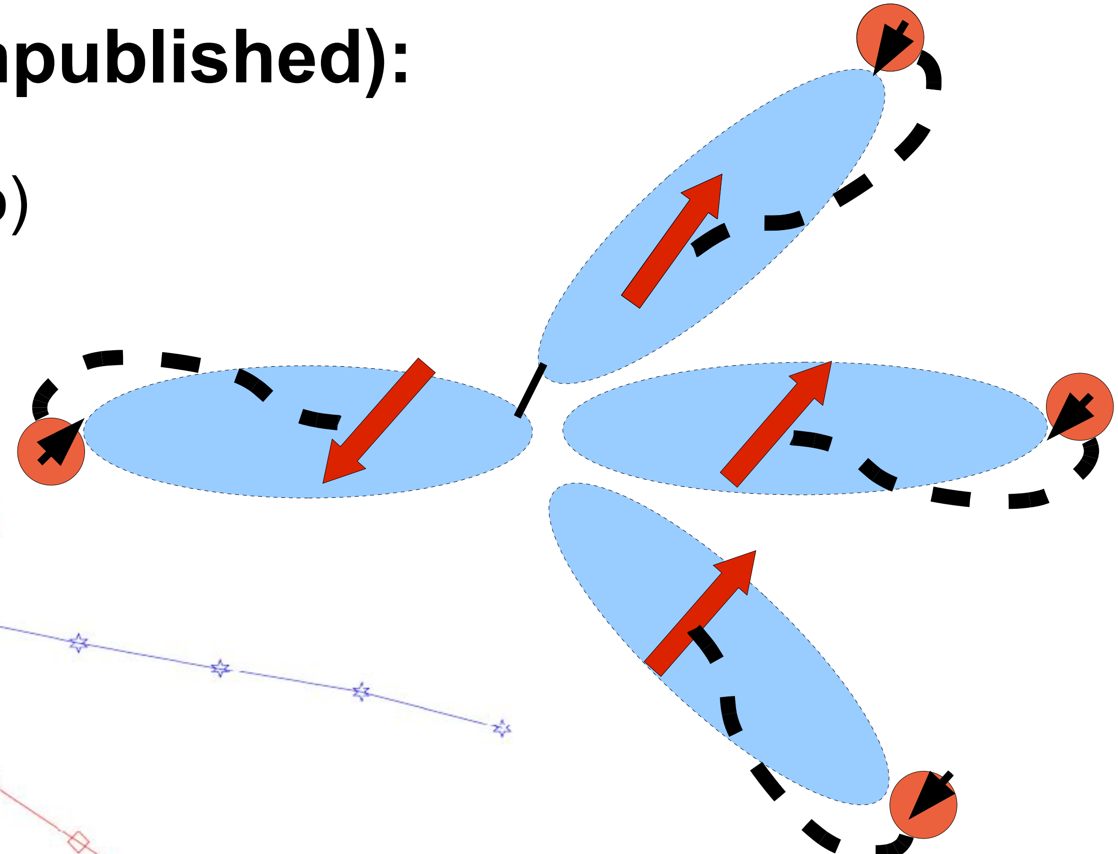
Useful Entanglement



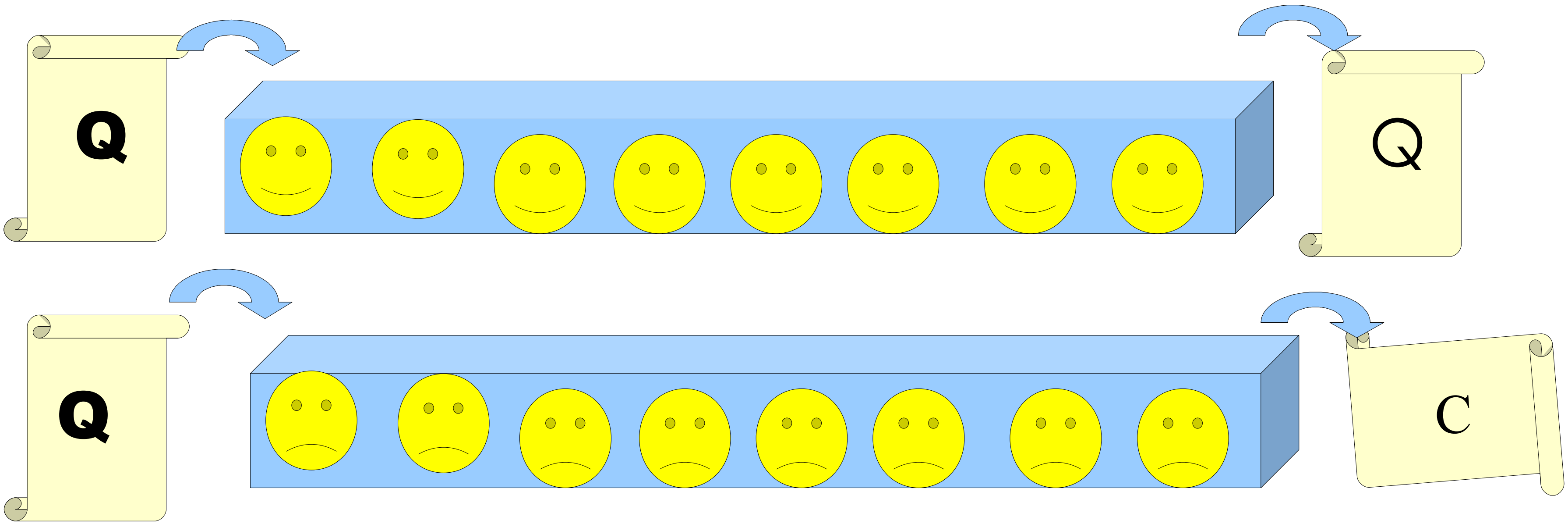
Kondo Routers (unpublished):

(with Bayat & Sodano)





One can also, of course, use a spin chain as a quantum channel – just like an optical fiber – (Reviewed: S. Bose, Contemp. Phys. **48**, 13 (2007).) But how does it depend on the **phase**?



