Electronic entanglement via quantum-Hall interferometry (in analogy to an optical method)

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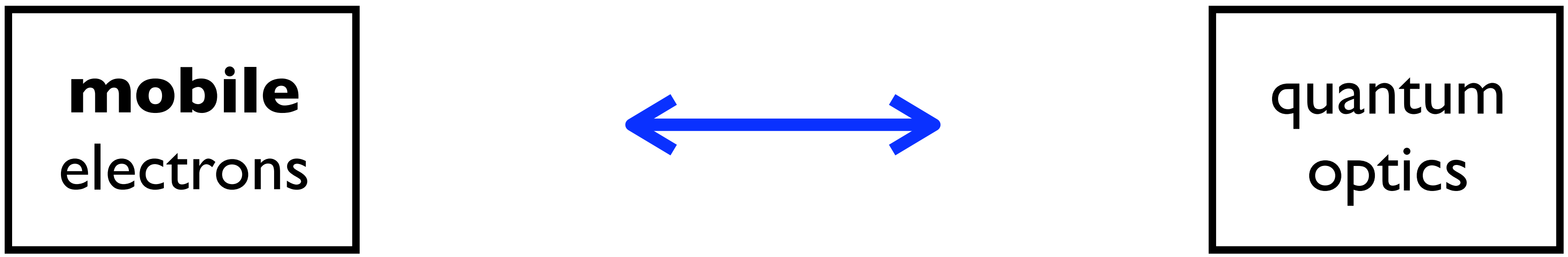
An active research program

translate successful **optical technologies** for quantum information into **quantum electronics**



Important differences:

- **statistics** (Fermions)

- **superselection rules** (particle-number conservation)

- **electric charge** (AB effect; decoherence)

Some examples of optical-like electronic interferometers

1. e-Mach-Zehnder interferometer

Ji *et al.*, Nature (2003)

(realized)

applications: decoherence, dephasing, anyonic interferometry.

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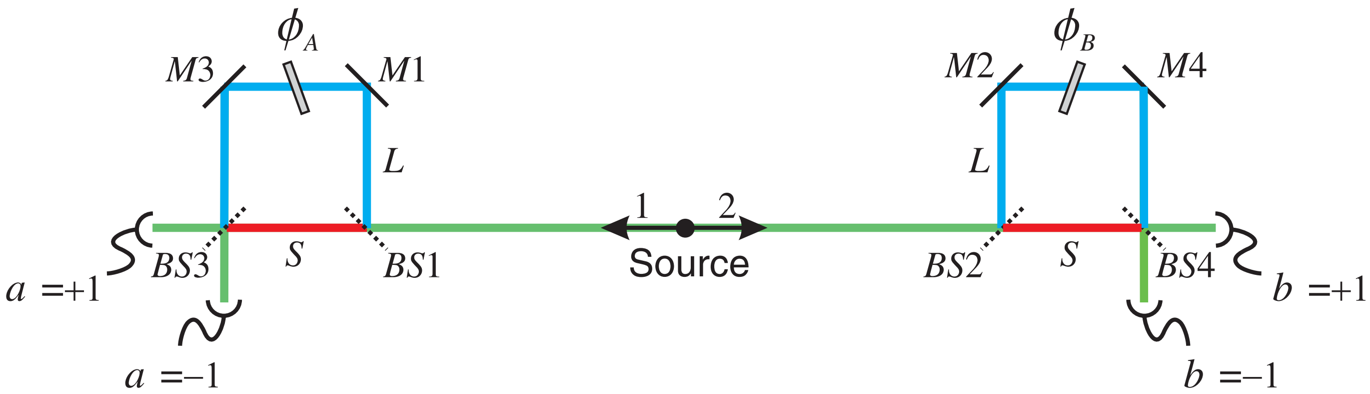
3. e-Hong-Ou-Mandel interferometer

Giovannetti, D. F., Taddei & Fazio, PRB (2006, 2007)

(proposed)

applications: mode and occupancy entanglement, e-bunching/antibunching transition.

Franson interferometer

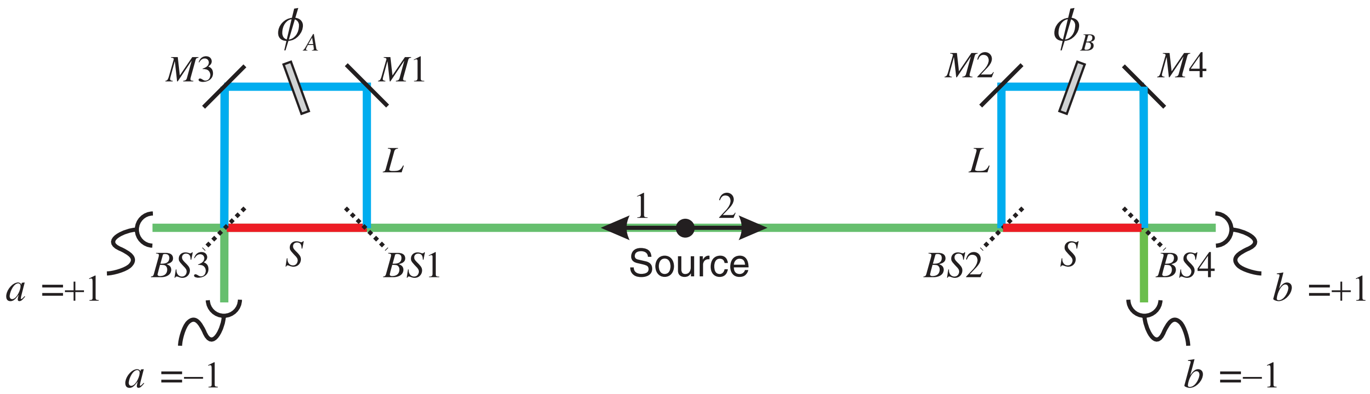


Franson, PRL (1989)

To prove violation of local realism (via postselective time-bin entanglement/ no orbital entanglement possible).

