ESF-FWF Conference on Quantum Engineering of States and Devices
Obergurgl, June 7, 2010

Two-Impurity Kondo Model with Spin-Orbit Interactions

in collaboration with
David Mross

UNIVERSITY OF GOTHENBURG

VETENSKAPSRÅDET
supported by the
Swedish Research Council

Outline

Basics on two-impurity Kondo physics

Outline

Basics on two-impurity Kondo physics

Adding spin-orbit (SO) interactions

Outline

Basics on two-impurity Kondo physics

Adding spin-orbit (SO) interactions

SO effects in the “RKKY limit”

Outline

Basics on two-impurity Kondo physics

Adding spin-orbit (SO) interactions

SO effects in the “RKKY limit”

SO effects at quantum criticality

Outline

Basics on two-impurity Kondo physics

Adding spin-orbit (SO) interactions

SO effects in the “RKKY limit”

SO effects at quantum criticality

Summary

Outline

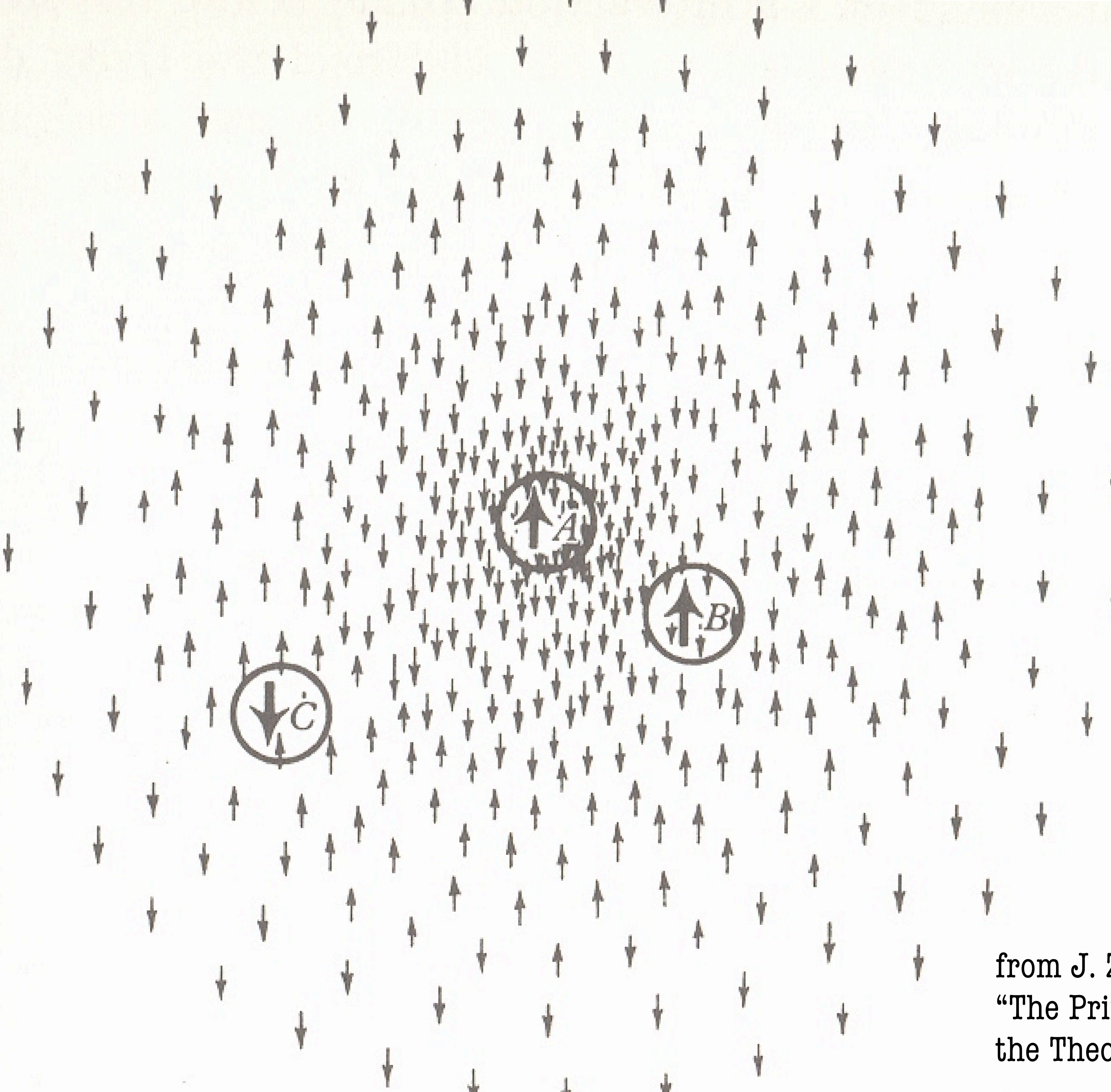
Basics on two-impurity Kondo physics

Adding spin-orbit (SO) interactions

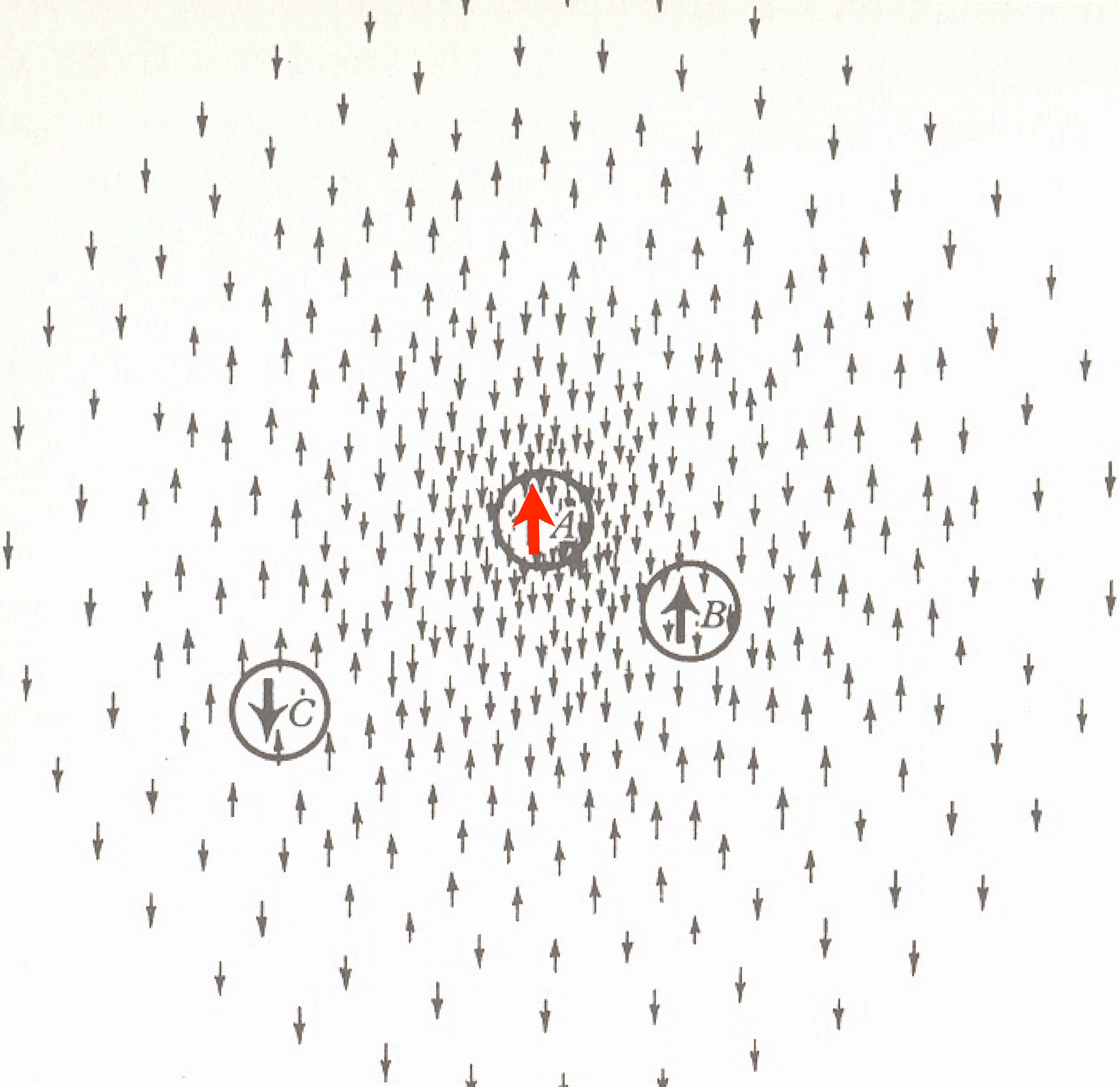
SO effects in the "RKKY limit"