An *Informational perspective* of a canonical condensed matter system

Transferring one half of an entangled state is a reasonably good unbiased comparison tool

Related work on transfer through general phases:

Spin-1 chain general phases:

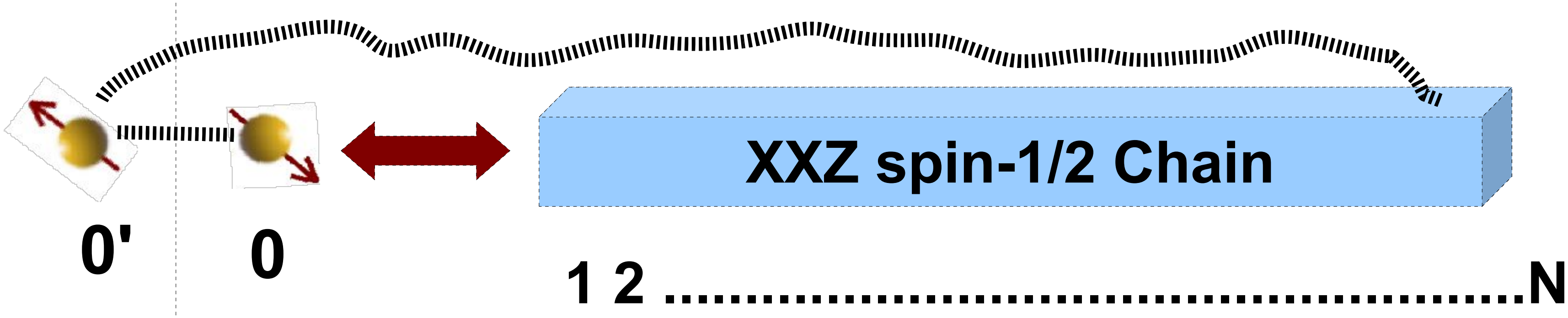
O. Romero-Isart, *et. al.*, Phys. Rev. A 75, 050303(R) (2007).