Widely used in **optical** Bell tests and Bell-inequality-based **quantum cryptography**.

Franson interferometer



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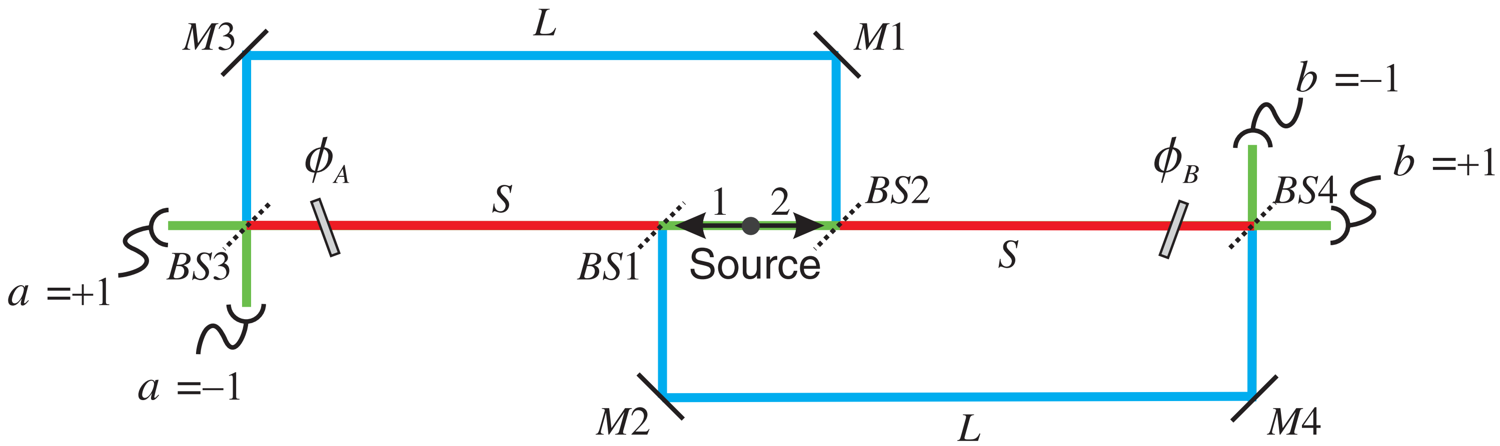
However, there is a **local hidden variable model** that simulates its results !! [Aerts *et al.*, PRL (1999)]

very recently...



theory:
Cabello *et al.*,
PRL (2009)

experiment:
Lima *et al.*,
PRB(R) (2010)

Overcomes all problems of Franson interferometer.

Allows for **both**, time-bin and orbital entanglement.

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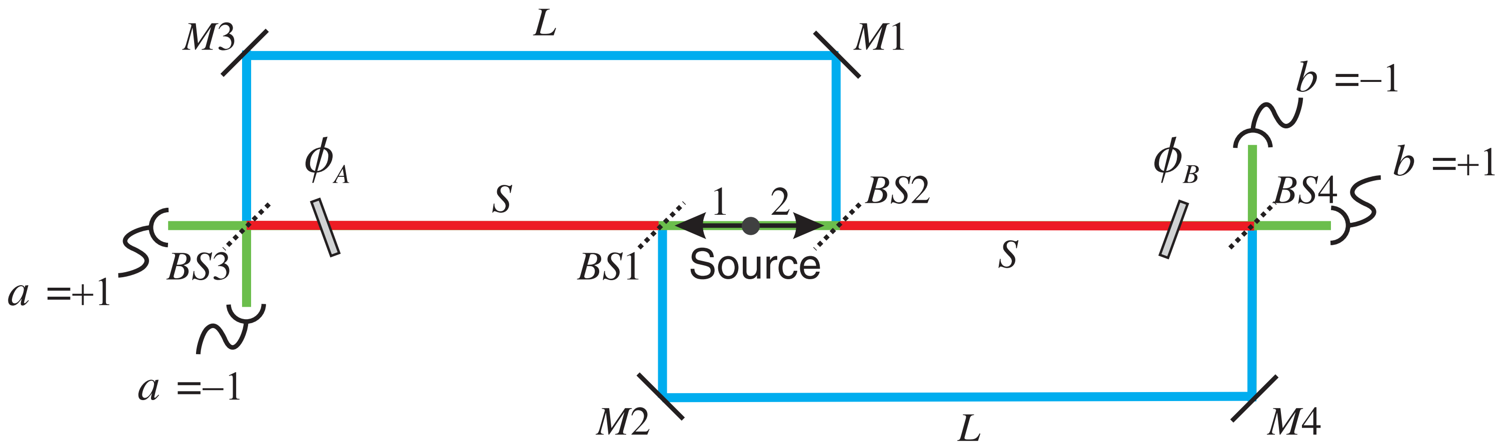

