Quantum control in 1D interacting lattice systems using edge-induced physics

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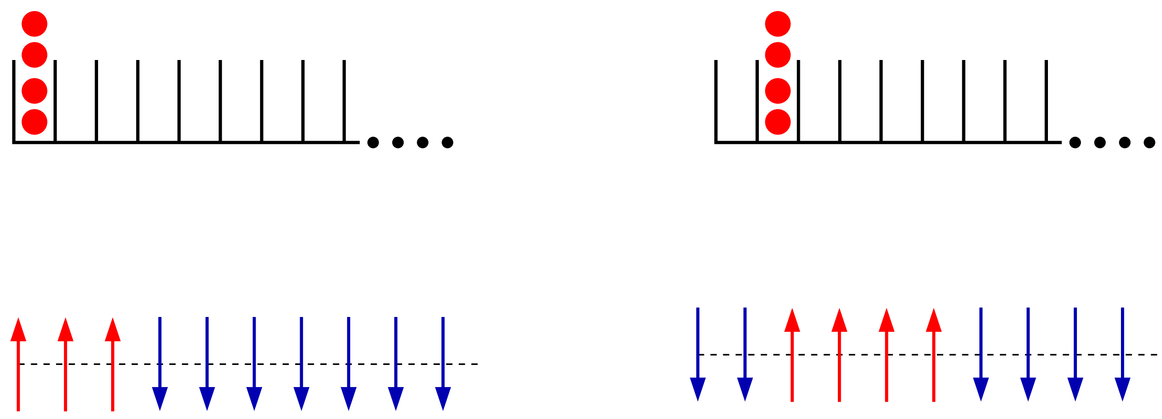
Edge-localization in 1D lattice models

Bose-Hubbard chain

spinless fermion model

XXZ chain



PHYSICS:

Eigenstates far from ground state

Far-from-equilibrium dynamics

Intricate structures in spectrum (FRACTAL)

QUANTUM CONTROL:

Locking and release of magnetization/state

Designing a quantum switch

For greater detail ....

R. A. Pinto, M. Haque, and S. Flach; Phys. Rev. A **79**, 052118 (2009).

Edge-localized states in quantum one-dimensional lattices.

M. Haque, arXiv:0906.0996.

Edge-locking and quantum control in highly polarized spin chains.

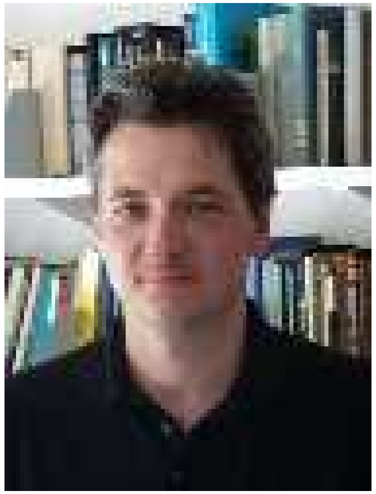
Nice people thank their collaborators

Ricardo Pinto

MPI-PKS Dresden

↓

Riverside



Sergej Flach

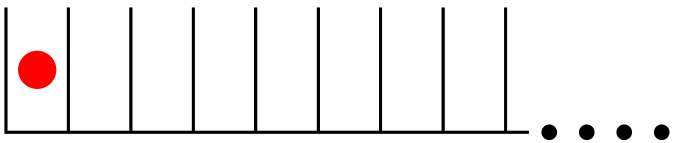
MPI-PKS Dresden

Start with some guessing games

1D Bose-Hubbard model in an OPEN chain (has edges)

$$\hat{H} = -t\sum_{j=1}^{L-1}\left(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j\right) + \frac{U}{2}\sum_{j=1}^{L} a_j^\dagger a_j^\dagger a_j a_j$$

I'm interested in large U/t.

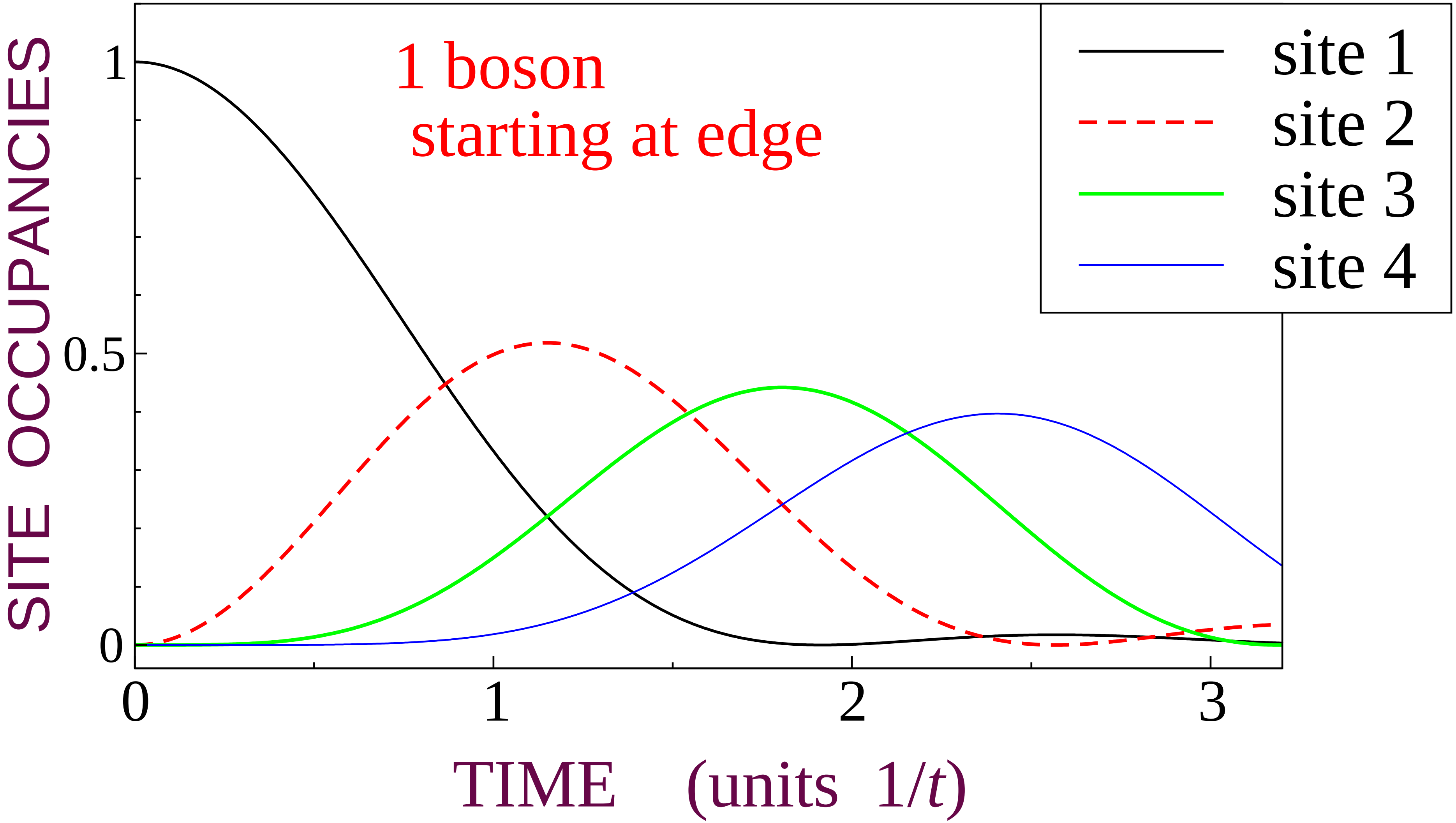


1 0 0 0 0 0 0 0 .....

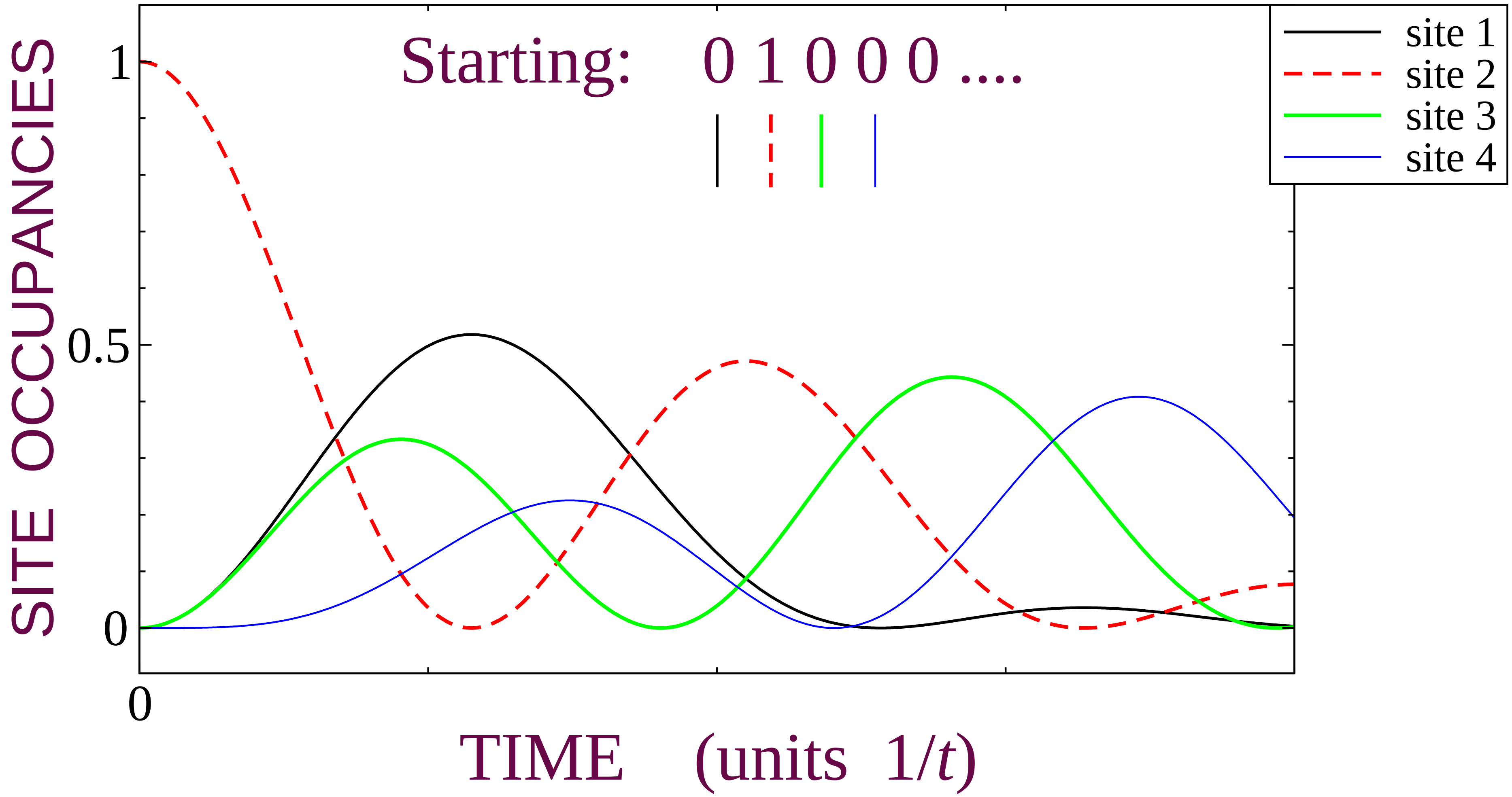
How does this evolve?

At timescales $\sim \hbar/t$

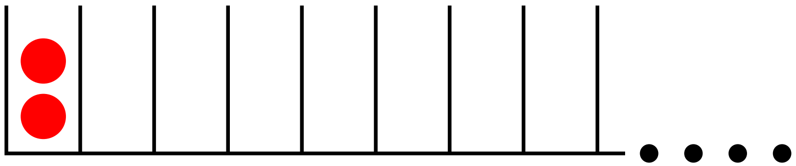
One boson starting at site 1



1 BOSON STARTING AT SITE 2 (NEXT-TO-EDGE)



Next: two bosons



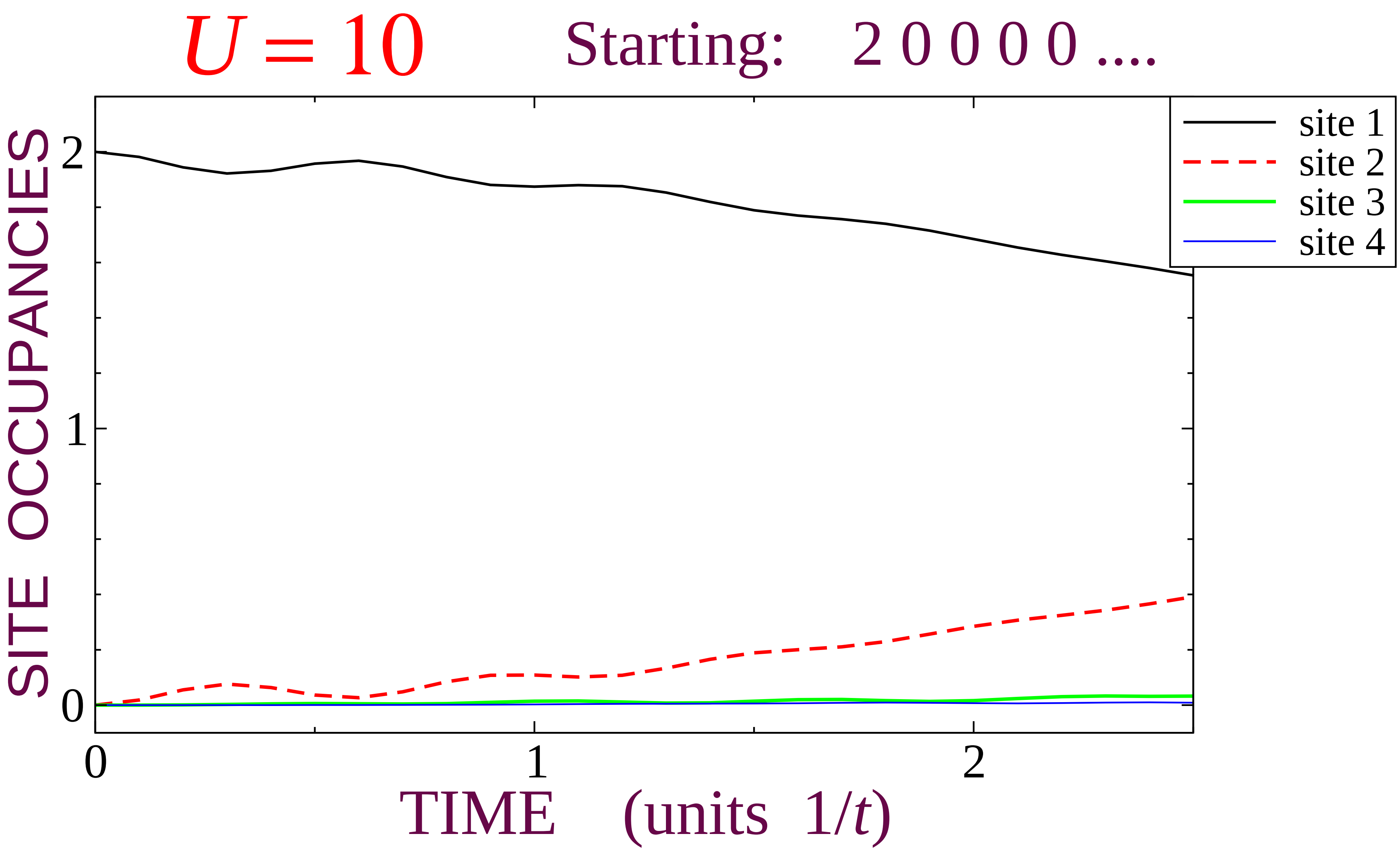
2 0 0 0 0 0 0 .....