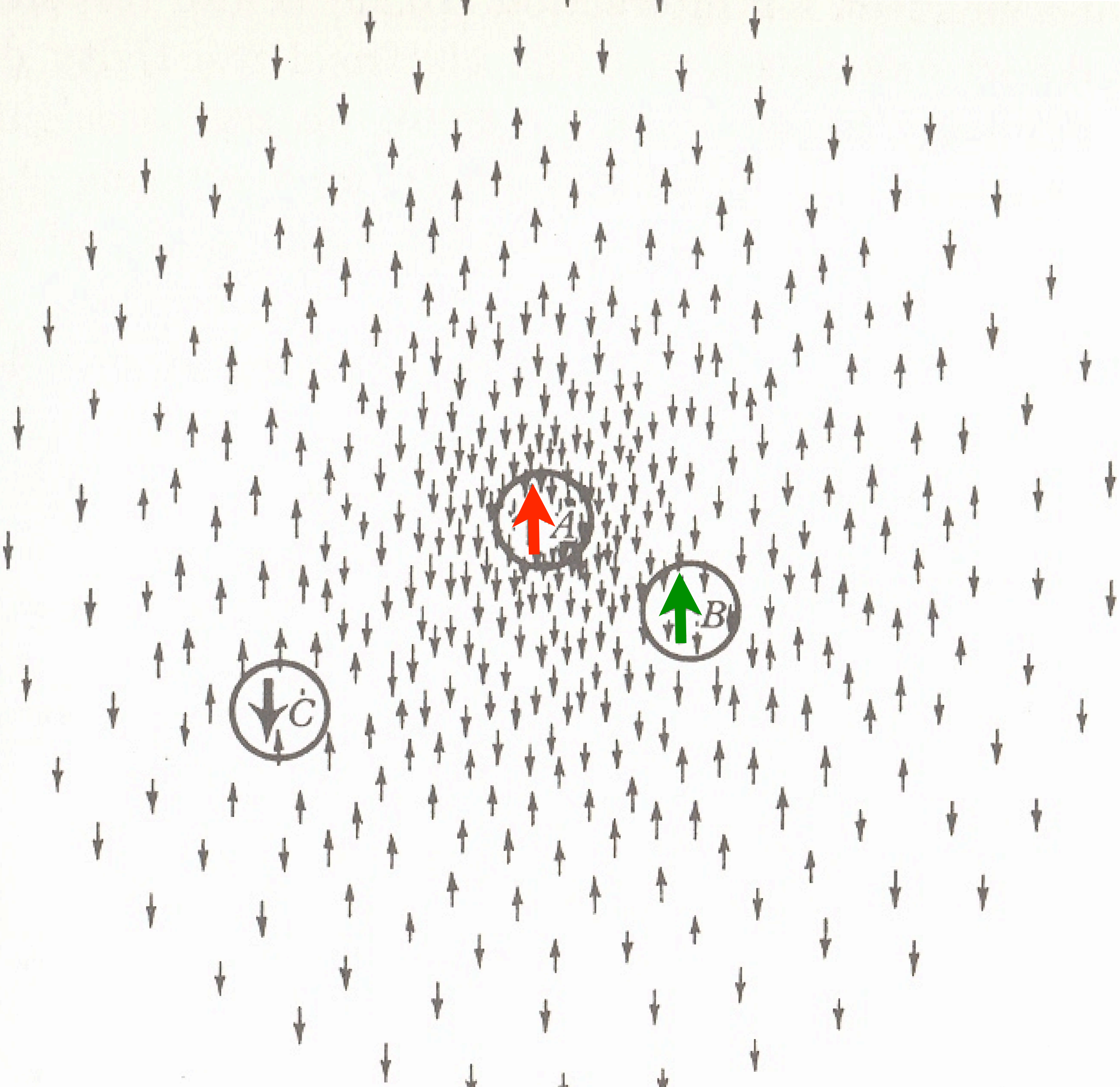
SO effects at quantum criticality

Summary

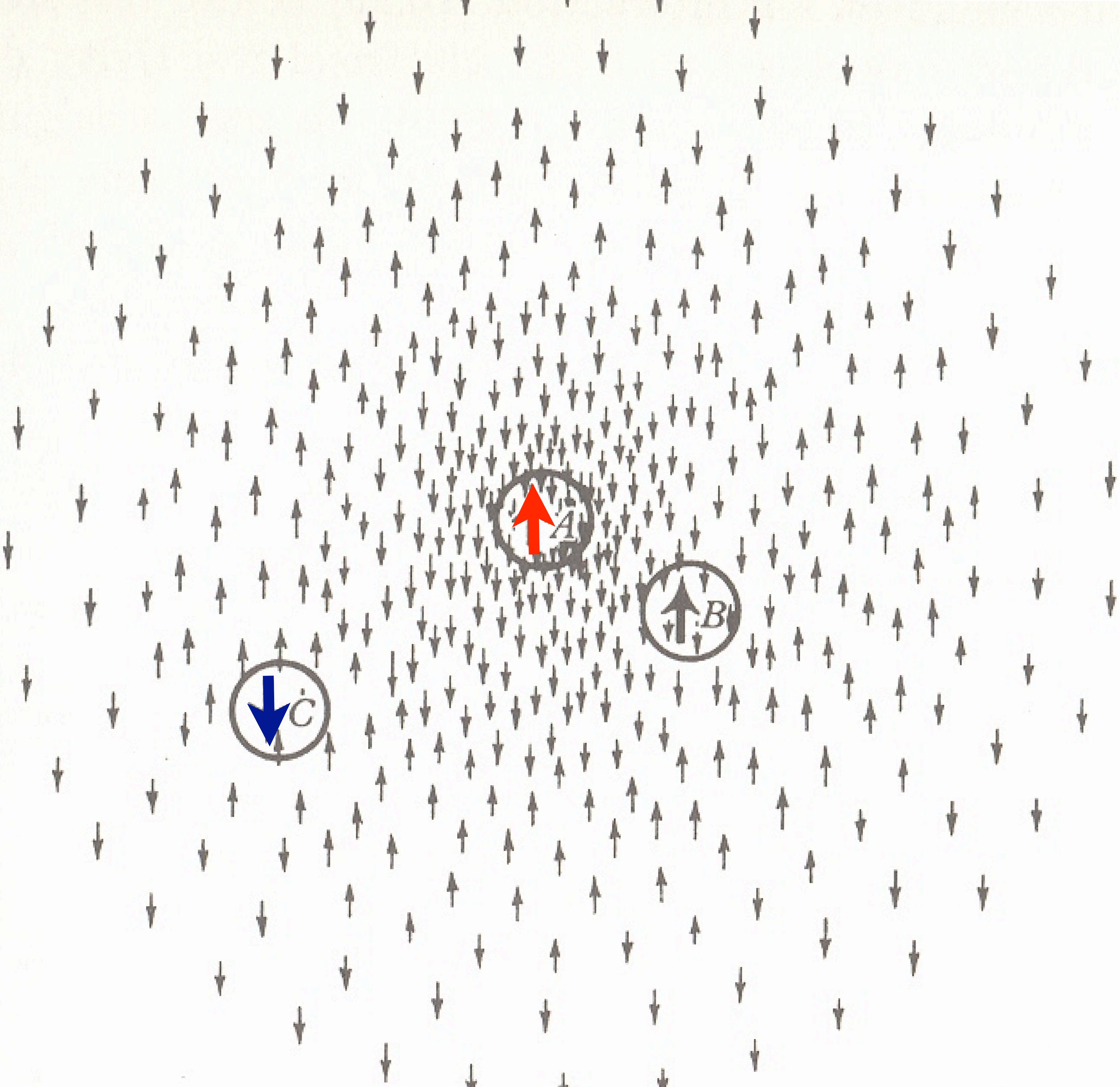


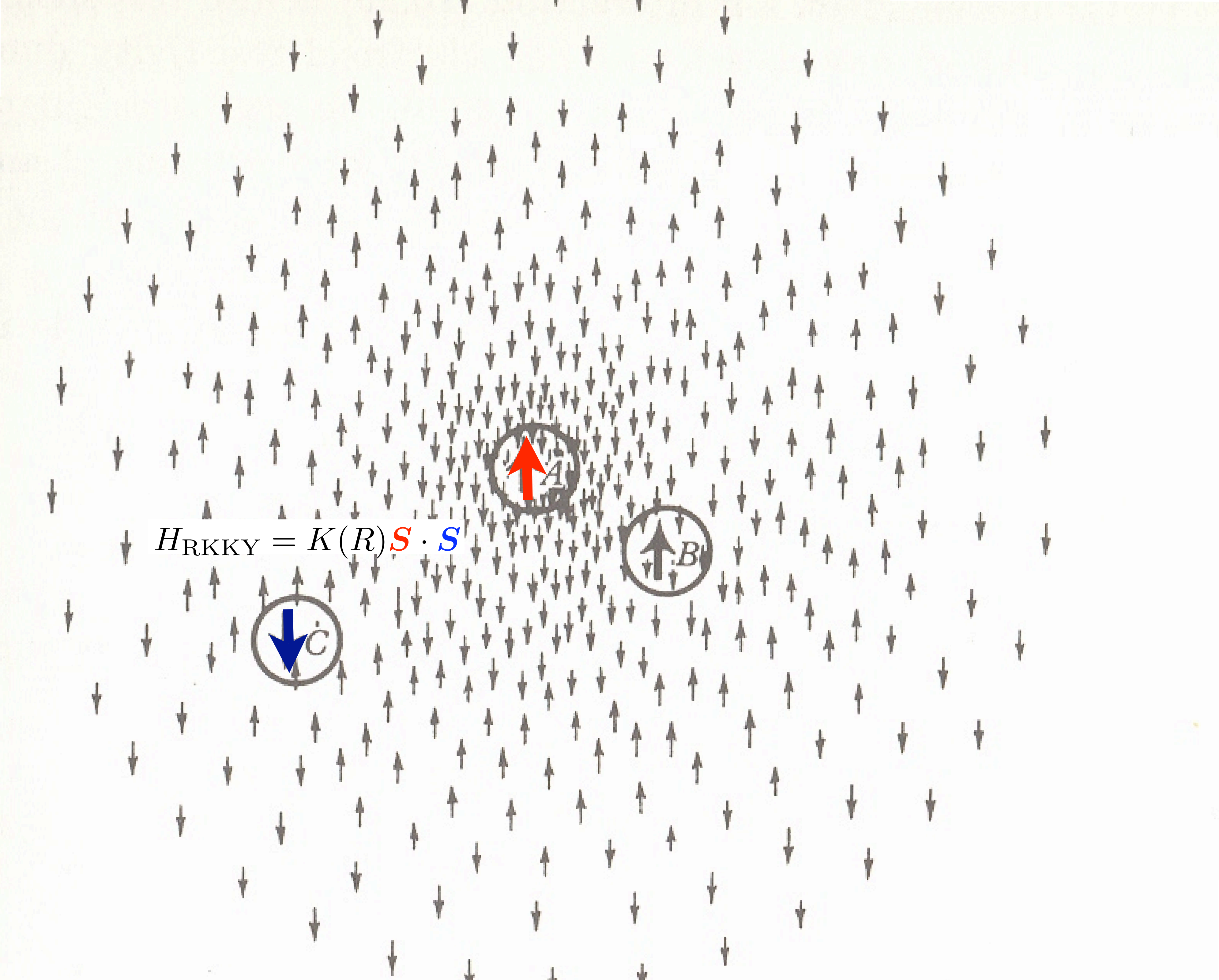
from J. Ziman,
"The Principles of
the Theory of Solids"