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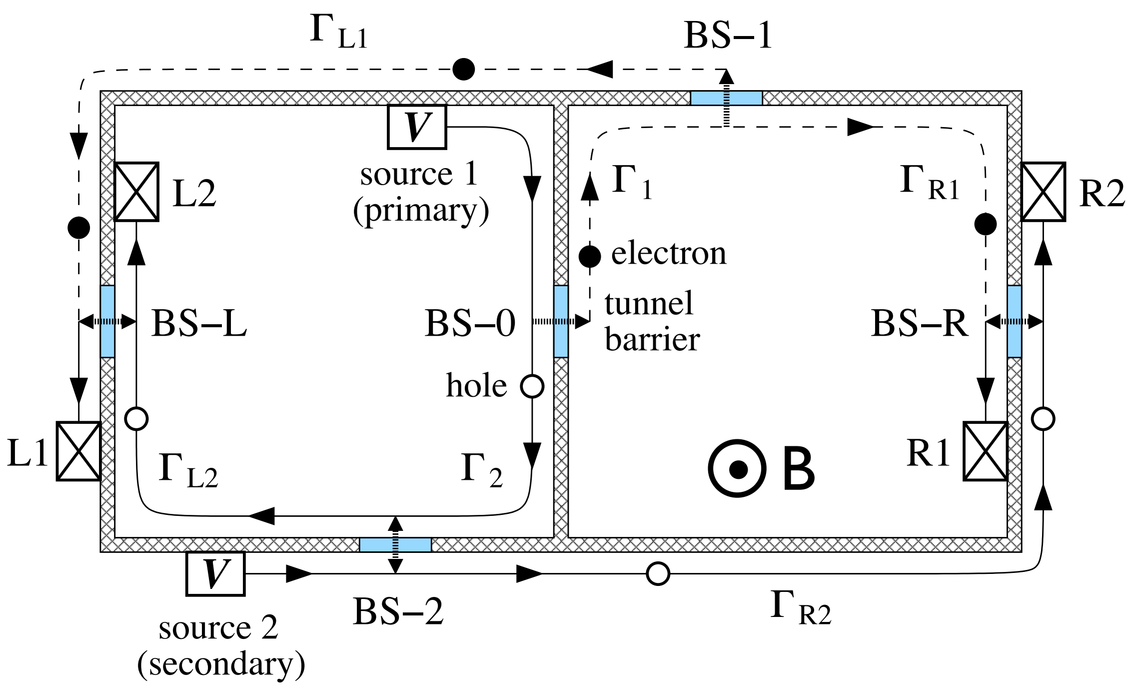
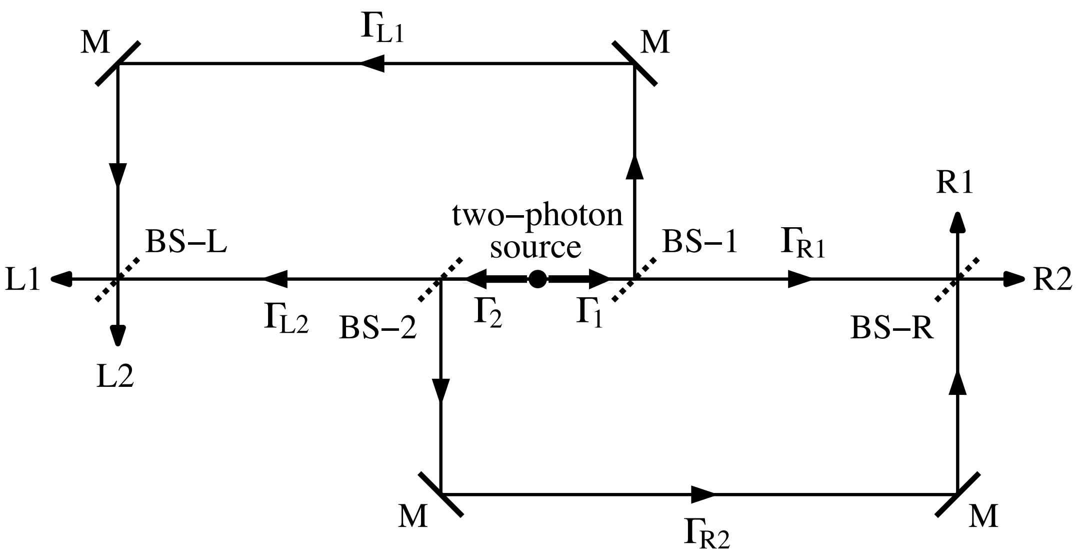
Here, we introduce its electronic version.