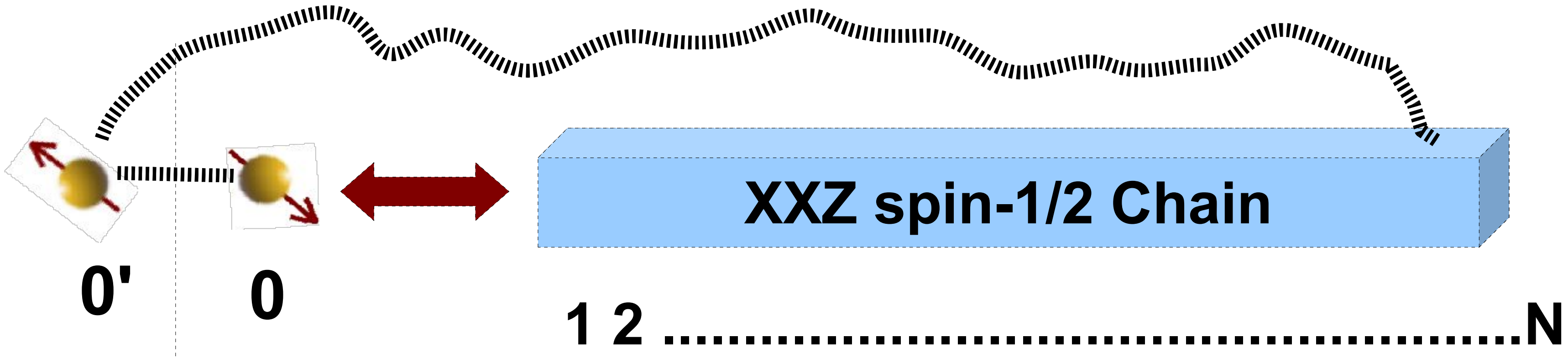
Antiferromagnetic phase for weak couplings:

L. Campos Venuti, Degli Esposti Boschi, & M. Roncaglia, PRL 99, 060401 (2007)

The general setting that we consider:

Bayat & Bose, Phys. Rev. A **81**, 012304 (2010)