How does this evolve?

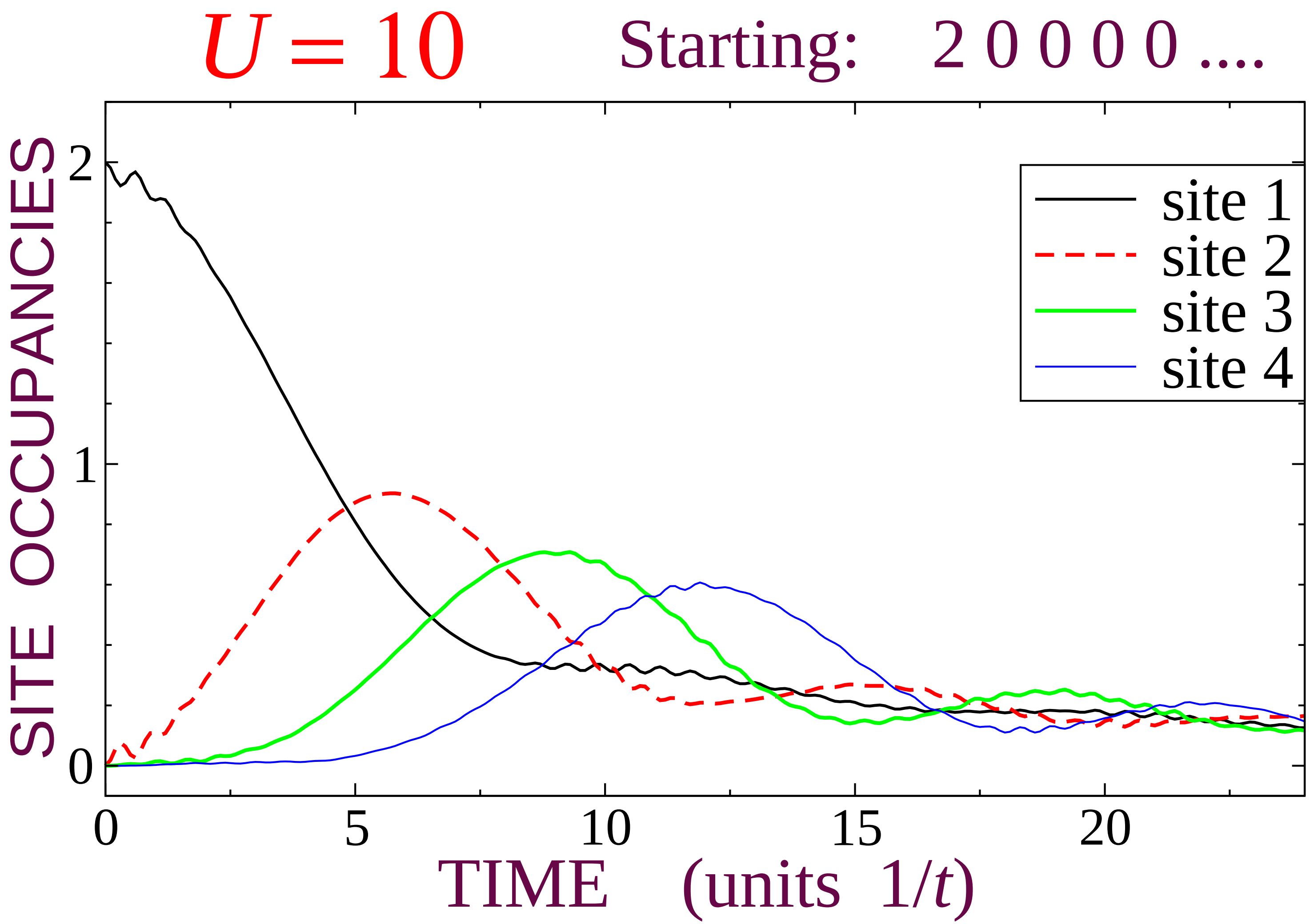
At timescales $\sim 1/t \sim 1$

At timescales $\sim 1/(t^2/U) \sim U$

TWO BOSONS AT EDGE: TIMESCALES $\sim 1/t \sim 1$



2 BOSONS AT EDGE: TIMESCALES $\sim 1/(t^2/U) \sim U$



Large U encourages correlated pair motion

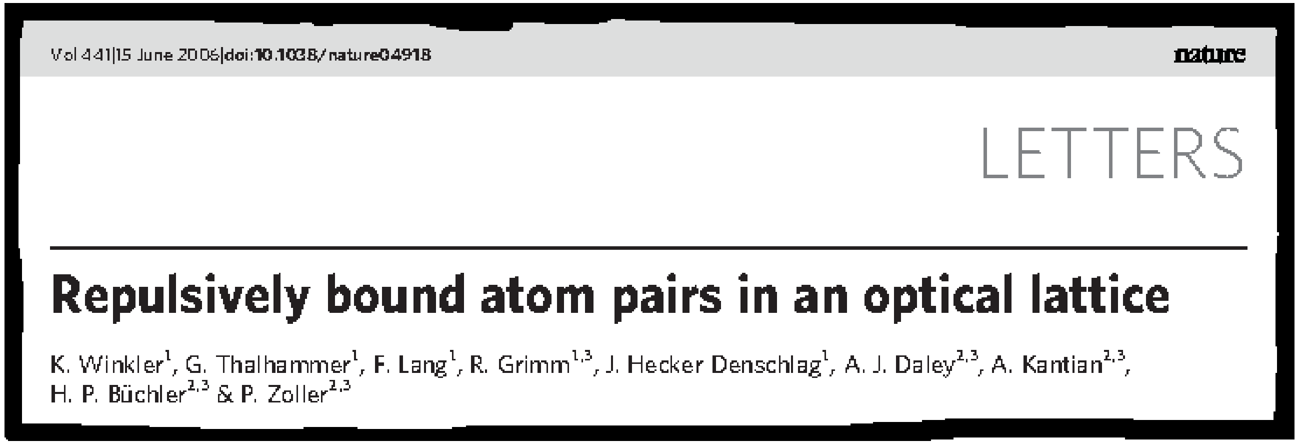
Single particle hopping timescale $\sim 1/t \sim 1$

Pair hopping time scale $\sim 1/\left(\frac{t^2}{U}\right) \sim U$

"Repulsively bound pairs"

Triplet hopping time scale $\sim 1/\left(\frac{t^3}{U^2}\right) \sim U^2$

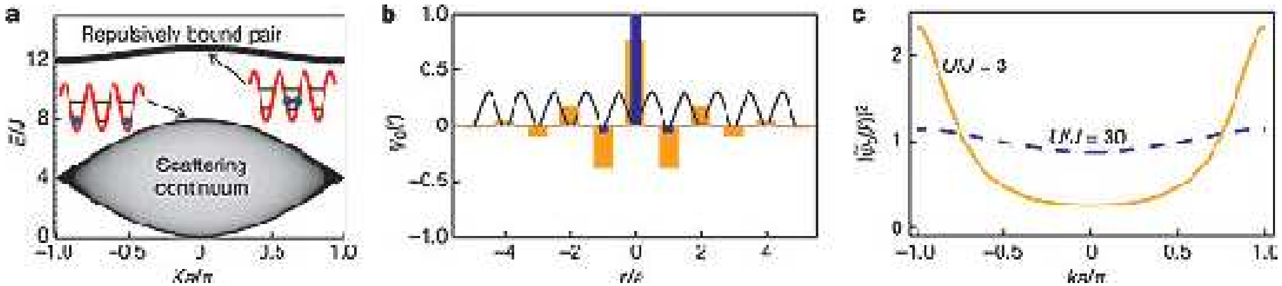
REPULSIVELY BOUND PAIRS

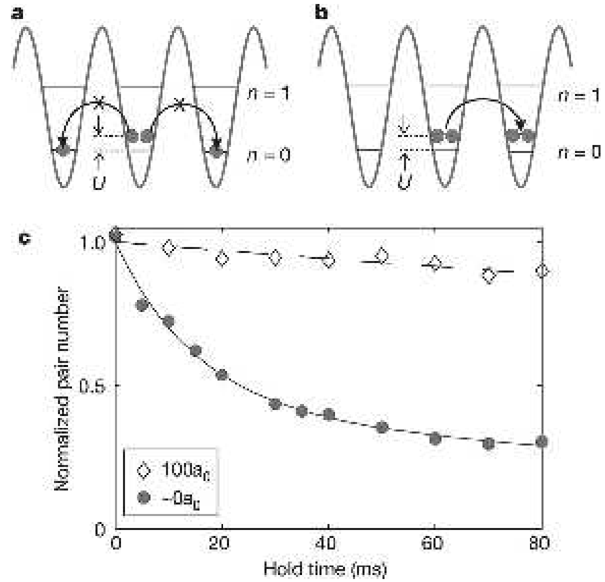

Vol 441|15 June 2006|doi:10.1038/nature04918 nature

LETTERS

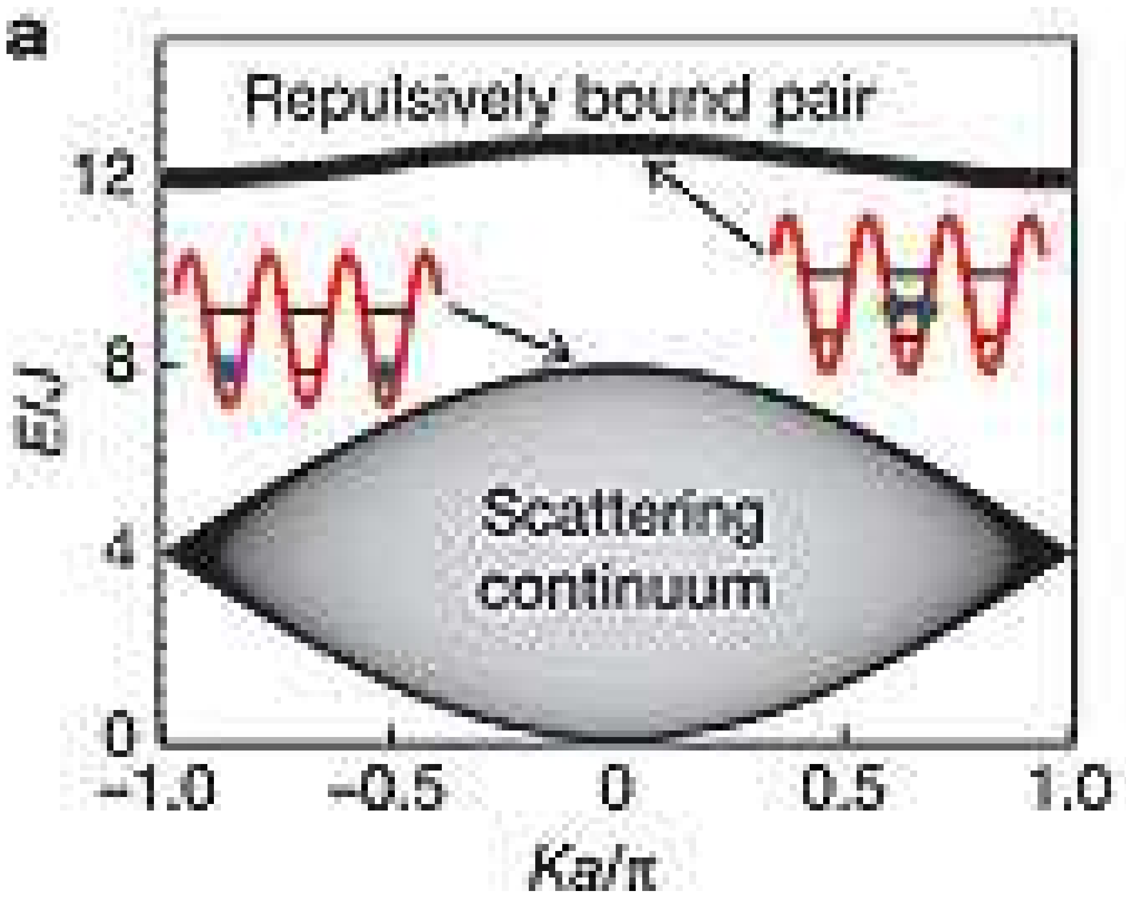
Repulsively bound atom pairs in an optical lattice

K. Winkler[1], G. Thalhammer[1], F. Lang[1], R. Grimm[1,3], J. Hecker Denschlag[1], A. J. Daley[2,3], A. Kantian[2,3], H. P. Büchler[2,3] & P. Zoller[2,3]





“Bands” in energy spectrum, 2 bosons



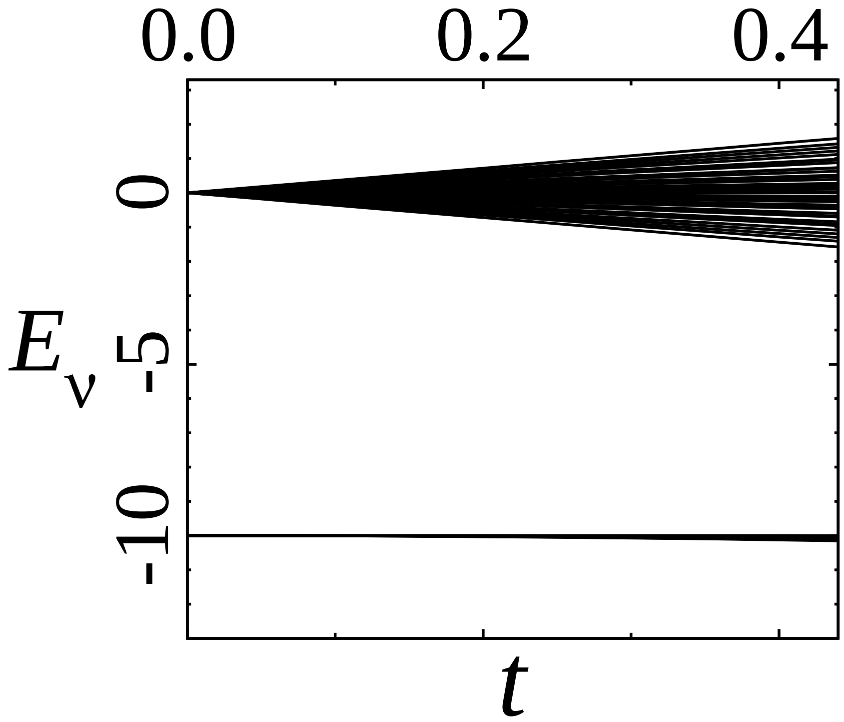
(translation-invariant picture)