Our proposal: (integer) quantum-Hall interferometer

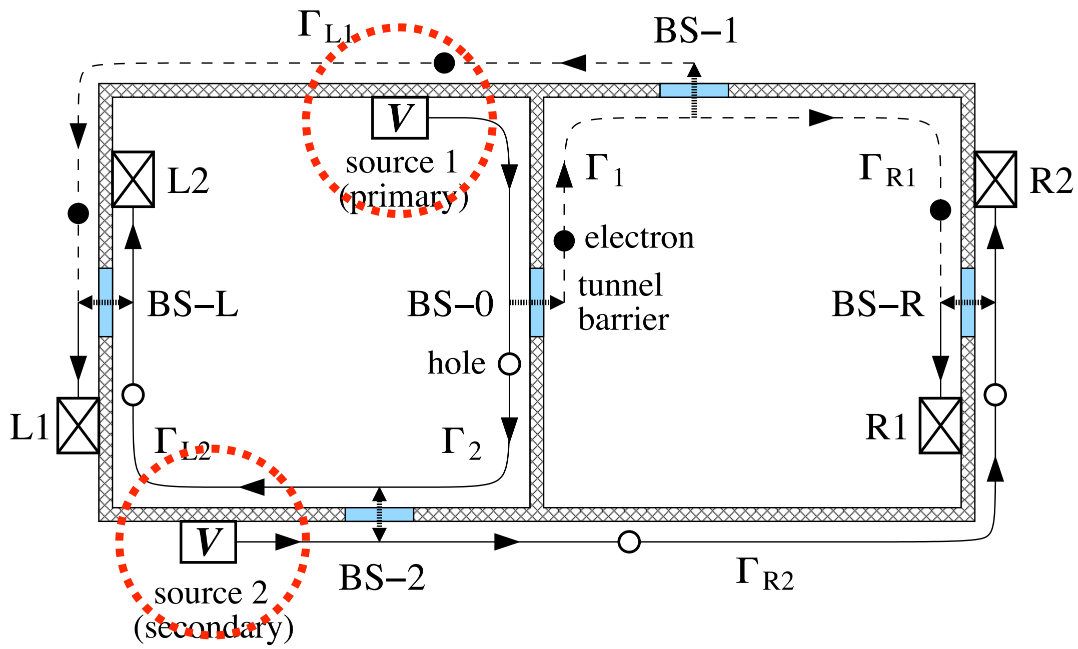


- single edge channel
- primary & secondary e-source
- BS: quantum point contacts
- BS-0: tunnel barrier
- topological constraints satisfied
- problems from Fermionic statistics overcome



optical-like
sketch

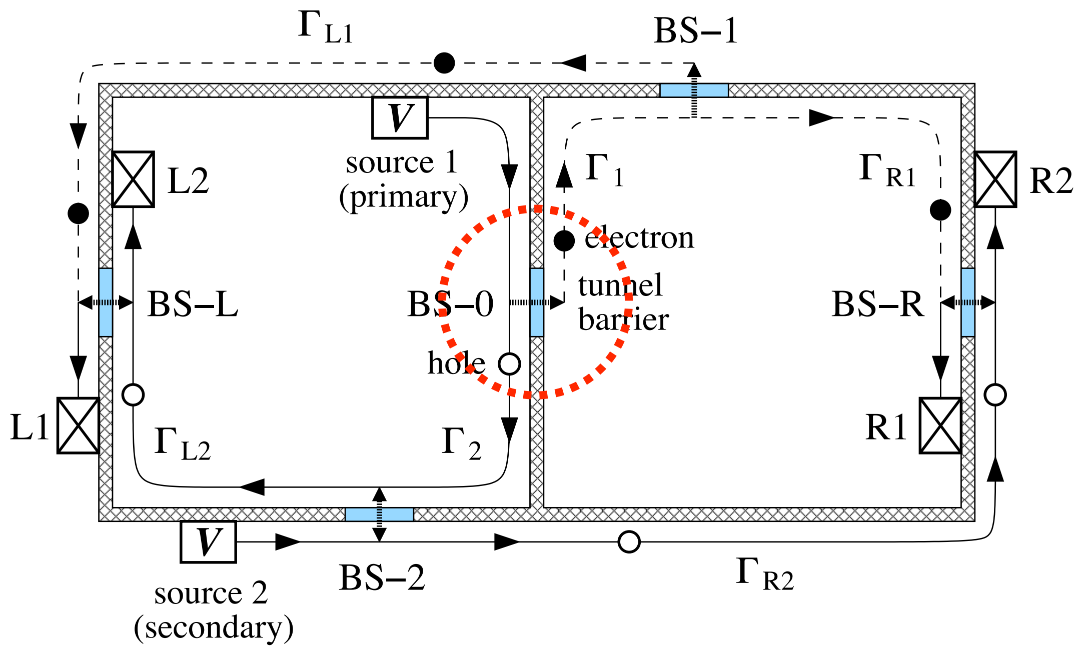
Entanglement production



$$|\Psi_{\rm in}\rangle = \prod_{\varepsilon}^{eV} a_1^\dagger(\varepsilon) a_2^\dagger(\varepsilon)|0\rangle$$

incoming state

Entanglement production



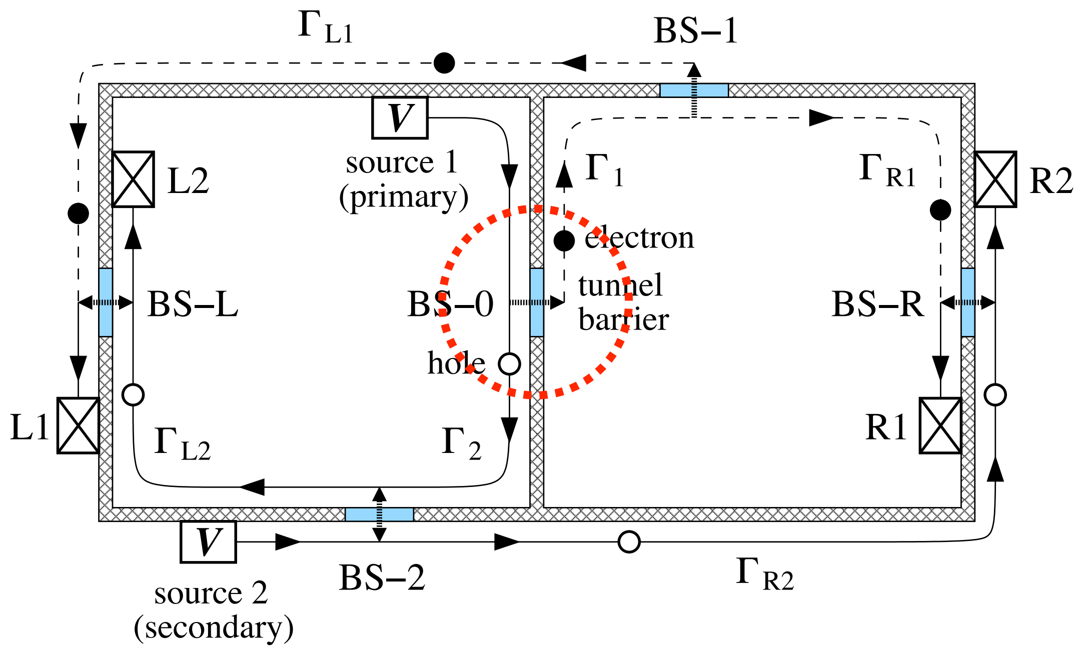
$$|\Psi_{\rm in}\rangle = \prod_{\varepsilon}^{eV} a_1^\dagger(\varepsilon) a_2^\dagger(\varepsilon)|0\rangle$$