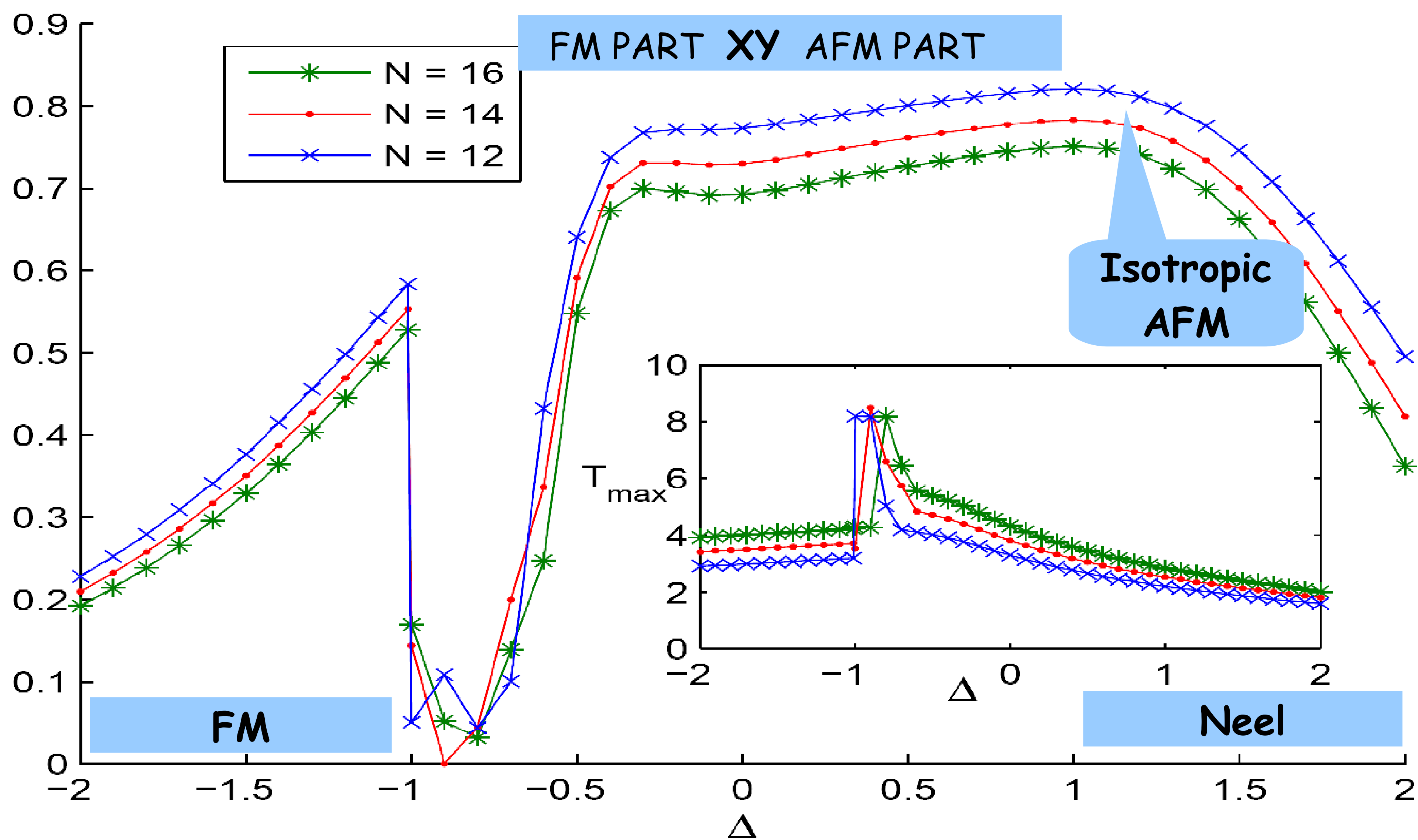


The XXZ spin chain

$$H_{ch} = J \sum_{i=1}^{N_{ch}-1} \{\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z\}$$

Entanglment in this case, when quantified by concurrence, is given by:

$$E_{0'j} = max(0, |\langle \sigma_0^x(0) \sigma_j^x(t) \rangle| - \frac{1}{2} \langle \sigma_0^z(0) \sigma_j^z(t) \rangle - \frac{1}{2})$$