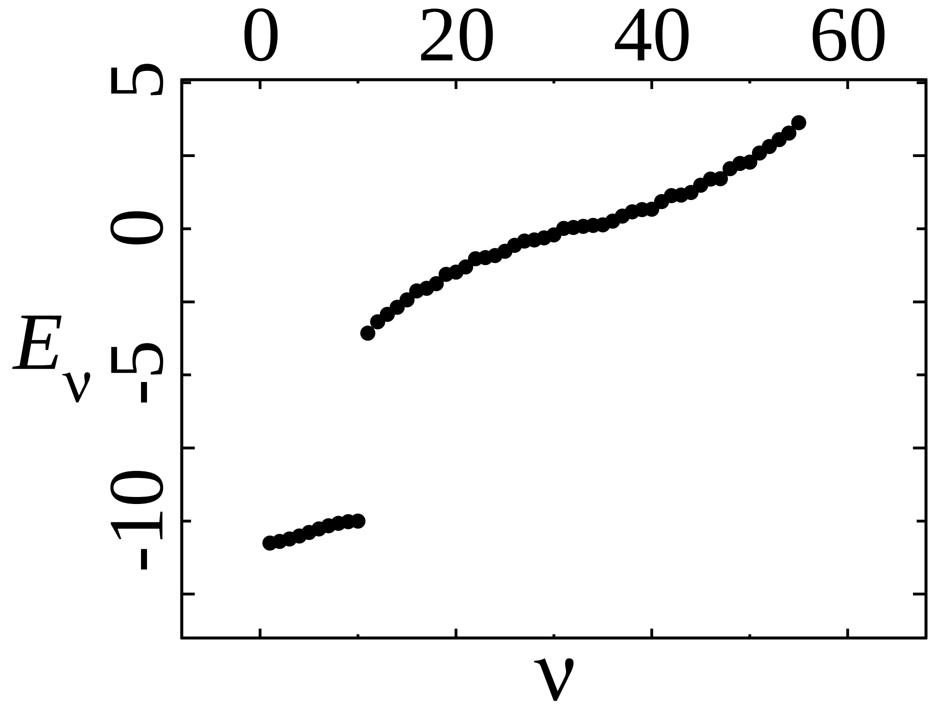
Pairs cannot break without losing energy,

$\Longrightarrow$ without energy relaxation mechanism.

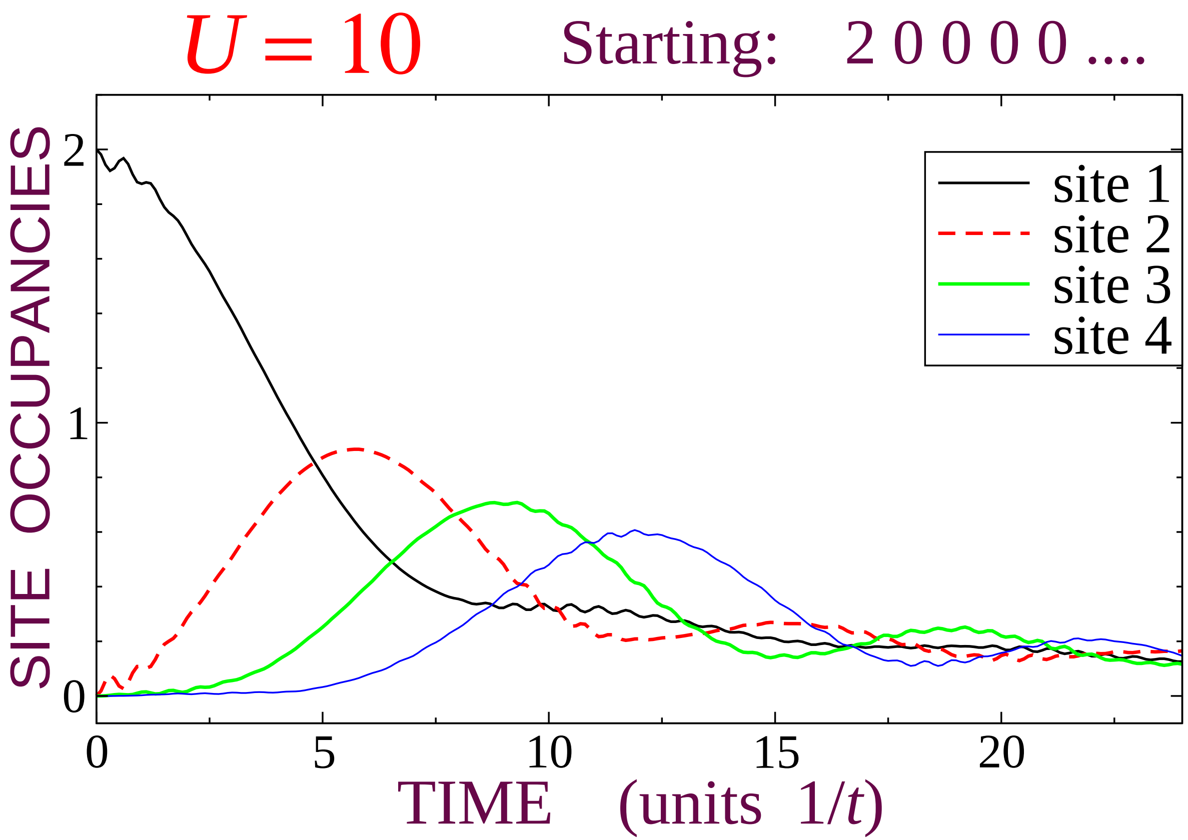
2-BOSON SPECTRUM, BANDS

2 Bosons in 10-site open chain. Negative U !! $U = -10$





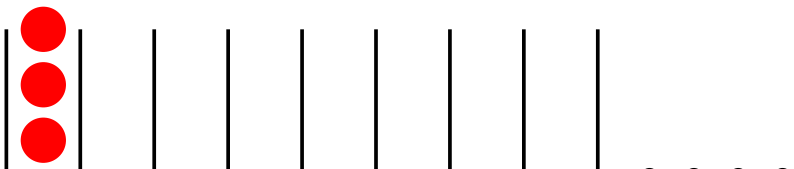
Two bosons



Long time-scale → hopping mostly within bound-pair band.

High-frequency oscillations → inter-band processes.

Let's move on: THREE bosons



3 0 0 0 0 0 0 ....

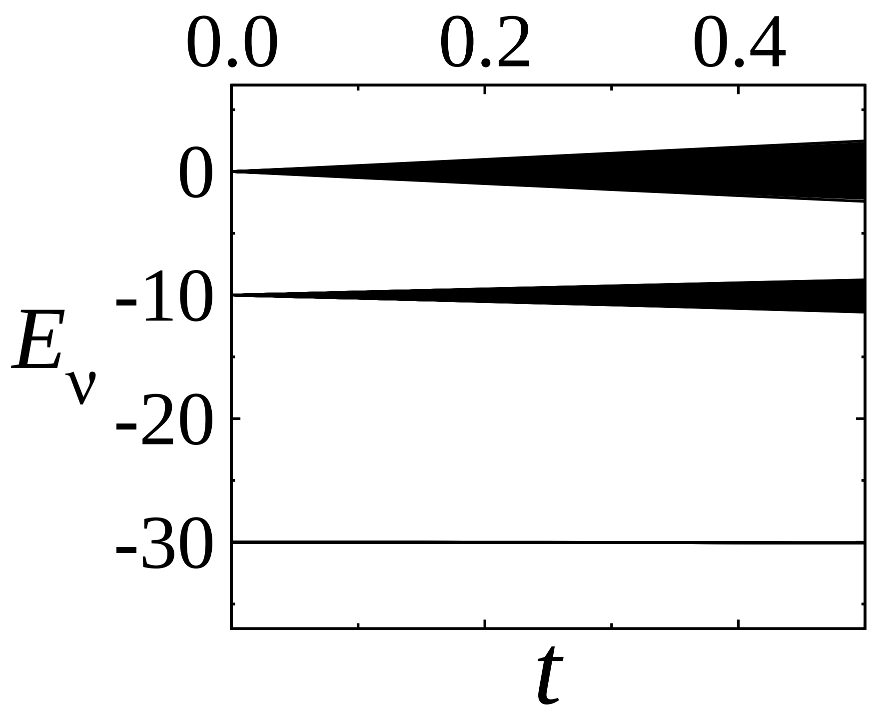
How does this evolve?