incoming state

$$|\Psi'\rangle = \prod_{\varepsilon}^{eV} \left[t_0 b_1^\dagger(\varepsilon) + r_0 b_2^\dagger(\varepsilon)\right] a_2^\dagger(\varepsilon)|0\rangle$$

scattering at BS-0 $\left(|t_0|^2 \ll 1\right)$

Entanglement production



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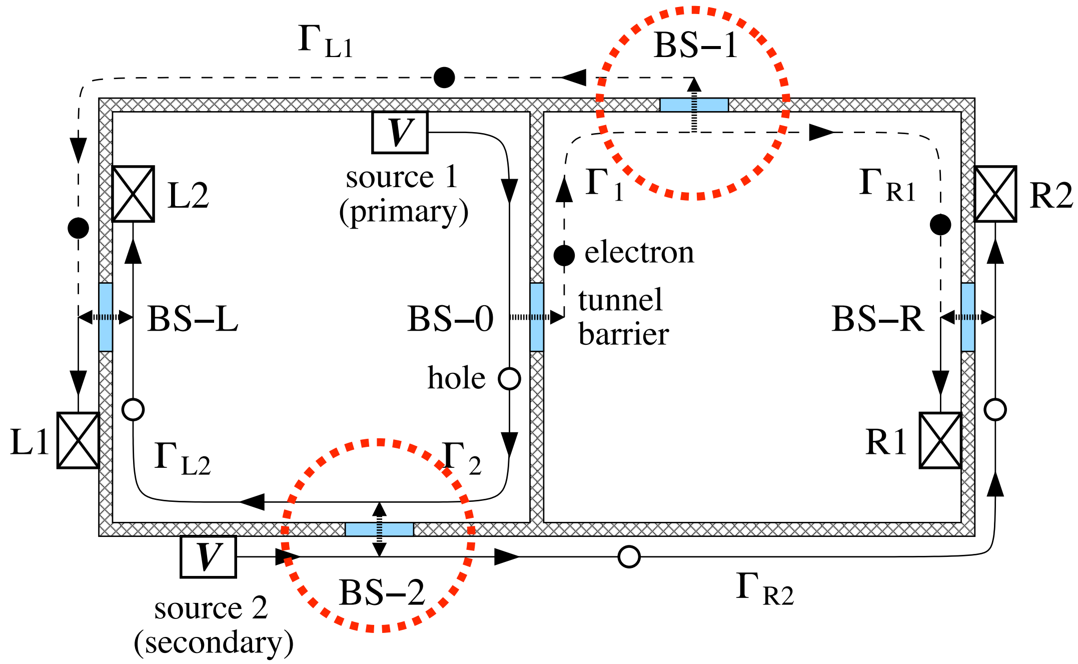
scattering at BS-0 $\left(|t_0|^2 \ll 1\right)$

$$\approx \left[1 - t_0 \int_0^{eV} \mathrm{d}\varepsilon' \, b_2(\varepsilon') b_1^\dagger(\varepsilon')\right] \prod_{\varepsilon}^{eV} b_2^\dagger(\varepsilon) a_2^\dagger(\varepsilon)|0\rangle$$