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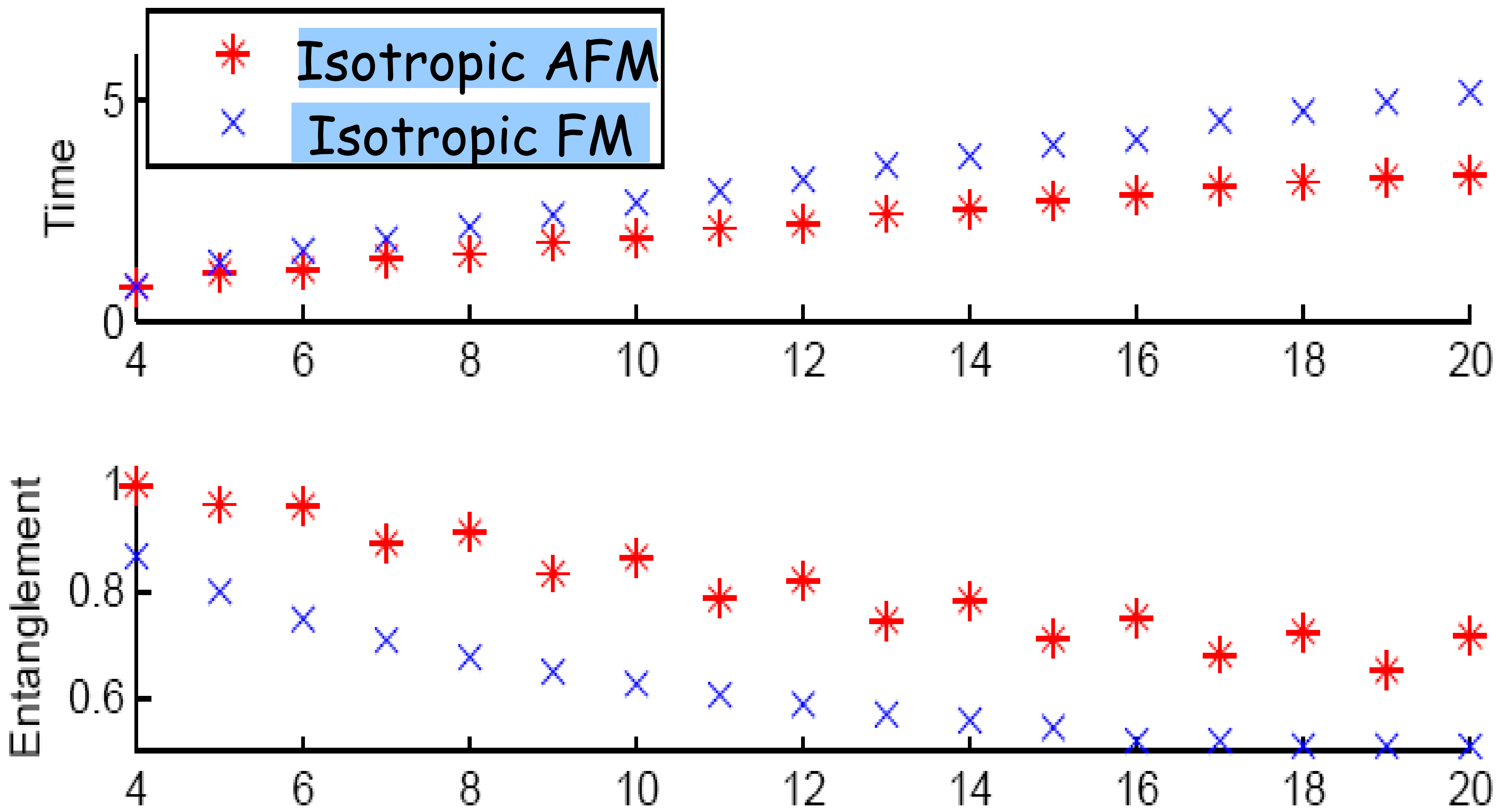
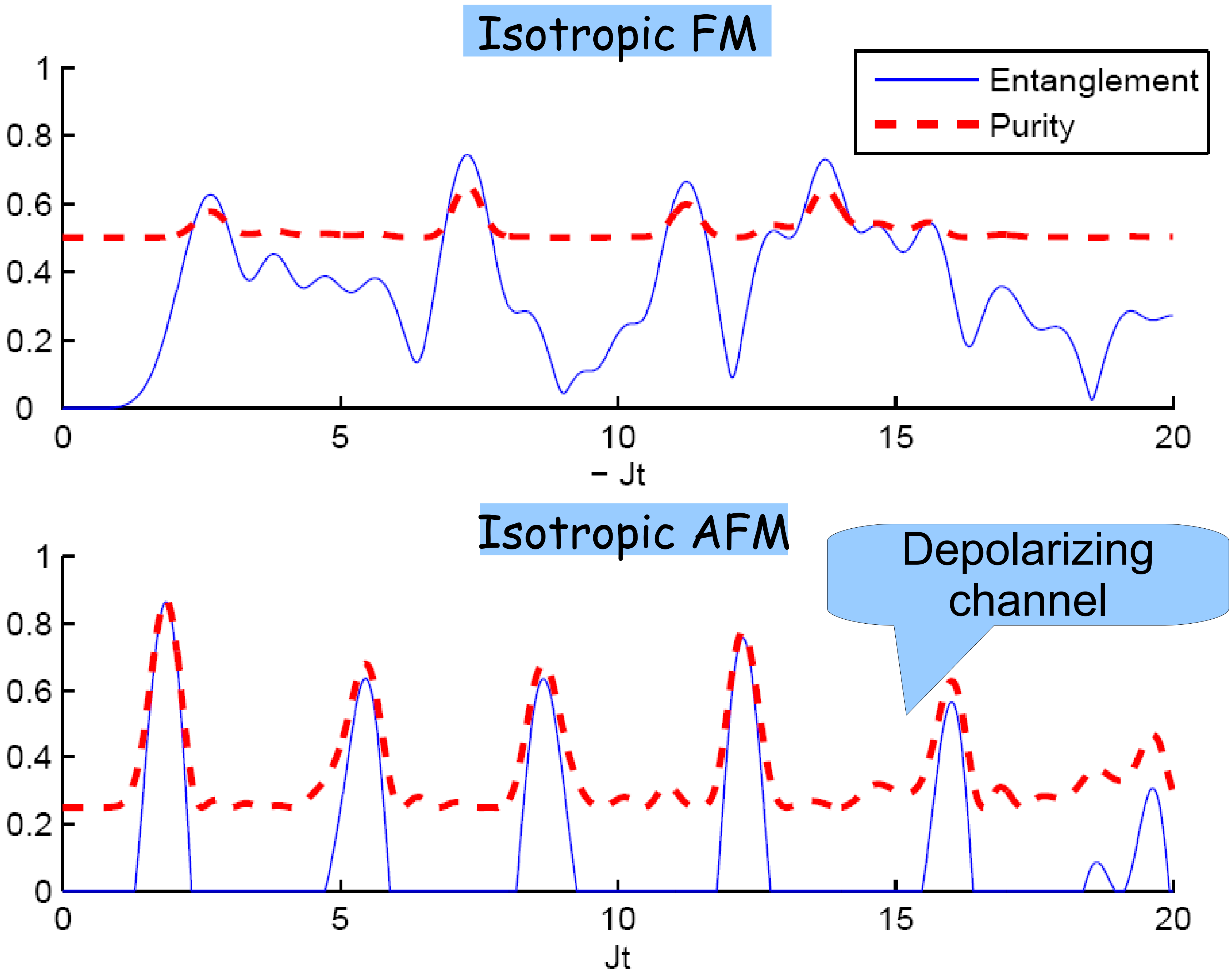