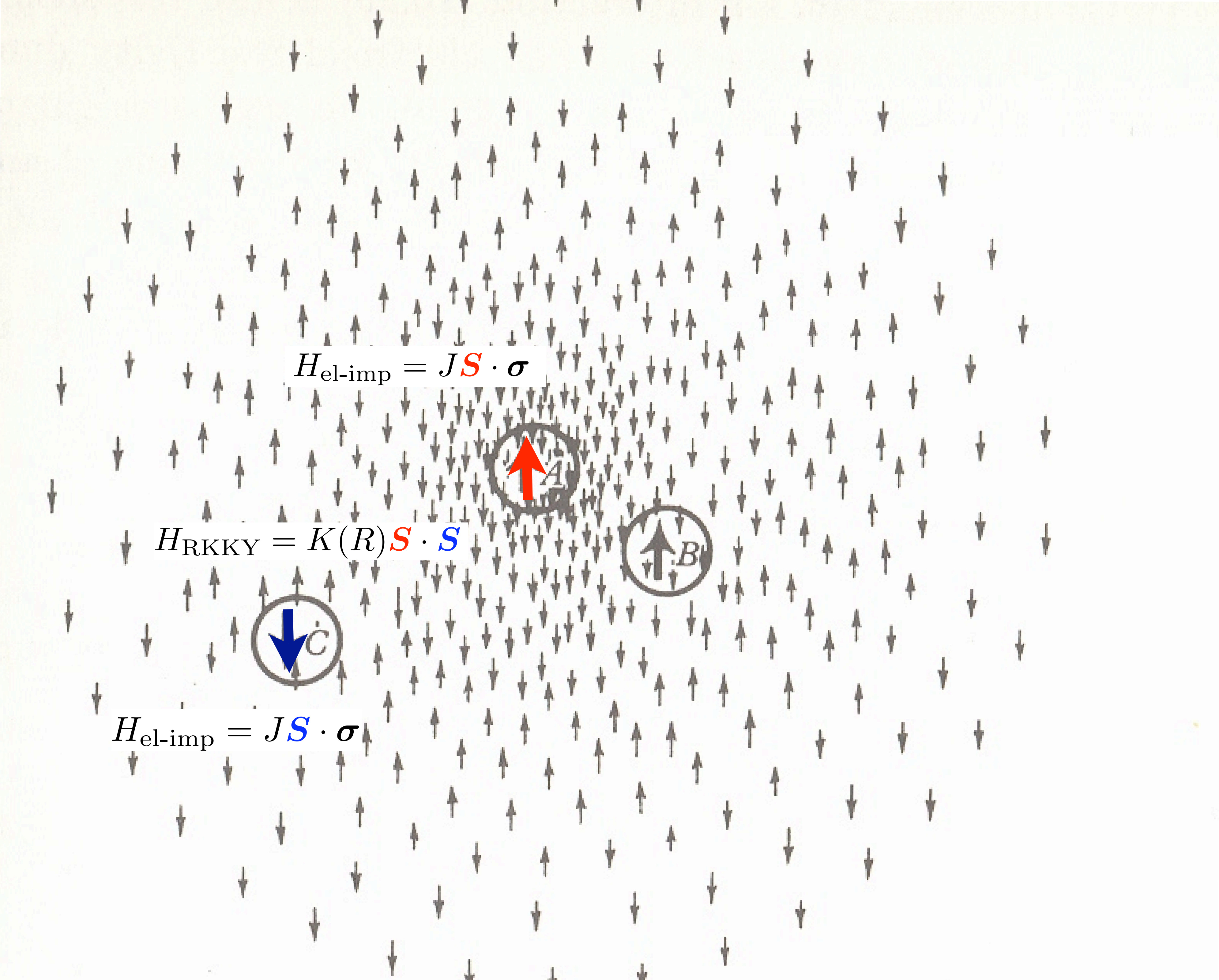

A
B
C



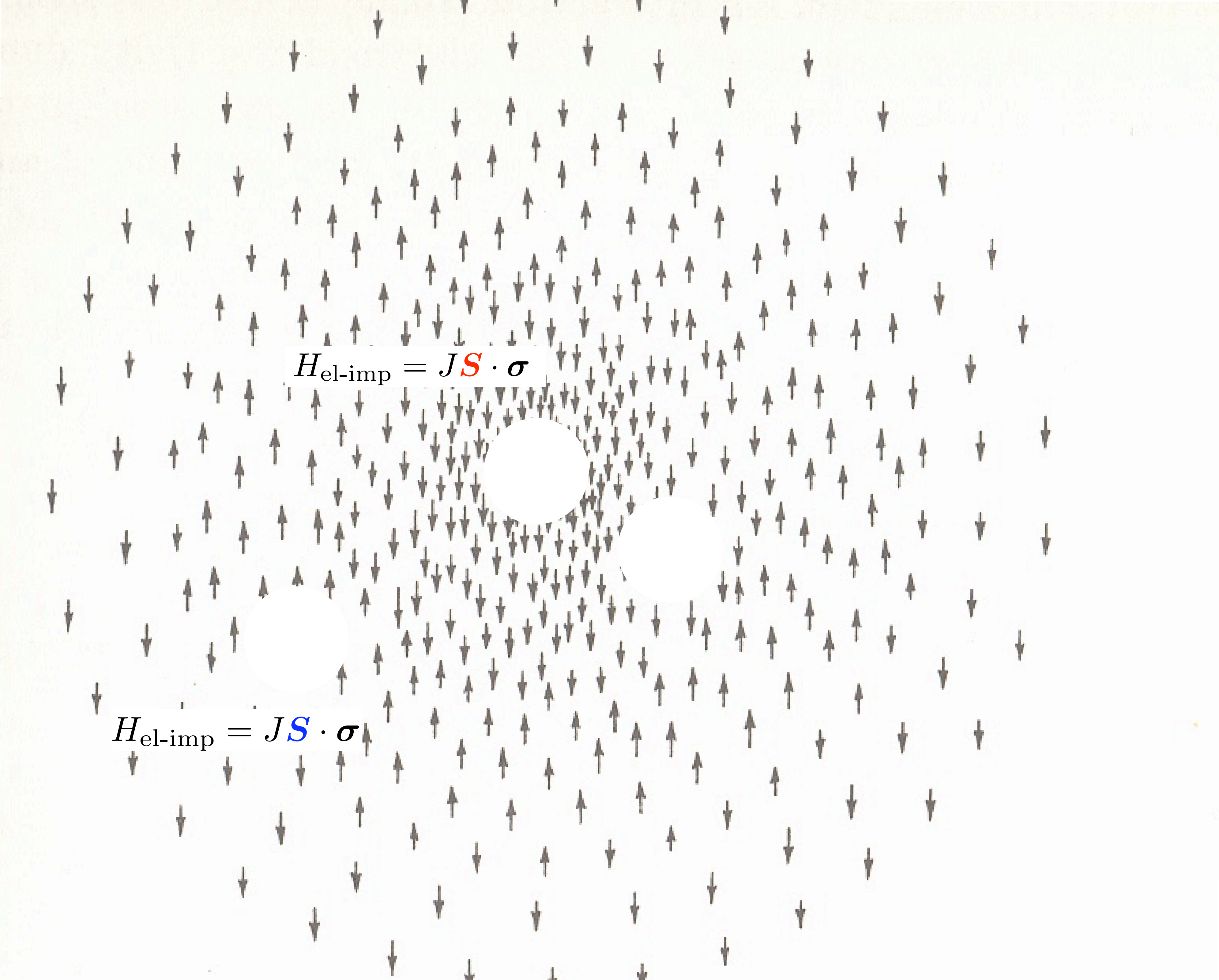
A
B
C


A
B
C


A
B
C
$H_{\text{RKKY}} = K(R)\boldsymbol{S}\cdot\boldsymbol{S}$



$H_{\text{el-imp}} = J\boldsymbol{S}\cdot\boldsymbol{\sigma}$
A
B
$H_{\text{RKKY}} = K(R)\boldsymbol{S}\cdot\boldsymbol{S}$
C
$H_{\text{el-imp}} = J\boldsymbol{S}\cdot\boldsymbol{\sigma}$


$H_{\text{el-imp}} = J\boldsymbol{S}\cdot\boldsymbol{\sigma}$
$H_{\text{el-imp}} = J\boldsymbol{S}\cdot\boldsymbol{\sigma}$

$$H = H_{\mathrm{kin}} + J\boldsymbol{S}_1 \cdot \boldsymbol{\sigma} + J\boldsymbol{S}_2 \cdot \boldsymbol{\sigma} + K(R)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

VOLUME 47, NUMBER 10 PHYSICAL REVIEW LETTERS 7 SEPTEMBER 1981

Two-Impurity Kondo Problem

C. Jayaprakash
Nordisk Institut for Teoretisk Atomfsyik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Cornell University, Ithaca, New York 14853

and

H. R. Krishna-murthy
Nordisk Institut for Teoretisk Atomfsyik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Indian Institute of Science, Bangalore, India

and

J. W. Wilkins
Nordisk Institut for Teoretisk Atomfsyik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Cornell University, Ithaca, New York 14853
(Received 28 May 1981)

The two-impurity Kondo problem is studied by use of perturbative scaling techniques. The physics is determined by the interplay between the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between the two impurity spins and the Kondo effect. In particular, for a strong ferromagnetic RKKY interaction the susceptibility exhibits three structures as the temperature is lowered, corresponding to the ferromagnetic locking together of the two impurity spins followed by a two-stage freezing out of their local moments by the conduction electrons due to the Kondo effect.

competition between RKKY-interaction and Kondo screening

$$H = H_{\text{kin}} + J\boldsymbol{S}_1 \cdot \boldsymbol{\sigma} + J\boldsymbol{S}_2 \cdot \boldsymbol{\sigma} + K(R)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

VOLUME 47, NUMBER 10 PHYSICAL REVIEW LETTERS 7 SEPTEMBER 1981

Two-Impurity Kondo Problem

C. Jayaprakash
Nordisk Institut for Teoretisk Atomfysik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Cornell University, Ithaca, New York 14853

and

H. R. Krishna-murthy