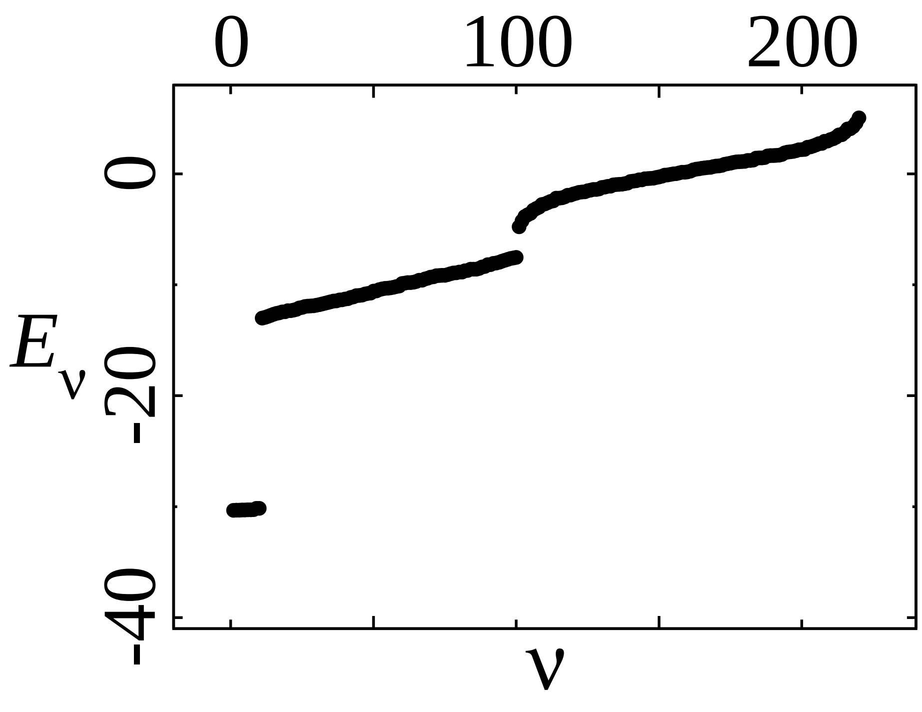
At timescales $\sim 1/t$

At timescales $\sim 1/(t^3/U^2) \sim U^2/t^3$

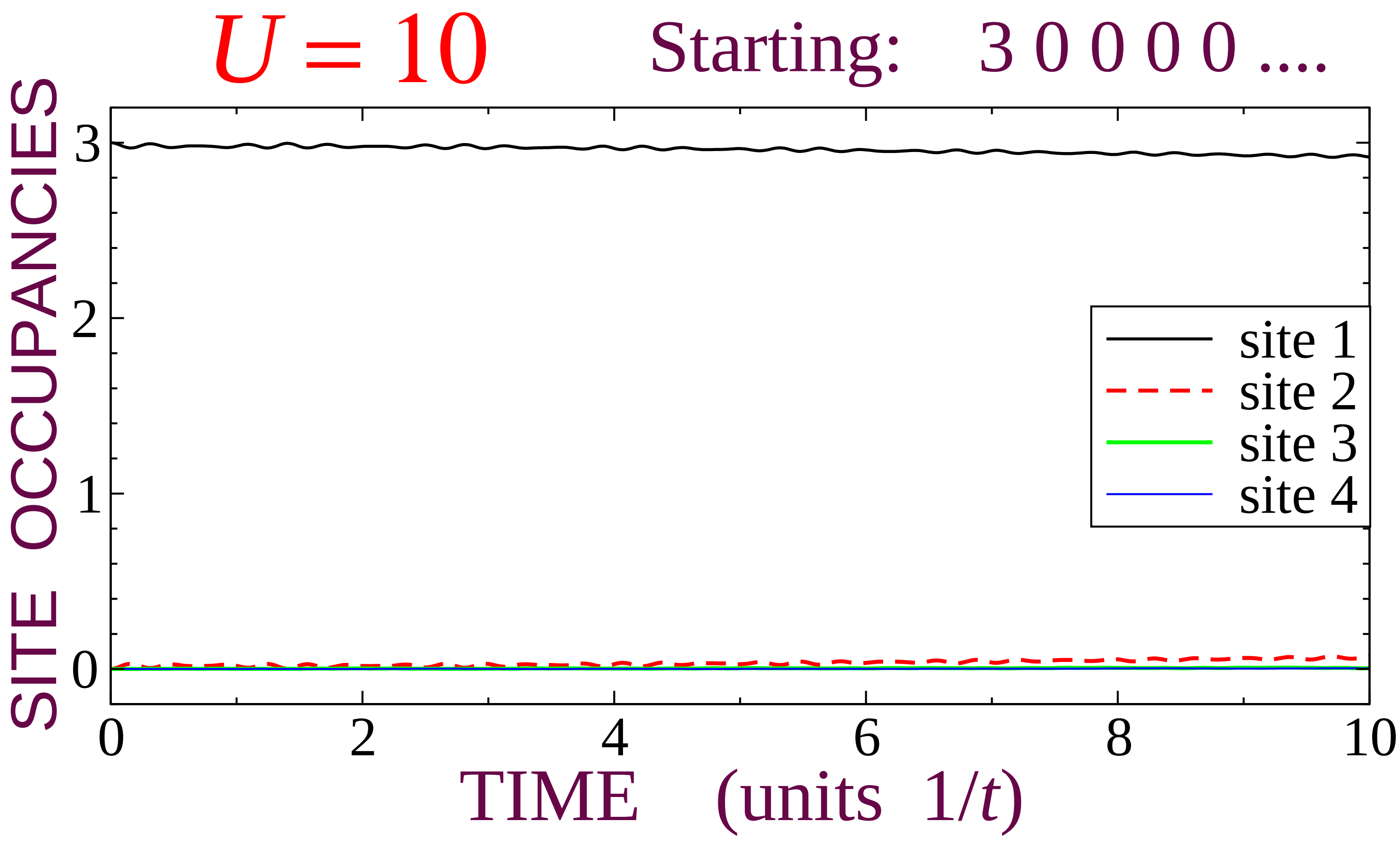
3-BOSON SPECTRUM, BANDS

3 Bosons in 10-site open chain. Negative U; $U = -10$



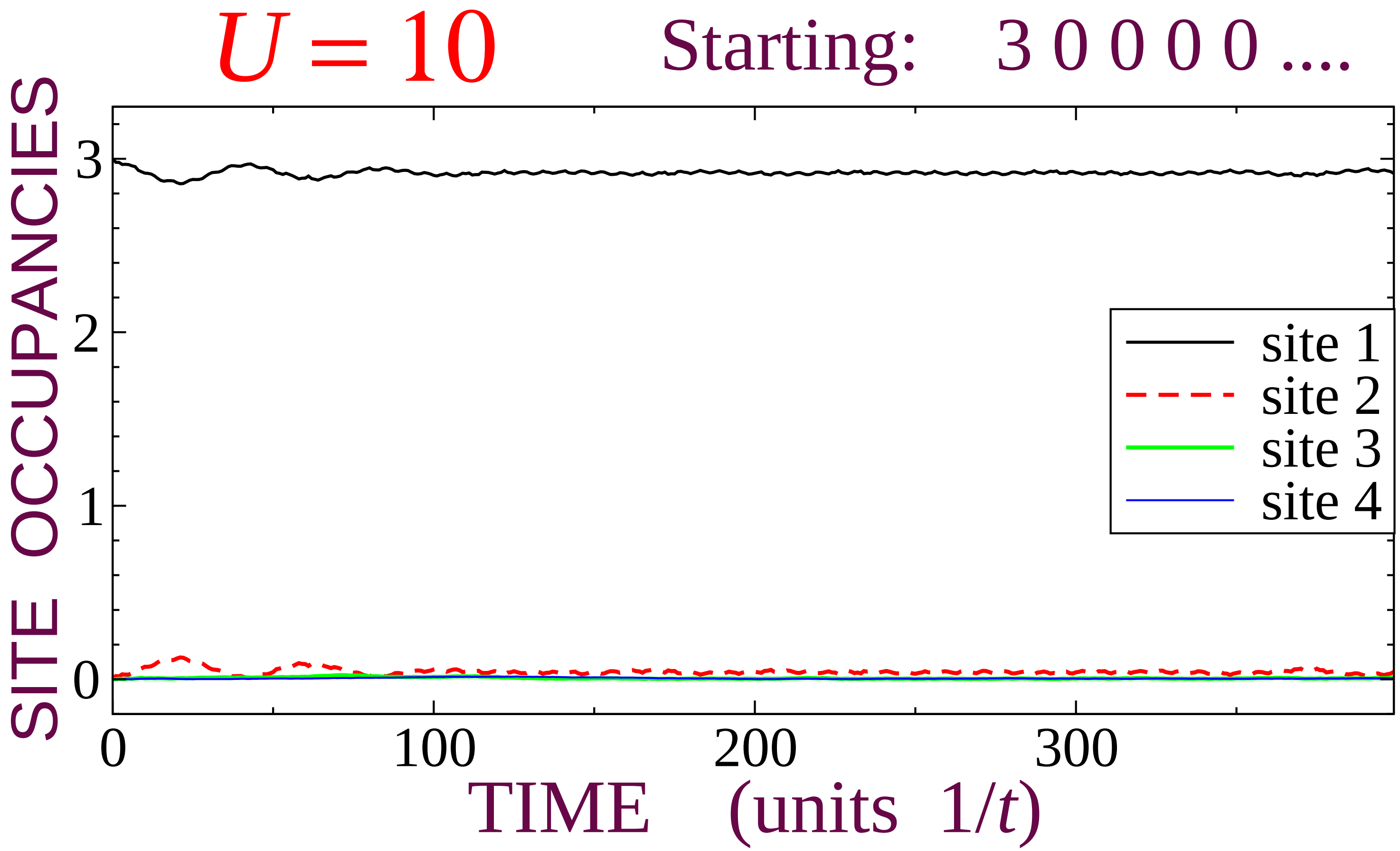


THREE BOSONS AT EDGE: TIMESCALES $\sim 1/t$



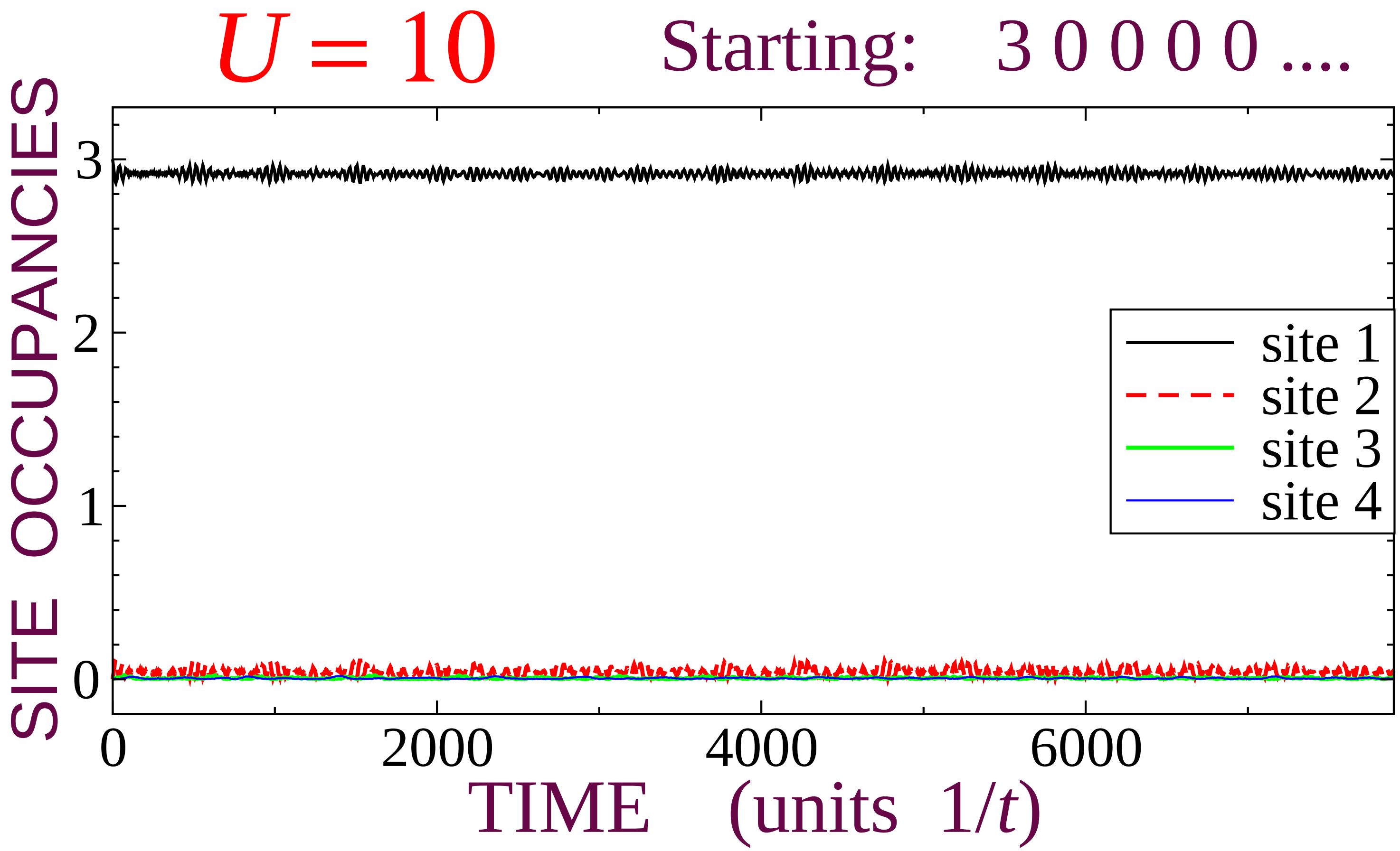
No big surprise.

THREE BOSONS AT EDGE: TIMESCALES $\sim U^2$



CONFUSED: TRY TIMESCALES $\gg \sim U^2$

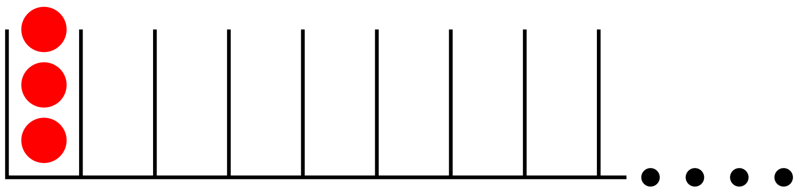
TRYING TIMESCALES $\gg\sim U^2$



? ? ? ? ? ? ? ? ?

You should be surprised

WE'VE FOUND A STABLE STATE

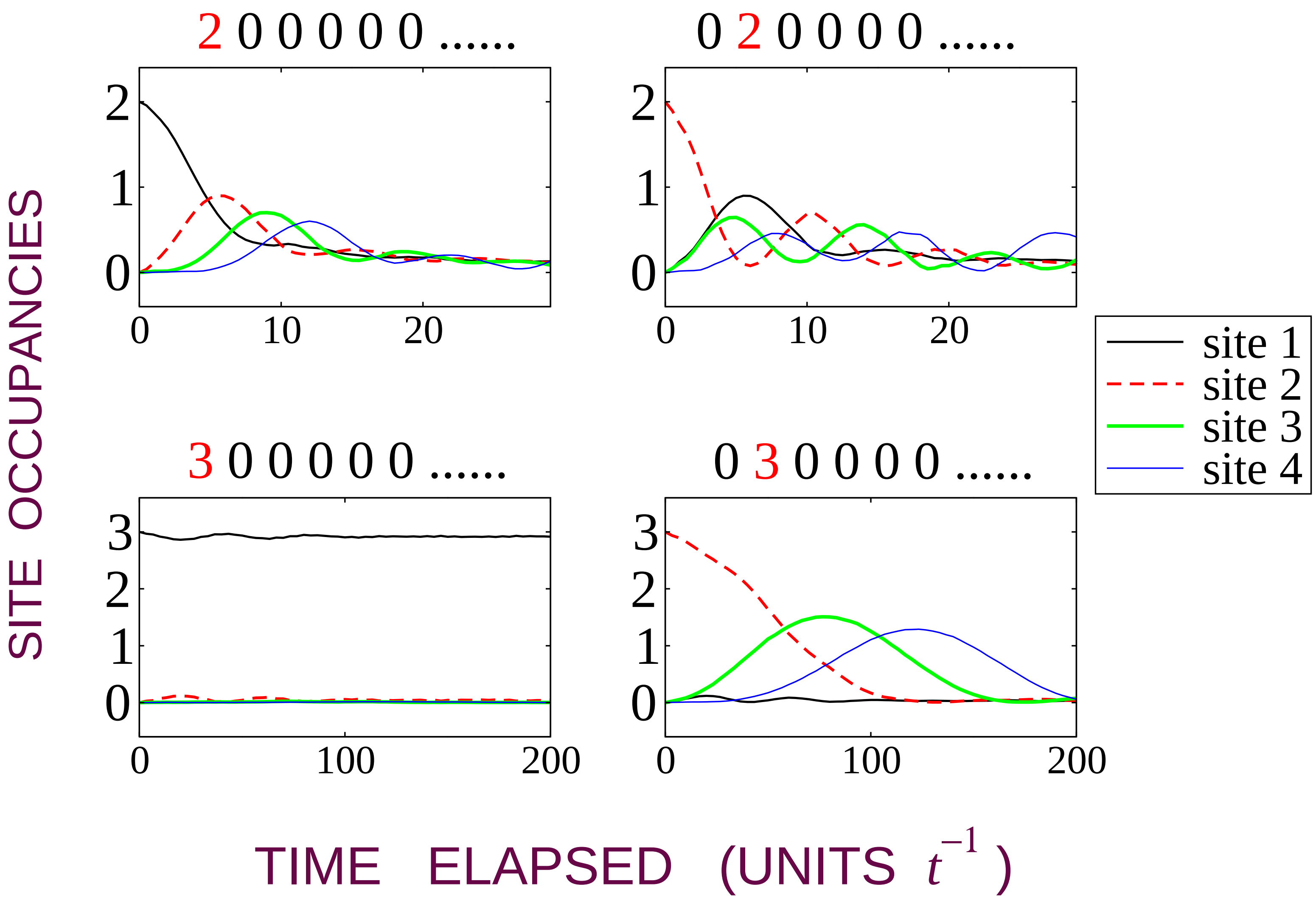


3 0 0 0 0 0 0 ....

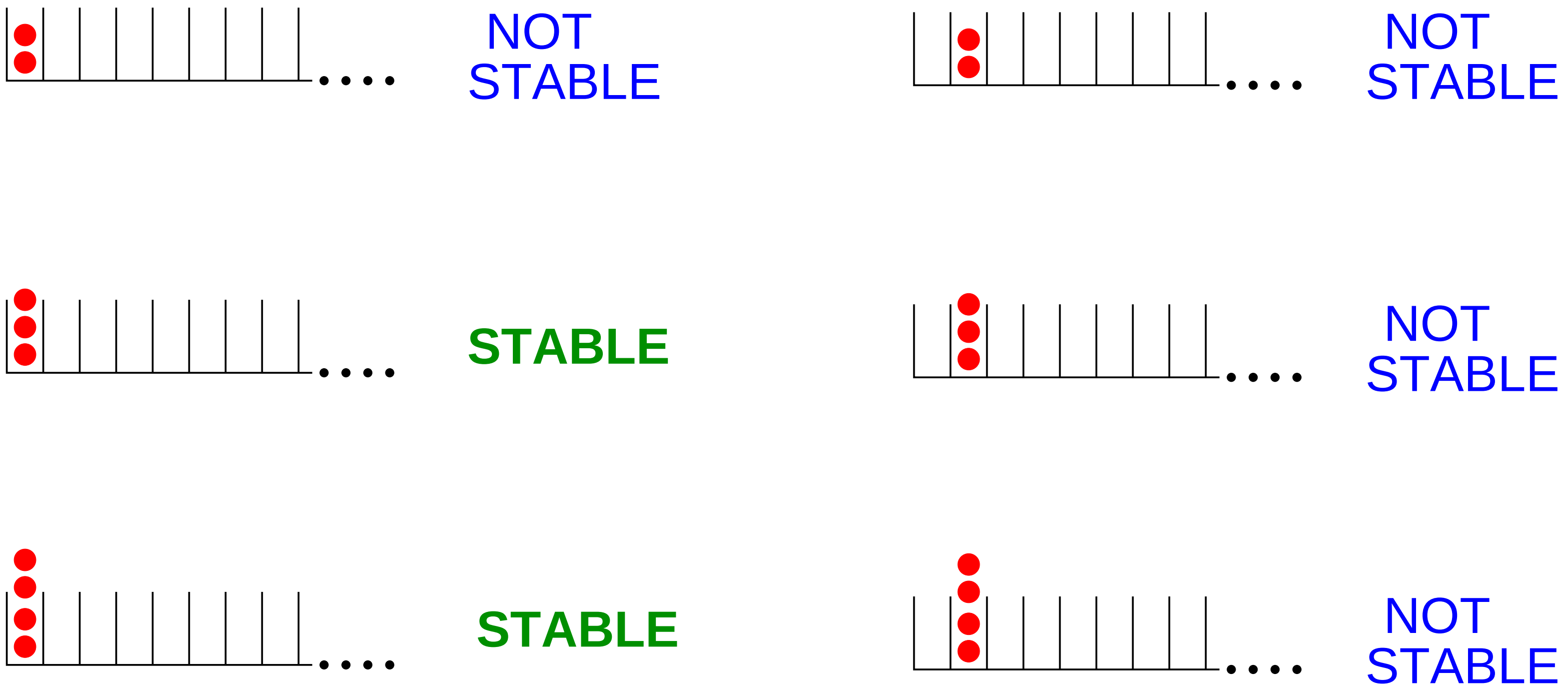
For $n \geq 3$ bosons, edge states are stable.