A. Bayat & S. Bose, Phys. Rev. A **81**, 012304 (2010)



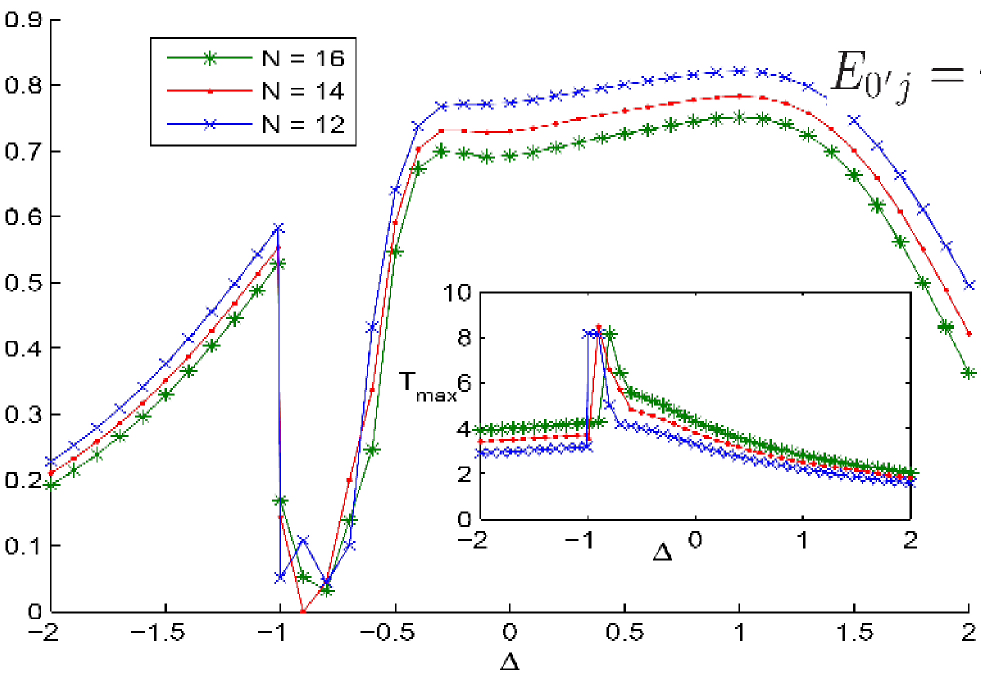
Isotropic FM
Entanglement
Purity
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
− Jt
Isotropic AFM
Depolarizing channel
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
Jt

