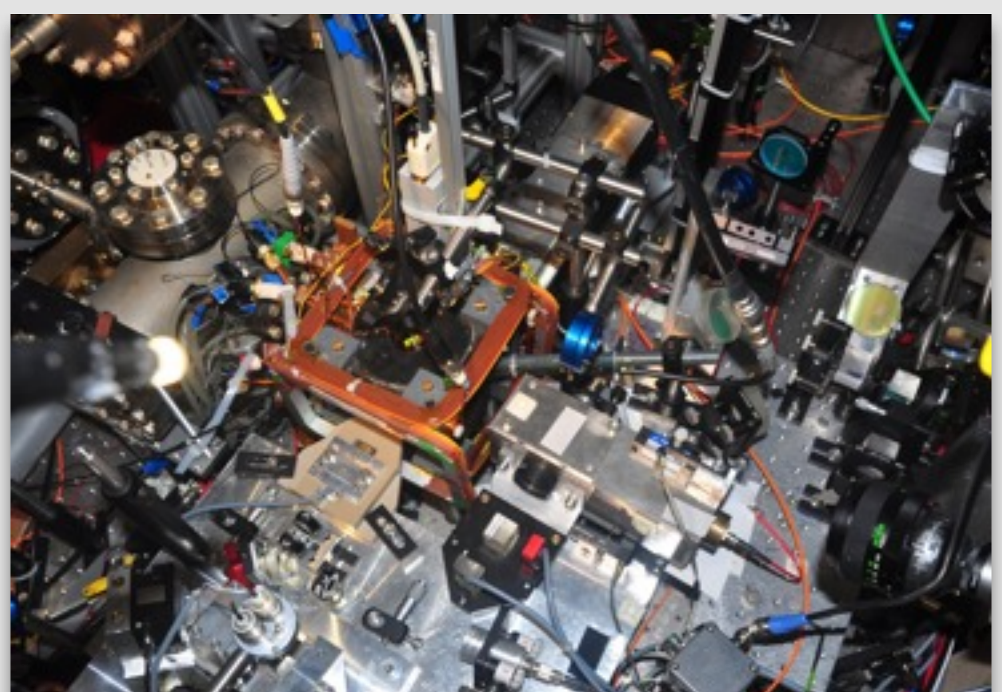
Nonlinear atom interferometry beyond the standard quantum limit

Christian Groß

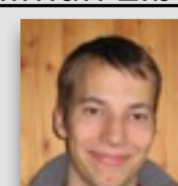
Kirchhoff Institute for Physics
University of Heidelberg

The BEC machine



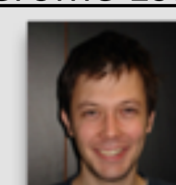
The BEC team

Tilman Zibold



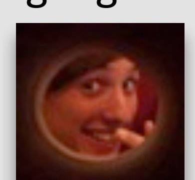
Eike Nicklas

Jerome Esteve



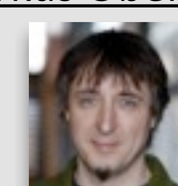
Helmut Strobel

Wolfgang Müssel

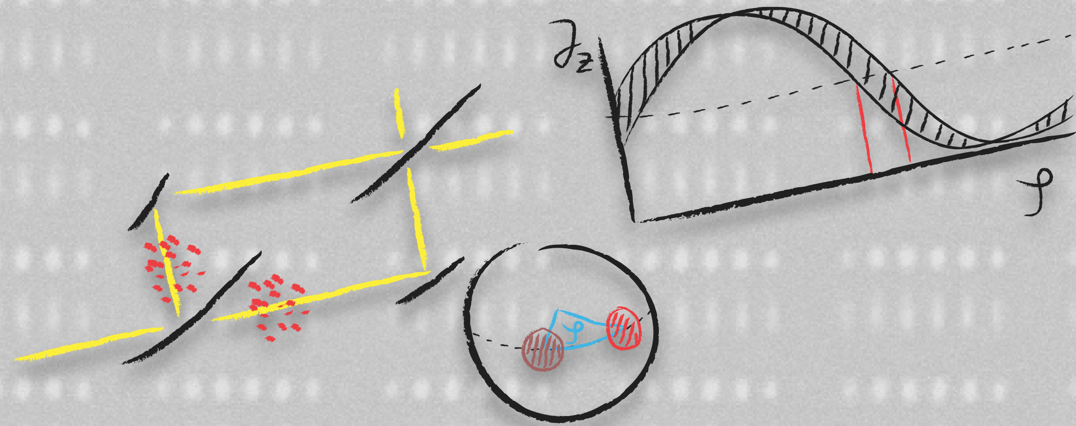


Ion Stroescu

Markus Oberthaler



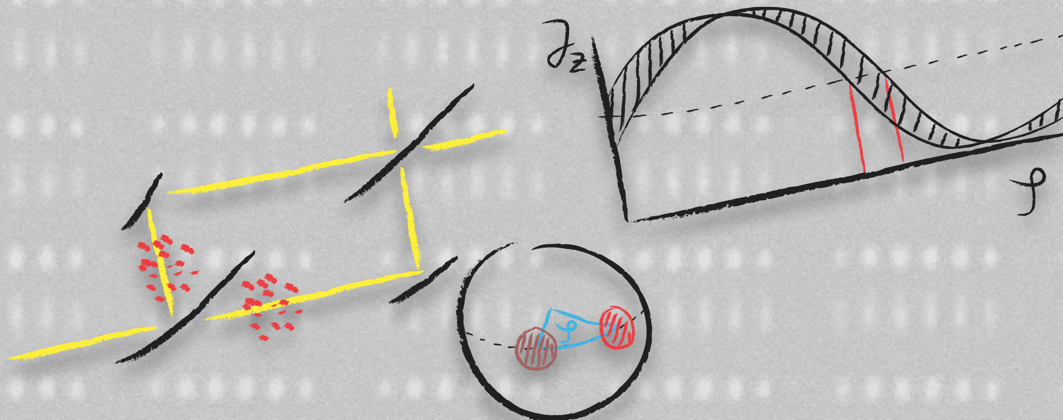
Outline



- Linear Ramsey interferometry

  - "classical", standard quantum limit -

Outline



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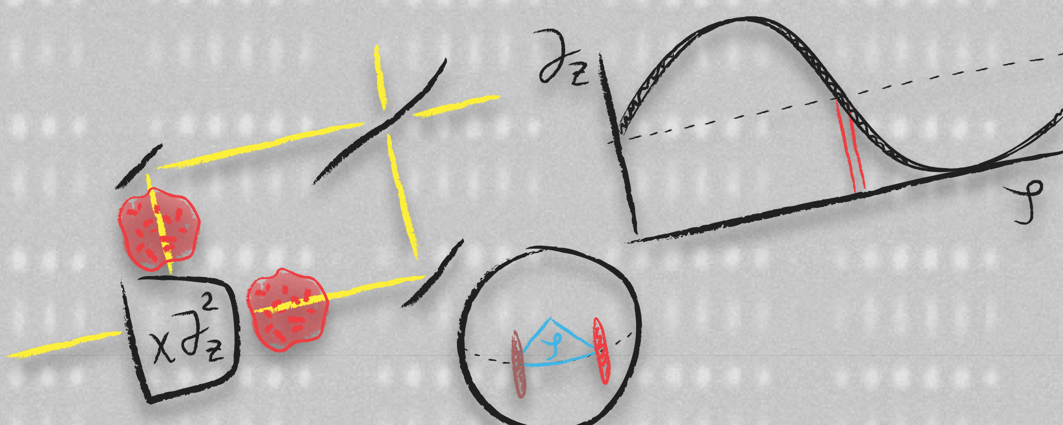

- A nonlinear atom interferometer

  - phase precision beyond the “classical” limit -

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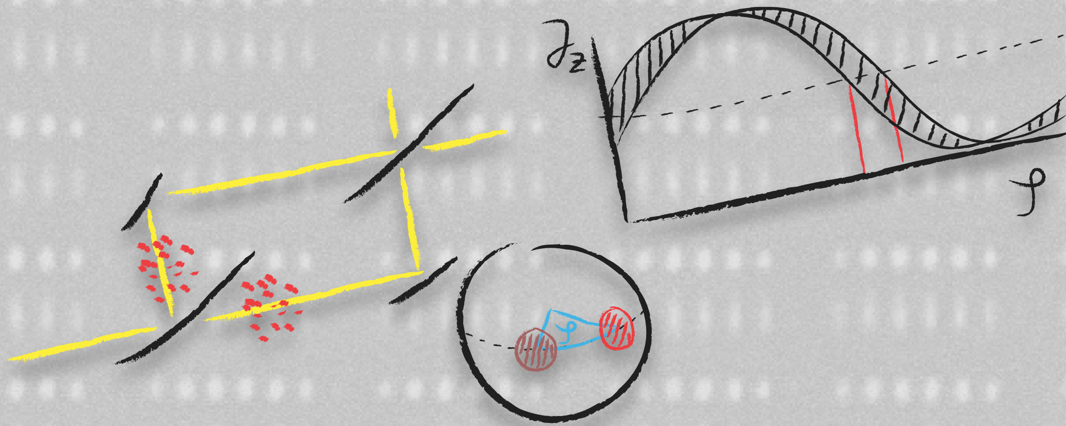

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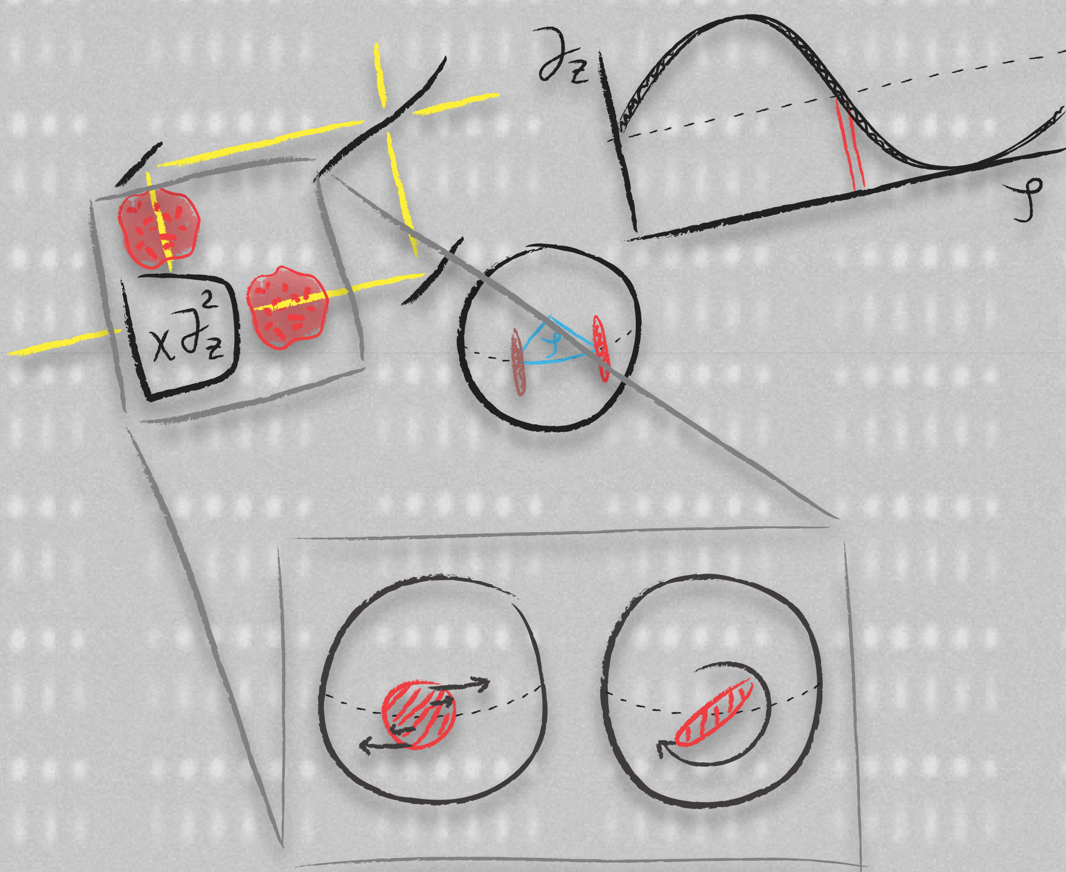


- A nonlinear atom interferometer

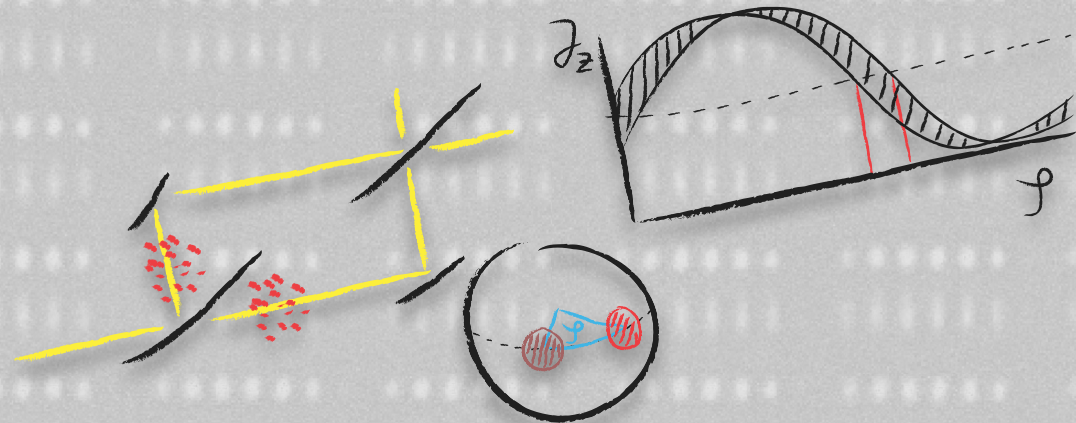
  - phase precision beyond the "classical" limit -

- The nonlinear beamsplitter

  - Noise tomography reveals spin squeezing -

  - Detection of a large entangled state -

Outline

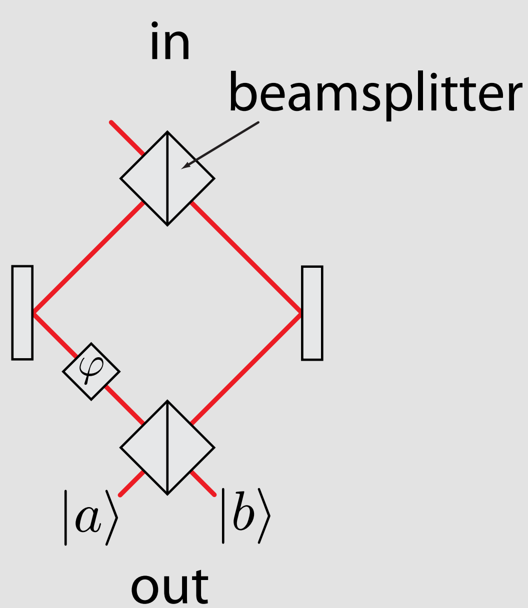


- Linear Ramsey interferometry

  - “classical”, standard quantum limit -

Classical Ramsey interferometry

- **Photons:** Mach-Zehnder interferometer



Classical Ramsey interferometry

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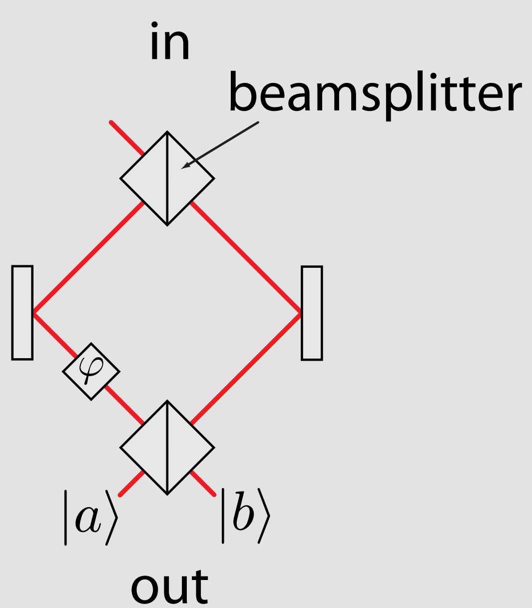

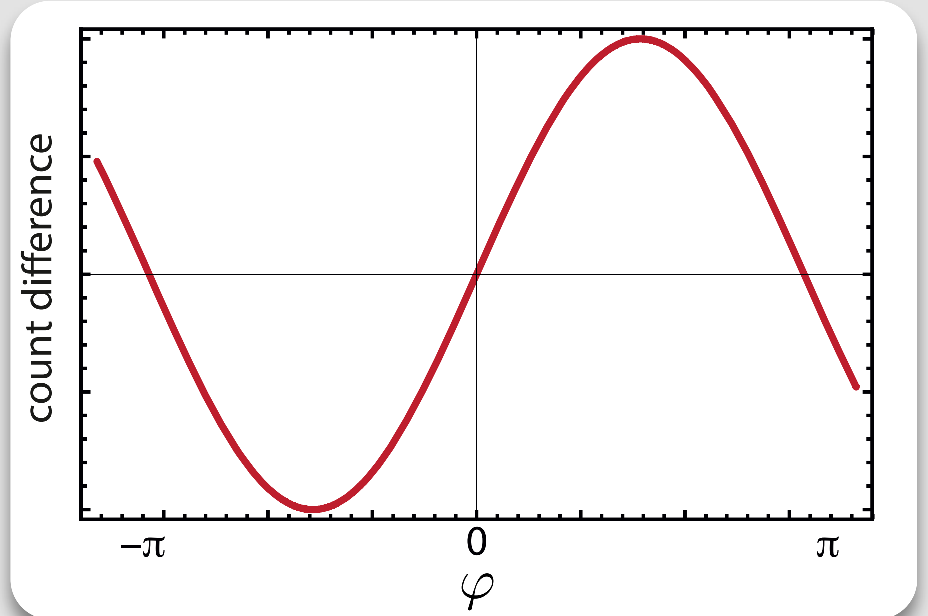

Classical Ramsey interferometry

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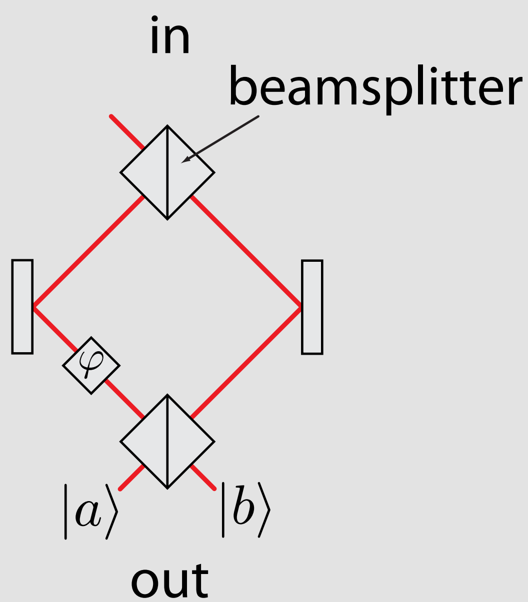
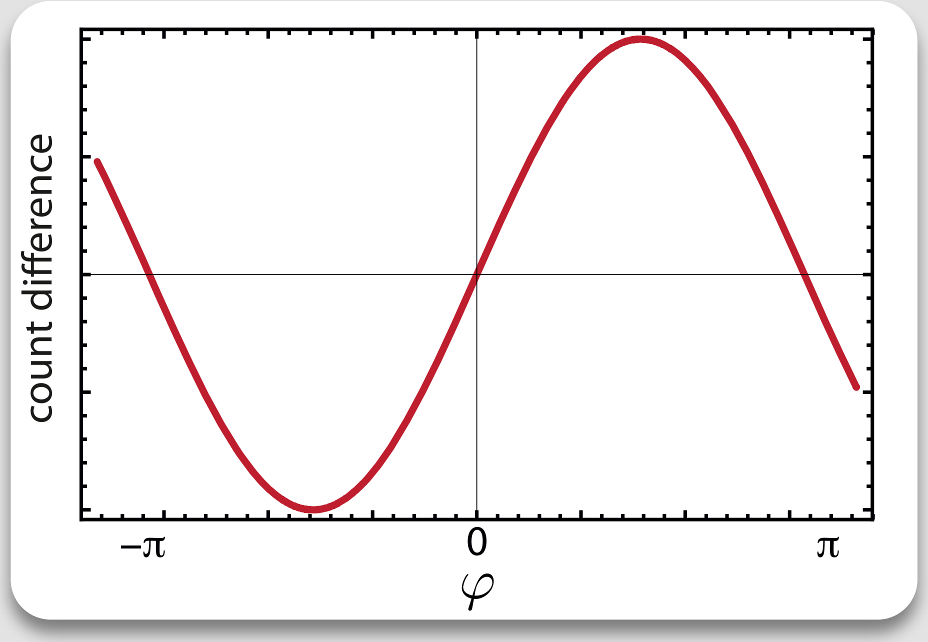
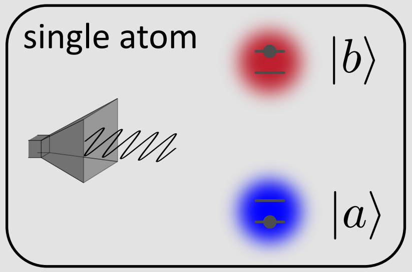
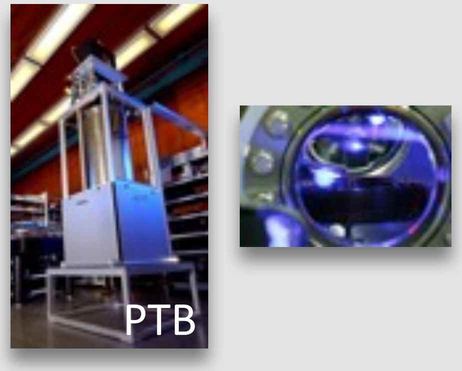


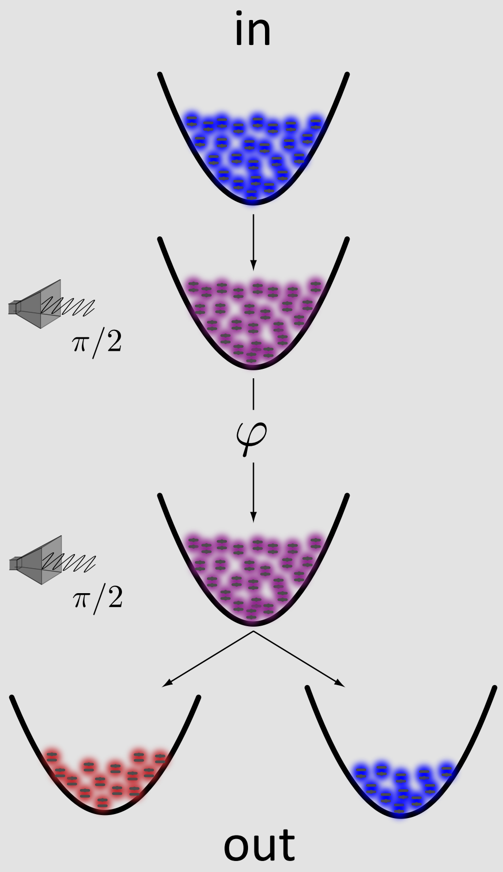


- **Atoms:** Ramsey interferometer





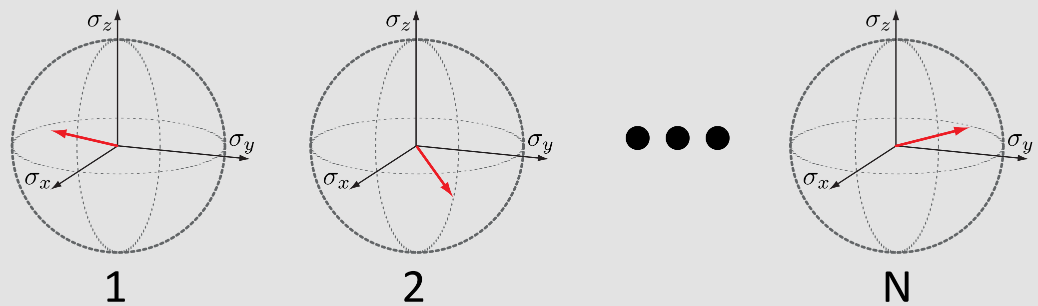
Cs atomic clock



What is the precision limit for interferometry?