Stable should mean "close" to an eigenstate?

Time evolution summary



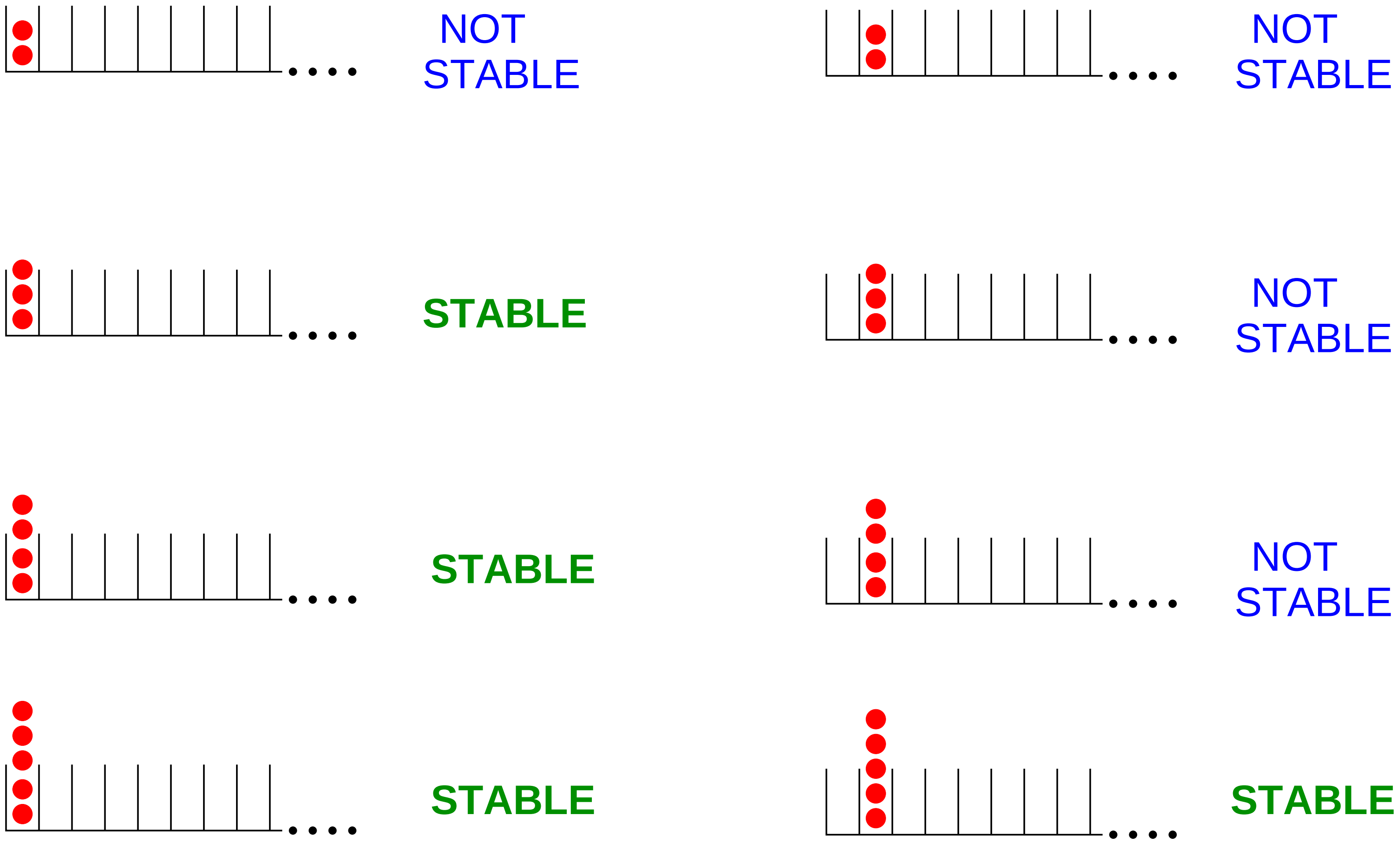
Edge-localized configurations



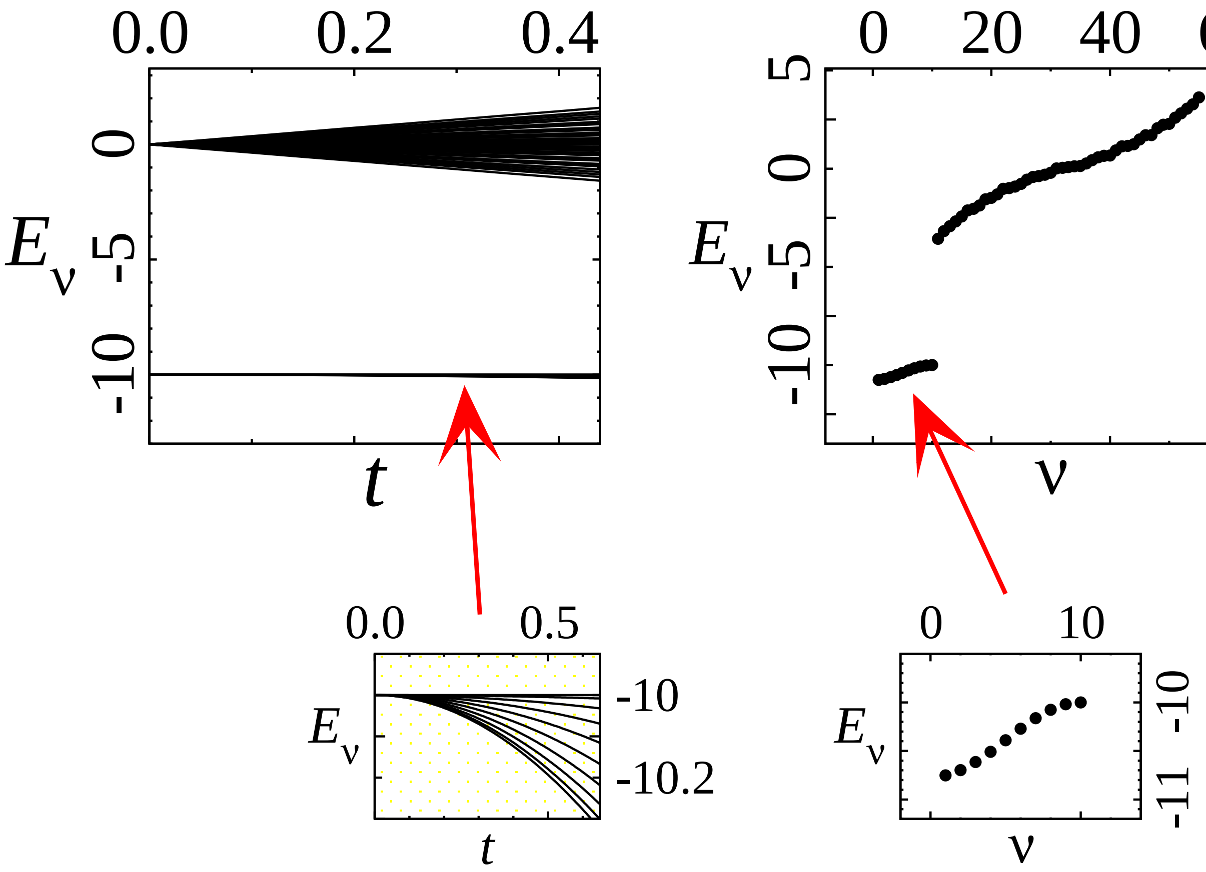
..... beginning of a ‘hierarchy’

‘Stable’ means almost an eigenstate at large U/t.

Hierarchy of edge-locked states



STRUCTURE OF ‘BOUND’ BAND: TWO BOSONS



Linear combinations of

$|20000.....000\rangle$

$|02000.....000\rangle$

$|00200.....000\rangle$

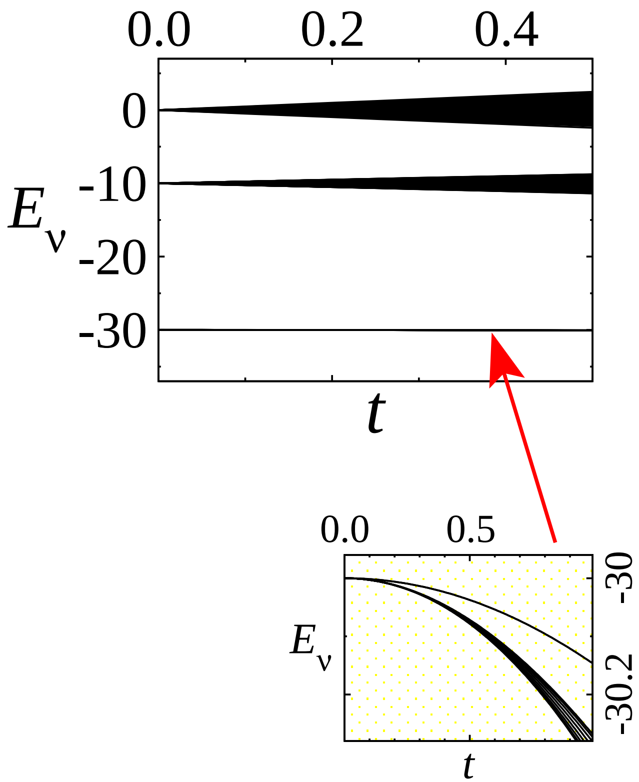
$|00020.....000\rangle$

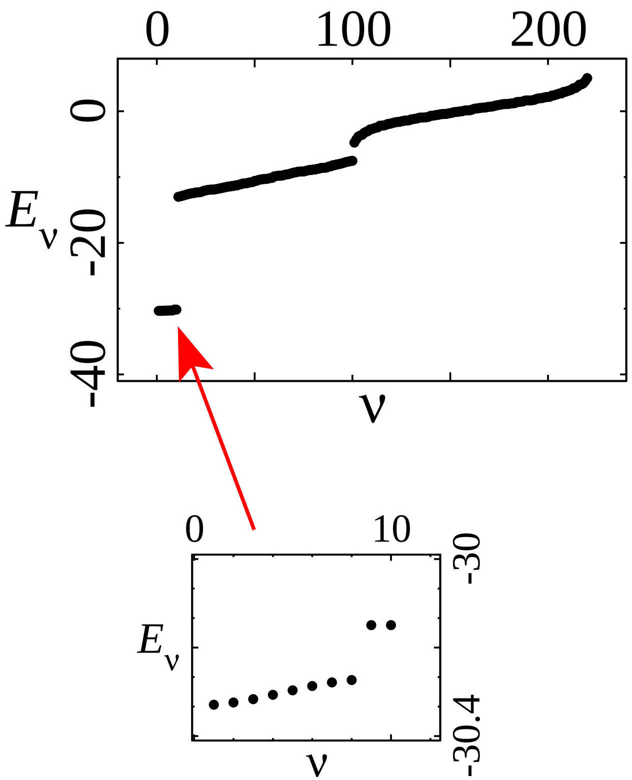
..

..

$|0000.....002\rangle$

‘Bound’ band: three bosons





Linear combinations of

$|03000.....000\rangle$

$|00300.....000\rangle$

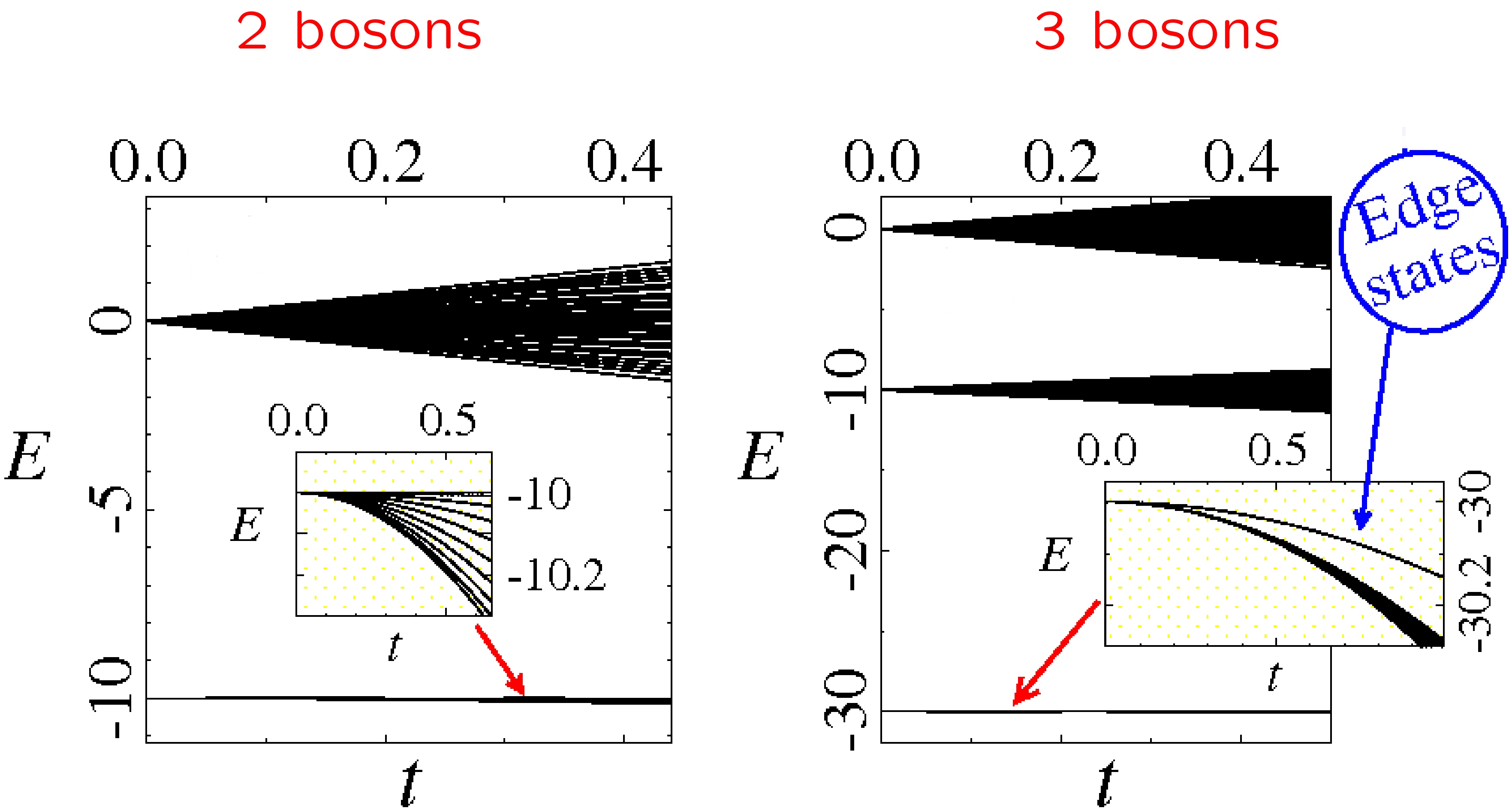
$|00030.....000\rangle$

..

..

$|0000.....030\rangle$

Separated out from the rest: $|30000.....000\rangle$ and $|0000.....003\rangle$.



$|L> =$ ....