Nordisk Institut for Teoretisk Atomfysik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Indian Institute of Science, Bangalore, India

and

J. W. Wilkins
Nordisk Institut for Teoretisk Atomfysik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Cornell University, Ithaca, New York 14853
(Received 28 May 1981)

The two-impurity Kondo problem is studied by use of perturbative scaling techniques. The physics is determined by the interplay between the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between the two impurity spins and the Kondo effect. In particular, for a strong ferromagnetic RKKY interaction the susceptibility exhibits three structures as the temperature is lowered, corresponding to the ferromagnetic locking together of the two impurity spins followed by a two-stage freezing out of their local moments by the conduction electrons due to the Kondo effect.

competition between RKKY-
interaction and Kondo screening!

RKKY-coupled spin-singlet,
no Kondo screening

$K(R) \to -\infty$ $K(R) \to \infty$

$$H = H_{\mathrm{kin}} + J\boldsymbol{S}_1 \cdot \boldsymbol{\sigma} + J\boldsymbol{S}_2 \cdot \boldsymbol{\sigma} + K(R)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

VOLUME 47, NUMBER 10 PHYSICAL REVIEW LETTERS 7 SEPTEMBER 1981

Two-Impurity Kondo Problem

C. Jayaprakash
Nordisk Institut for Teoretisk Atomfsyik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Cornell University, Ithaca, New York 14853

and

H. R. Krishna-murthy
Nordisk Institut for Teoretisk Atomfsyik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Indian Institute of Science, Bangalore, India

and

J. W. Wilkins
Nordisk Institut for Teoretisk Atomfsyik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Cornell University, Ithaca, New York 14853
(Received 28 May 1981)

The two-impurity Kondo problem is studied by use of perturbative scaling techniques. The physics is determined by the interplay between the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between the two impurity spins and the Kondo effect. In particular, for a strong ferromagnetic RKKY interaction the susceptibility exhibits three structures as the temperature is lowered, corresponding to the ferromagnetic locking together of the two impurity spins followed by a two-stage freezing out of their local moments by the conduction electrons due to the Kondo effect.

competition between RKKY-interaction and Kondo screening!