$$\langle \sigma_j^\alpha(0)\sigma_k^\alpha(t)\rangle \sim (-1)^{|j-k|}\frac{1}{\left(|j-k|^2 - v_F^2 t^2\right)^{1/2\eta_\alpha}},$$

$$\eta_x = \eta_y = 1/\eta_z = 1 - \frac{\cos^{-1}\Delta}{\pi}.$$

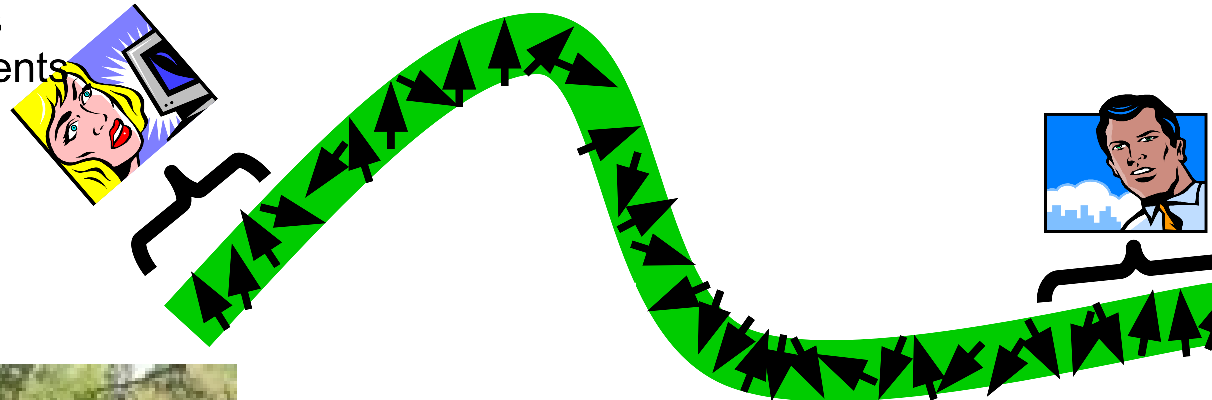
$$v_F \propto \frac{\sin(\cos^{-1}\Delta)}{\cos^{-1}\Delta}.$$

$$E_{0'j} = max\left(0, |\langle\sigma_0^x(0)\sigma_j^x(t)\rangle| - \tfrac{1}{2}\langle\sigma_0^z(0)\sigma_j^z(t)\rangle - \tfrac{1}{2}\right)$$



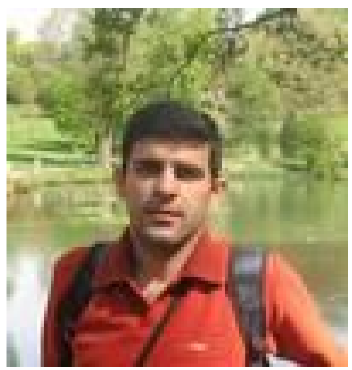
*We have investigated what kind of **long-range entanglement** exists and/or what kind of long range entanglement can be created by **non-equilibrium dynamics** in spin chain systems.*

- Analytics/justf for non-compl. blocks
- Relating the above to critical exponents
- Measuring such entanglement
- Is the Kondo quench entanglement truly distance independent?
- Physical Implementations?



Collaborators:

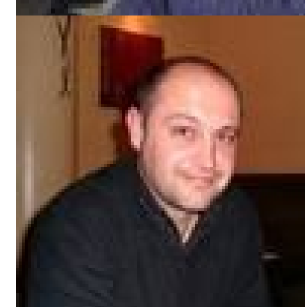
Abolfazl Bayat,



Pasquale Sodano

Hannu Wichterich

Javi Molina



Funding:

EPSRC, Royal Society & Wolfson Foundation