.... $|R> =$

Edge eigenstates: $|L> + |R>$ and $|L> - |R>$

Tunnel to other edge?

$$|L\rangle = |3000.....00\rangle \qquad \text{and} \qquad |R\rangle = |00.....0003\rangle$$

Question: Why doesn't $|L\rangle$ tunnel to $|R\rangle$?

Answer: It will. After some astronomically long time.

$|L\rangle \leftrightarrow |R\rangle$ tunneling exponentially suppressed.

Splitting between $|L\rangle + |R\rangle$ and $|L\rangle - |R\rangle$ exponentially small.

Spectral separation explains stability of edge states

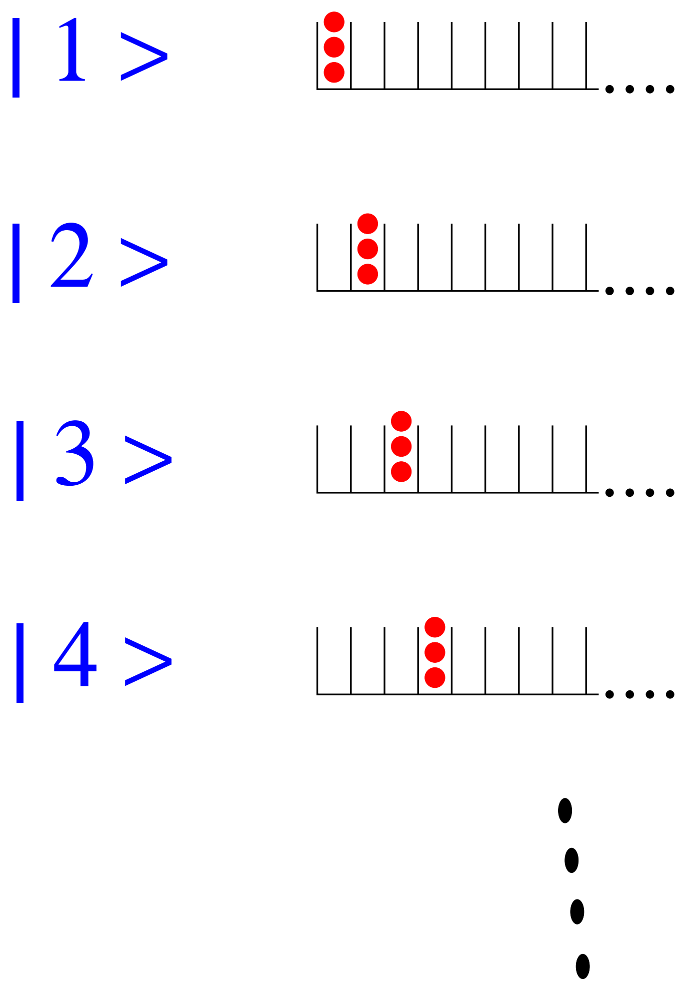
Spectral separation explains stability of edge states

Who ordered the spectral separations?

Degenerate perturbation theory.

Competition between energy shifts at $\mathcal{O}(t^2)$ and manifold mixing at $\mathcal{O}(t^n)$.

Degenerate perturbation theory



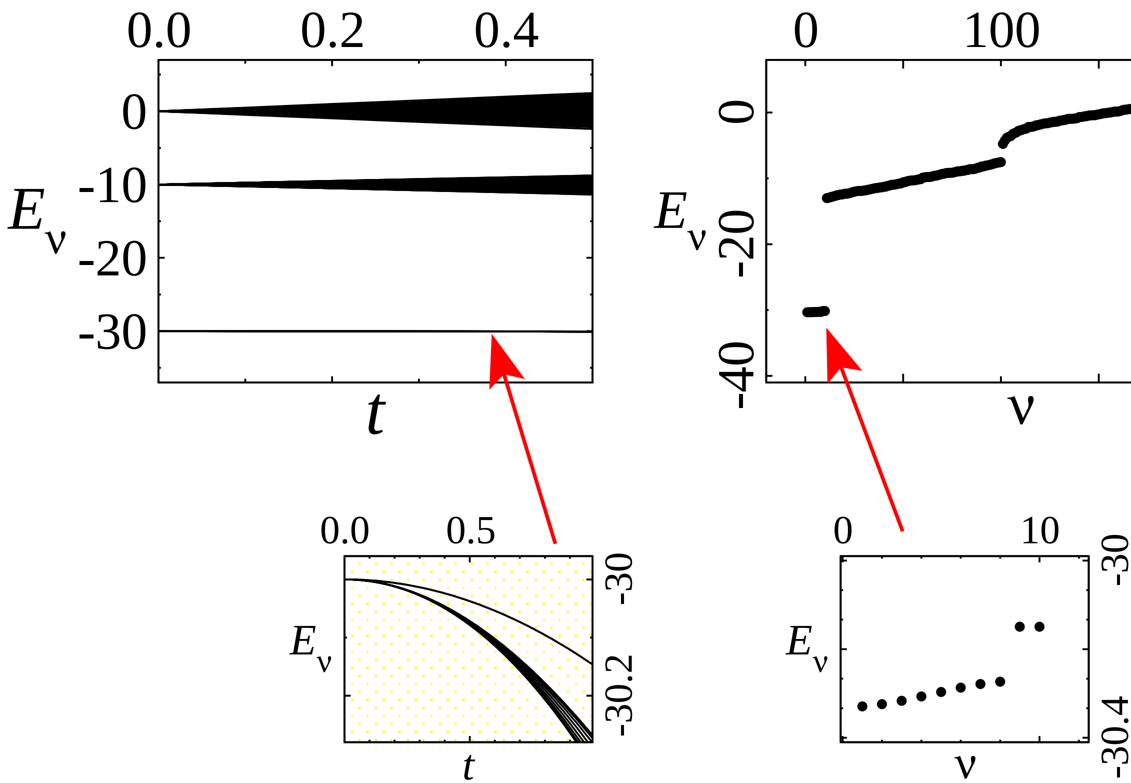
Degenerate manifold at $t/U = 0$.

States $|j\rangle$ and $|j+1\rangle$ connect at $\mathcal{O}(t^n)$.

State $|1\rangle$ acquires different shift at $\mathcal{O}(t^2)$.

State $|2\rangle$ acquires different shift at $\mathcal{O}(t^4)$.

THREE BOSONS: $\mathcal{O}(t^2)$ VERSUS $\mathcal{O}(t^3)$



Linear combinations of

$|03000.....000\rangle$

$|00300.....000\rangle$

$|00030.....000\rangle$

..

..

$|0000.....030\rangle$

Separated out from the rest: $|30000.....000\rangle$ and $|0000.....003\rangle$.

Spinless fermion model: similar hierarchy

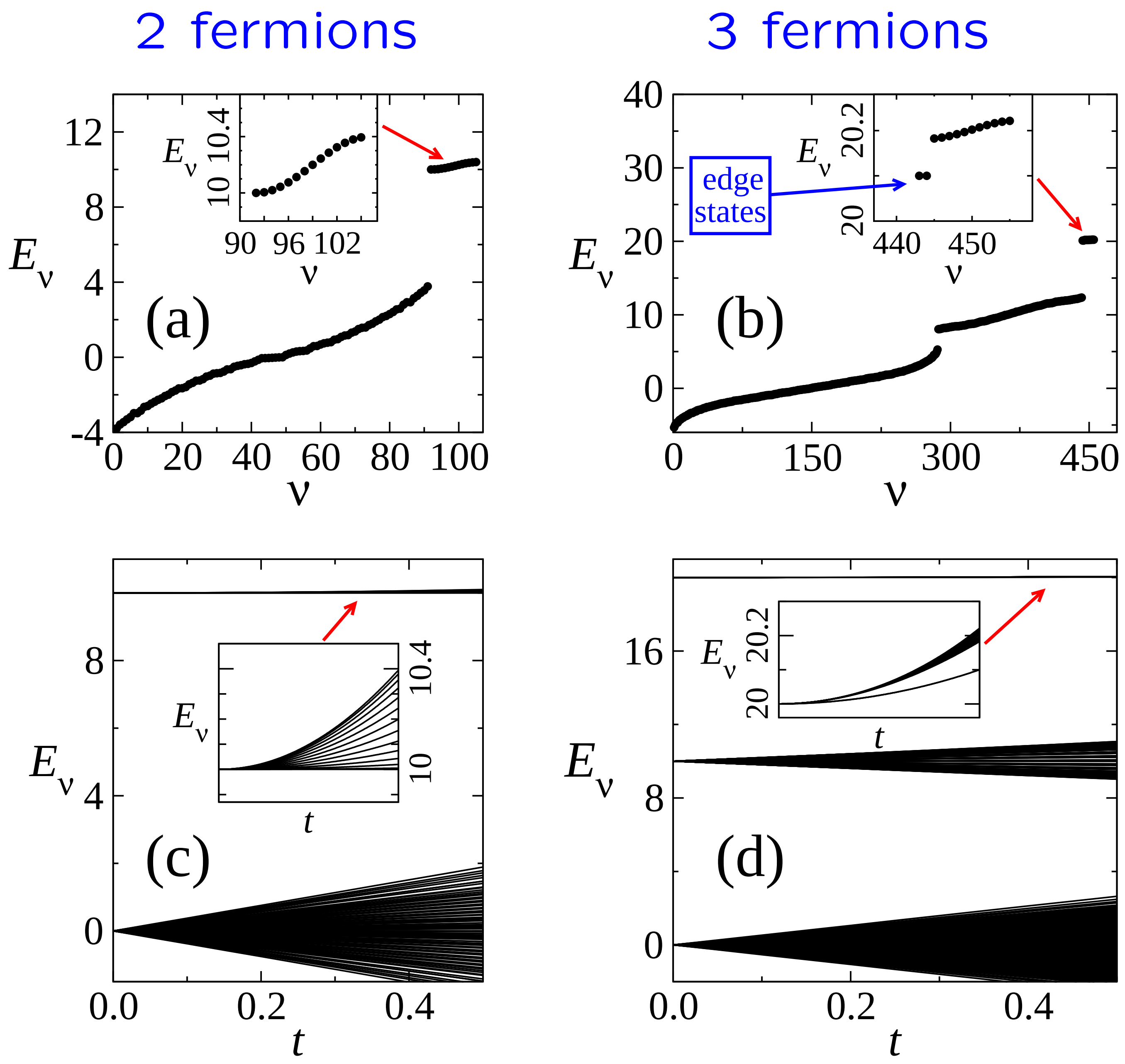
$$\hat{H} \;=\; -\, t \sum_{j=1}^{L-1} \left(c_j^\dagger c_{j+1} \;+\; c_{j+1}^\dagger c \right) \;+\; V \sum_{j=1}^{L-1} c_j^\dagger c_{j+1}^\dagger c_{j+1} c_j$$

Sometimes called t-V model or Heisenberg-Ising model.

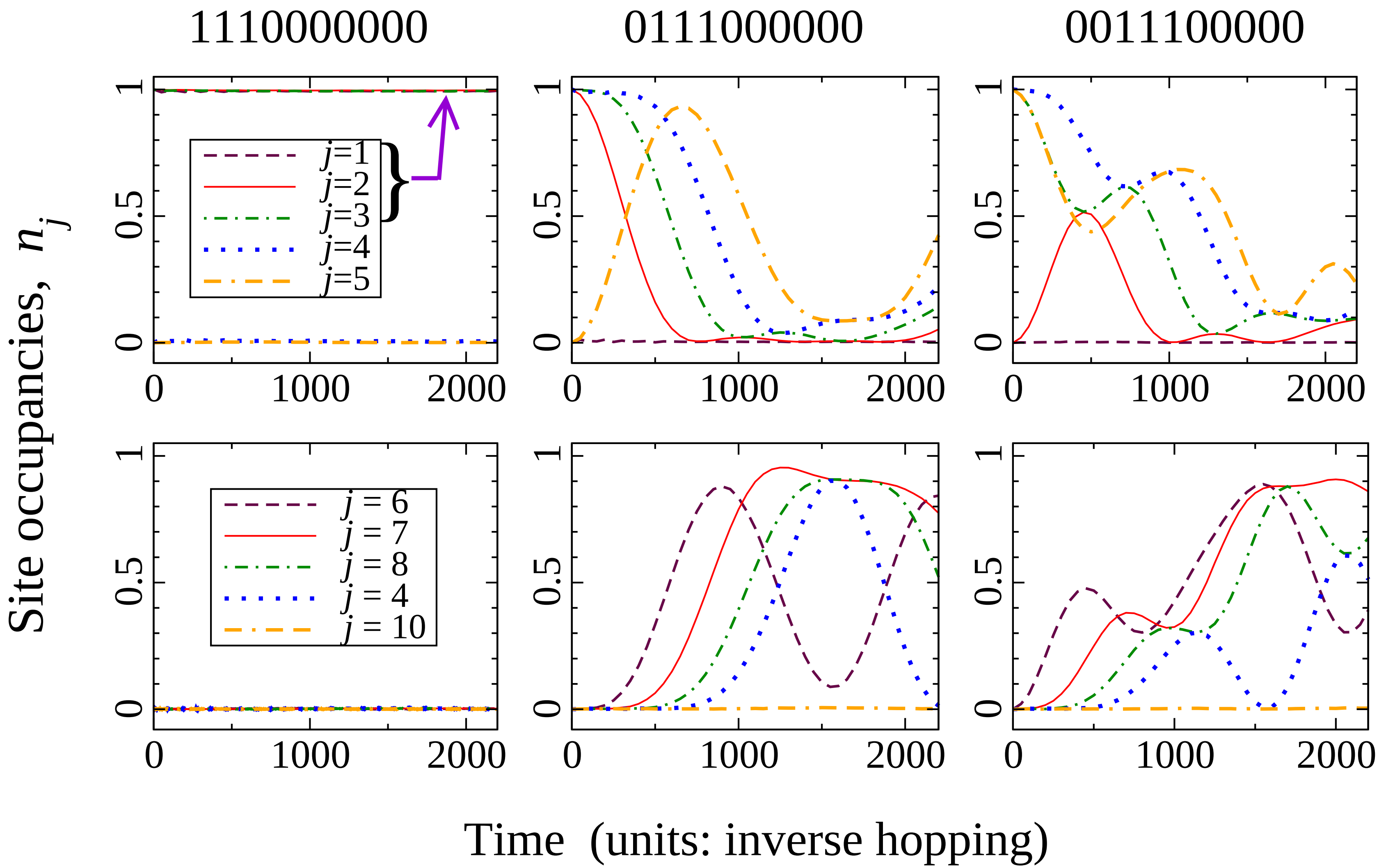
1 1 1 0 0 0 0 0 0 0 0	1 0 0 0 0 0 0 0 0 0 0 ......
1 1 1 1 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0 0 ......
1 1 1 1 1 0 0 0 0 0 0	0 1 0 0 0 0 0 0 0 0 0 ......
1 1 1 1 1 1 0 0 0 0 0	0 1 1 0 0 0 0 0 0 0 0 ......
0 1 1 1 1 1 0 0 0 0 0	0 1 1 1 0 0 0 0 0 0 0 ......
0 1 1 1 1 1 1 0 0 0 0	0 1 1 1 1 0 0 0 0 0 0 ......