RKKY-coupled spin-triplet,
Kondo screened by conduction electrons

RKKY-coupled spin-singlet,
no Kondo screening

$K(R) \to -\infty$ $K(R) \to \infty$

$$\delta = \pi/2$$

P. Nozières and A. Blandin,
J. Phys. (Paris) **41**, 193 (1980)

$$\delta = 0$$

RKKY-coupled spin-triplet,
Kondo screened by conduction electrons

RKKY-coupled spin-singlet,
no Kondo screening

$K(R) \to -\infty$

$K(R) \to \infty$

particle-hole symmetry ⟶ $\delta = 0$ *or* $\delta = \pi/2$

A. Millis et al.
Field Theories in Condensed Matter Physics
ed. Z. Tesanovic, 1990

$$\delta = \pi/2$$

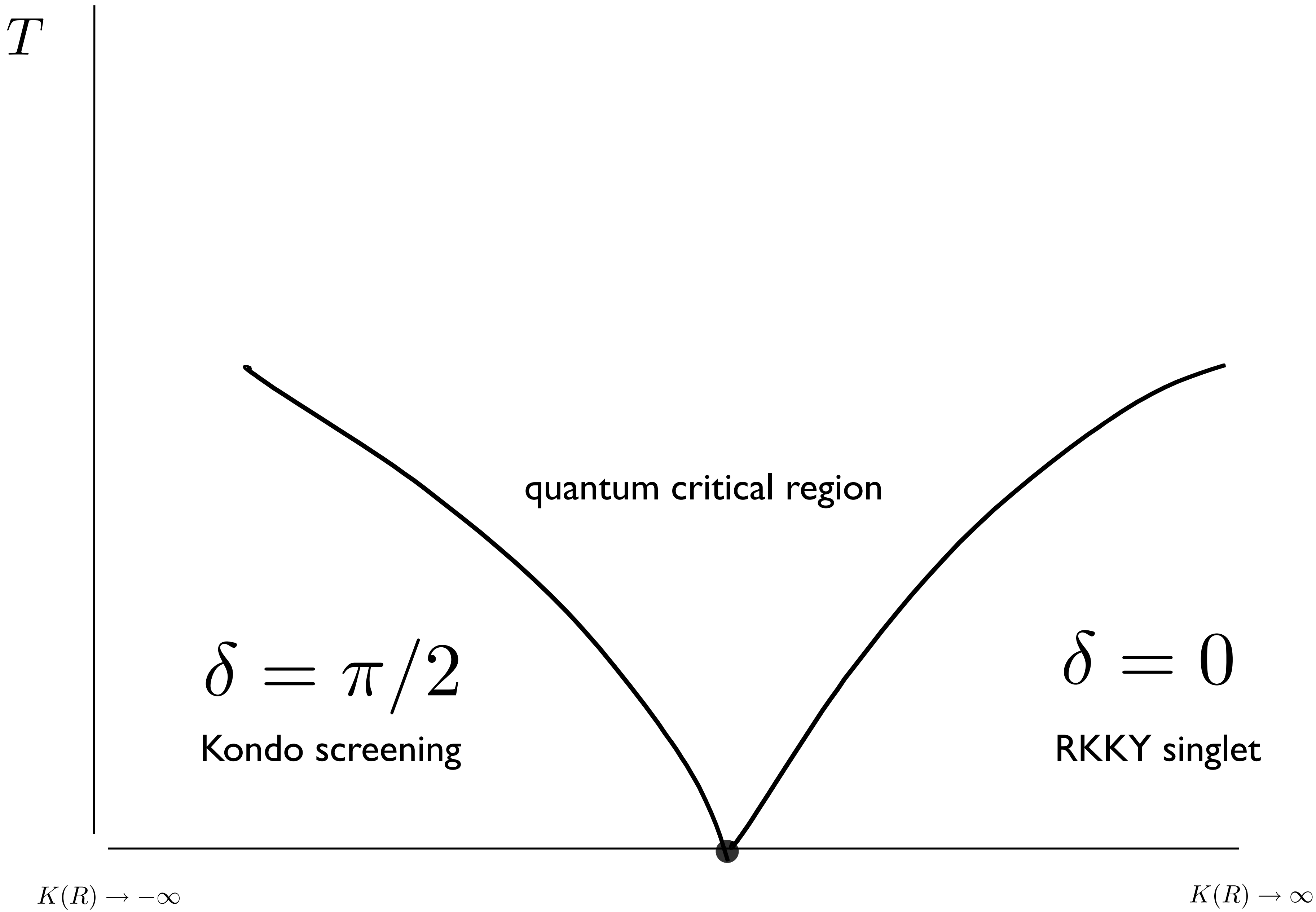
$$\delta = 0$$

RKKY-coupled spin-triplet,
Kondo screened by conduction electrons

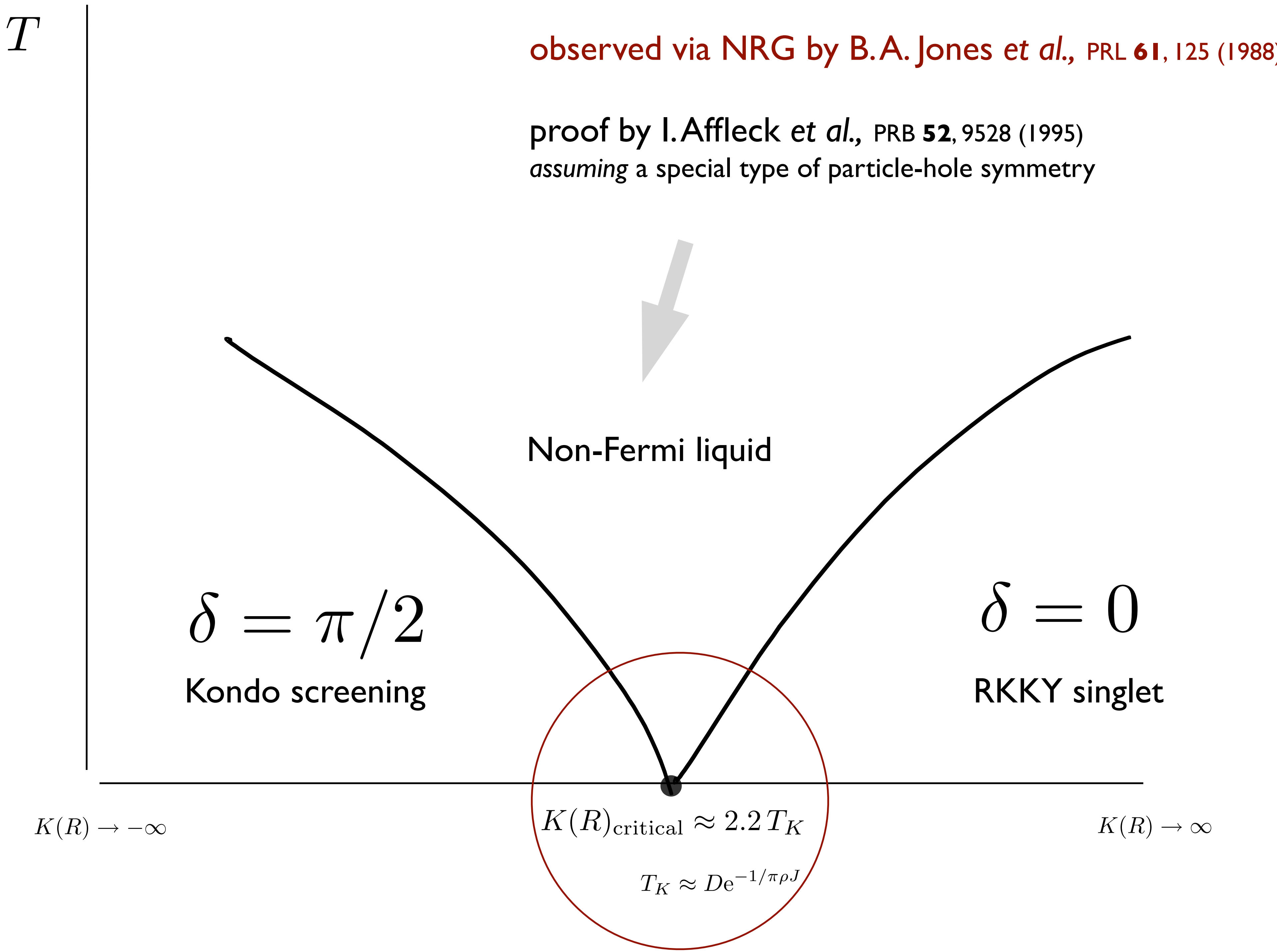
RKKY-coupled spin-singlet,
no Kondo screening