Collective spins and interferometry

- Interferometry: N atoms in two modes ⟶ N elementary spin 1/2



$$\hat{J}_i = \sum_{k=1}^{N} \hat{\sigma}_i^{(k)}$$

collective spin

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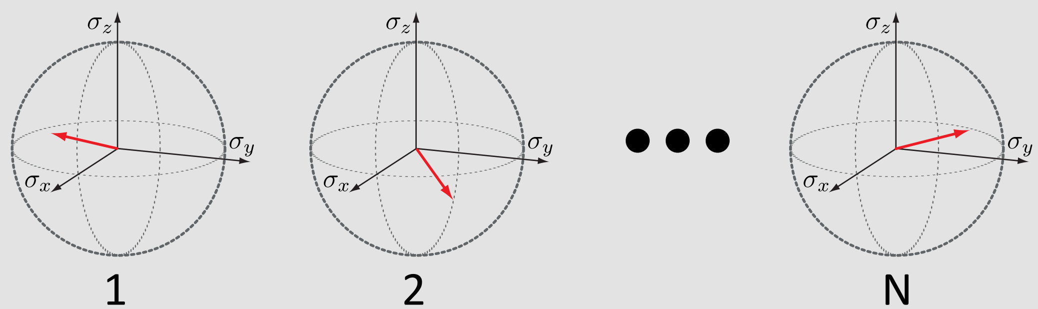

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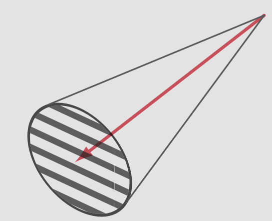
collective spin

- Heisenberg uncertainty relation $\Delta J_z^2 \, \Delta J_y^2 \geq \langle J_x \rangle^2 / 4$



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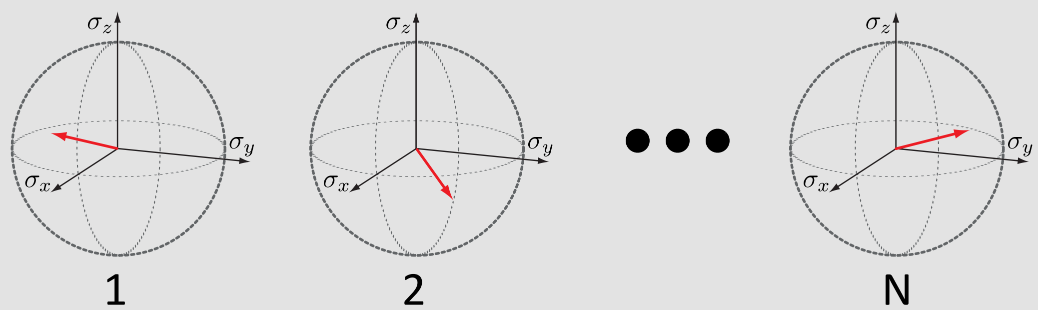

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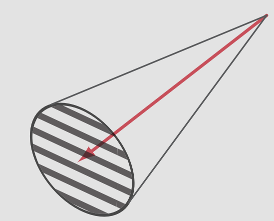

- Two-mode Bose-Einstein condensate: N Bosons in two modes

Schwinger spin representation:

$$\hat{J}_x = (\hat{b}^\dagger \hat{a} + \hat{a}^\dagger \hat{b})/2$$

$$\hat{J}_y = (\hat{b}^\dagger \hat{a} - \hat{a}^\dagger \hat{b})/2i$$

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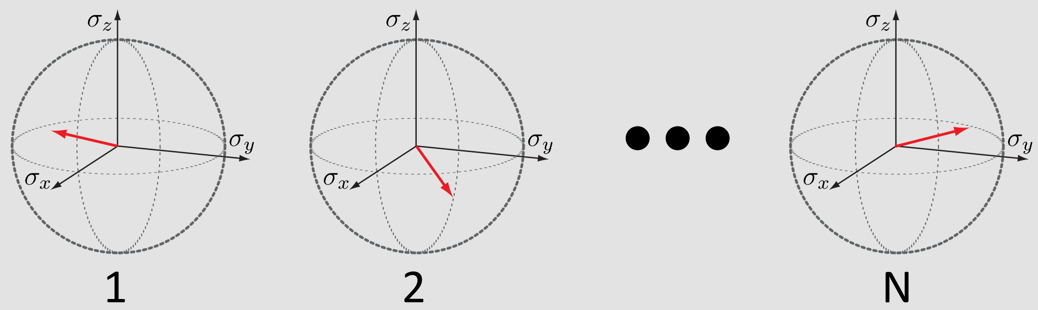

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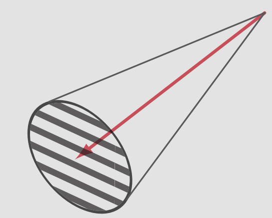

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$$\hat{J}_x = (\hat{b}^\dagger \hat{a} + \hat{a}^\dagger \hat{b})/2 \quad \propto \cos(\varphi)$$

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$$\hat{J}_z = (\hat{b}^\dagger \hat{b} - \hat{a}^\dagger \hat{a})/2 \quad = (n_b - n_a)/2$$

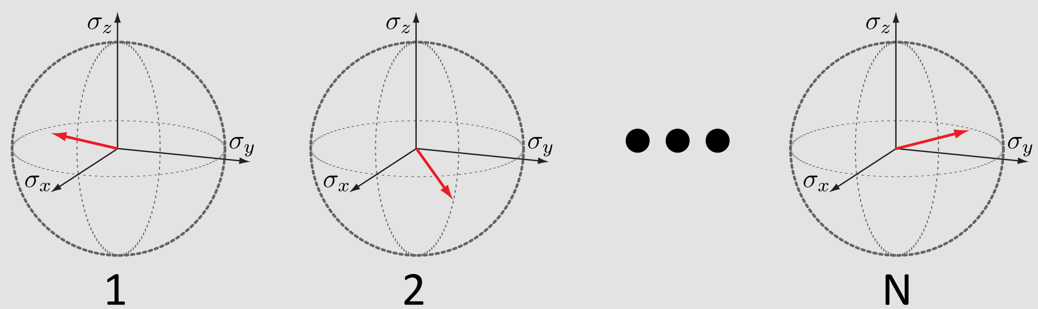
$N \gg 1$: $\hat{a} \rightarrow \sqrt{n_a} e^{i\varphi_a}$, $\hat{b} \rightarrow \sqrt{n_b} e^{i\varphi_b}$

relative phase

occupation number difference

Collective spins and interferometry

- Interferometry: N atoms in two modes $\longrightarrow$ N elementary spin 1/2



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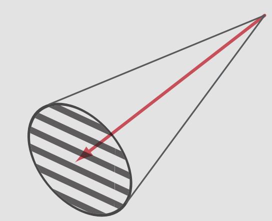

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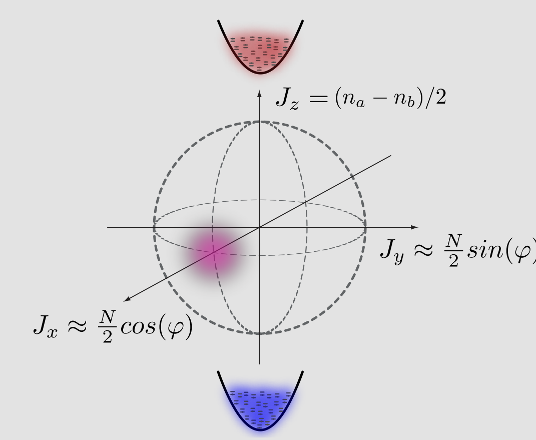
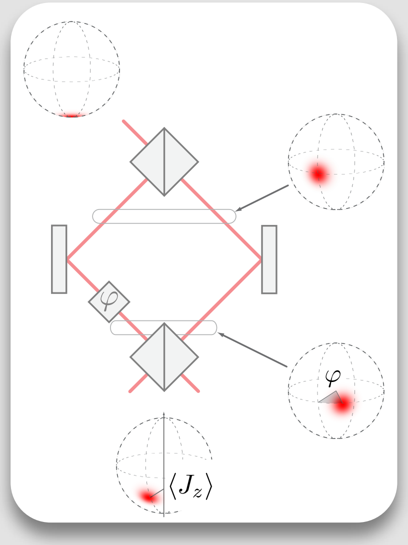


“Classical” interferometric precision limit

- linear interferometer



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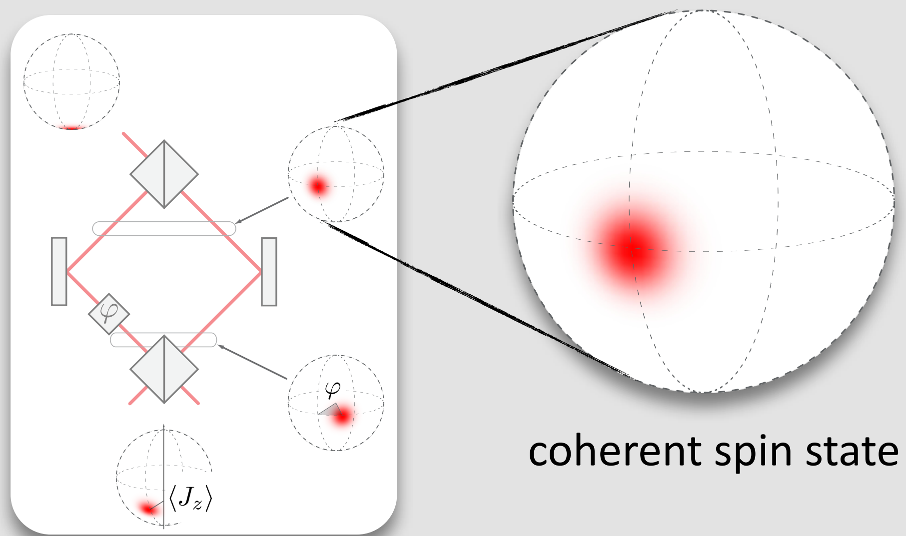

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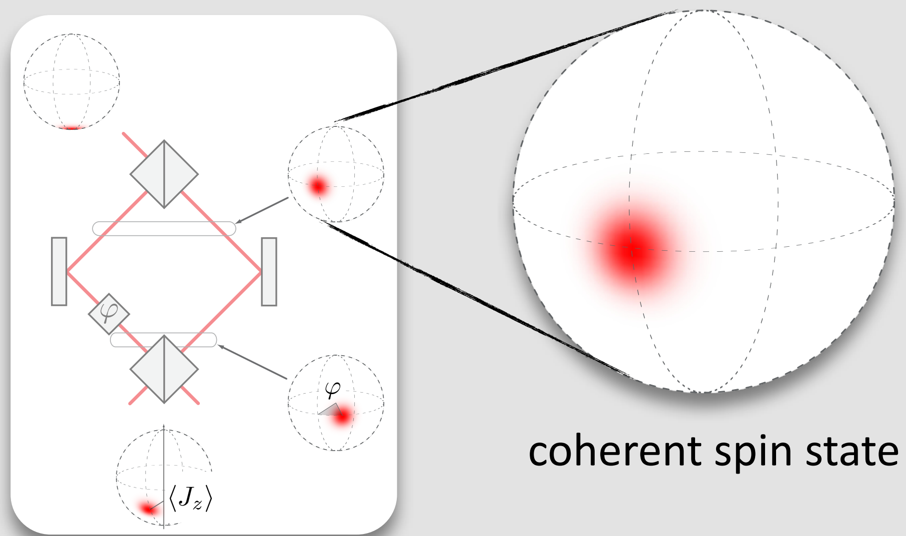
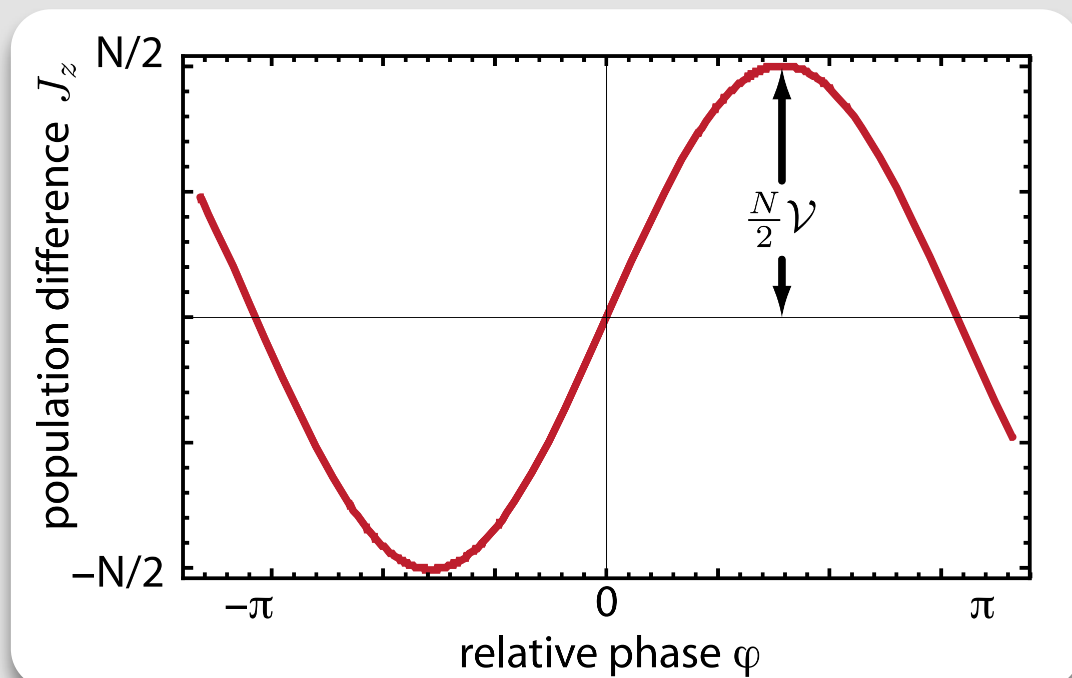


coherent spin state



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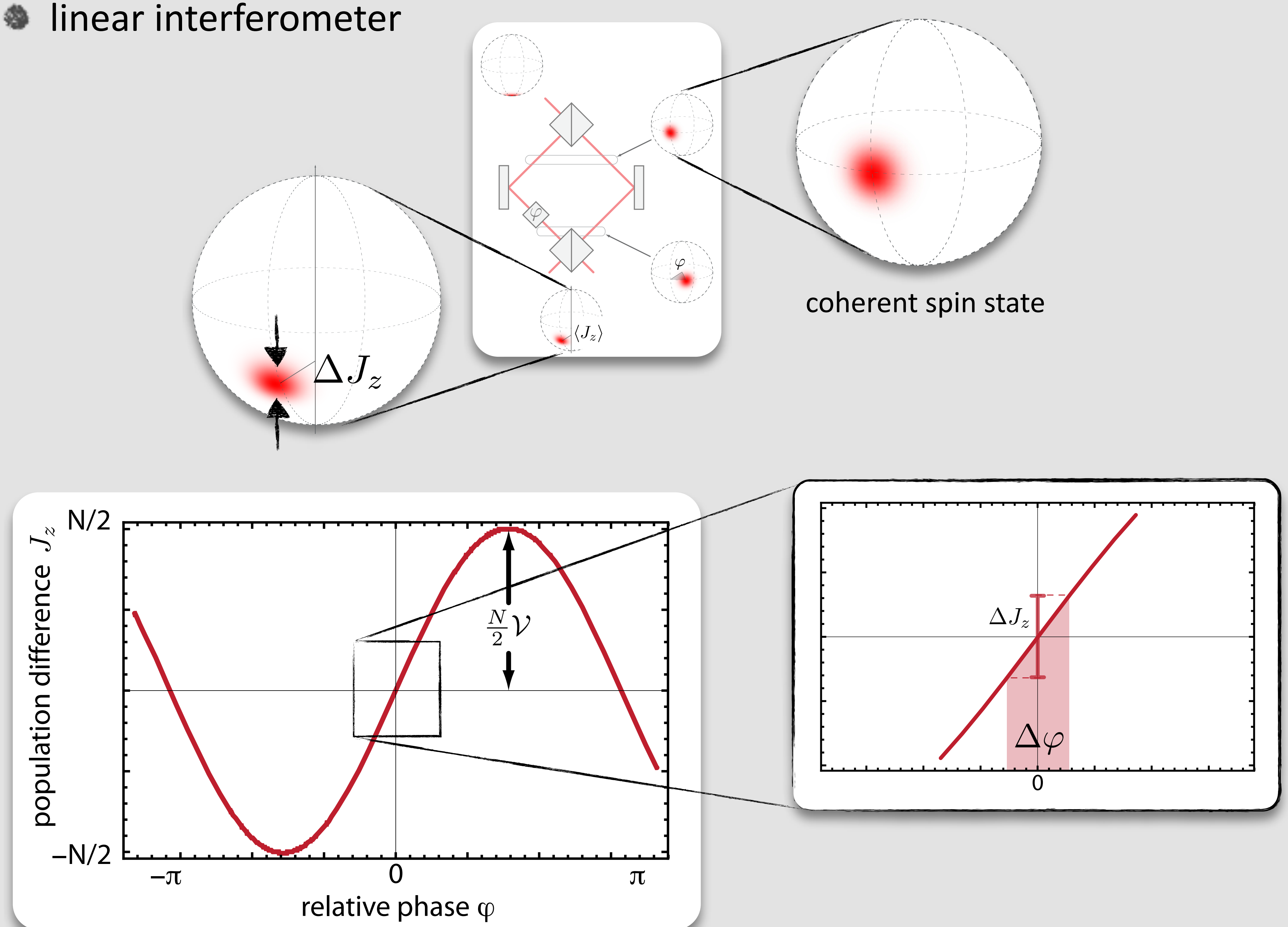

▷ phase precision:

$$\Delta\varphi = \frac{\Delta J_z}{\frac{N}{2}\mathcal{V}}$$

"Classical" interferometric precision limit

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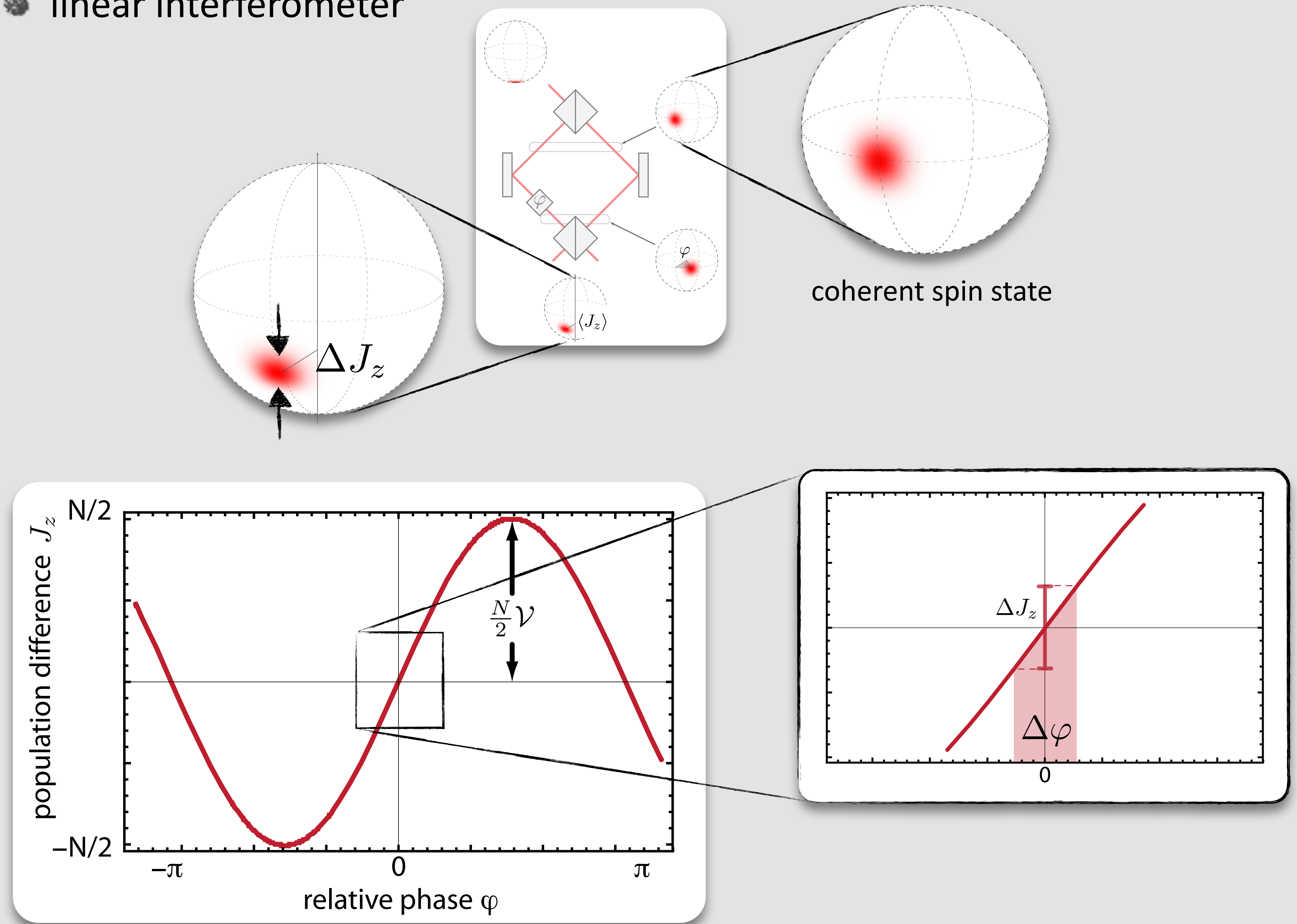

for a "classical" coherent spin state

$\sqrt{N/2}$ 1

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“Classical” interferometric precision limit

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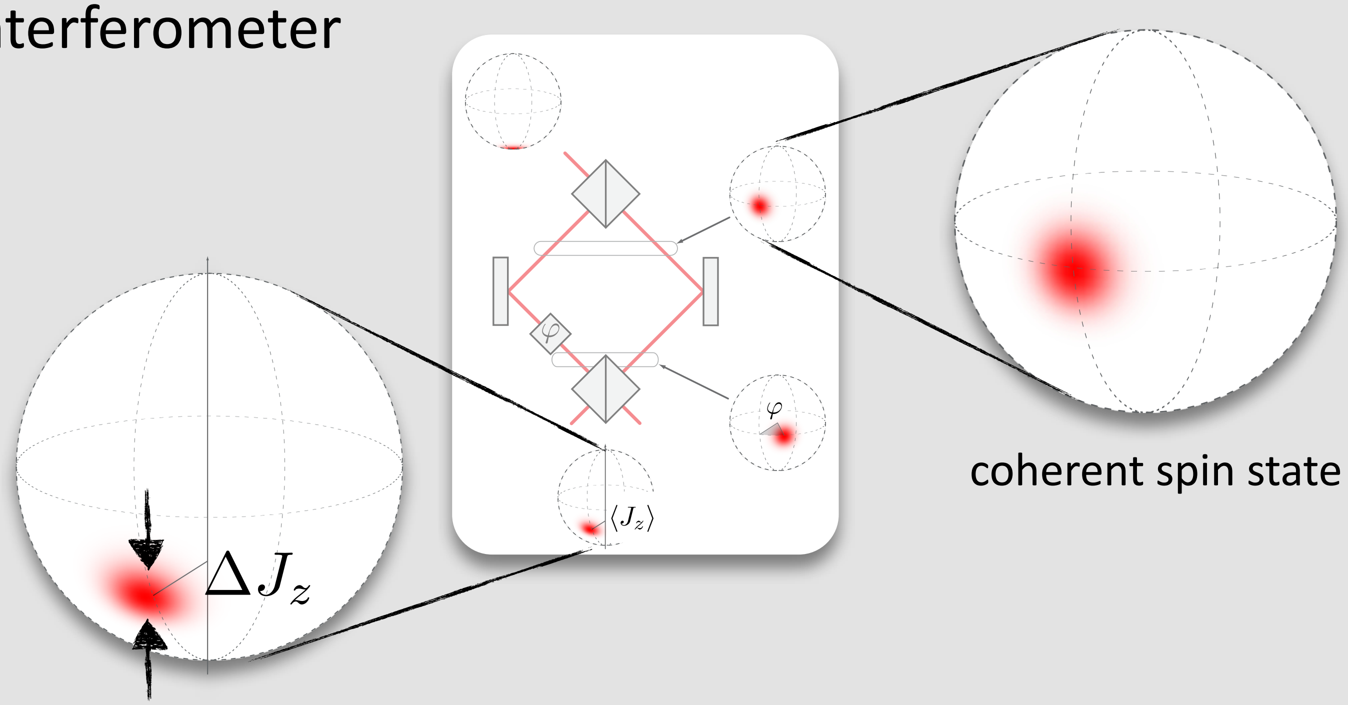