Spinless fermions: spectrum

Two, three fermions in 15 sites



Spinless fermions: dynamics

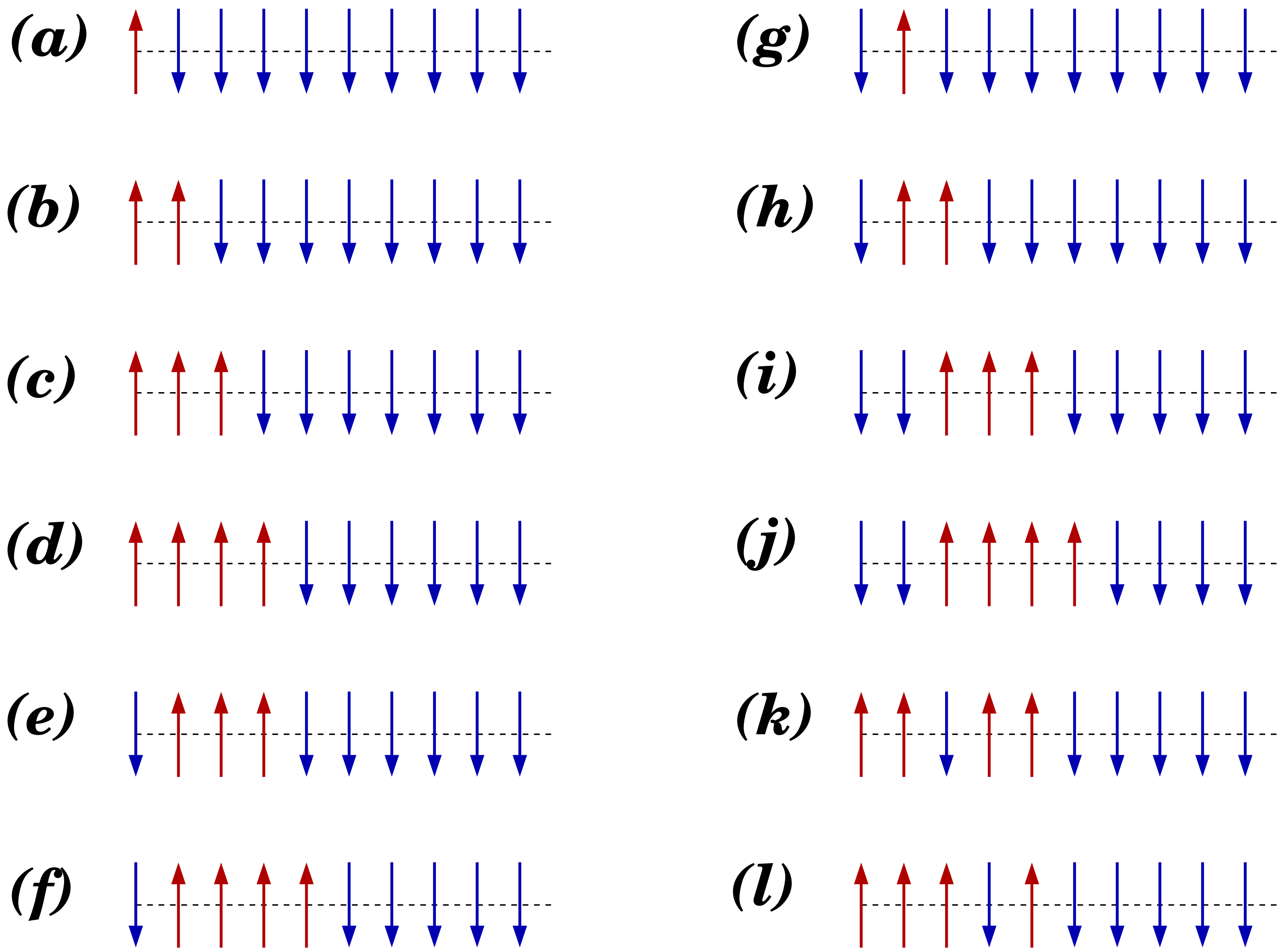


Anisotropic Heisenberg (XXZ) chain

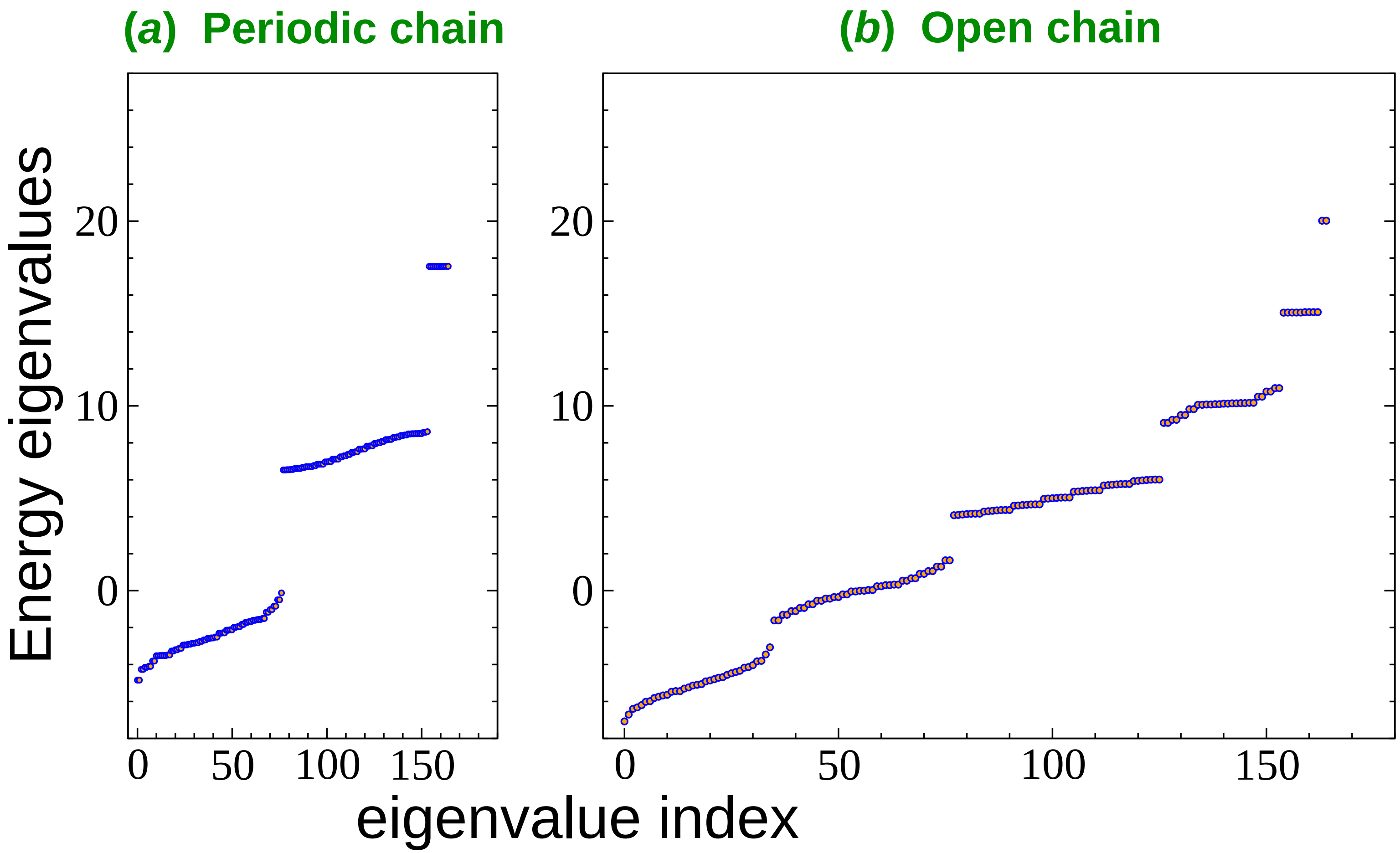
$$H = J_x \sum_{j=1}^{L-1} \left[S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \right]$$

Edge-locking hierarchy $\longrightarrow$ surprisingly different from t-V model.

t-V model does not have empty-empty or empty-occupied energy.
(Only occupied-occupied energy.)


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)

XXZ CHAIN: PERIODIC VERSUS OPEN SPECTRA

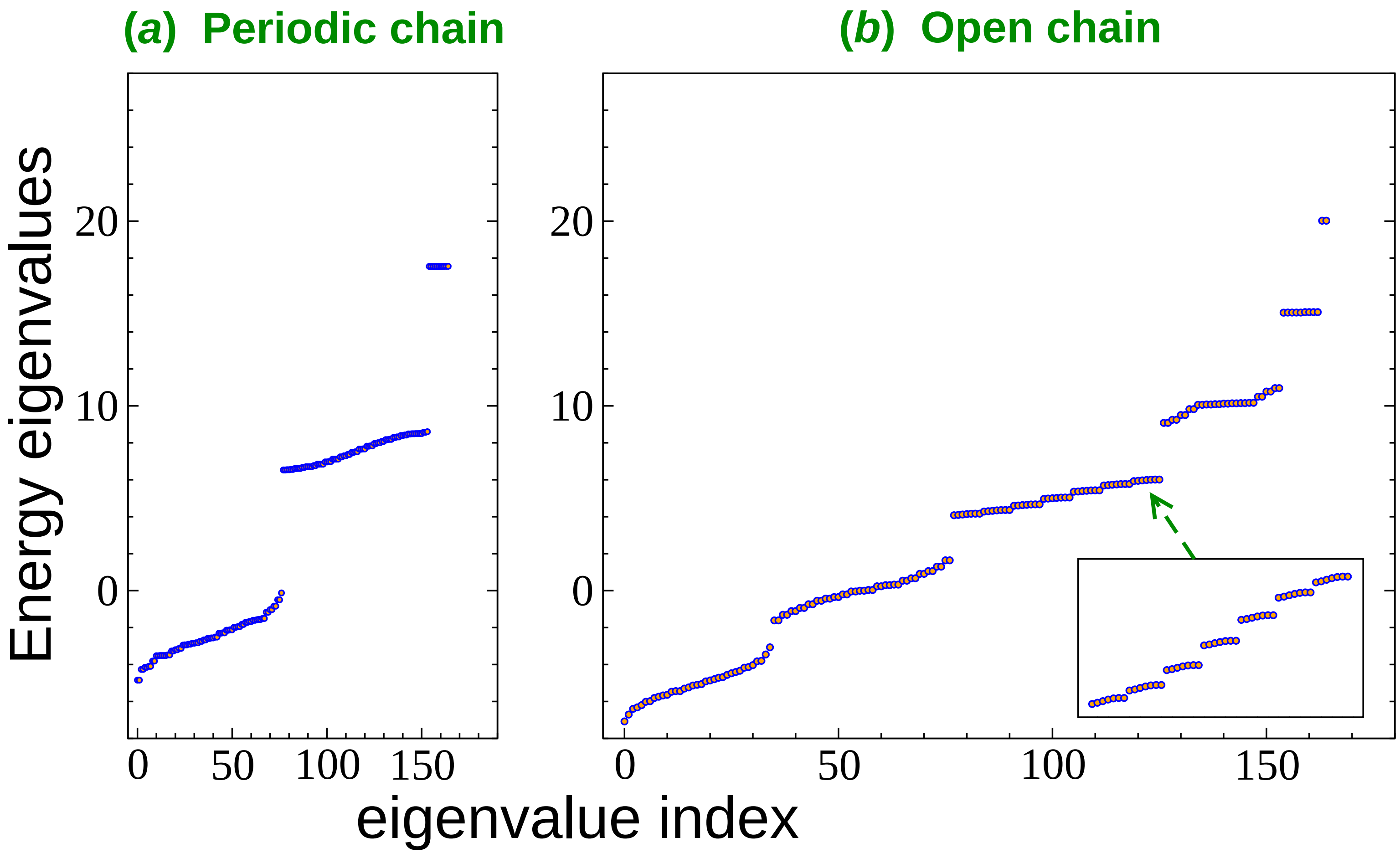


$N_{\uparrow} = 3$

11 sites

Many extra spectral features in open chain

XXZ CHAIN: PERIODIC VERSUS OPEN SPECTRA



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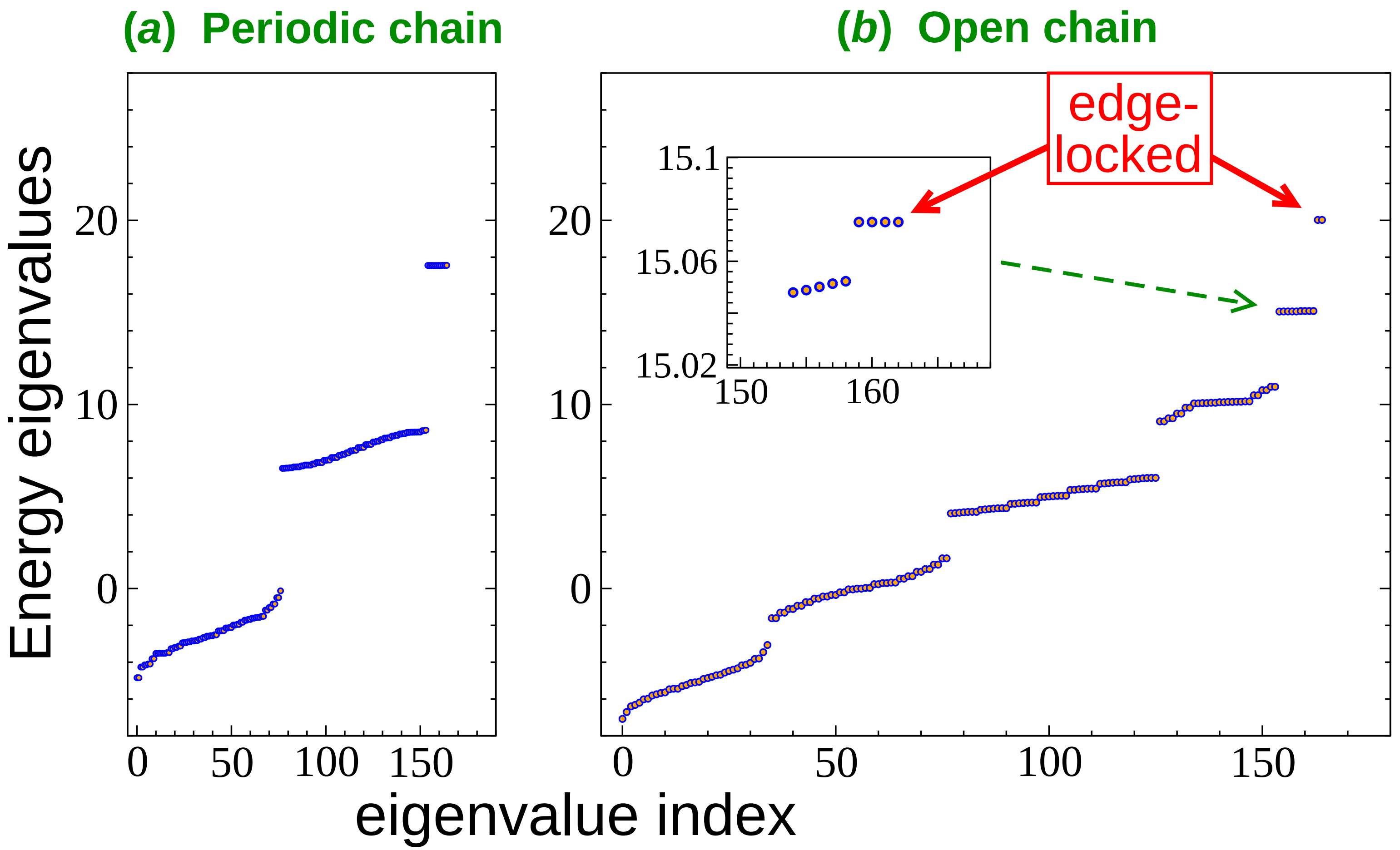
XXZ CHAIN: PERIODIC VERSUS OPEN SPECTRA