electron-hole pair packet emitted

Entanglement production



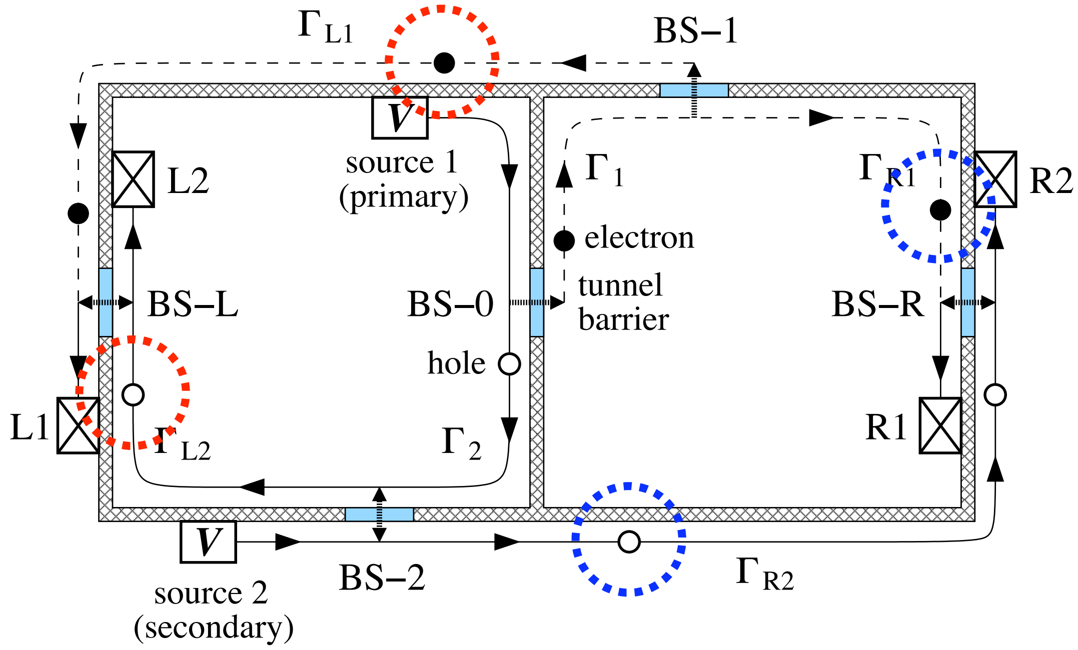
$$|\Psi_{\rm out}\rangle = |\bar{0}\rangle + |\bar{\Psi}\rangle$$ scattering at BS-1 and BS-2

$$|\bar{\Psi}\rangle = t_0 e^{i(\phi_1-\phi_2)} \int_0^{eV} \mathrm{d}\varepsilon' \quad \left[t_1 t_2^* C^\dagger_{\rm L1}(\varepsilon') C_{\rm R2}(\varepsilon') - r_1 r_2^* C_{\rm L2}(\varepsilon') C^\dagger_{\rm R1}(\varepsilon') + t_1 r_2^* C^\dagger_{\rm L1}(\varepsilon') C_{\rm L2}(\varepsilon') + r_1 t_2^* C^\dagger_{\rm R1}(\varepsilon') C_{\rm R2}(\varepsilon')\right] |\bar{0}\rangle$$

$$|\bar{0}\rangle = \prod_{\varepsilon}^{eV} C^\dagger_{\rm L2}(\varepsilon) C^\dagger_{\rm R2}(\varepsilon) |0\rangle$$ redefined vacuum: noiseless stream

(thanks to secondary source !!)

Entanglement production



orbital entanglement
(electron-hole swapping)

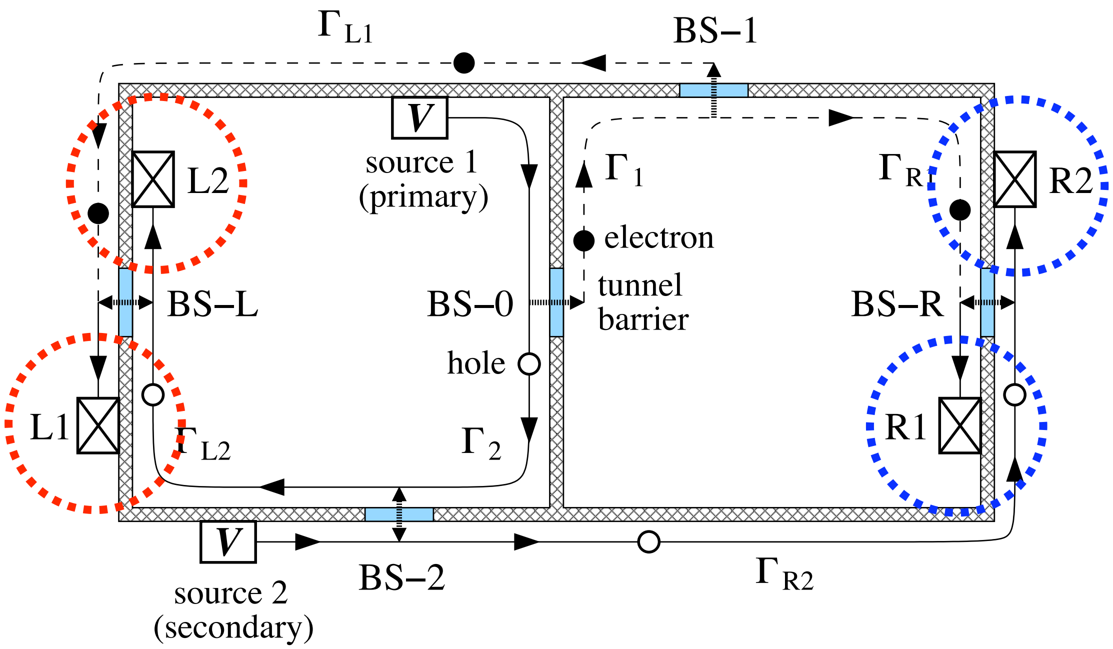
maximum for $T_1 T_2 = R_1 R_2$

$$|\bar{\Psi}\rangle = t_0 e^{i(\phi_1-\phi_2)} \int_0^{eV} \mathrm{d}\varepsilon' \quad \overbrace{[t_1 t_2^* C_{\mathrm{L1}}^\dagger(\varepsilon') C_{\mathrm{R2}}(\varepsilon') - r_1 r_2^* C_{\mathrm{L2}}(\varepsilon') C_{\mathrm{R1}}^\dagger(\varepsilon') +}$$