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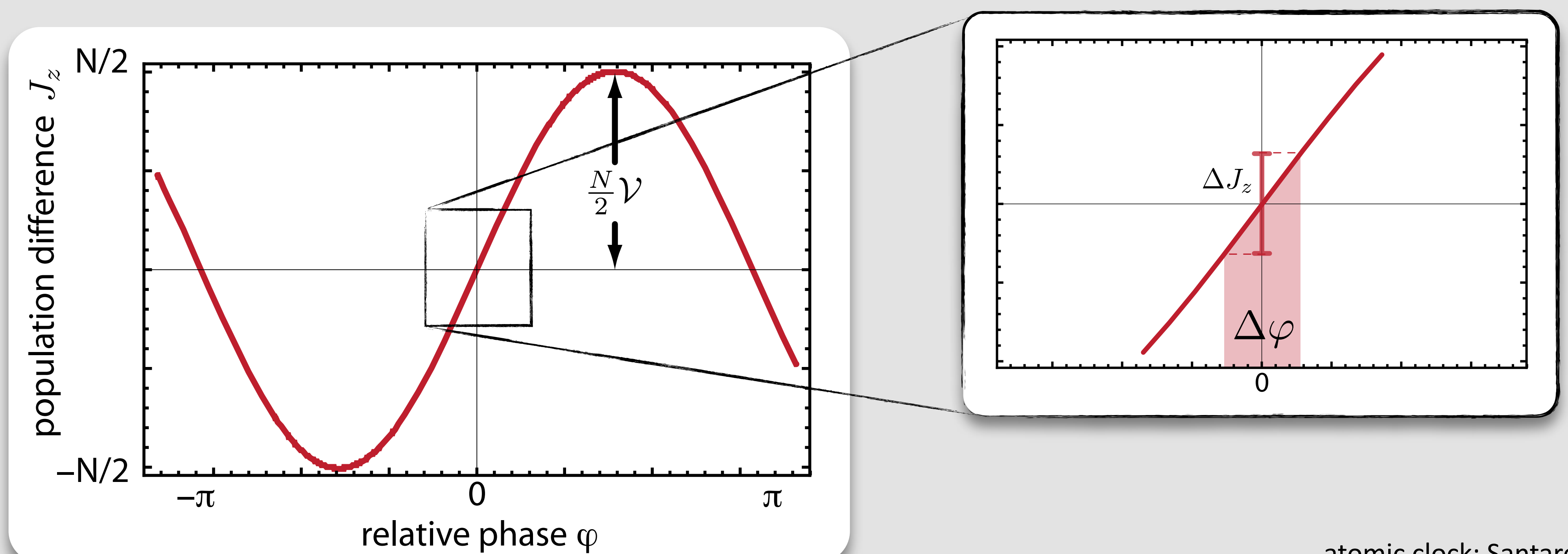
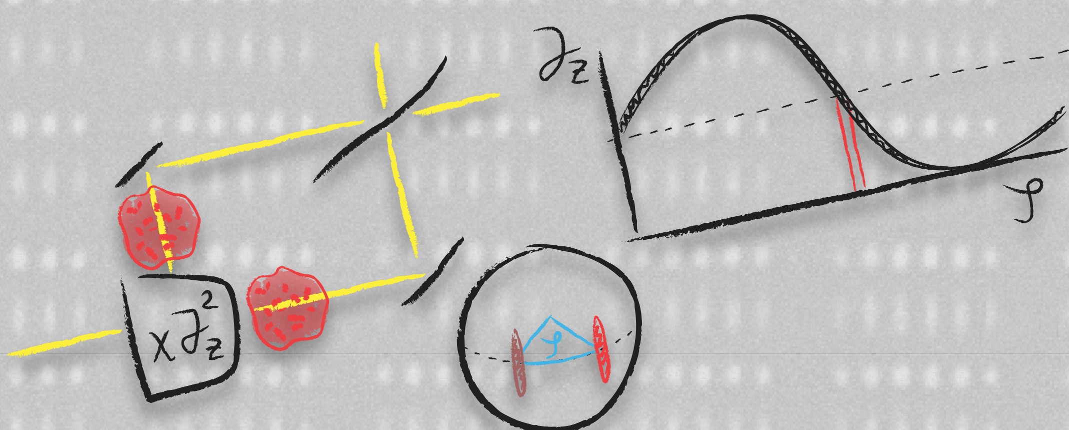


$$\Delta\varphi = \frac{1}{\sqrt{N}}$$

standard quantum limit

atomic clock: Santarelli et al., PRL 82, 4619 (1999)
magnetic field sensing: Budker & Romalis, Nat. Phys. 3, 227 (2007)

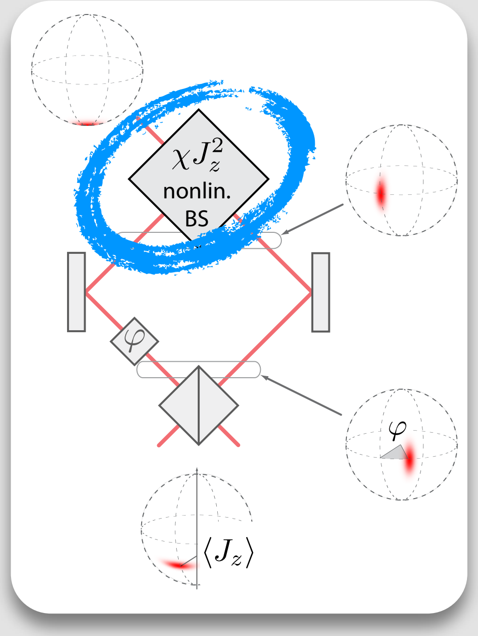
Outline



- A nonlinear atom interferometer
  - phase precision beyond the “classical” limit -

Spin squeezing and nonlinear interferometry

- **Nonlinear** interferometer



Spin squeezing and nonlinear interferometry

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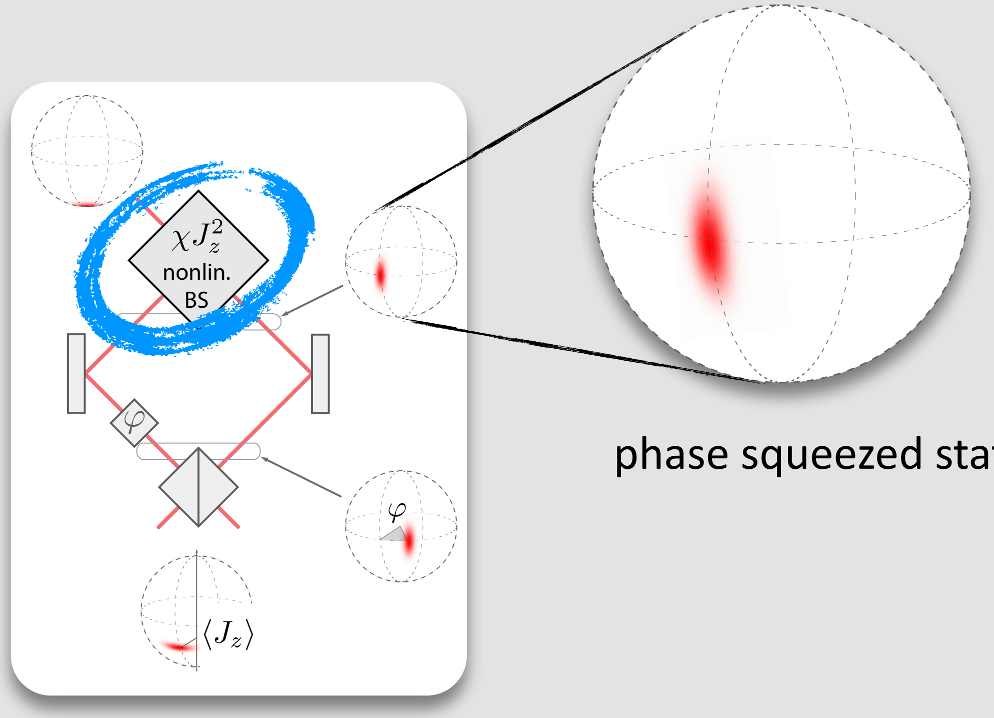

phase squeezed state

Spin squeezing and nonlinear interferometry

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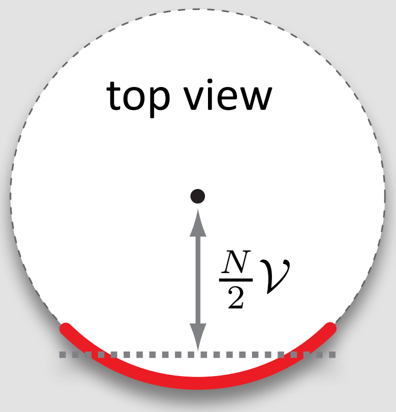
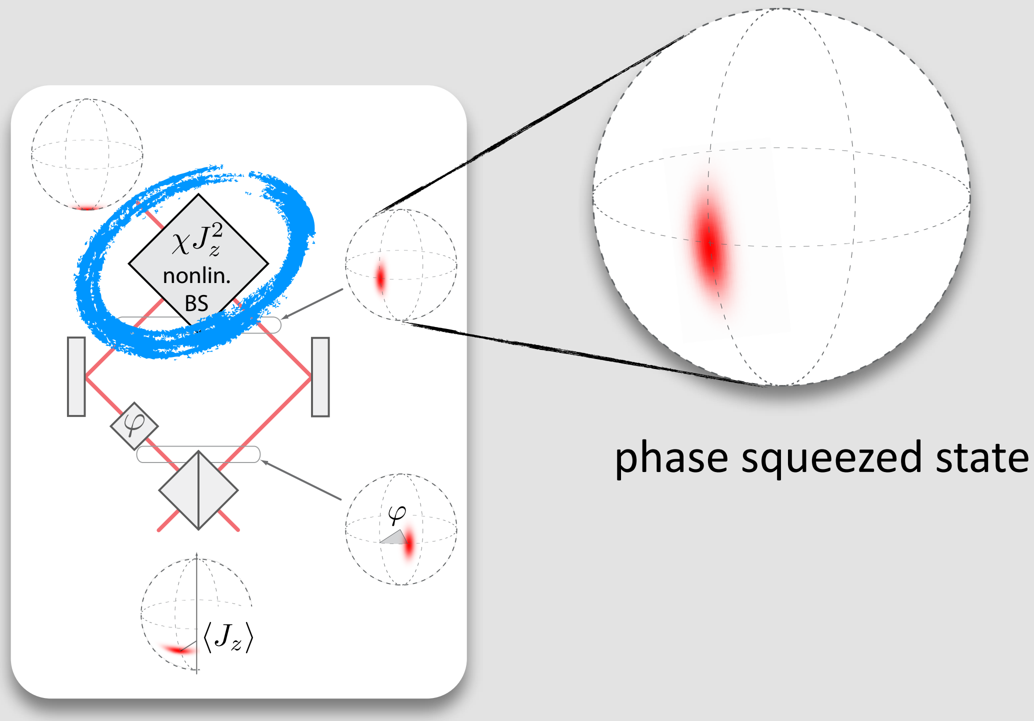
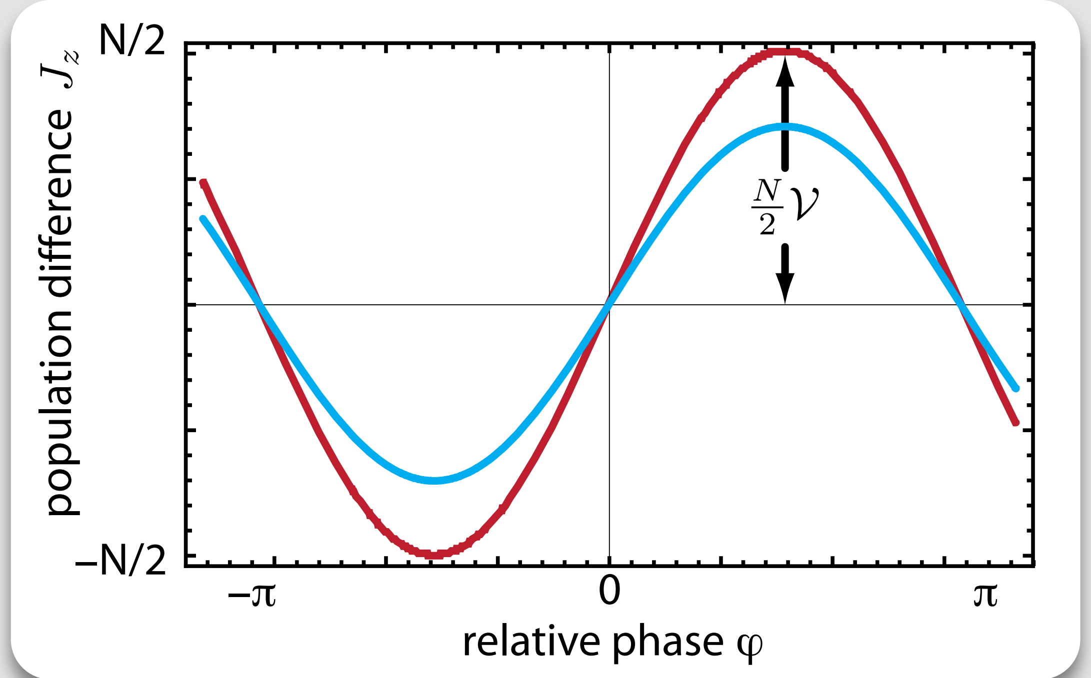




- Phase estimation precision:



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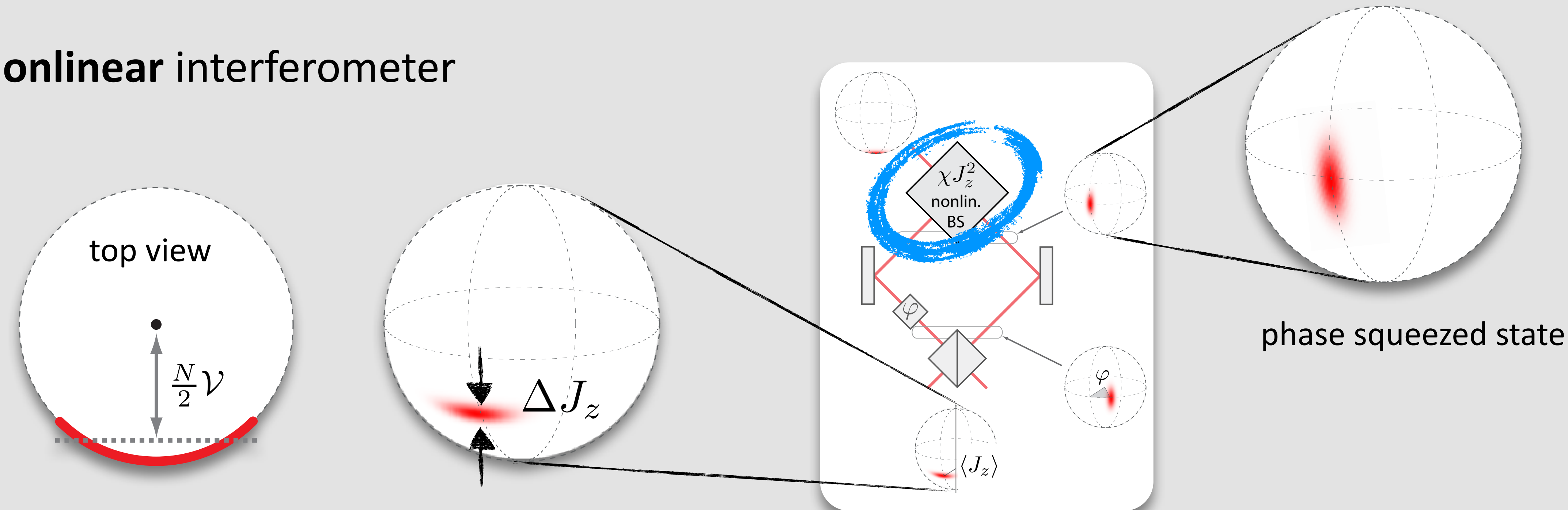

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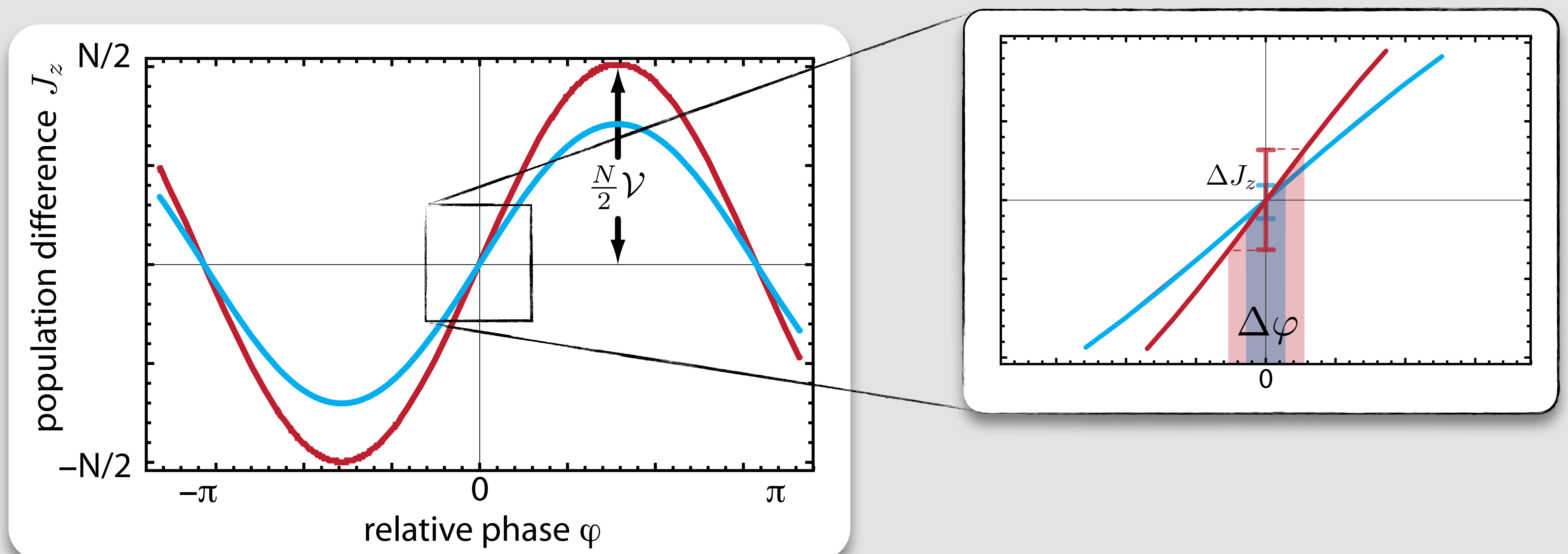

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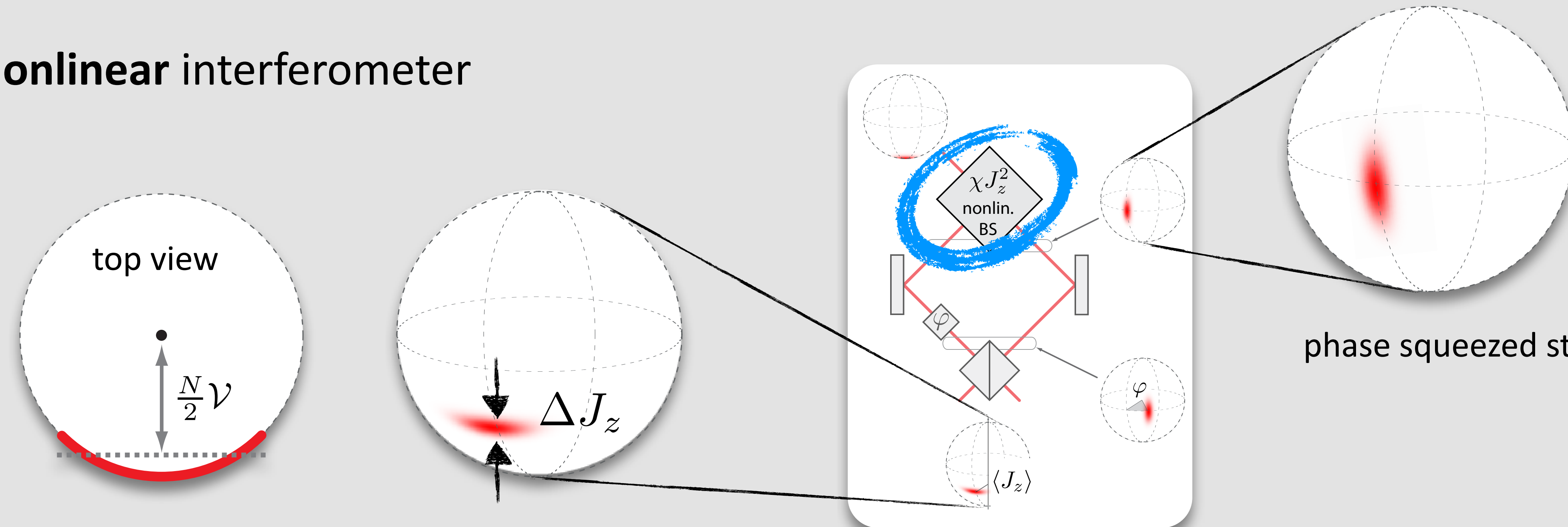

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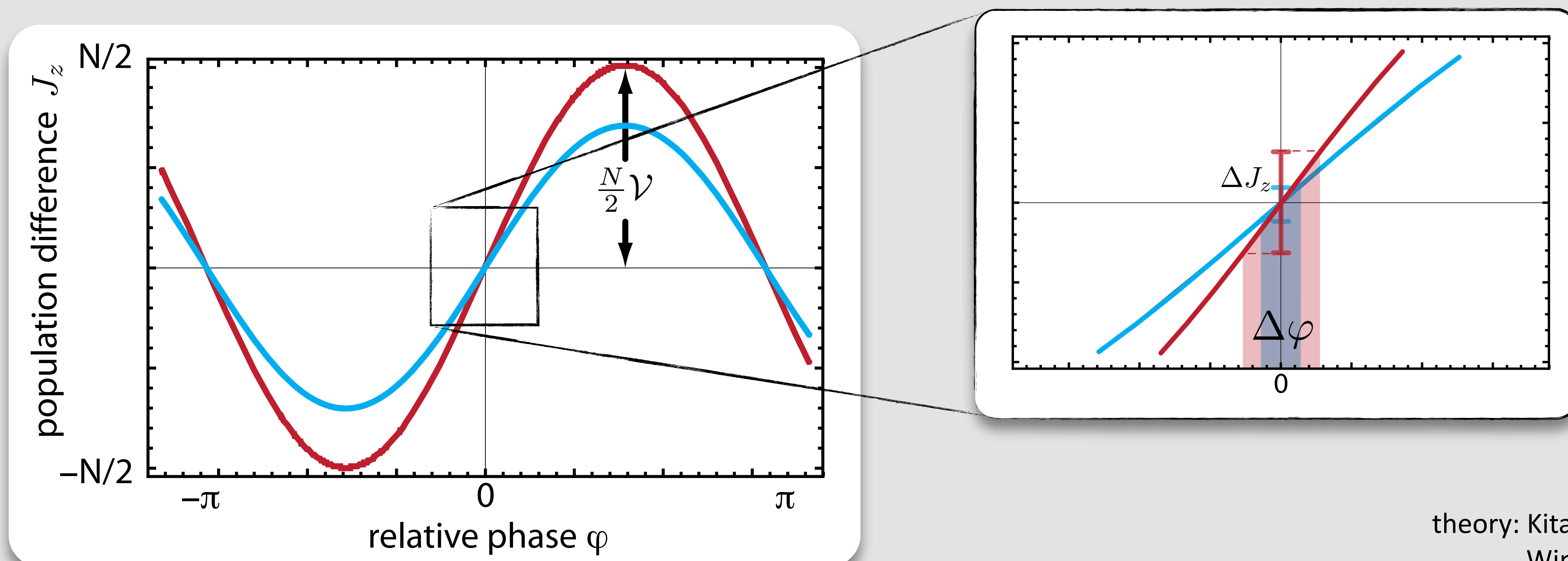


▹ phase precision with spin squeezed states:

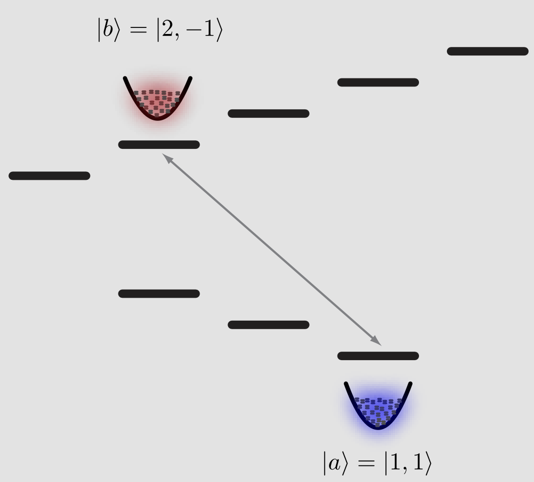
$$\Delta\varphi = \xi_S \frac{1}{\sqrt{N}}$$

theory: Kitagawa & Ueda, PRA 47, 5138 (1993)
Wineland et al., PRA 50, 67 (1994)

Experiments with Bose-Einstein condensates...

