BF superconductors

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BF Theory in (d+1)-dimensions

$$S_{TM} = \int_{M_{d+1}} \frac{k}{2\pi} A_p \wedge dB_{d-p} \; (S_{BF})$$

$$\frac{-1}{2e^2} dA_p \wedge *dA_p + \frac{(-1)^{d-1}}{2g^2} dB_{d-p} \wedge *dB_{d-p}$$

where k is a dimensionless parameter.

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- Phase transition a T = 0 between the superconducting and insulating phase.
- Perfect agreement with the phase diagram of JJA.

Ground State Degeneracy (Torus)

- Electric condensation:

$$Z_{EC} = \int \mathcal{D}A\mathcal{D}B\mathcal{D}Q \exp -S(A_1, B_1, Q_1) \longrightarrow$$

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- Define:

$$A_i^R = (A_i + B_i^c) \; ; A_i^L = (A_i - B_i^c)$$

.

- gauge transformations:

$$A_i^R \rightarrow A_i^R + d_i\lambda \; ; A_i^L \rightarrow A_i^L + d_i\chi$$

$$\lambda(x_i + P_i) = \lambda(x_i) + 2\pi n_i;$$

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  $P_i (i = 1, 2)$ periods of the torus.
- There is only one generators of large gauge transformations per homology cycle.

- The action of the generators of large gauge transformations on physical states gives (semi)periodic conditions solved by:
  - $(k_1 k_2)^2$ theta functions if both topological defects are dense (not allowed dynamically);
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BF versus conventional superconductors

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- BCS: gap arises from SSB $U(1) \rightarrow Z_N$, Z_2 for Cooper pairs:
  - N^2 ground state degeneracy; 4 for Z_2.
  - residual Aharonov-Bohm interaction between charges and vortices (Bais et al.);
  - effective BF theory with $k = N$;
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- $1 \leftrightarrow 1$ correspondences between generators of large gauge transformation of BF model and generators of FFZ algebra.
- Gapless excitations on manifold with boundaries described by a (R)CFT .

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- BF superconductors can be distiguished from conventional (BCS) superconductors by their respective topological entanglement entropy.