$$\underbrace{t_1 r_2^* C_{\mathrm{L1}}^\dagger(\varepsilon') C_{\mathrm{L2}}(\varepsilon') + r_1 t_2^* C_{\mathrm{R1}}^\dagger(\varepsilon') C_{\mathrm{R2}}(\varepsilon')]}\ |\bar{0}\rangle$$

occupancy entanglement

Entanglement detection



violation of **Bell-like inequality** upon cross **current-noise** correlator

$$S_{ij} = \lim_{\mathcal{T}\to\infty} \frac{h\nu}{\mathcal{T}^2} \int_0^{\mathcal{T}} \mathrm{d}t_1 \mathrm{d}t_2 \langle \delta I_{\mathrm{L}i}(t_1) \delta I_{\mathrm{R}j}(t_2) \rangle = -e^3 V/h |(t_{\mathrm{L}} t_{\mathrm{R}}^{\dagger})_{ij}|^2 \propto T_0$$

finite **only** when there is one **tunneling** excitation on **each** side

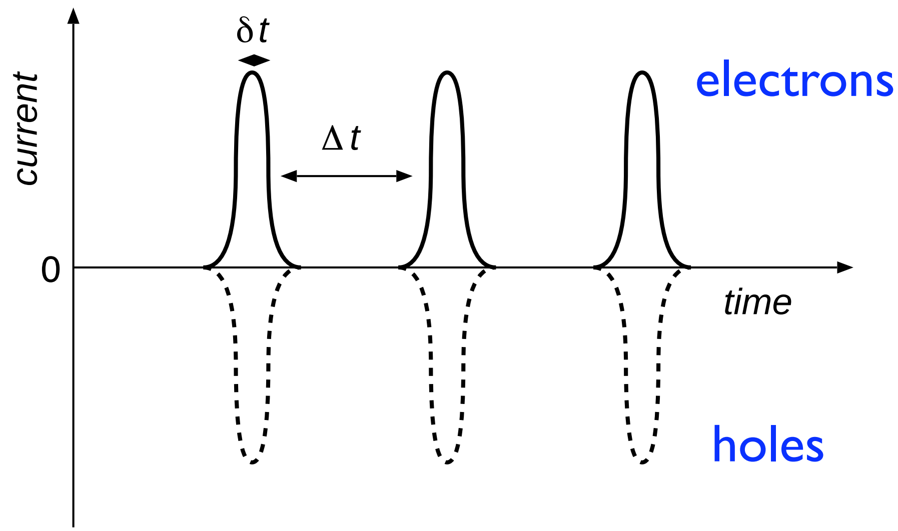
⟶ **orbitally** entangled component **only** (post-selection)

Entanglement detection

S_{ij} proportional to **joint-detection** probabilities thanks to **time-scale separation** via **tunneling**

Samuelsson, Sukhorukov & Büttiker, PRL (2003)

review: Beenakker, Proc. Fermi School (2006)

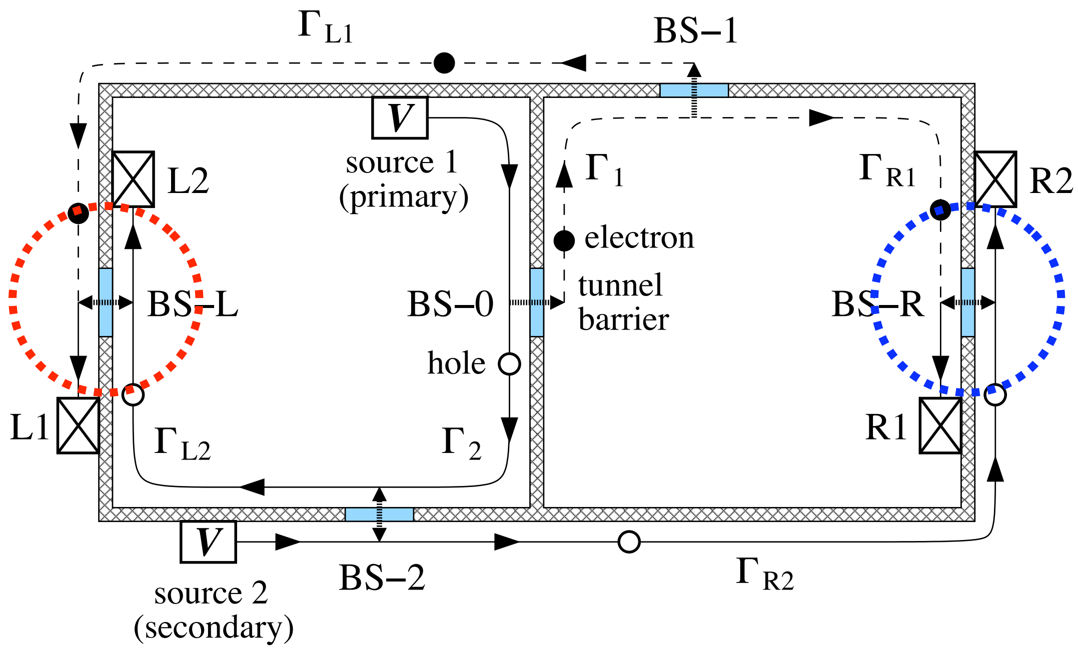


$$\delta t = h/eV$$

$$\Delta t = \delta t/T_0$$

$$\boxed{\delta t \ll \Delta t}$$

Entanglement detection



$$\mathcal{E} = E(U_\mathrm{L}, U_\mathrm{R}) + E(U'_\mathrm{L}, U_\mathrm{R}) + E(U_\mathrm{L}, U'_\mathrm{R}) - E(U'_\mathrm{L}, U'_\mathrm{R})$$ Bell-CHSH parameter