Optical trapping of Rb BEC’s in two internal states

- Rubidium hyperfine manifold



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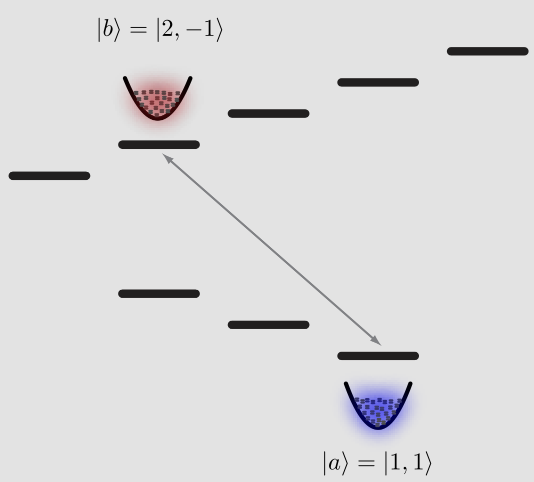
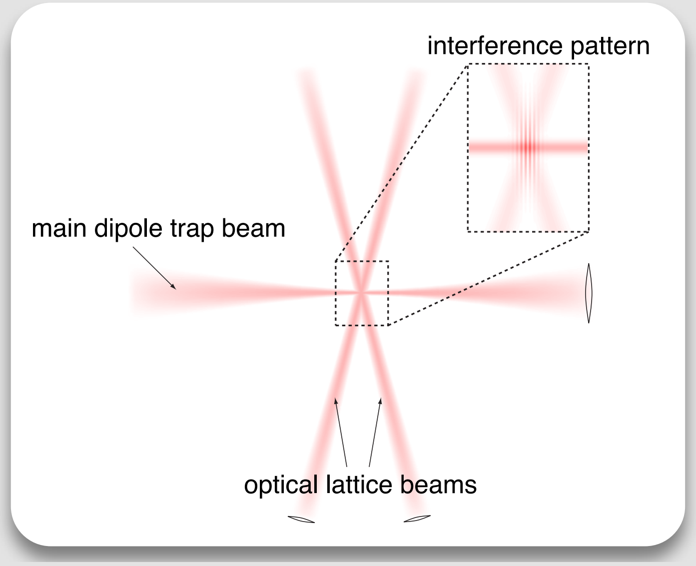


- optical trapping in six-well lattice



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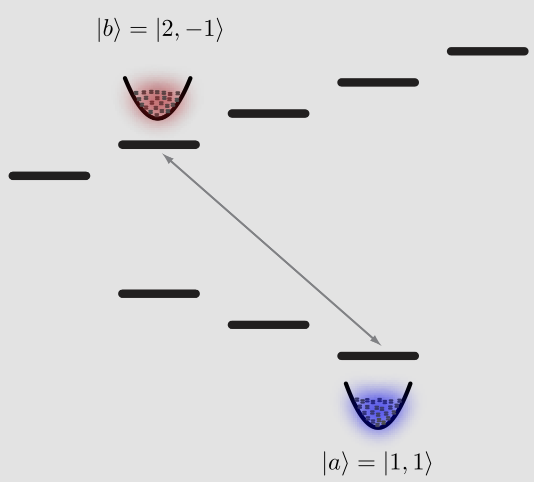

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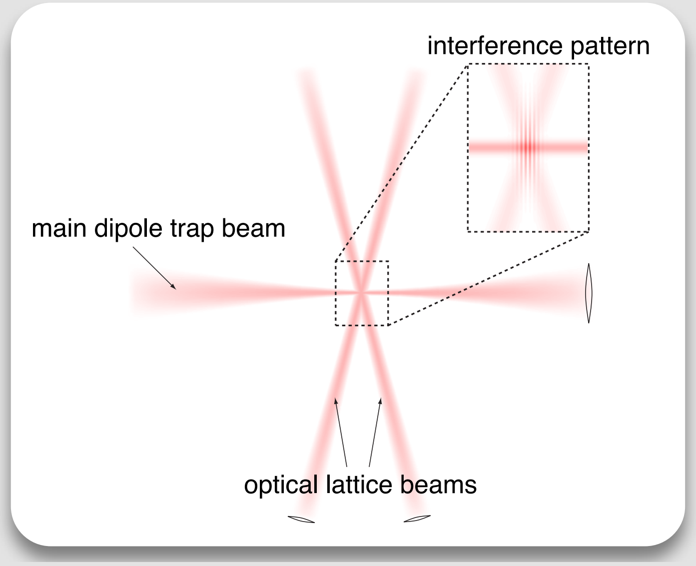
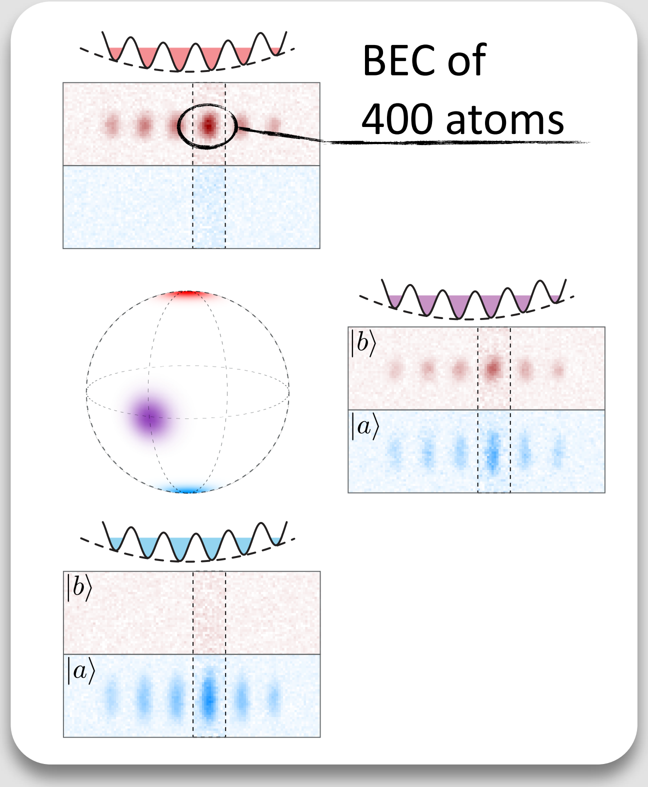


- High resolution state selective detection



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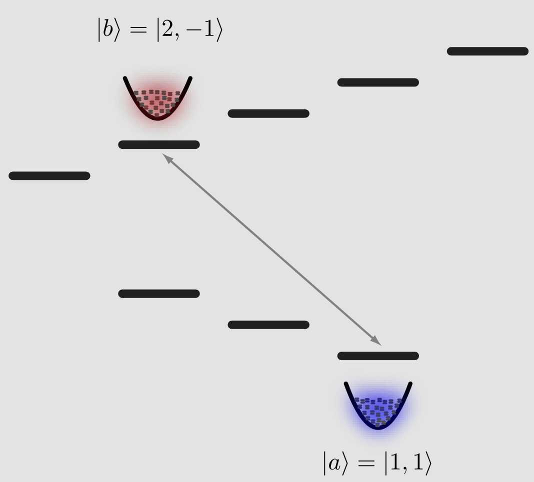

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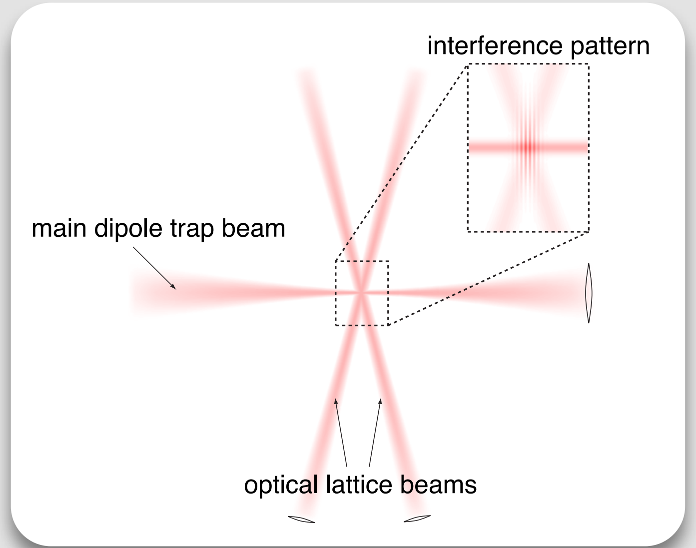

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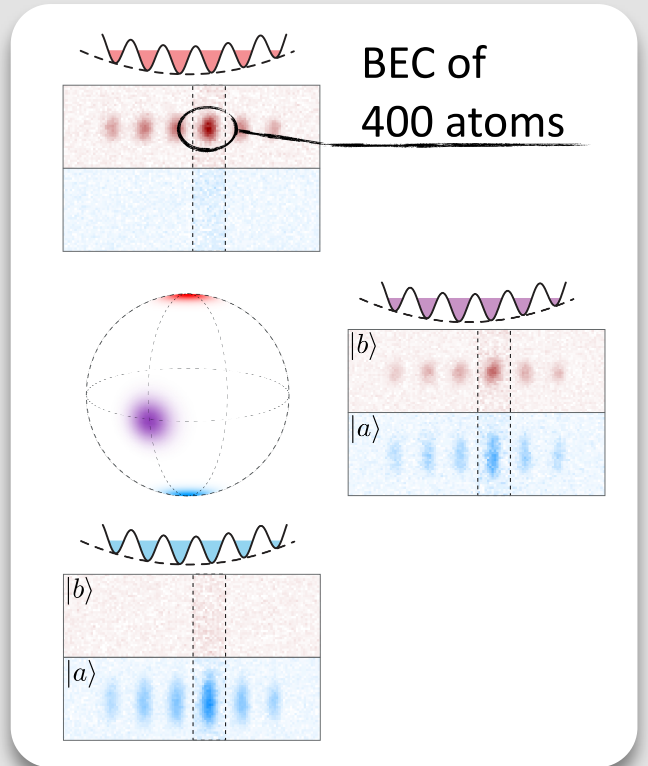
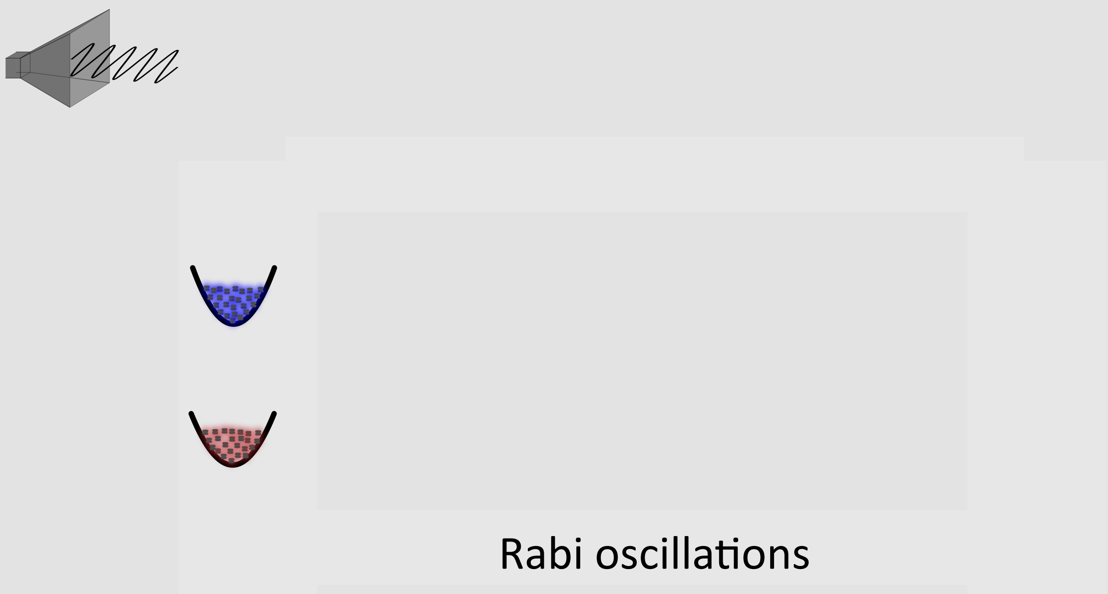


- 2-Photon microwave + radio-frequency coupling



Sonntag, 13. Juni 2010

Optical trapping of Rb BEC's in two internal states

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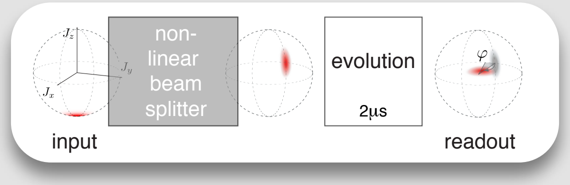
- High resolu... adio-frequency coupling

...bi oscillations

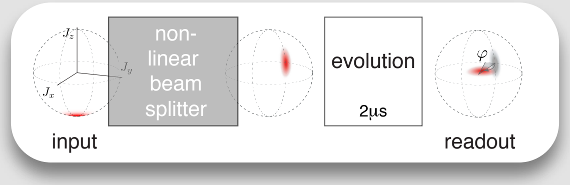
Nonlinear interferometer for Bose-Einstein condensates

- Non-linear interferometric sequence

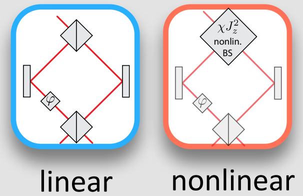


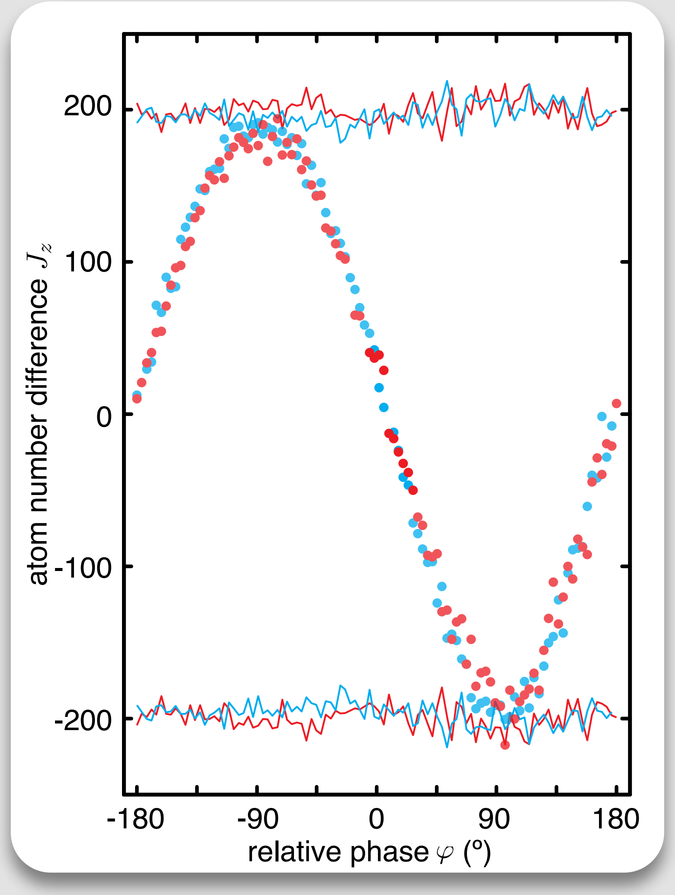
Nonlinear interferometer for Bose-Einstein condensates

- Non-linear interferometric sequence



- direct comparison: linear vs nonlinear interferometer





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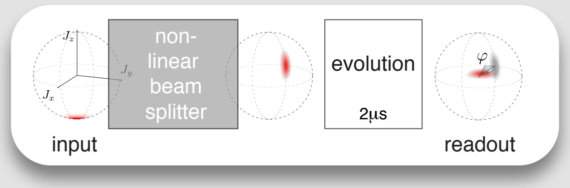

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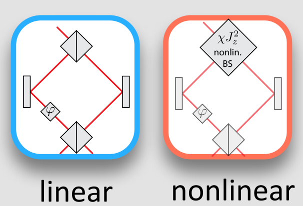

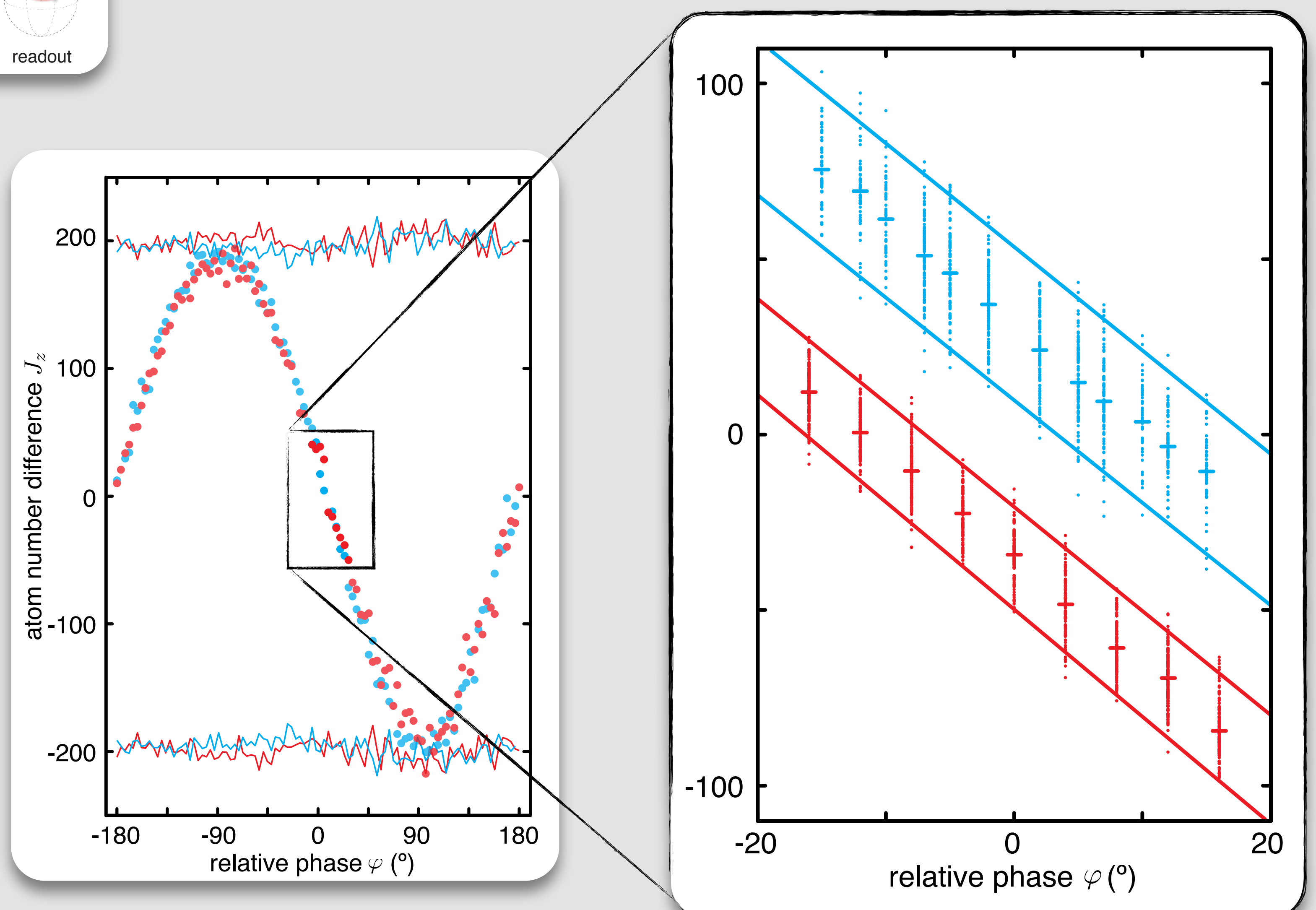

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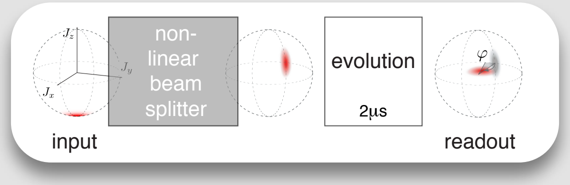

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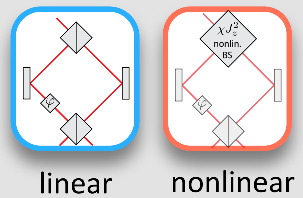

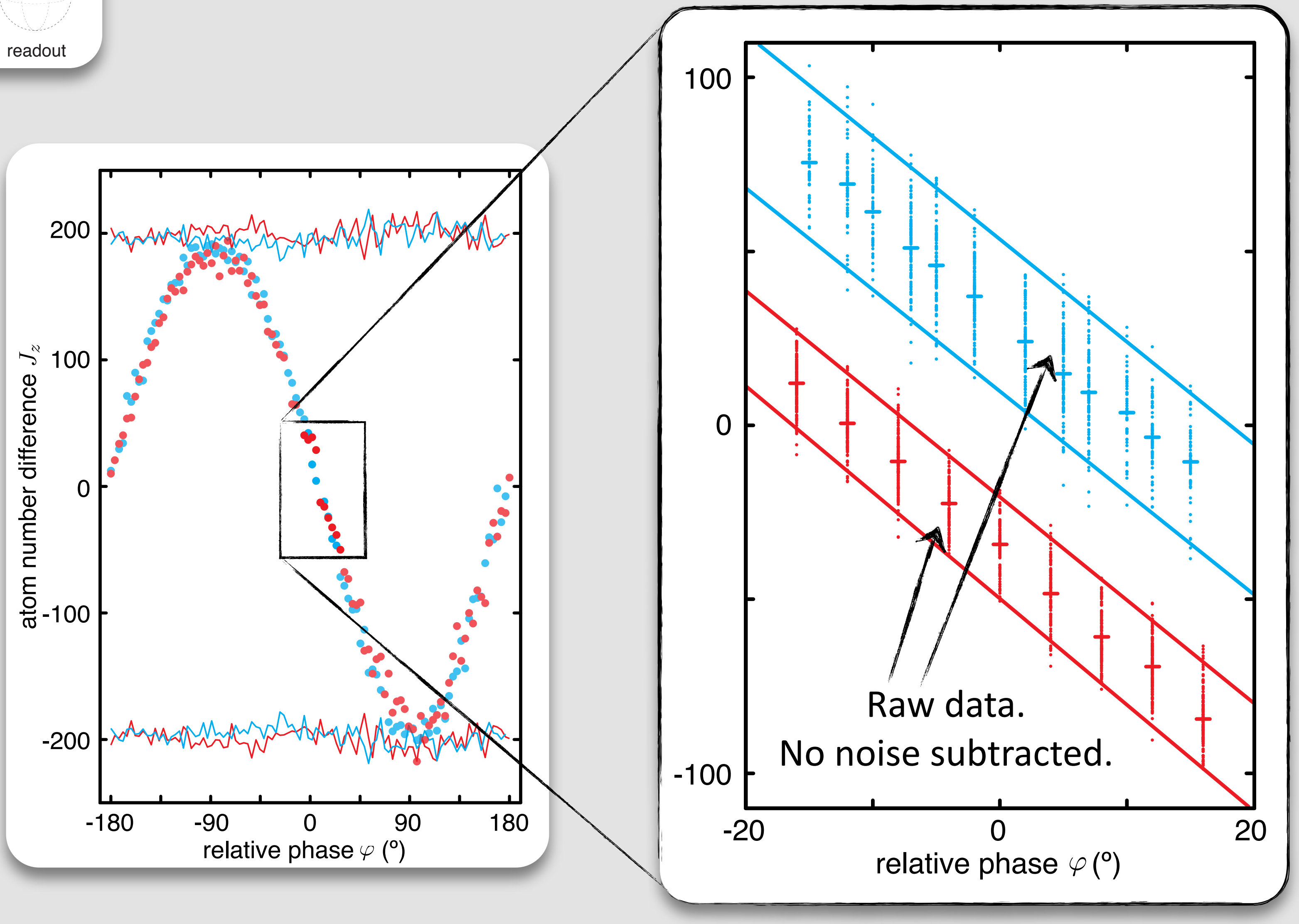

Nonlinear interferometer for Bose-Einstein condensates

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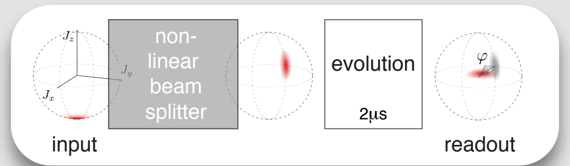