$K(R) \to -\infty$

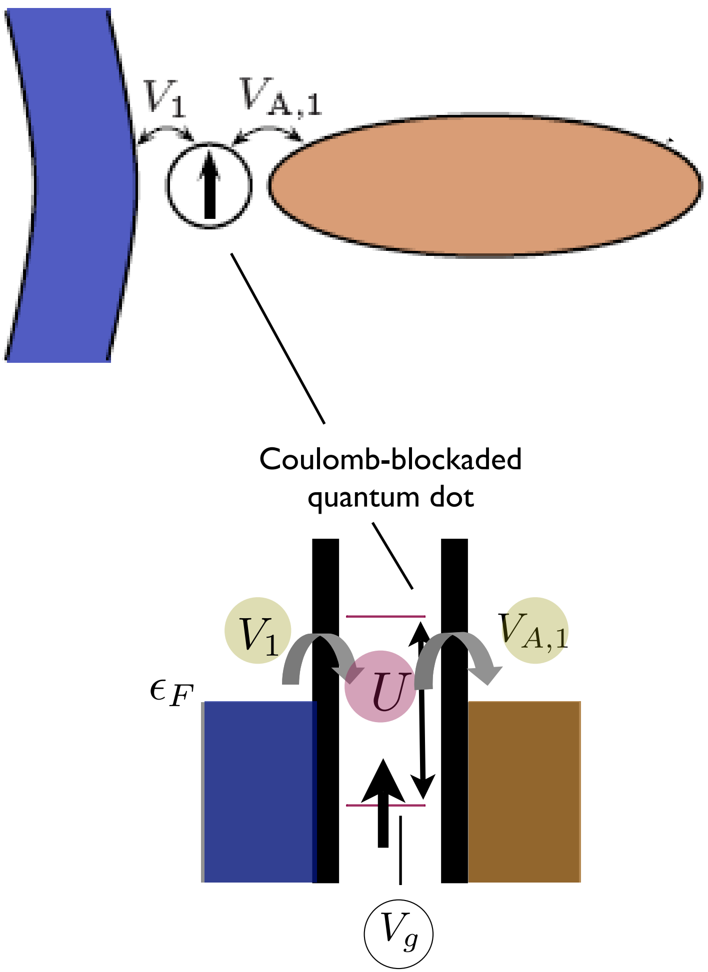
$K(R) \to \infty$



T
quantum critical region
$\delta = \pi/2$
Kondo screening
$\delta = 0$
RKKY singlet
$K(R) \to -\infty$
$K(R) \to \infty$

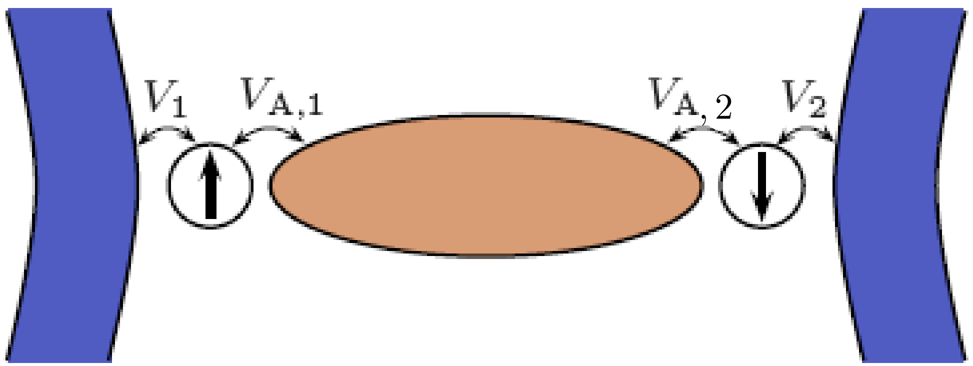

T
observed via NRG by B.A. Jones *et al.*, PRL **61**, 125 (1988)
proof by I. Affleck *et al.*, PRB **52**, 9528 (1995)
assuming a special type of particle-hole symmetry
Non-Fermi liquid
$\delta = \pi/2$
Kondo screening
$\delta = 0$
RKKY singlet
$K(R) \to -\infty$
$K(R)_{\mathrm{critical}} \approx 2.2\, T_K$
$K(R) \to \infty$
$T_K \approx D\mathrm{e}^{-1/\pi\rho J}$

Realization in double quantum-dot systems

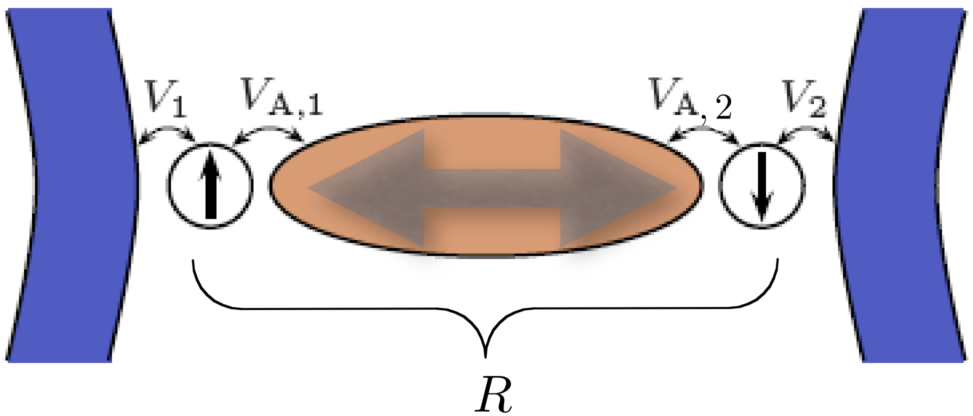


spin exchange $J \propto V^2/U$

Realization in double quantum-dot systems



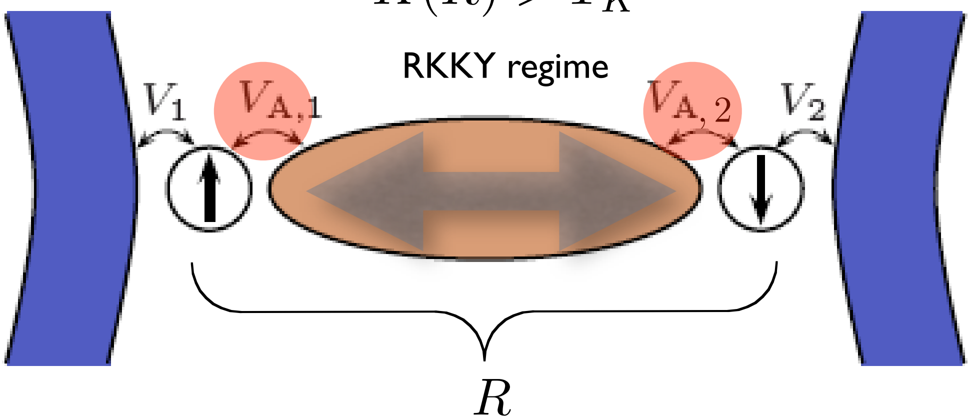
Realization in double quantum-dot systems



RKKY coupling $K(R) \propto (J^2/R^2)\cos(k_F R)$

Realization in double quantum-dot systems

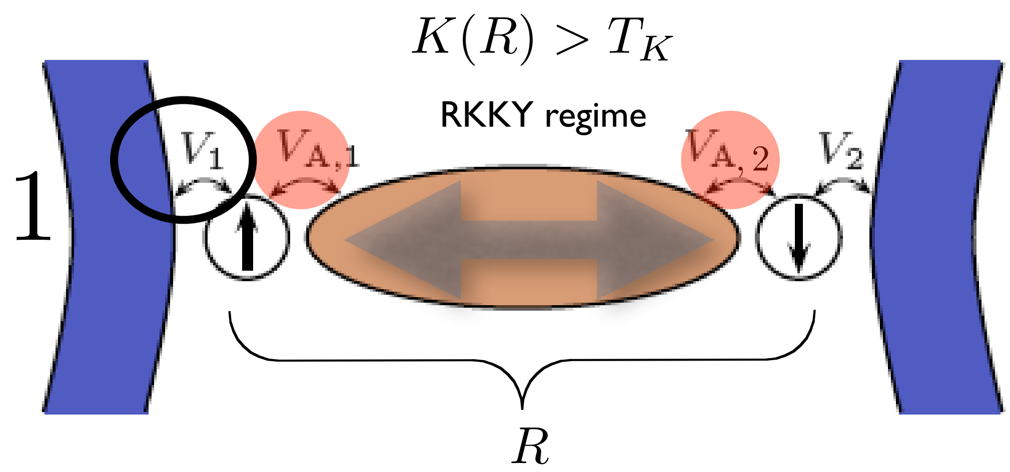
$K(R) > T_K$



RKKY coupling $K(R) \propto (J^2/R^2)\cos(k_F R)$

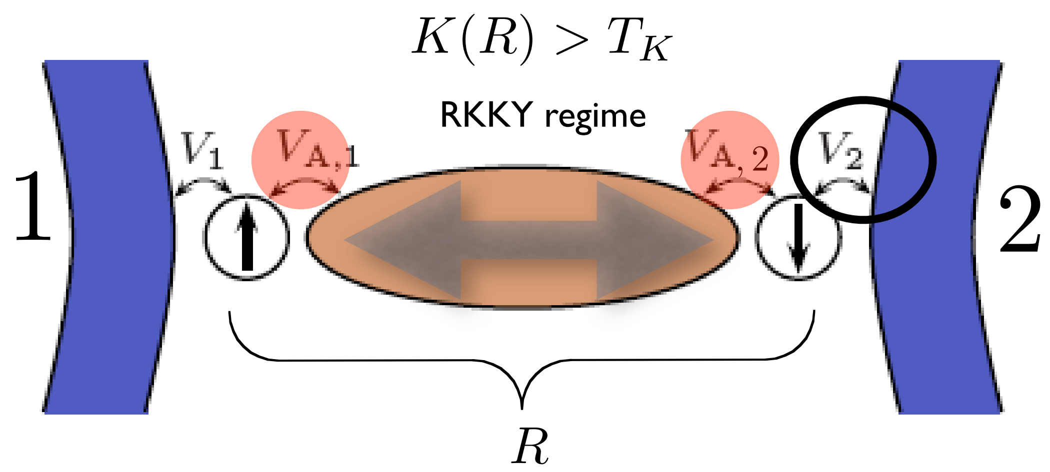
Kondo temperature $T_K \propto D \exp(-1/\pi\rho J)$