$$E(U_\mathrm{L}, U_\mathrm{R}) = \frac{S_{11} + S_{22} - S_{12} - S_{21}}{S_{11} + S_{22} + S_{12} + S_{21}} = \frac{\mathrm{tr}\left[U_\mathrm{L}^\dagger \sigma_z U_\mathrm{L} t_\mathrm{L} t_\mathrm{R}^\dagger U_\mathrm{R}^\dagger \sigma_z U_\mathrm{R} t_\mathrm{R} t_\mathrm{L}^\dagger\right]}{\mathrm{tr}\left[t_\mathrm{L}^\dagger t_\mathrm{L} t_\mathrm{R}^\dagger\, t_\mathrm{R}\right]}$$

entanglement if $|\mathcal{E}| > 2$ for some $\{U_\mathrm{L}, U_\mathrm{R}, U'_\mathrm{L}, U'_\mathrm{R}\}$

Entanglement detection

$$\mathcal{E}_{\max} = 2\sqrt{1 + \frac{4(1-\lambda_+)(1-\lambda_-)\lambda_+\lambda_-}{(\lambda_+ + \lambda_- - \lambda_+^2 - \lambda_-^2)^2}}$$

$\lambda_{+/-}$:

eigenvalues of $t_R^\dagger t_R$

$$= 2\sqrt{1+\mathcal{C}^2}$$ (tunneling regime)

$$\mathcal{C} = 2\frac{\sqrt{T_1 T_2 R_1 R_2}}{T_1 T_2 + R_1 R_2}$$

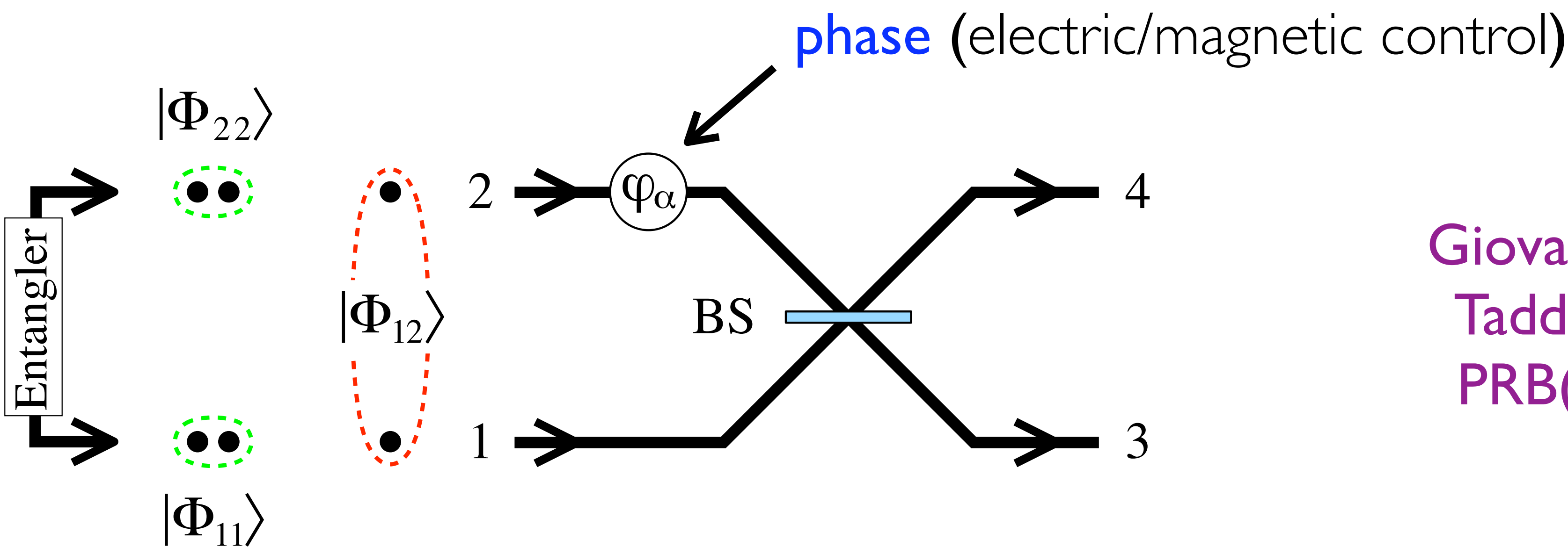
concurrence of

$$t_1 t_2^* C_{L1}^\dagger C_{R2} - r_1 r_2^* C_{L2} C_{R1}^\dagger$$

$0 \leq \mathcal{C} \leq 1$ maximum for $T_1 T_2 = R_1 R_2$

$\mathcal{C}$ **as a measurable quantity**

Alternative detection scheme



Giovannetti, D. F., Taddei & Fazio, PRB(R) (2007)

states: undefined local occupancy, multichannel (orbital/spin modes α)

$$|\Psi\rangle = \sin\theta(\cos\phi|\Phi_{11}\rangle + \sin\phi|\Phi_{22}\rangle) + \cos\theta|\Phi_{12}\rangle$$

observable: $\langle I_i I_j \rangle$ instead of $\langle \delta I_i \delta I_j \rangle$

use: discriminate/quantify **mode** and **occupancy** entanglement

Summary

quantum optics into **quantum electronics**:
possible and **worthy**

e-interferometer for **entanglement** production/detection
with practical and conceptual **advantages**

feasible with present technology