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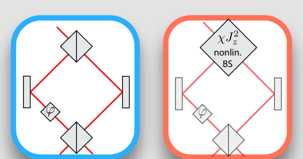

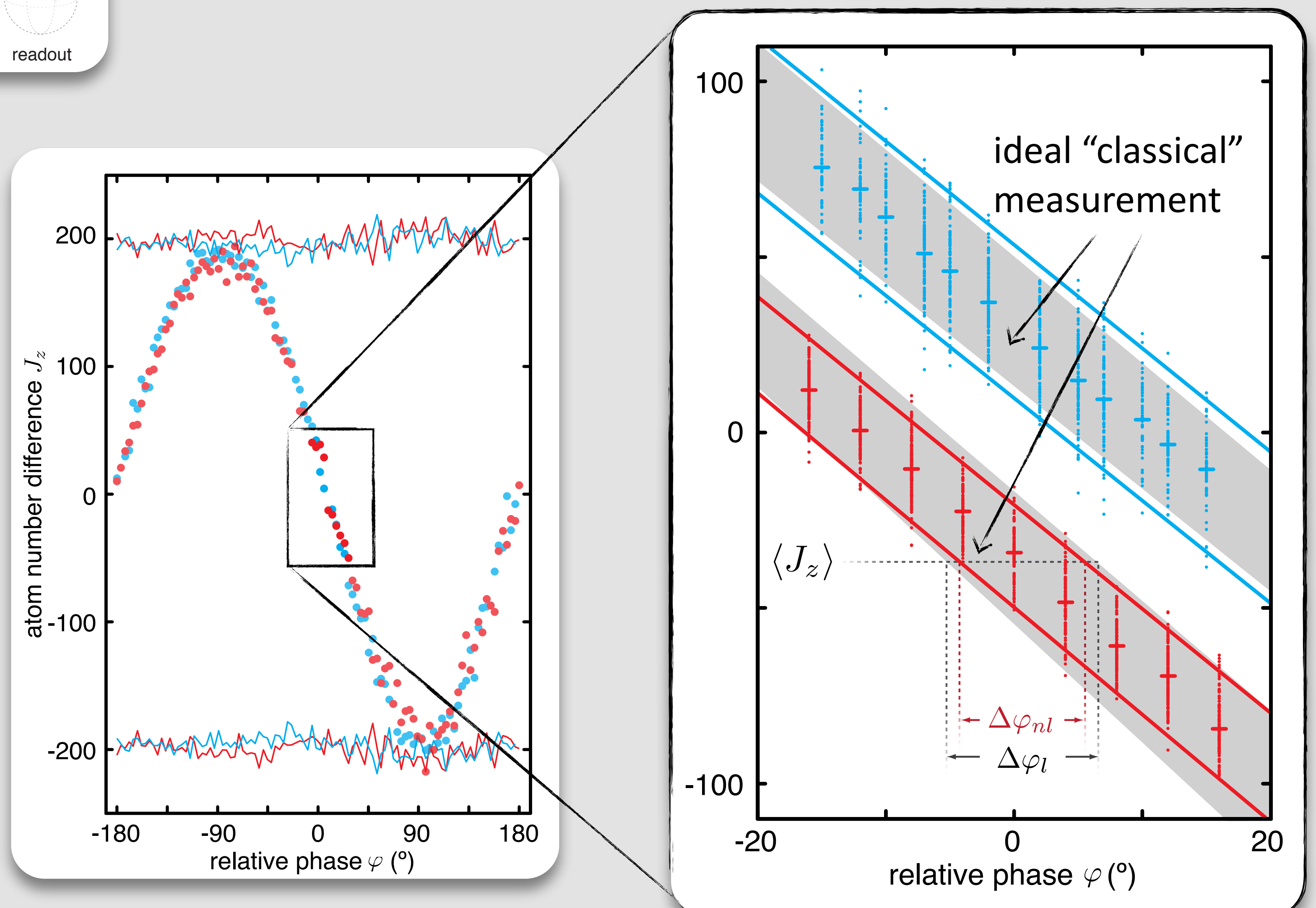

Nonlinear interferometer for Bose-Einstein condensates

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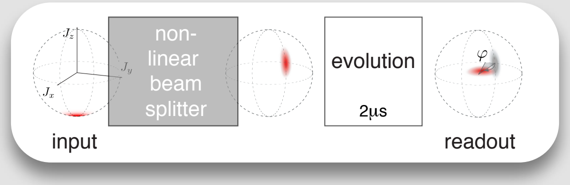

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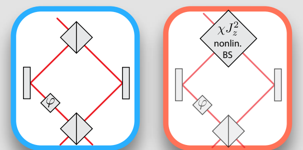
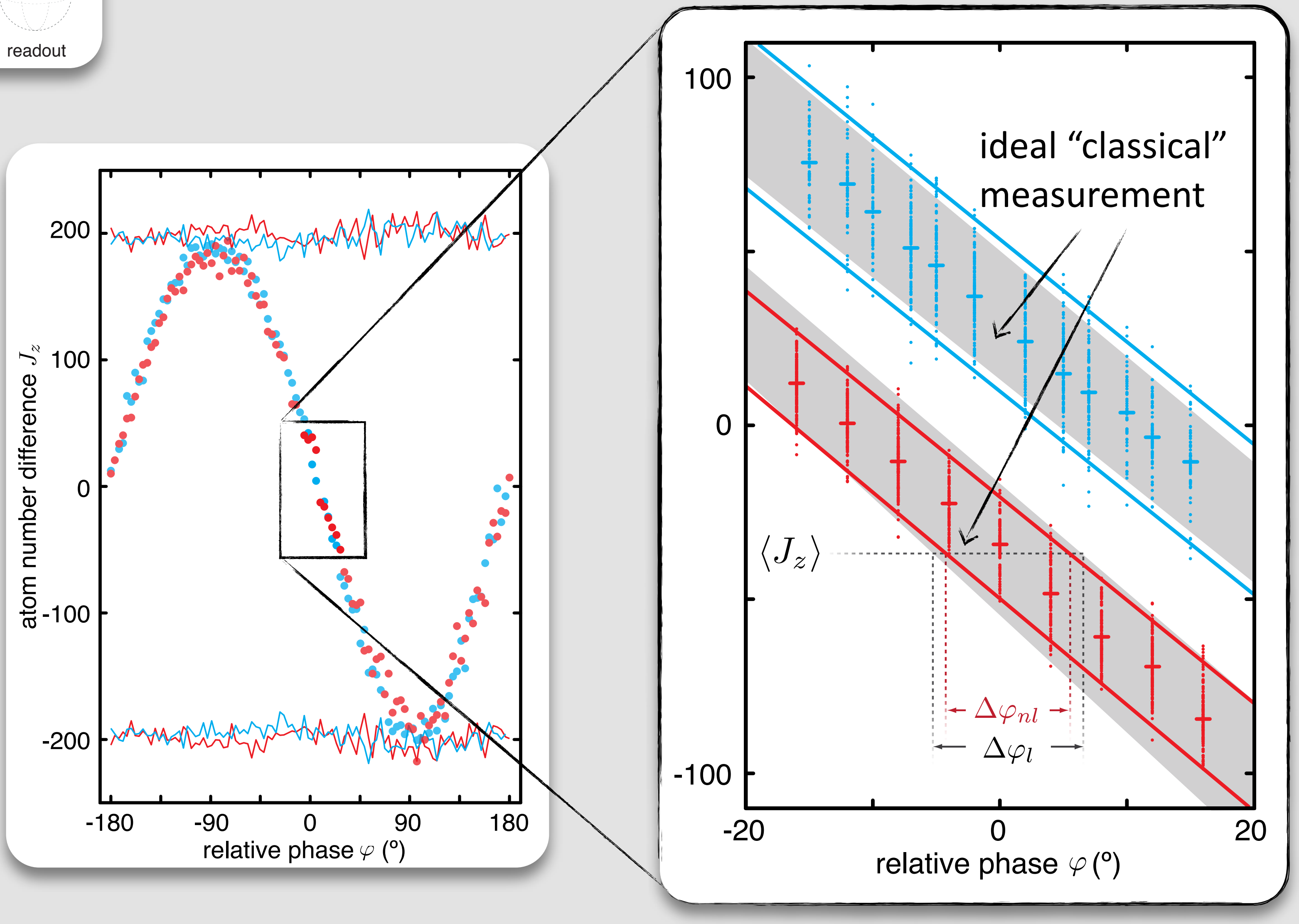
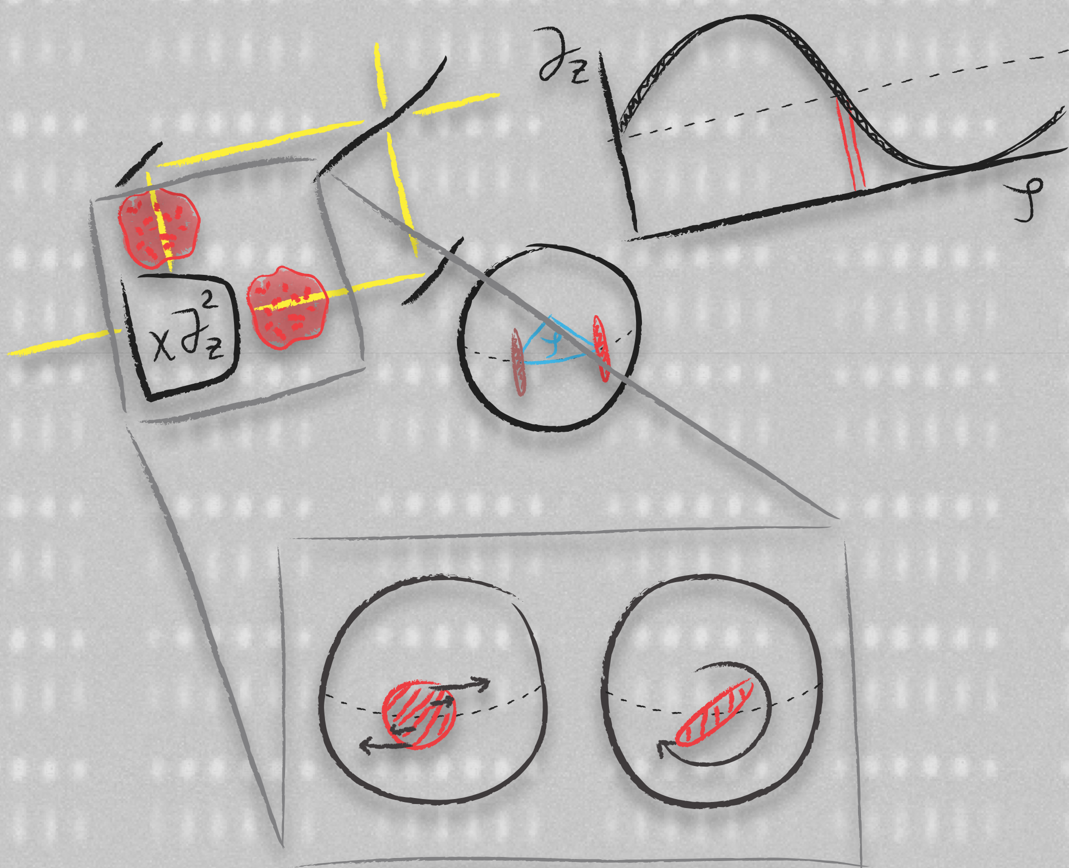




phase precision 15% higher than for an <u>ideal</u> linear interferometer

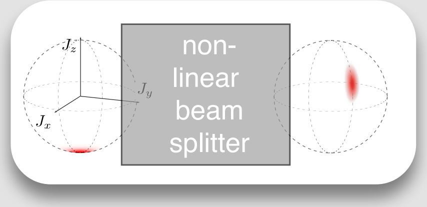
see also: Louchet-Chauvet et al., arXiv:0912.3895v2 (2010)

Outline

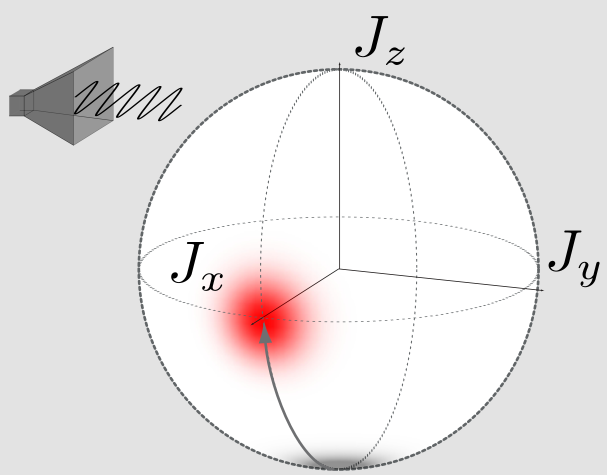


- The nonlinear beamsplitter
  - Noise tomography reveals spin squeezing -
  - Detection of a large entangled state -

The nonlinear beamsplitter



- Spin squeezing via "one axis twisting"

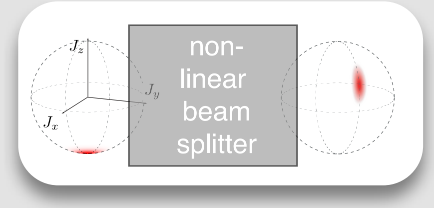


$$H = -\Omega J_y$$

- rotation around J_y

Sonntag, 13. Juni 2010

The nonlinear beamsplitter



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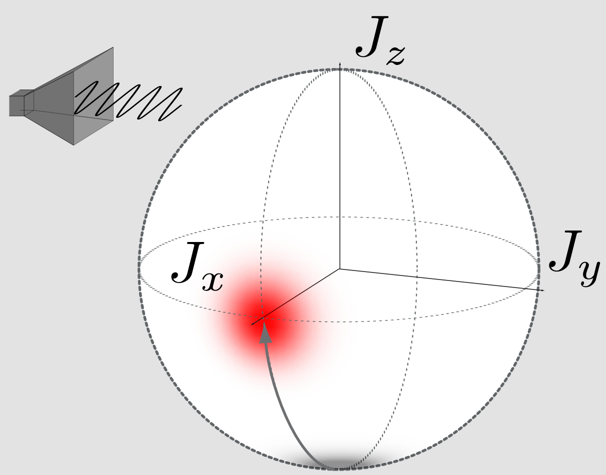

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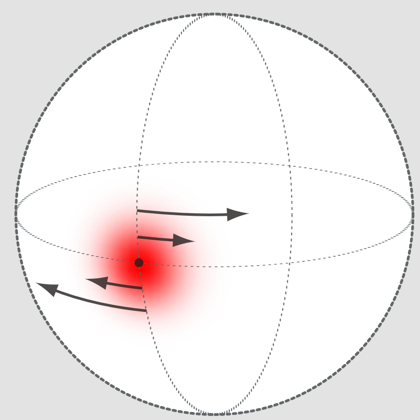
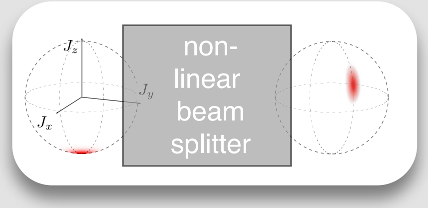


$$H = \chi J_z^2$$

- non-linear - J_z dependent - rotation around J_z

The nonlinear beamsplitter



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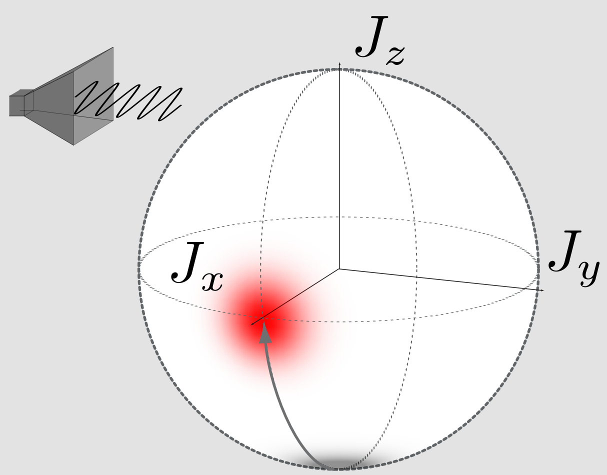

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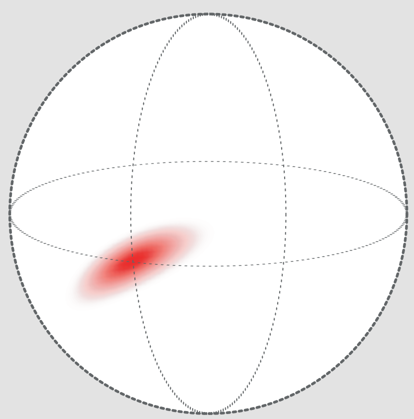

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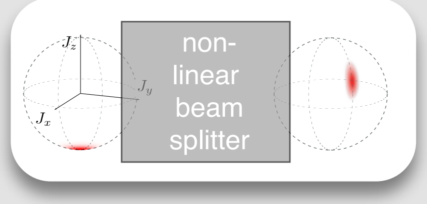
The nonlinear beamsplitter



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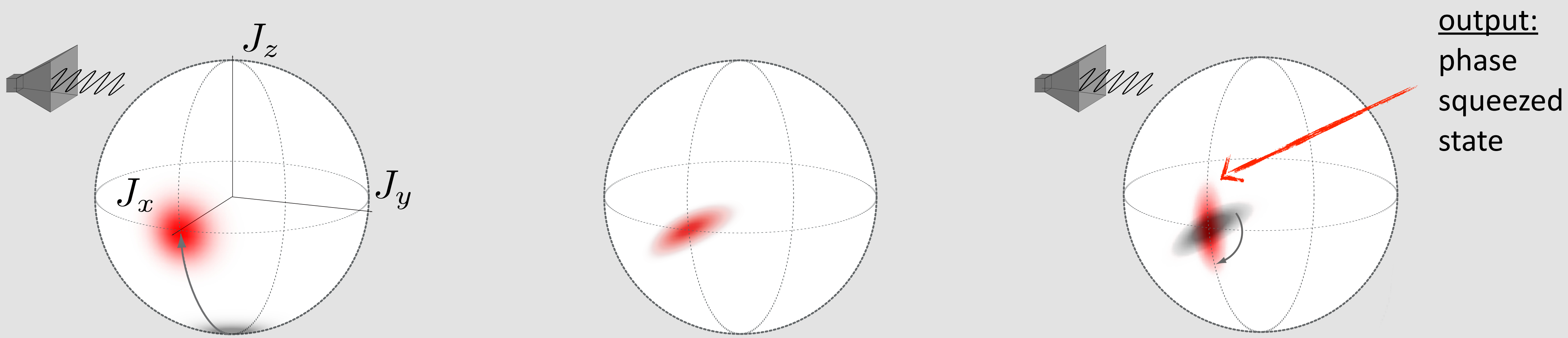
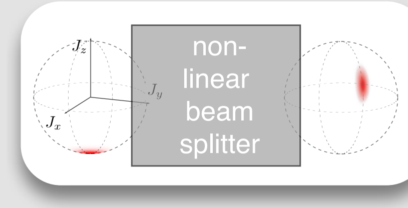


$H = -\Omega J_y$	$H = \chi J_z^2$	$H = -\Omega J_x$
▷ rotation around J_y	▷ non-linear - J_z dependent - rotation around J_z	▷ rotation around the center of the spin state (J_x)

The nonlinear beamsplitter



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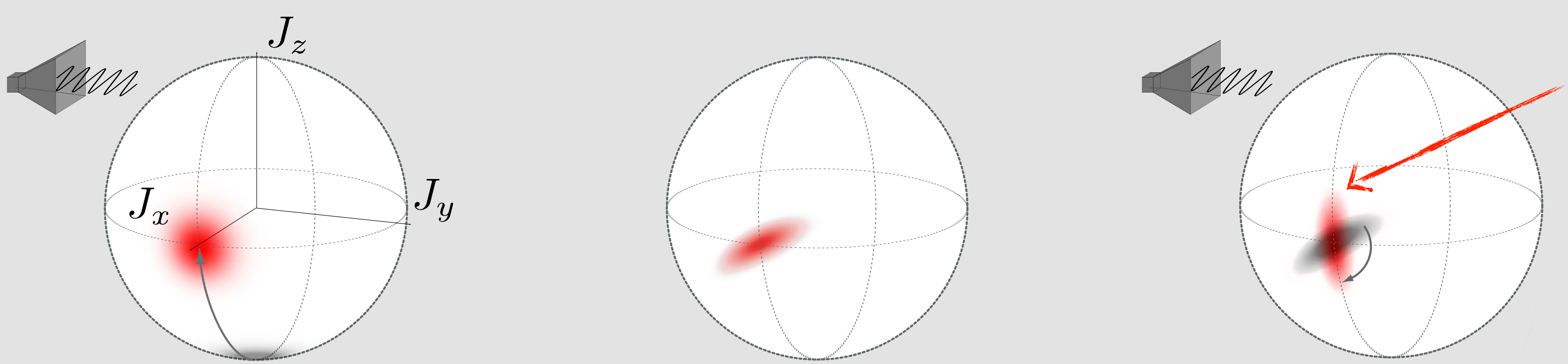


output:
phase
squeezed
state

$$H = -\Omega J_y$$

- rotation around J_y

$$H = \chi J_z^2$$

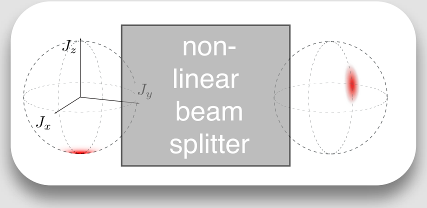
- non-linear - J_z dependent - rotation around J_z

$$H = -\Omega J_x$$

- rotation around the center of the spin state (J_x)

Interaction χ needed!