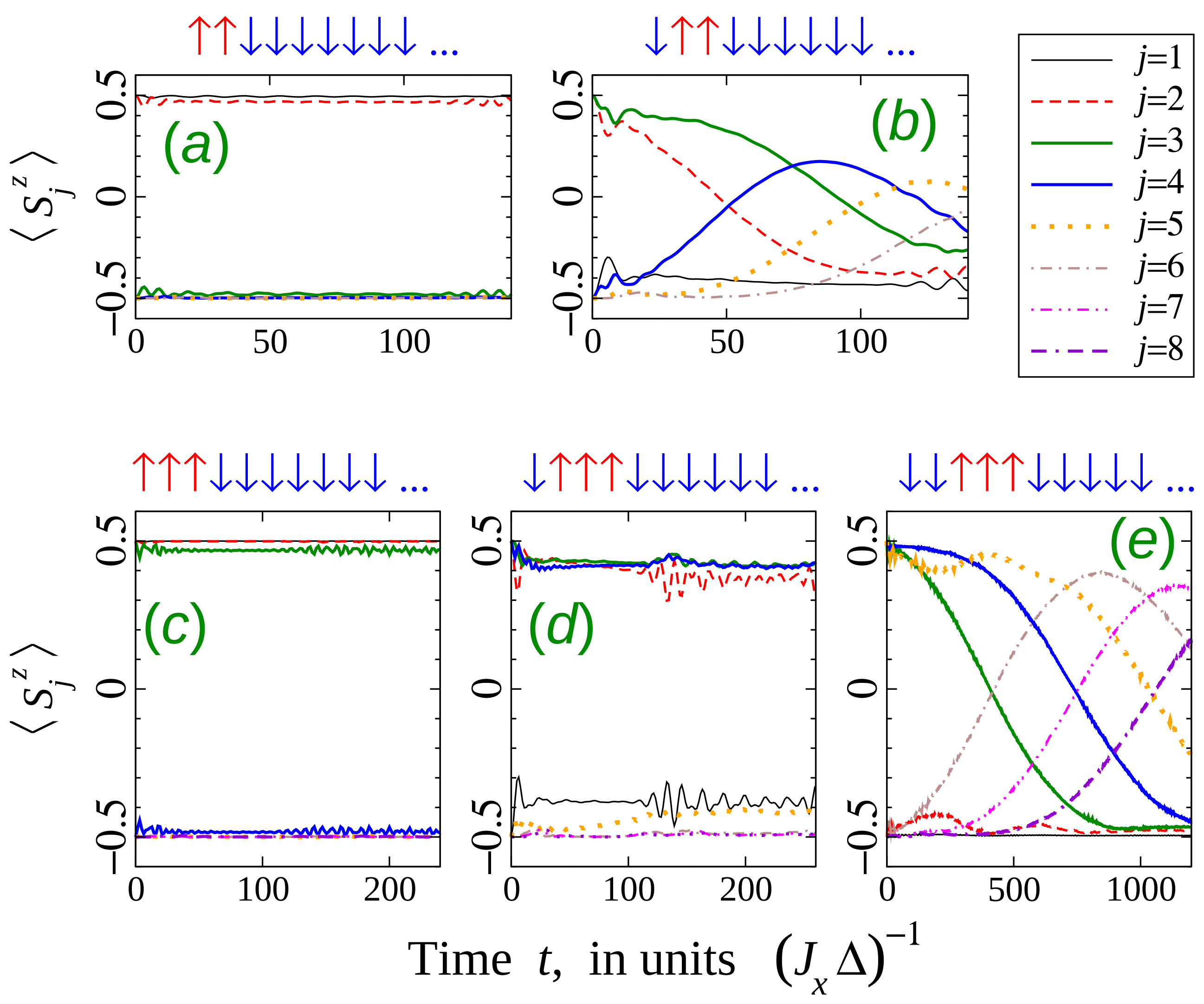


$N_\uparrow = 3$

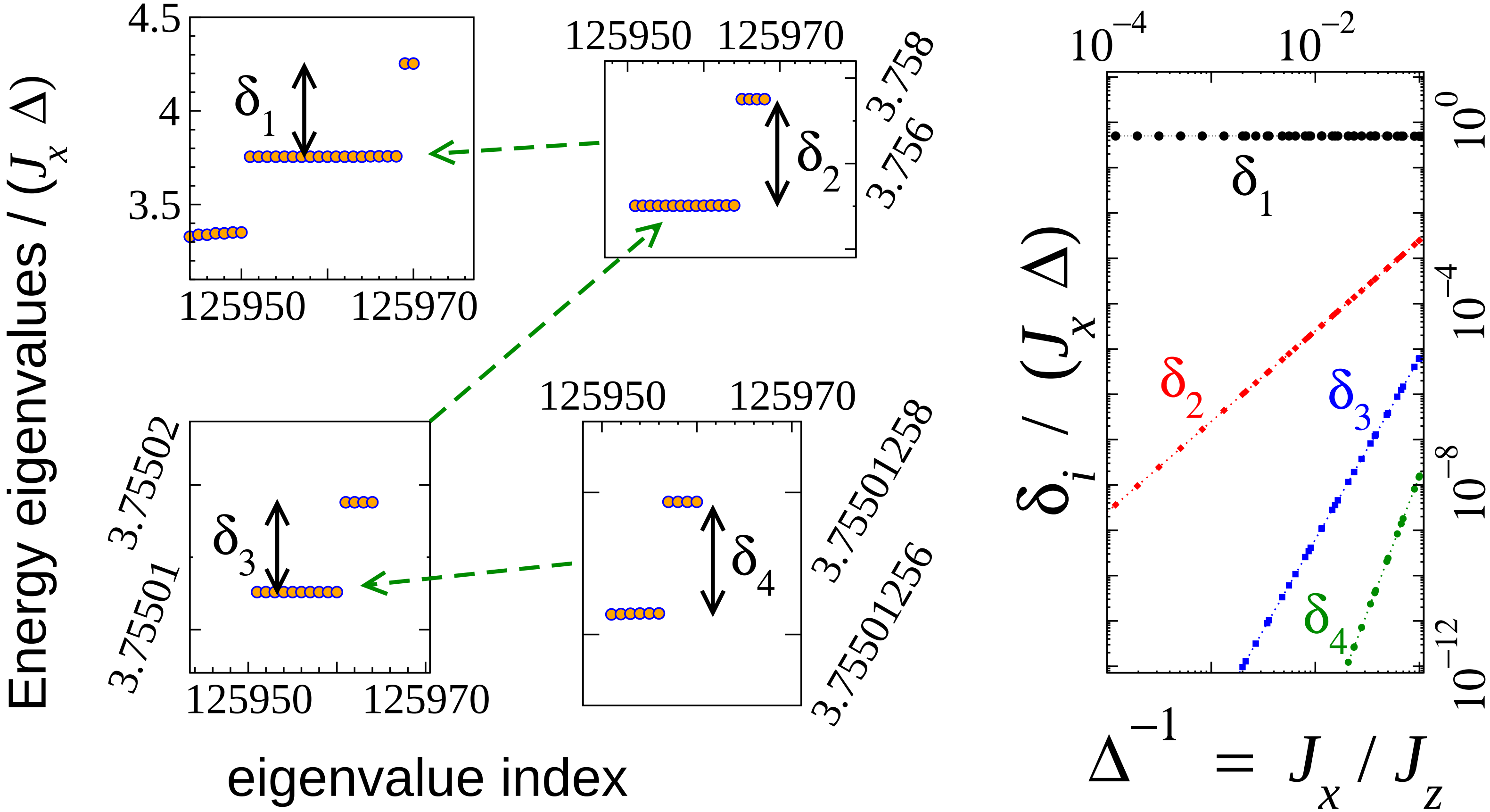
11 sites

Many extra spectral features in open chain

XXZ CHAIN: DYNAMICS



XXZ chain: hierarchy



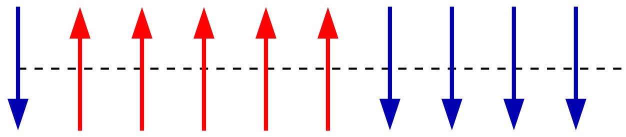
$N_\uparrow = 8$; 20 sites. $\delta_1 \sim \Delta^0$ $\delta_2 \sim \Delta^{-2}$ $\delta_3 \sim \Delta^{-4}$

Hierarchy of edge-localization

Energy spectrum contains structures at
many different scales.

FRACTAL structure in spectrum

“Quantum control” of magnetization transport

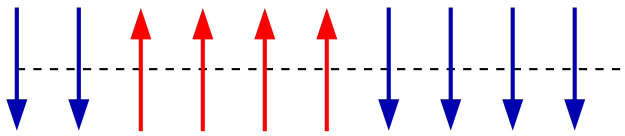


Single-site π-pulse

$\Downarrow$

quantum switch

for unlocking magnetization.



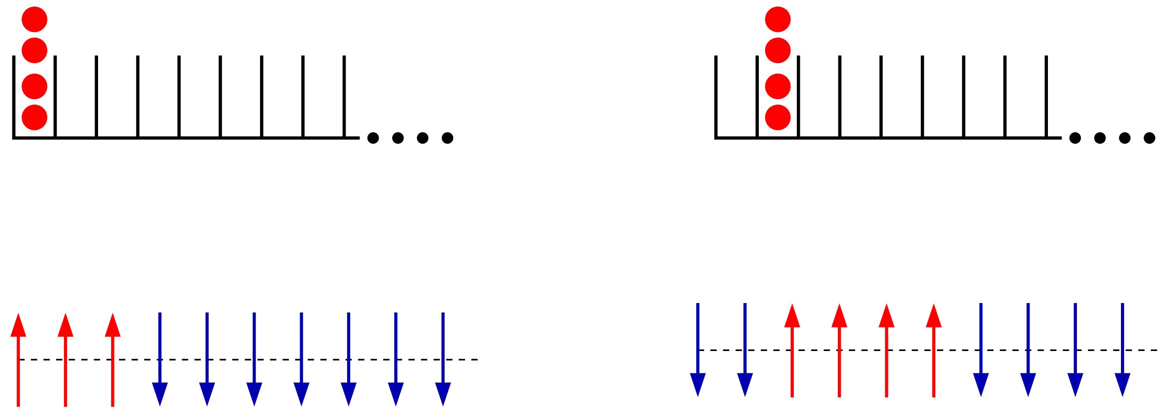
Many other control protocols....

Edge-localization in 1D lattice models

Bose-Hubbard chain

spinless fermion model

XXZ chain



PHYSICS:

Eigenstates far from ground state

Far-from-equilibrium dynamics

Intricate structures in spectrum (FRACTAL)

QUANTUM CONTROL:

Locking and release of magnetization/state

Designing a quantum switch

Edge localization: more bosons

For more bosons, a hierarchy of localization patterns.

$n \geq 5$ bosons $\longrightarrow$ can also be bound in site 2

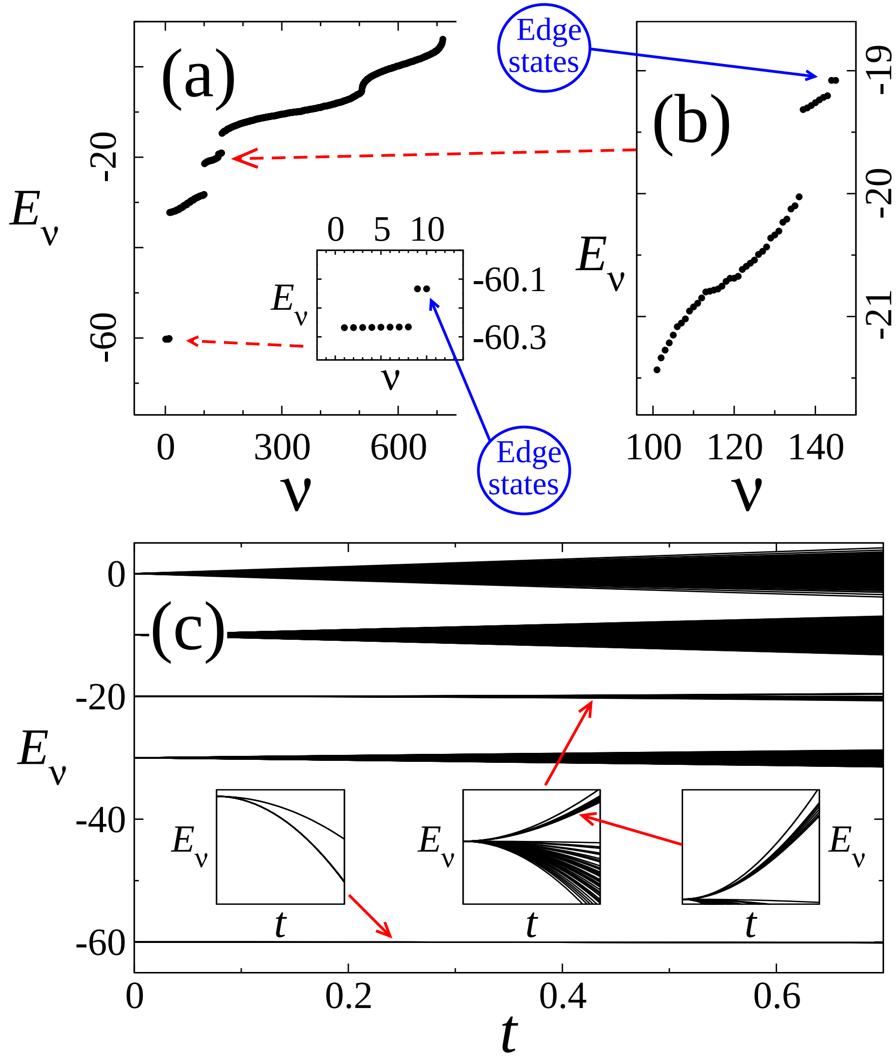
$n \geq 7$ bosons $\longrightarrow$ can also be bound in site 3

... ...etc

Actually, several hierarchies, with other localization patterns:

2 2 0 0 0 0 ......

EDGE-LOCALIZATION: SPECTRAL PICTURE (4 BOSONS)



Isn't this just self-trapping?

Question from
nonlinear dynamics
and/or BEC
community

Isn't this just self-trapping?

NO