Realization in double quantum-dot systems



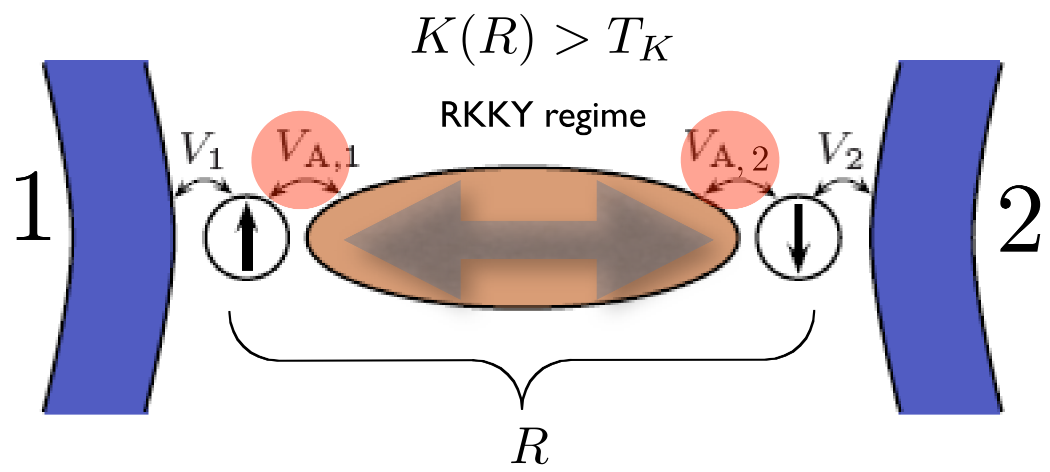
$$H_{\rm int} = J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}_2 \cdot \boldsymbol{\sigma}_2 + K(R) \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

Realization in double quantum-dot systems



$$H_{\rm int} = J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}_2 \cdot \boldsymbol{\sigma}_2 + K(R) \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

Realization in double quantum-dot systems



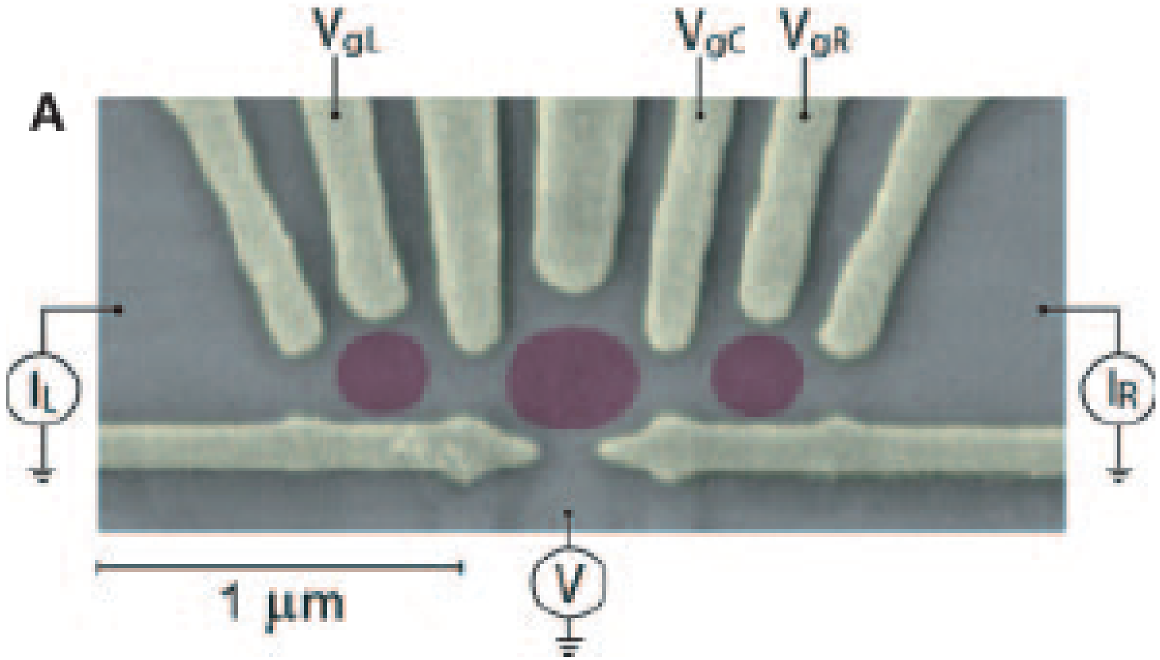
$$H_{\text{int}} = J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}_2 \cdot \boldsymbol{\sigma}_2 + K(R) \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

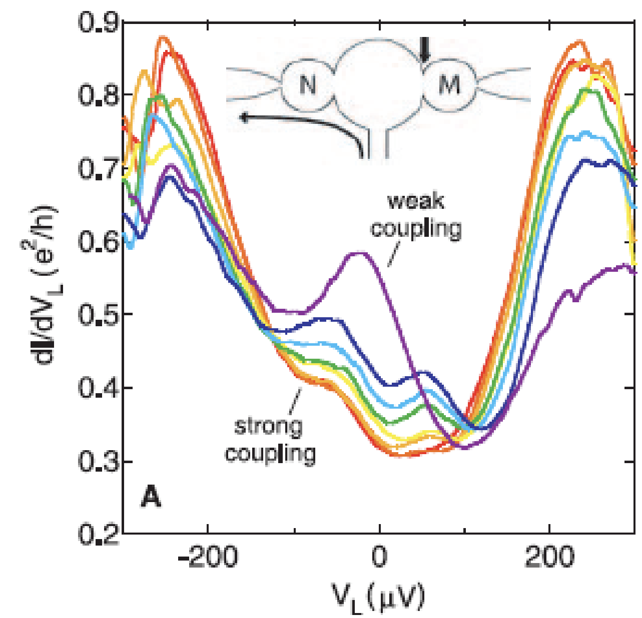
No transfer of electrons between 1 and 2: quantum critical point $K_c \approx 2.2 T_K$ is stable against electron-hole symmetry breaking *and breaking of parity*