The nonlinear beamsplitter



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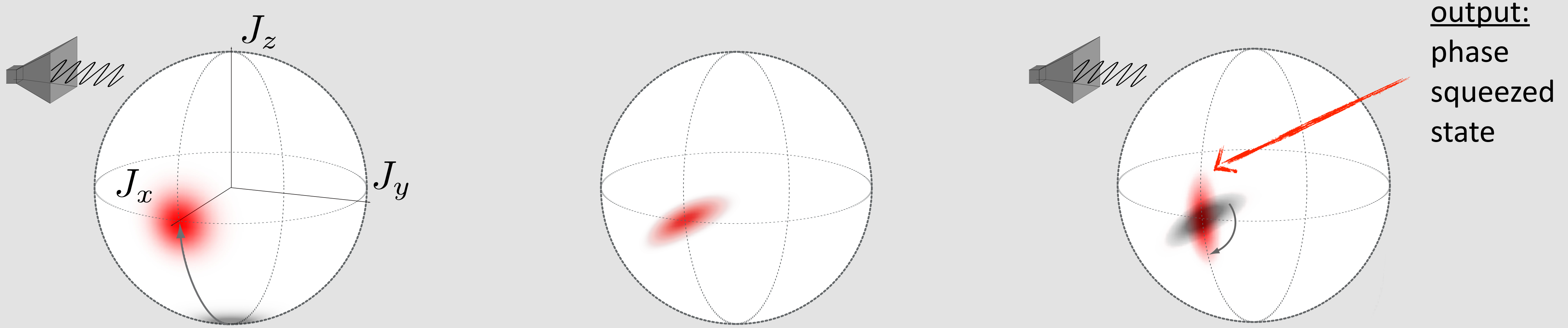

$H = -\Omega J_y$

- rotation around J_y

$H = \chi J_z^2$

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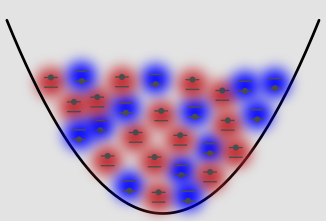
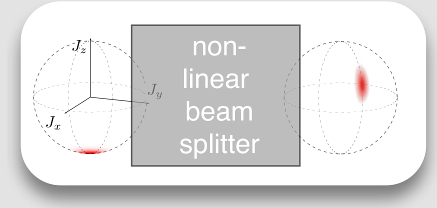


$$\chi \propto a_{aa} + a_{bb} - 2a_{ab}$$

for fully mixed sample
(single spatial mode approximation)

The nonlinear beamsplitter



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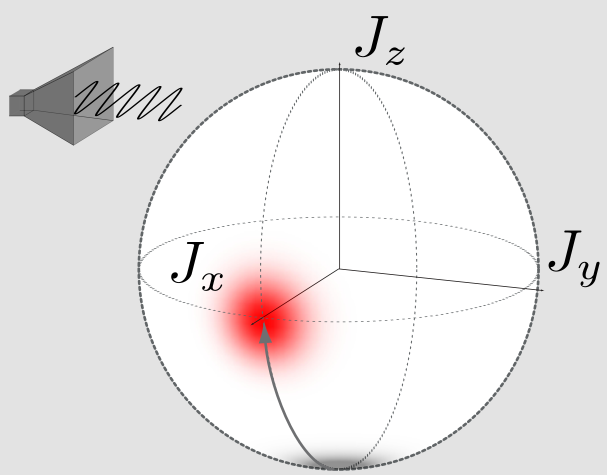

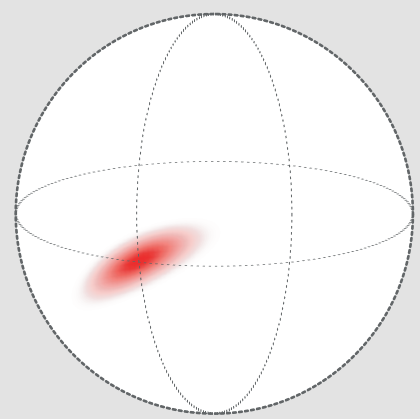

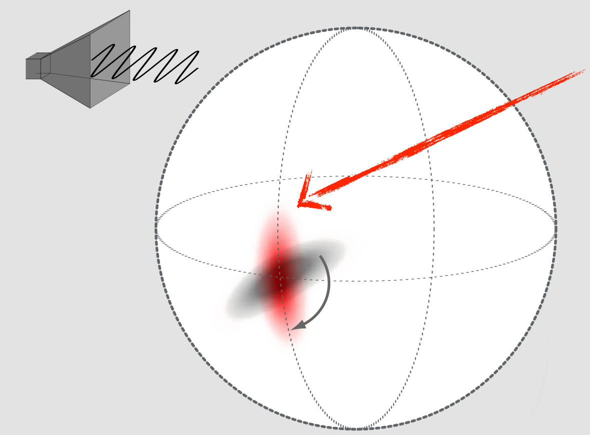

output:
phase
squeezed
state

$$H = -\Omega J_y$$

- rotation around J_y

$$H = \chi J_z^2$$

- non-linear - J_z dependent - rotation around J_z

$$H = -\Omega J_x$$

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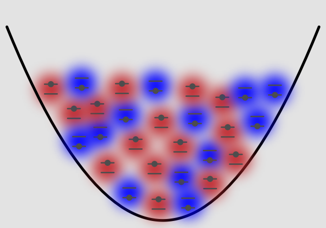
Interaction χ needed!



$$\chi \propto a_{aa} + a_{bb} - 2a_{ab}$$

for fully mixed sample
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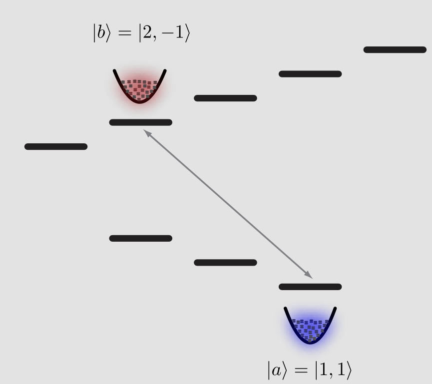
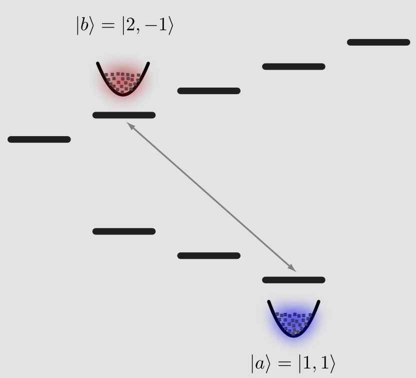
^{87}Rubidium

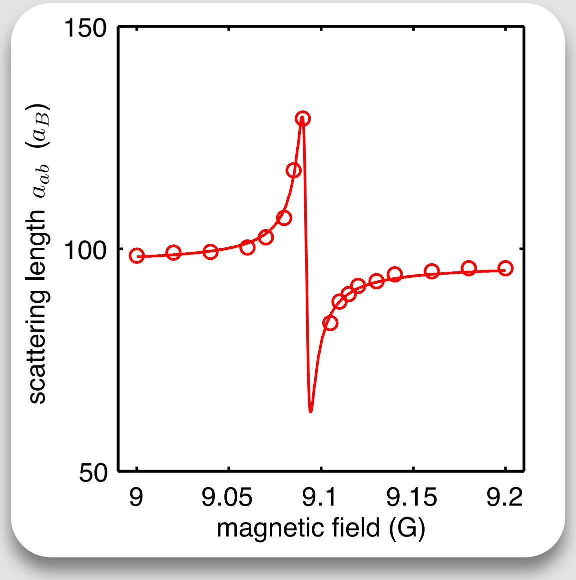


$$\chi \approx 0$$

Interaction tuning

- Interaction tuning - a narrow interspecies Feshbach resonance





- tuning of the interspecies scattering length
  - **increase** of the nonlinearity χ

Interaction tuning

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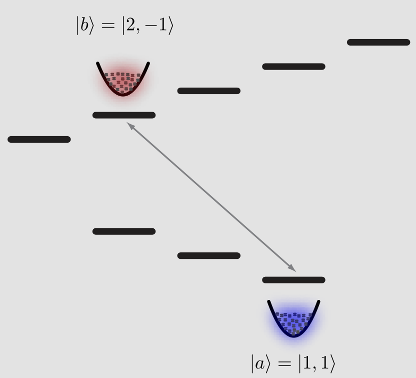

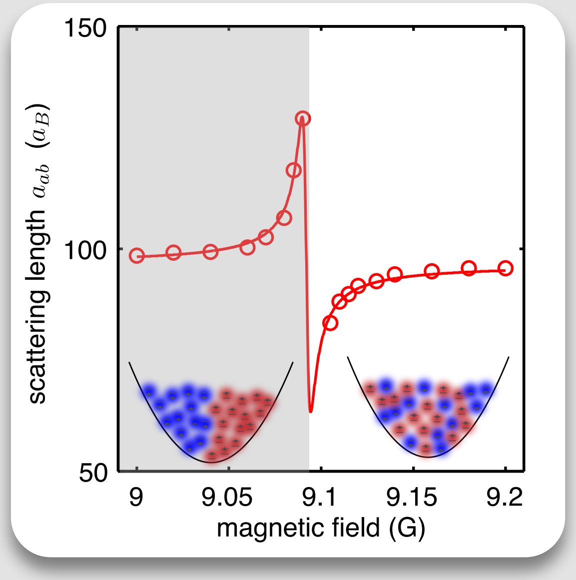

- tuning of the interspecies scattering length
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  - **miscible** regime accessible

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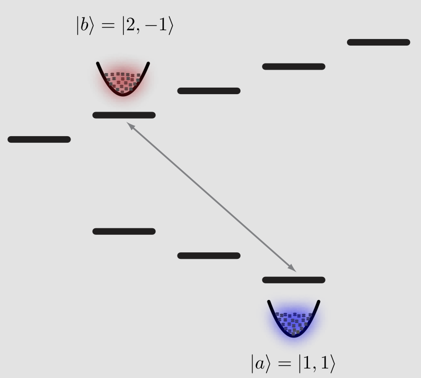

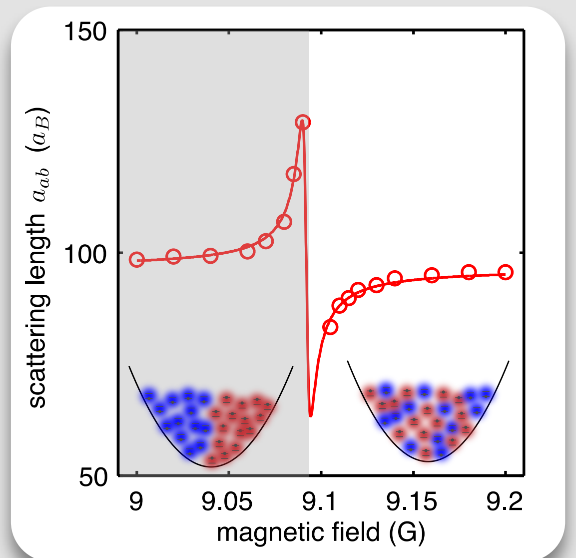

- tuning of the interspecies scattering length
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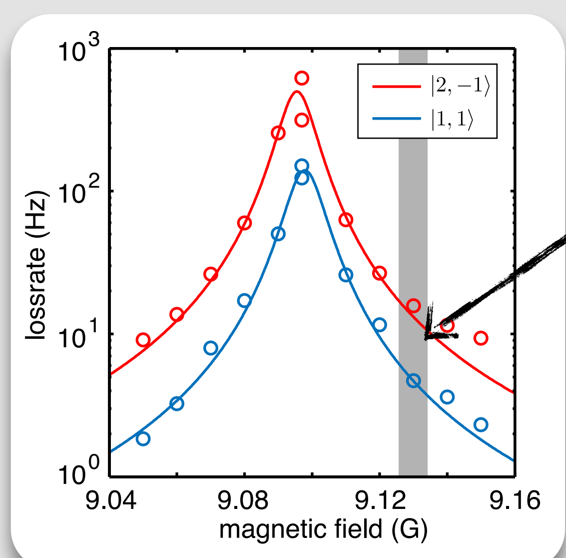
  - **miscible** regime accessible

- Increased nonlinearity but also higher loss rate



loss rate

$\tau^{-1} \approx 10\,\mathrm{Hz}$

expected loss during experimental sequence:

$10 - 15\,\%$

Interaction tuning

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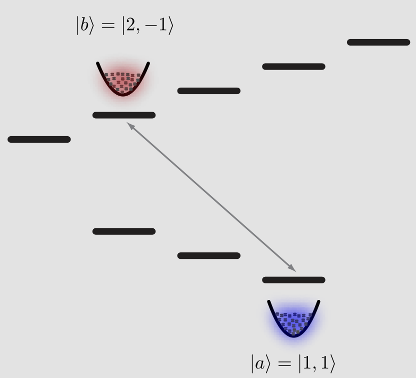

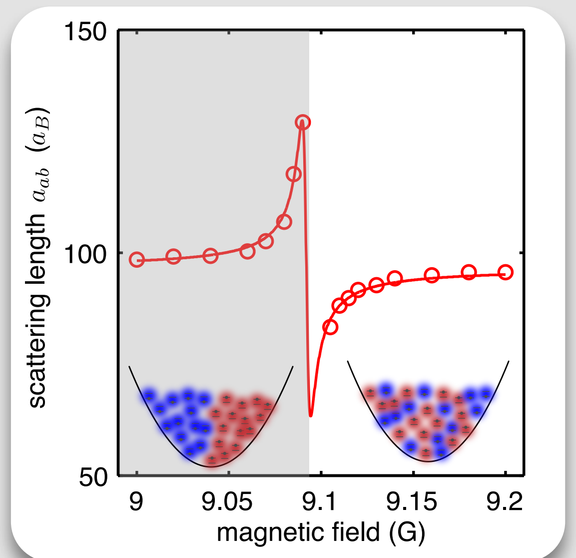

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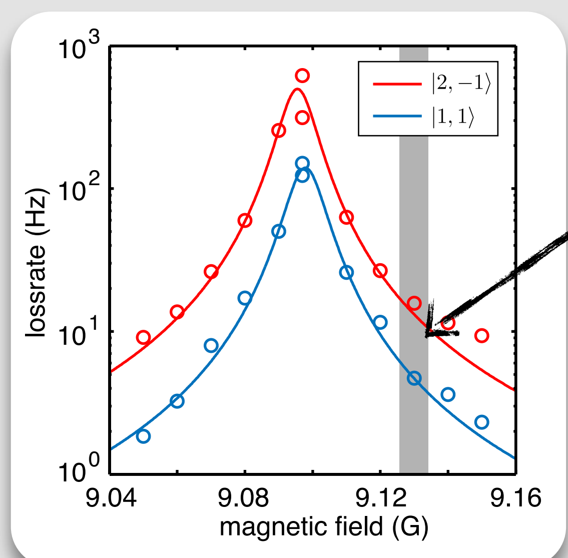

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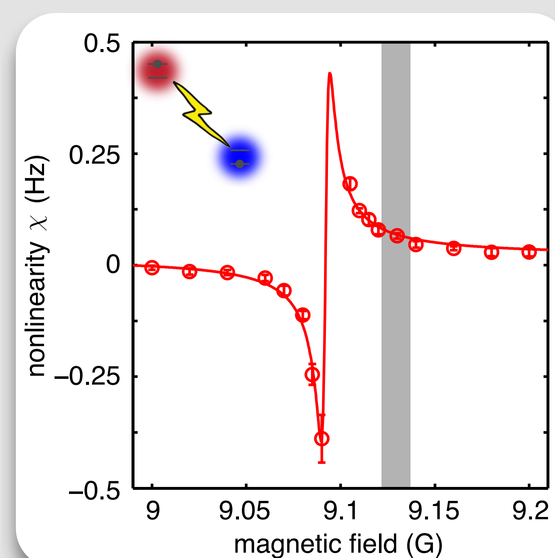
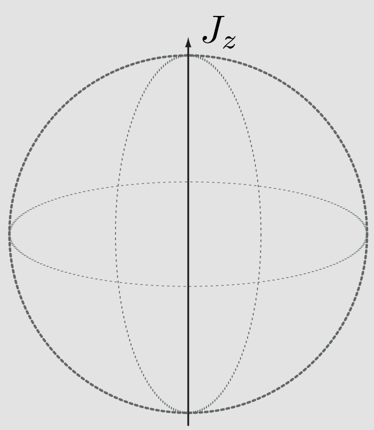


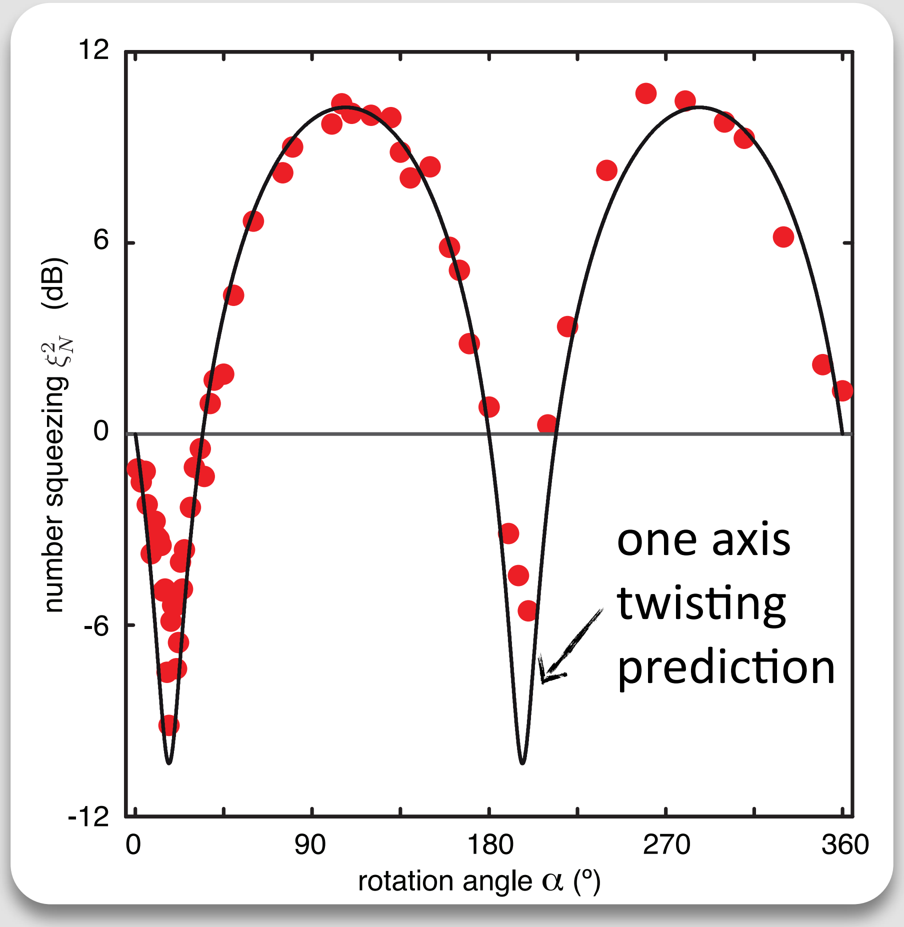
- effective nonlinearity at $B = 9.13\,\mathrm{G}$

$$\chi = 0.063\,\mathrm{Hz}$$

Noise tomography

- Mapping of the noise ellipse





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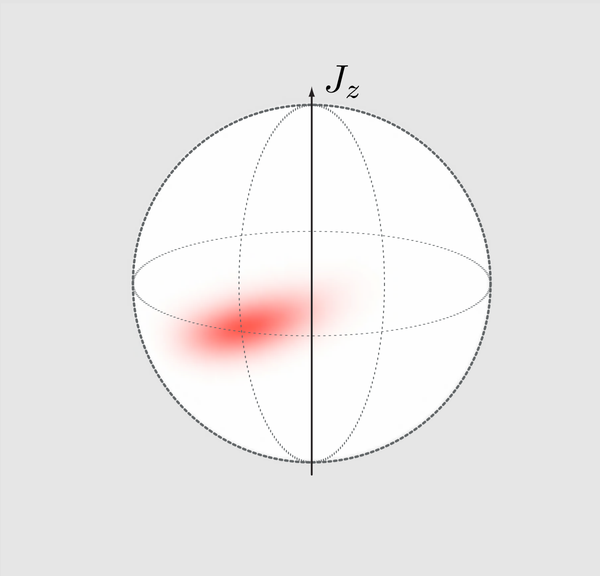

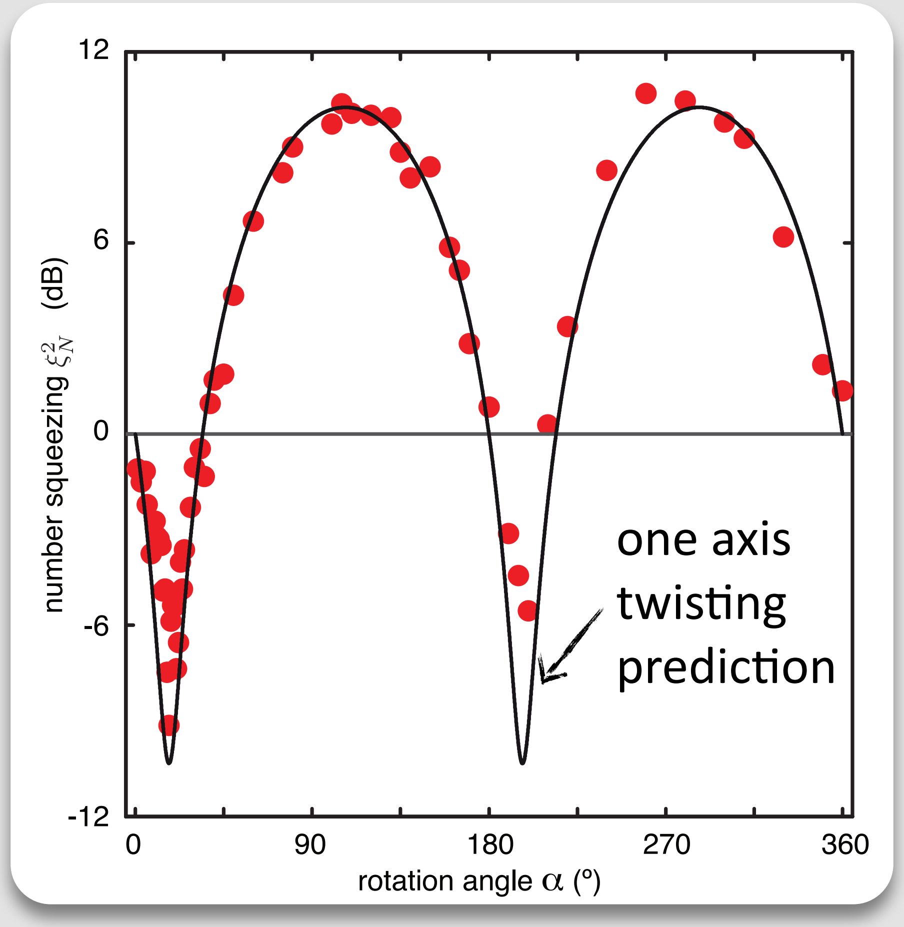

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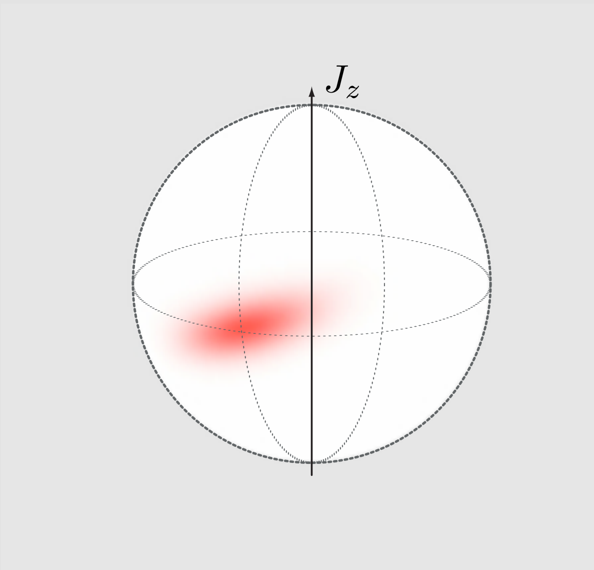

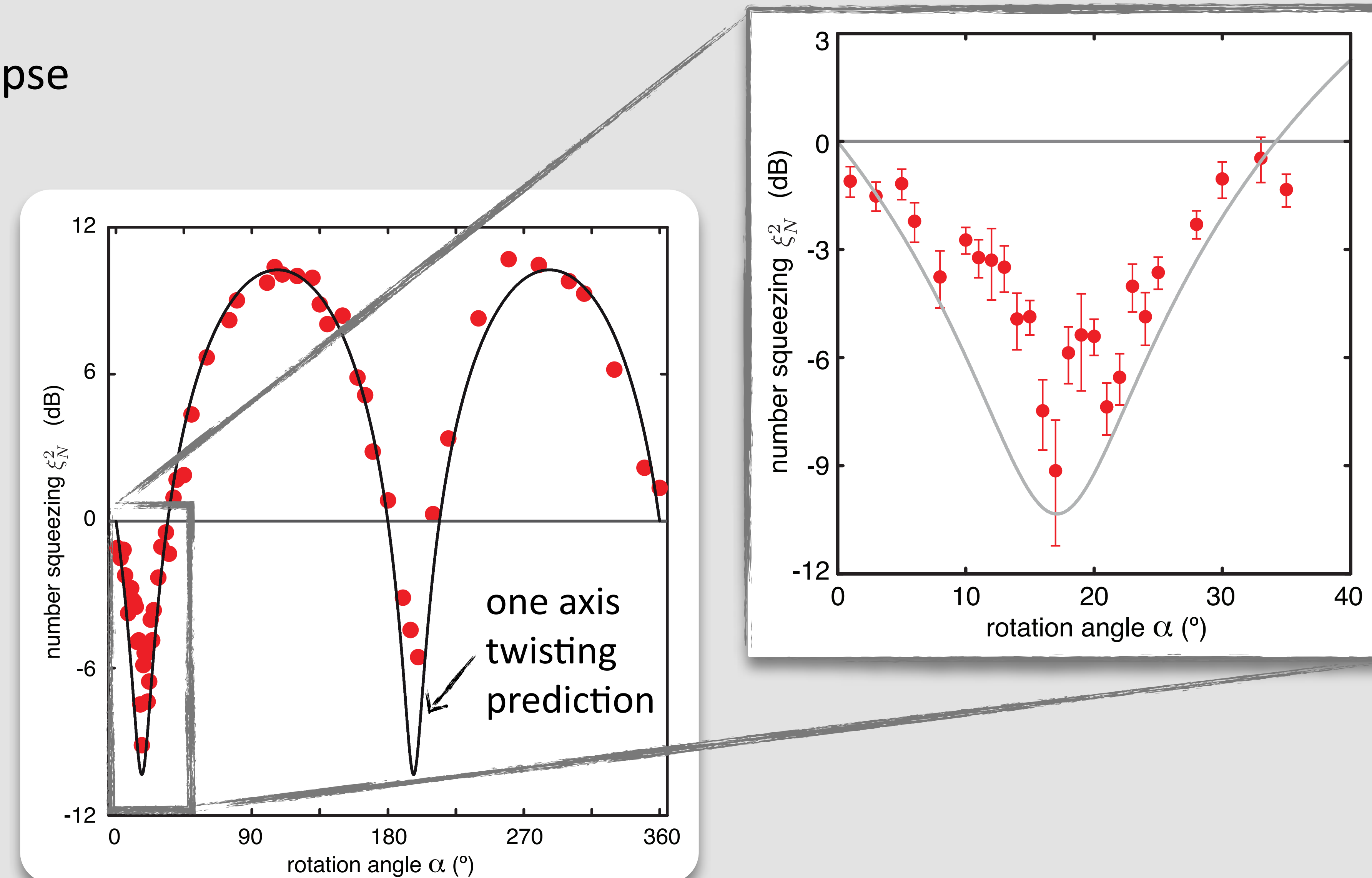

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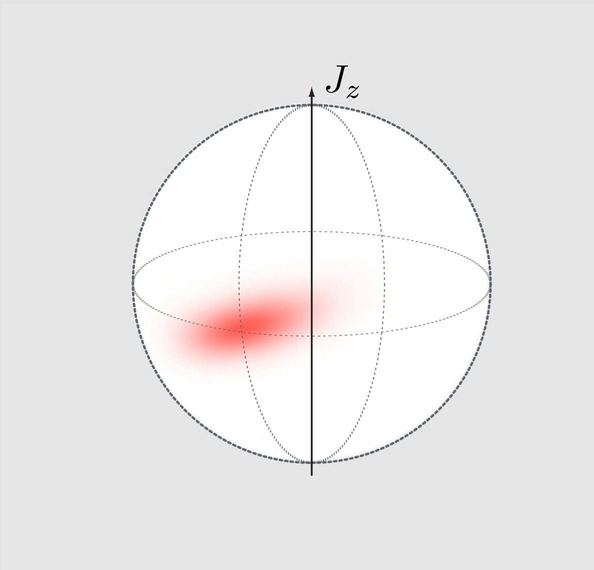
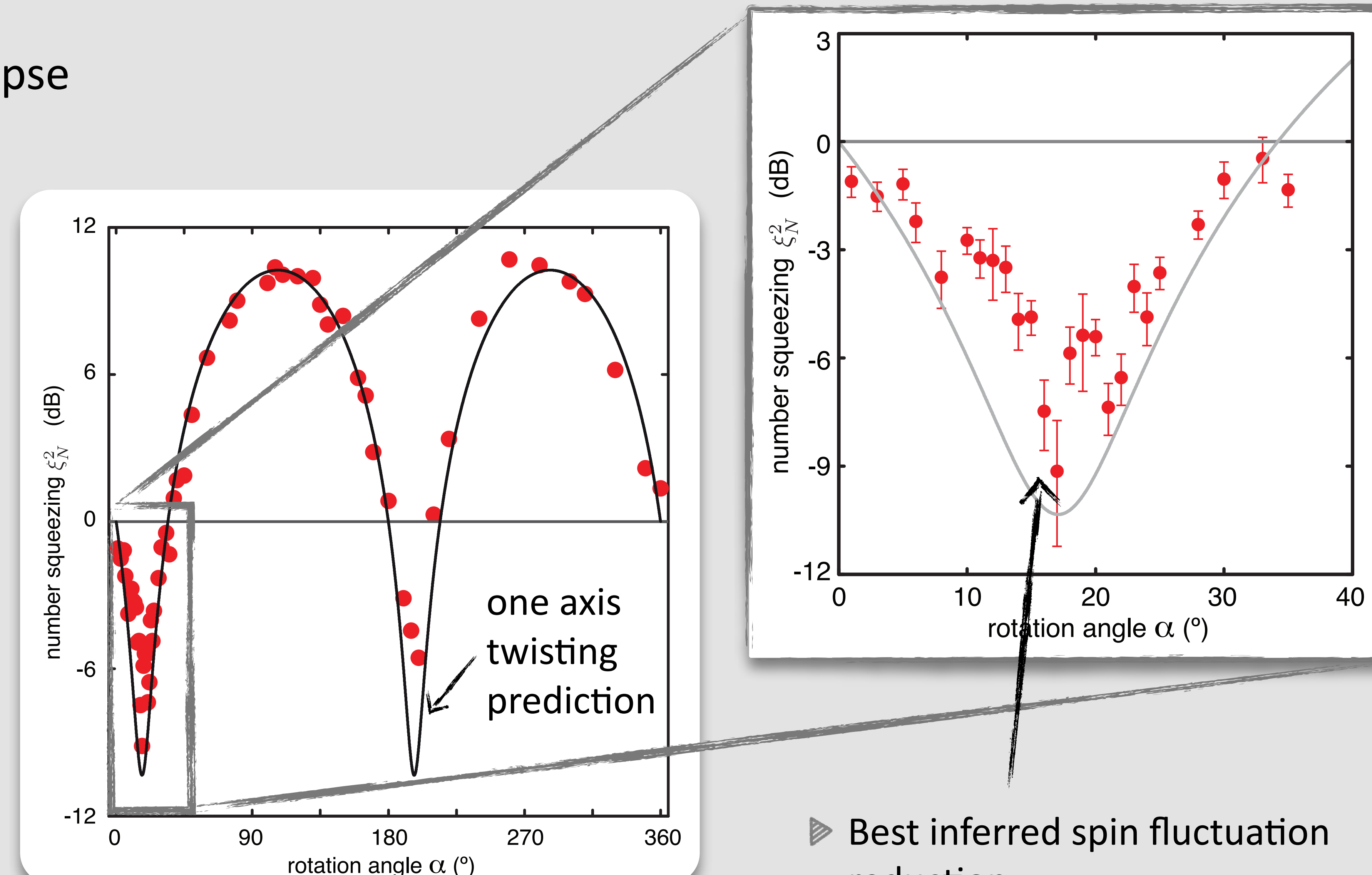


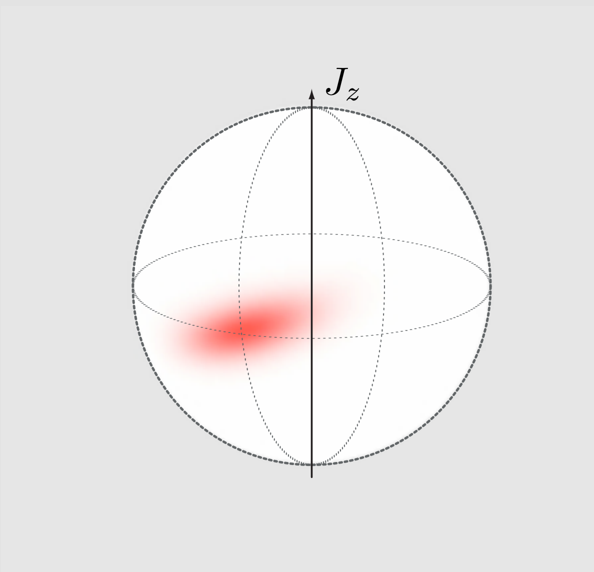


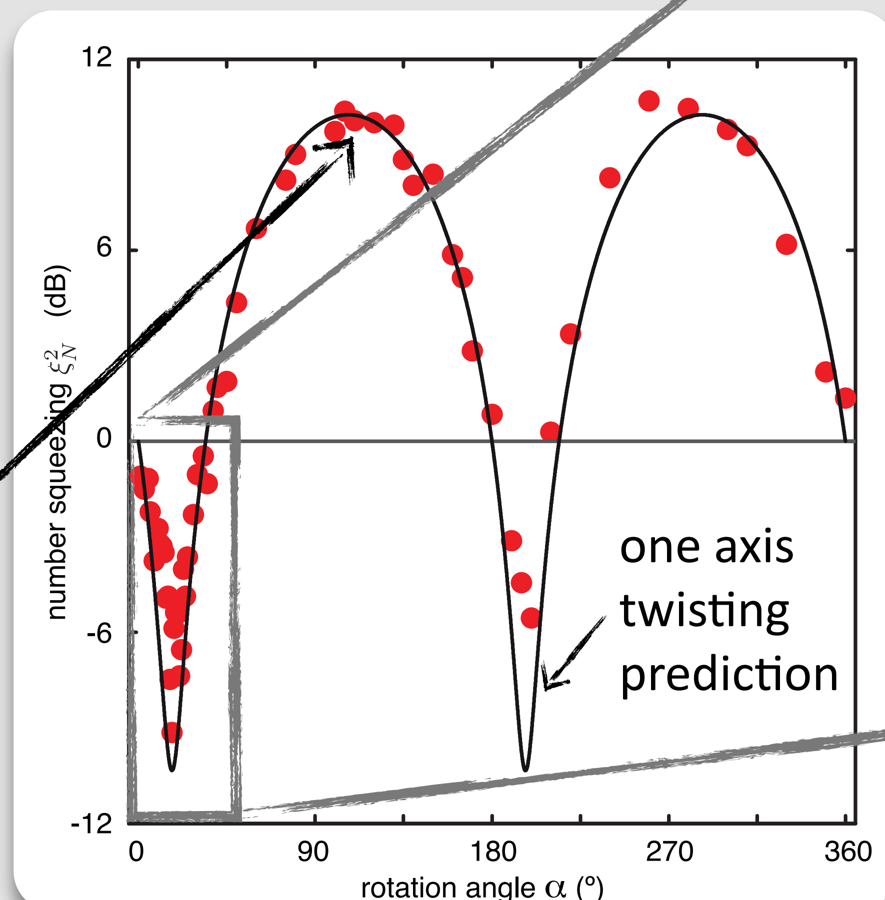
- Best inferred spin fluctuation reduction:

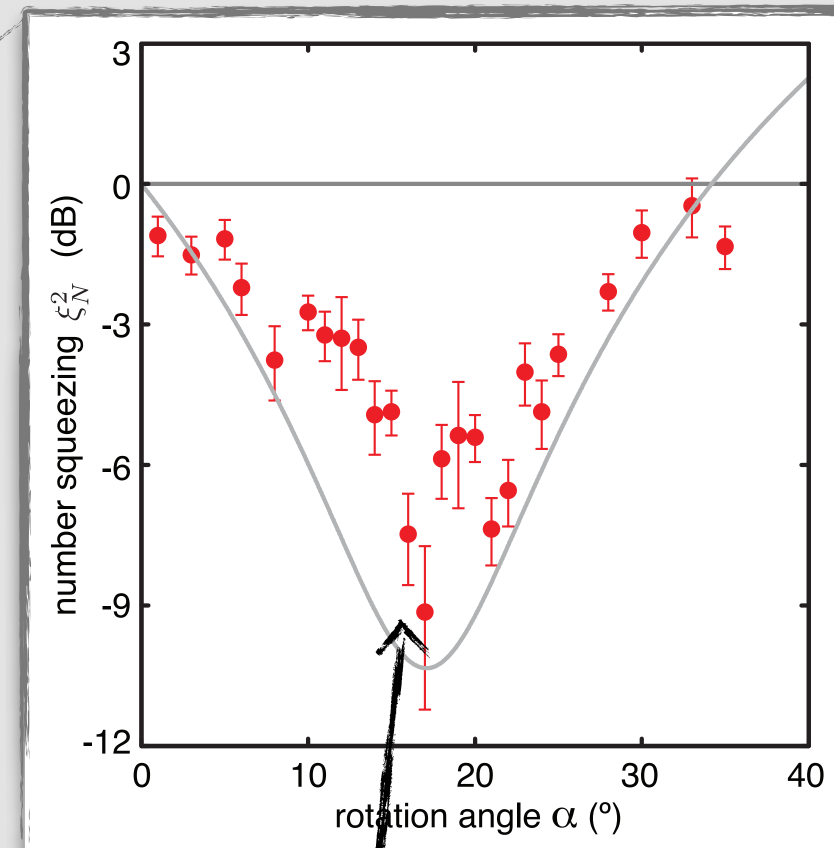
$$\xi_N^2 = \frac{4\Delta J_z}{N} = -8.2\,\mathrm{dB}$$

Noise tomography

- Mapping of the noise ellipse







- coherence in two-mode approximation:

$$\frac{2\langle J_x \rangle}{N} = 0.986$$

- Best inferred spin fluctuation reduction:

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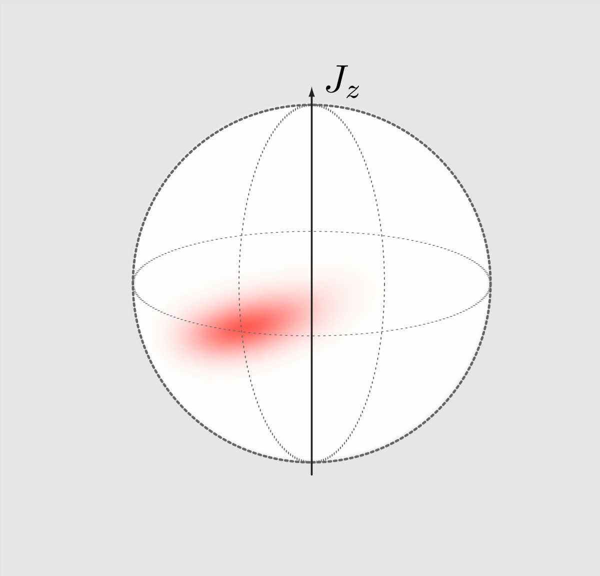
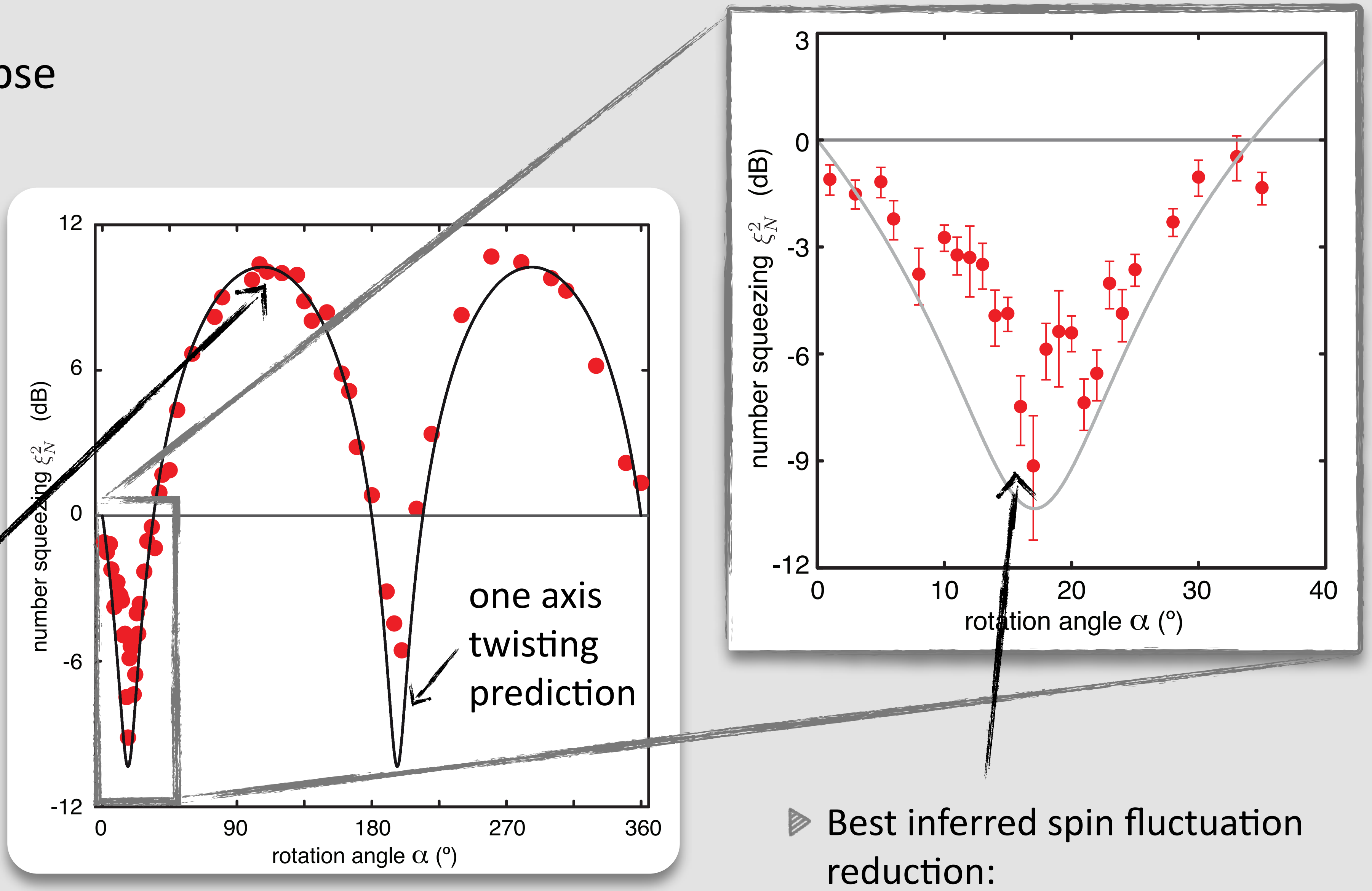
Noise tomography

- Mapping of the noise ellipse





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$$\frac{2\langle J_x \rangle}{N} = 0.986$$

- Best inferred spin fluctuation reduction:

$$\xi_N^2 = \frac{4\Delta J_z}{N} = -8.2\,\mathrm{dB}$$

- spin squeezing:

$$\xi_S^2 = \frac{N\Delta J_z^2}{\langle J_x \rangle^2} = -8.2^{+0.9}_{-1.2}\,\mathrm{dB}$$

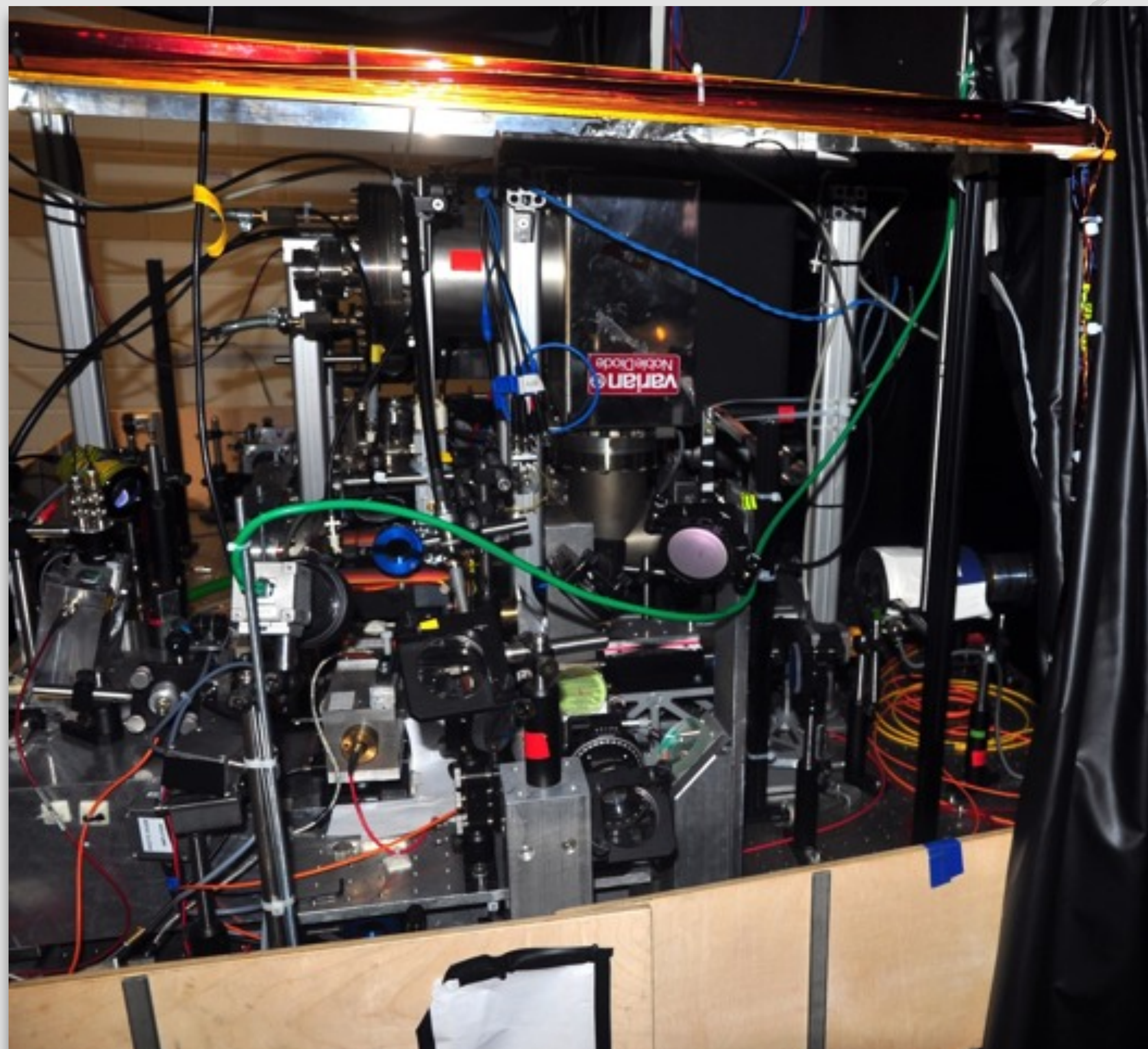
cold thermal atoms: Appel et al., PNAS 106, 10960 (2009)　　*Cold thermal atoms + cavity:* Leroux et al., PRL 104 073602 (2010)

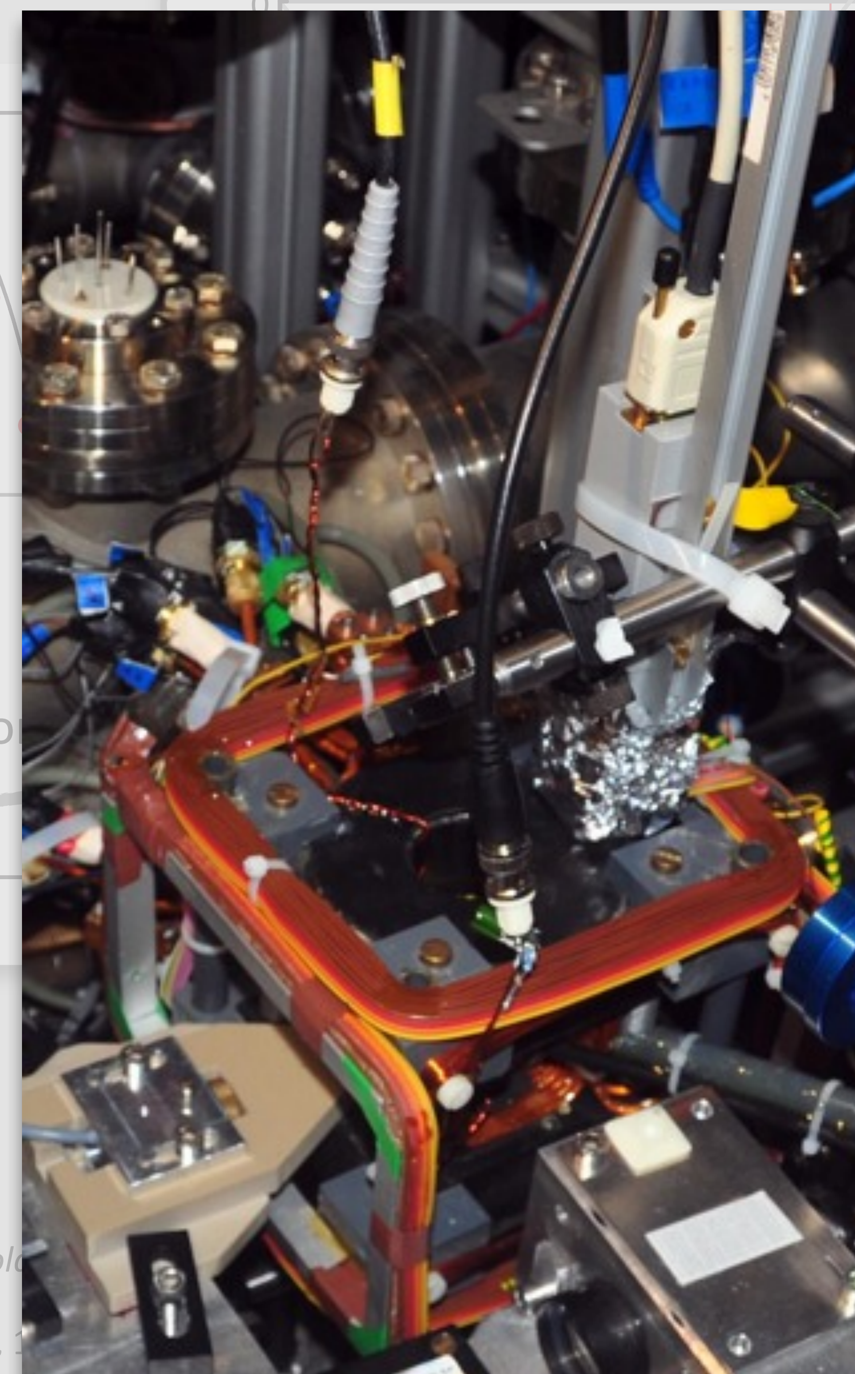
BEC on atomchip: Riedel et al., Nature 464, 1170 (2010)

Noise tomography

Magnetic field stability: $\mathcal{O}(500\,\mu\text{G})$ over a few hours

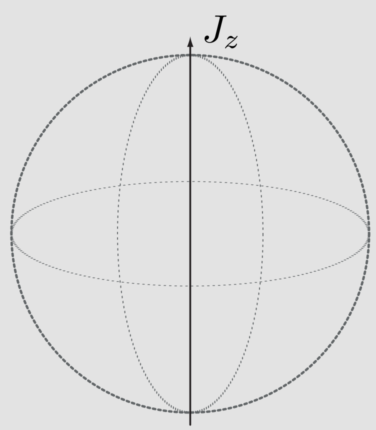
- Mapping of the noise ellip

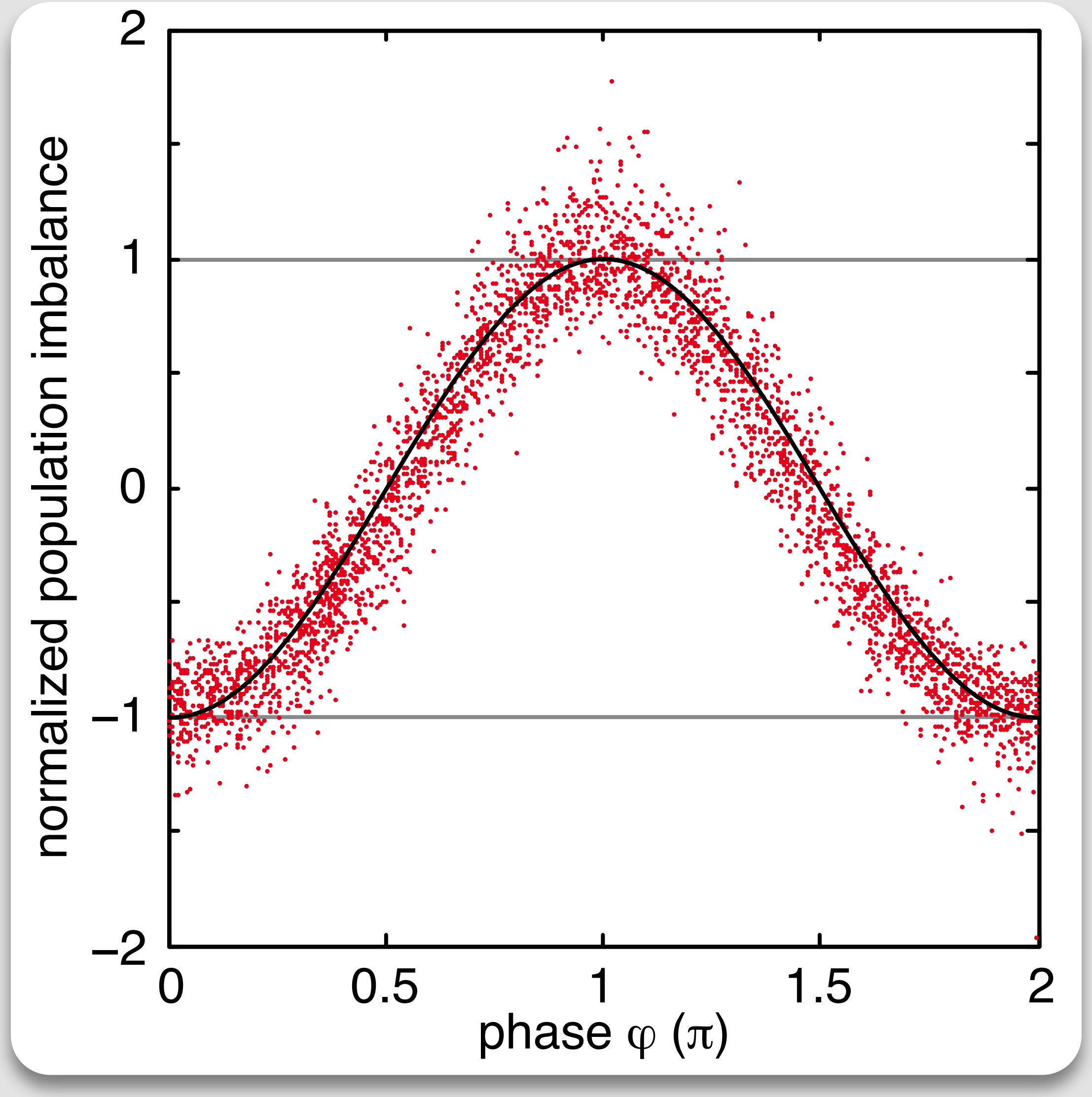




Two mode approximation?

- Ramsey fringe with the squeezed state after the non-linear beamsplitter:





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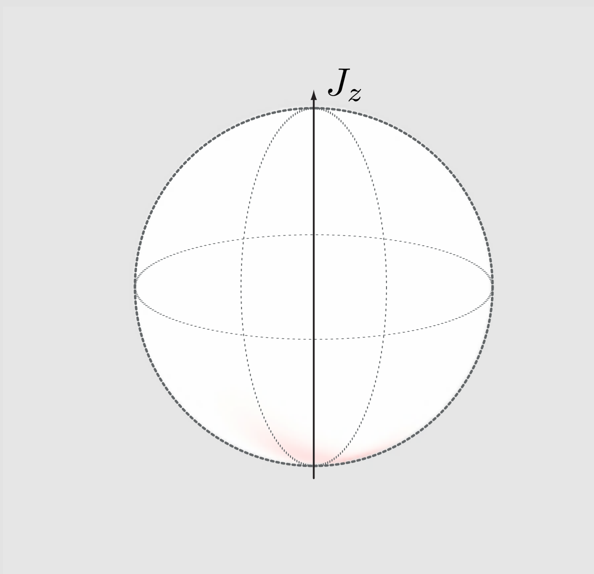

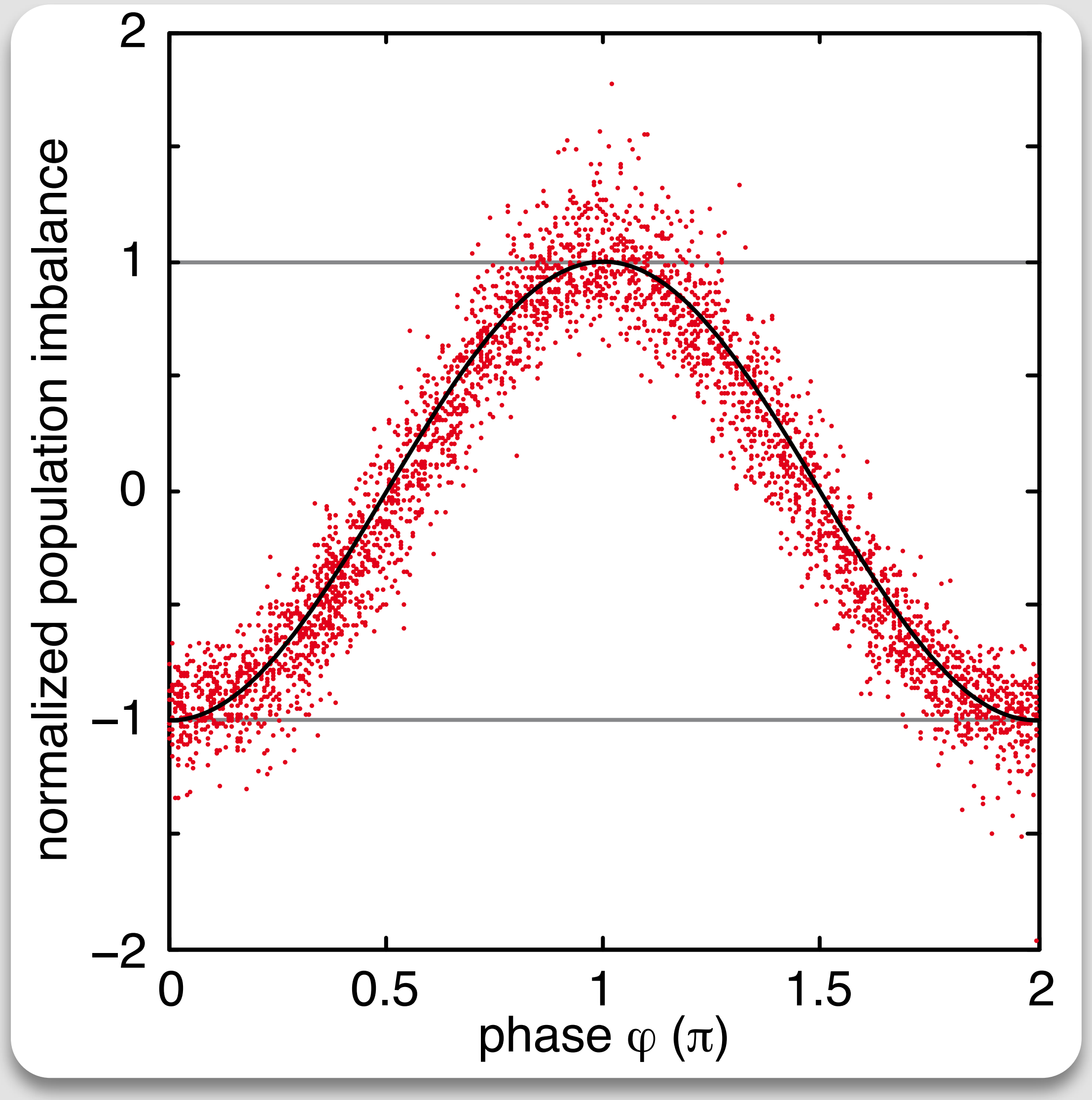

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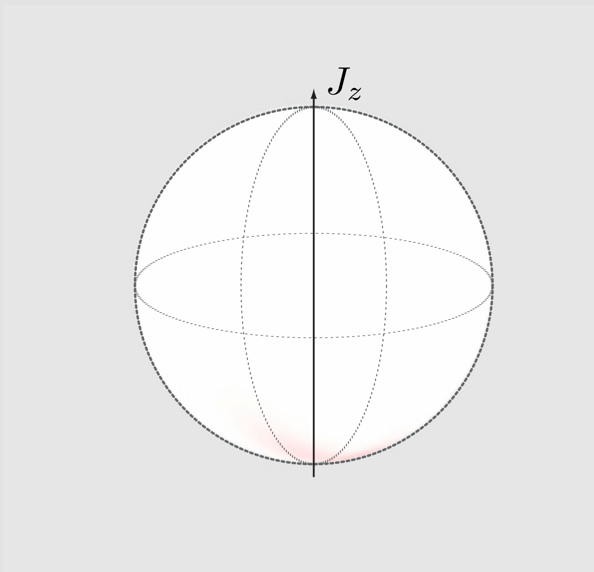

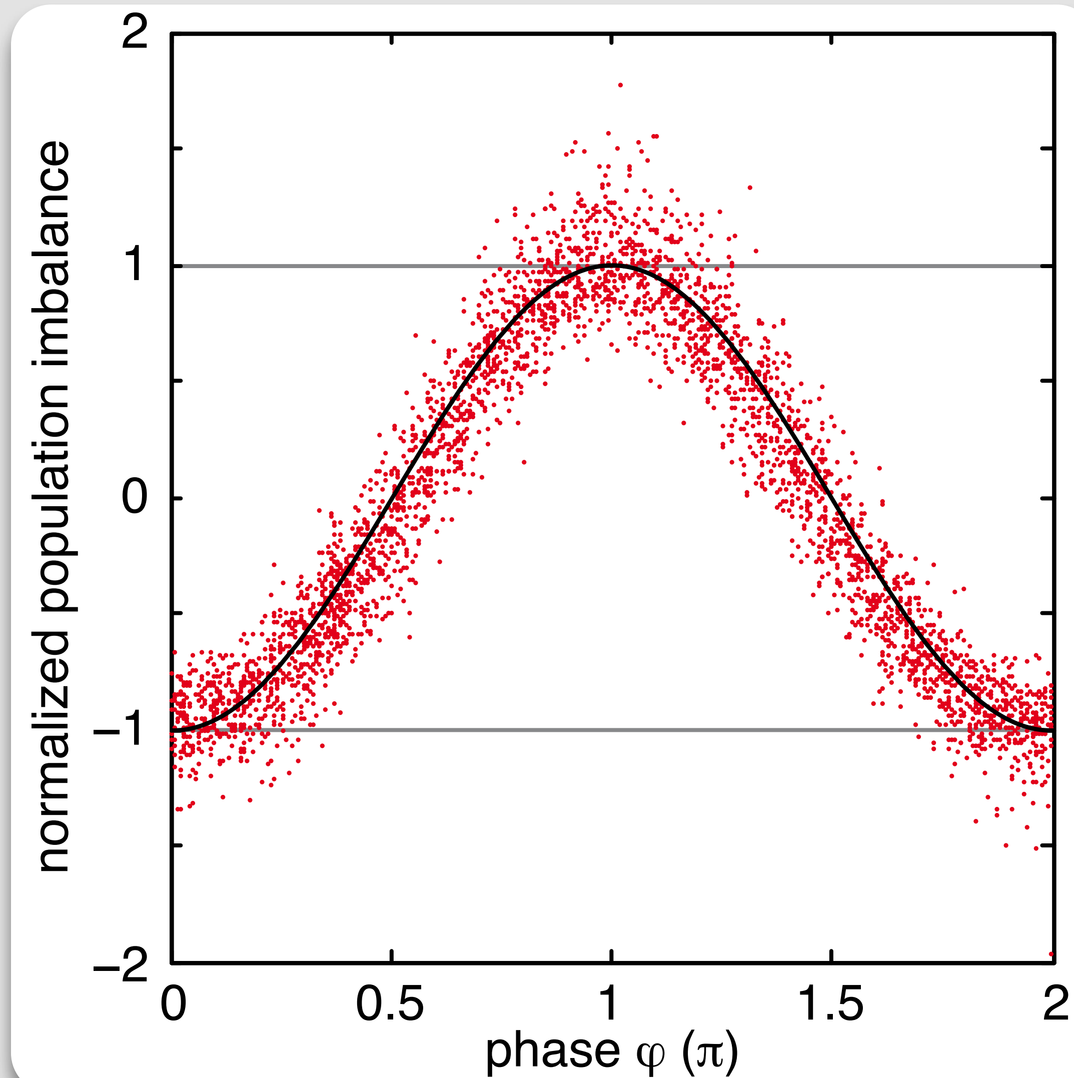

- fitted visibility:

$$\mathcal{V} = 1.00 \pm 0.02$$

confirms the two mode value of 0.986

Depth of entanglement

- Many body entanglement: $\rho \neq \sum_k P_k \rho_k^1 \otimes \rho_k^2 \otimes \cdots \otimes \rho_k^m \otimes \cdots \otimes \rho_k^N$

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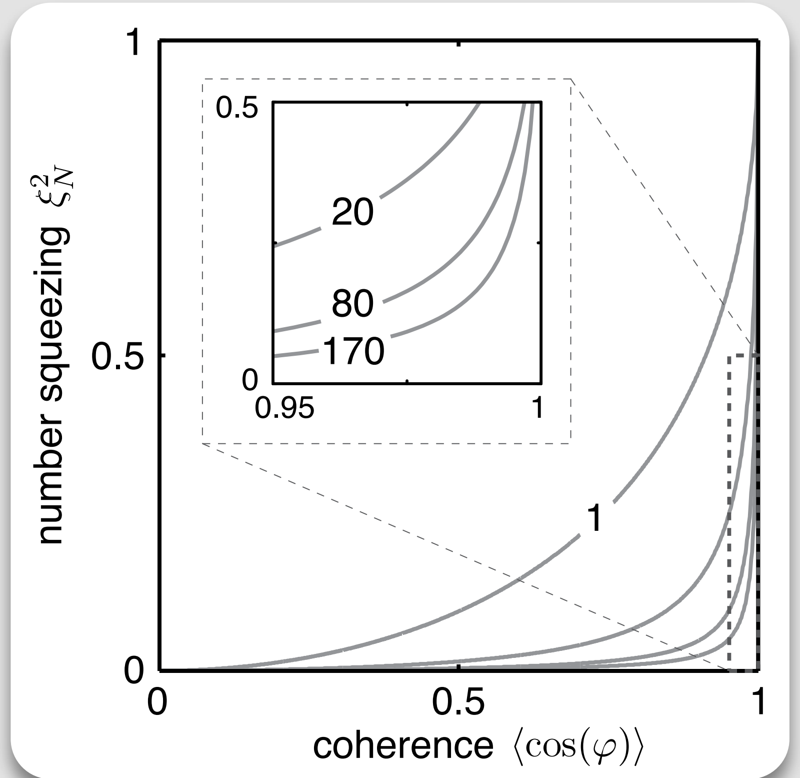

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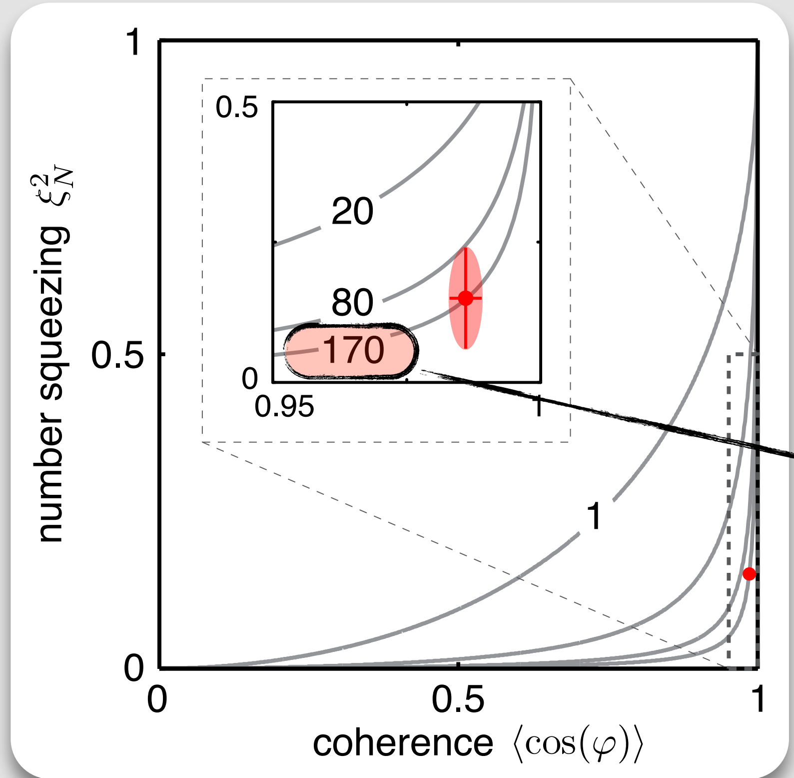
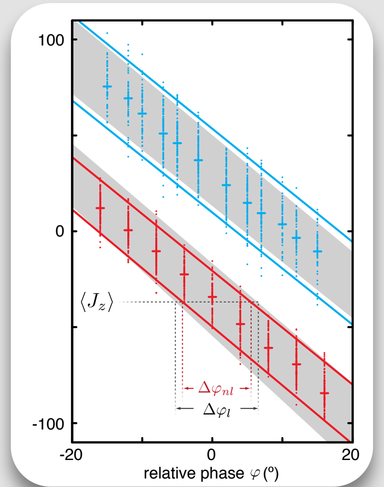


170 elementary spins (atoms) non-separable!

Summary



- Non-linear atom interferometer with phase precision 15% beyond the standard quantum limit

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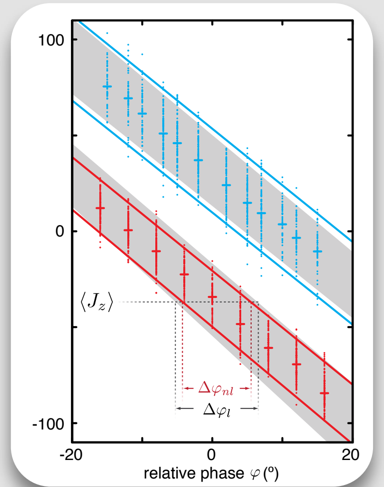

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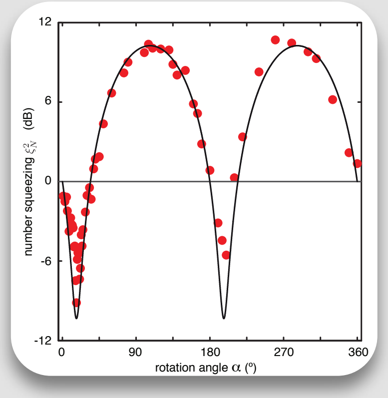

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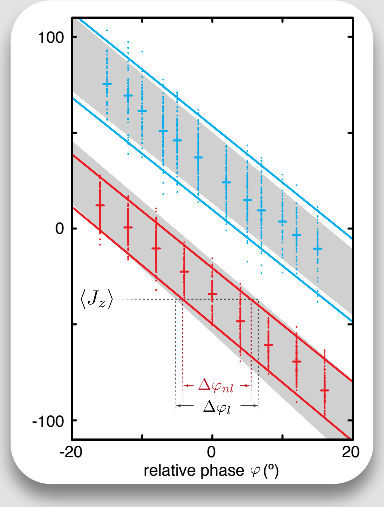

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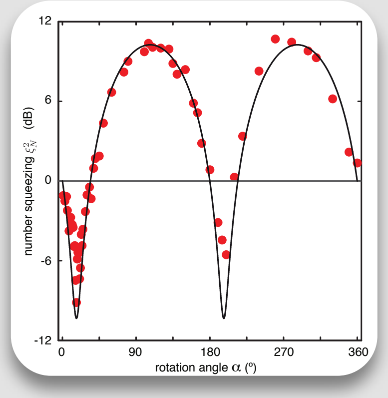

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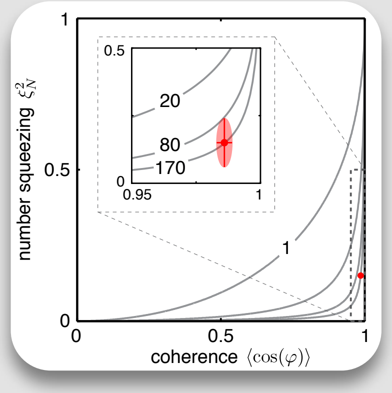

- Spin fluctuations and coherence imply 170 entangled atoms

→ Nature 464, 1165 (2010)