G. Zaránd *et al.*, PRL **97**, 166802 (2006)

Realization in double quantum-dot systems

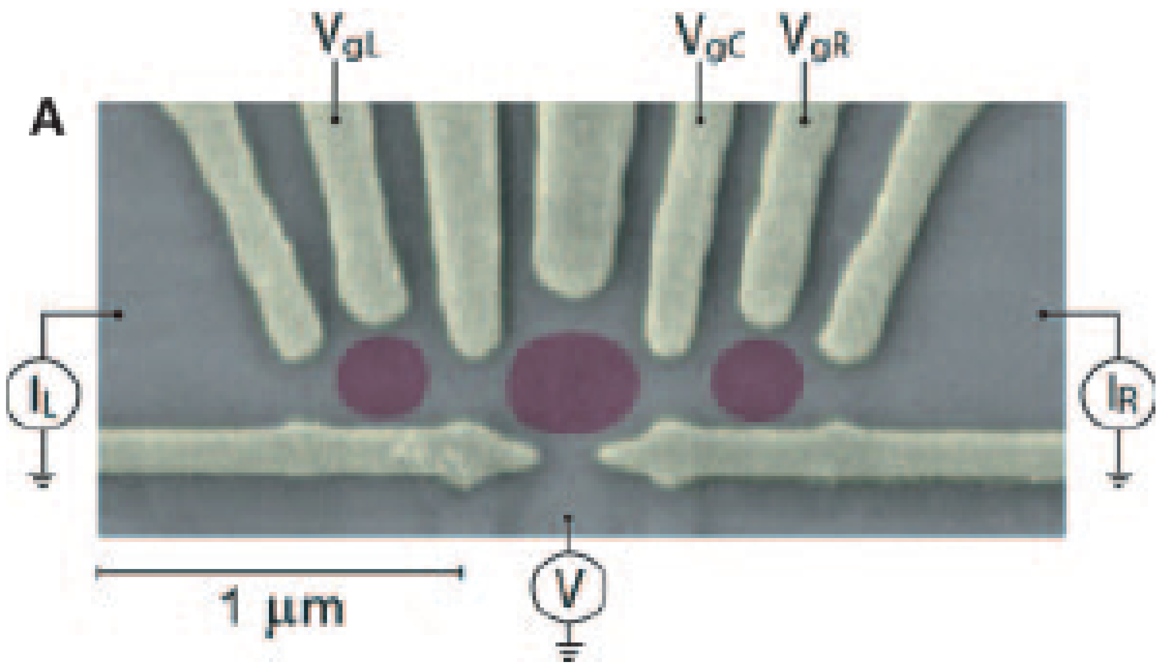
N. J. Craig *et al.*, Science **304**, 565 (2004)

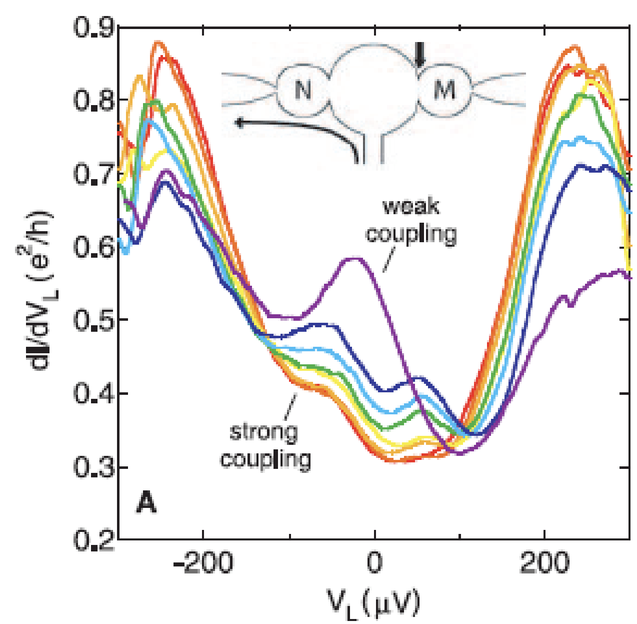




Realization in double quantum-dot systems

N. J. Craig *et al.*, Science **304**, 565 (2004)

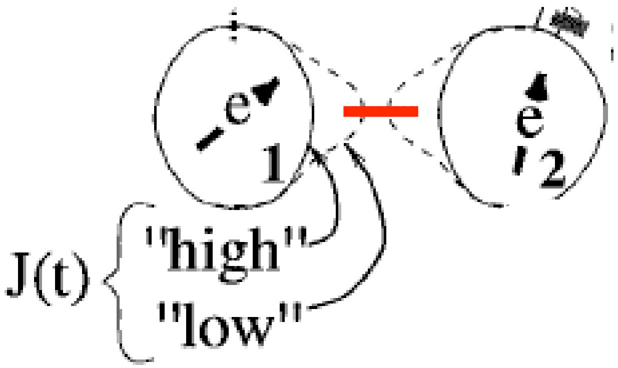




Nota Bene:
The central dot supports both RKKY and Kondo screening.
The experiment does *not* probe quantum criticality.

An aside: quantum dots, two-qubit gates, and all that...

Loss–DiVincenzo proposal for
spin-based quantum computing



from D. Loss and D.P. DiVincenzo,
PRA **57**, 120 (1998)

$$H_s(t) = J(t)\vec{S}_1 \cdot \vec{S}_2$$

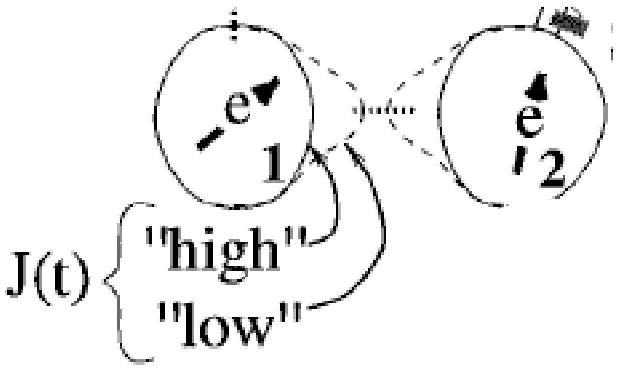
$$U_s(t) = T\exp\{-i\int_0^t H_s(t')dt'\}$$

$$U_s(\tau_s = \pi\hbar/J_0) = U_{sw}$$

$$U_{sw}\,|\,ij\,\rangle = |\,ji\,\rangle, \quad i,j = \uparrow,\downarrow$$

An aside: quantum dots, two-qubit gates, and all that...

Loss–DiVincenzo proposal for
spin-based quantum computing



from D. Loss and D.P. DiVincenzo,
PRA **57**, 120 (1998)

$$H_s(t) = J(t)\vec{S}_1 \cdot \vec{S}_2$$

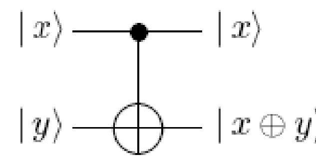
$$U_s(t) = T\exp\{-i\int_0^t H_s(t')dt'\}$$

$$U_s(\tau_s = \pi\hbar/J_0) = U_{sw}$$

$$U_{sw}\,|\,ij\,\rangle = |\,ji\,\rangle, \qquad i,j = \uparrow,\downarrow$$

$$U_{\rm CNOT} = e^{i\frac{\pi}{2}S_1^z}e^{-i\frac{\pi}{2}S_2^z}U_{sw}^{\frac{1}{2}}e^{i\pi S_1^z}U_{sw}^{\frac{1}{2}}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



$U_{\rm CNOT}$ + “single-qubit gates” (single-spin rotations)

→ “universal” quantum computer

An aside: quantum dots, two-qubit gates, and all that...

Using RKKY for nonlocal control of two-qubit gates...?

N. J. Craig *et al.*, Science **304**, 565 (2004)
M.G. Vavilov and L.I. Glazman, PRL **94**, 086805 (2005)

Using RKKY for nonlocal control of two-qubit gates…?

N. J. Craig *et al.*, Science **304**, 565 (2004)
M.G. Vavilov and L.I. Glazman, PRL **94**, 086805 (2005)

What about *spin decoherence* caused by the conduction electrons via RKKY?

GaAs/AlGaAs

$$T \approx 10 \text{ mK}$$
$$R \approx 10 \text{ nm}$$
$$K_0(R) \approx 5\ \mu\text{eV}$$
$$\tau_{\mathrm{dec}} \approx 60 \text{ ns}$$
$$\tau_{\mathrm{swap}} \approx 0.3 \text{ ns}$$

↓

~ 200 coherent $\sqrt{\mathrm{SWAP}}$ operations

Y. Rikitake and H. Imamura, PRB **72**, 033308 (2005)

Adding spin-orbit interactions...

Adding spin-orbit interactions...

relativistic correction in vacuum

$$H_{\mathrm{SO}} = \lambda_{\mathrm{vac}}(\nabla V \times \boldsymbol{k}) \cdot \boldsymbol{\sigma}$$

$$\lambda_{\mathrm{vac}} = \hbar^2/4m_0^2c^2 \approx 3.7 \times 10^{-6}\text{Å}^2$$

Adding spin-orbit interactions...

relativistic correction in vacuum

$$H_{\text{SO}} = \lambda_{\text{vac}}(\nabla V \times \boldsymbol{k}) \cdot \boldsymbol{\sigma}$$

$$\lambda_{\text{vac}} = \hbar^2/4m_0^2c^2 \approx 3.7 \times 10^{-6}\text{Å}^2$$

relativistic correction in a semiconductor

$$H_{\text{SO}} = \lambda_{\text{crystal}}(\nabla V \times \boldsymbol{k}) \cdot \boldsymbol{\sigma}$$

$$\lambda_{\text{crystal}} \approx \hbar^2/4m^* E_g \approx 10^6 \lambda_{\text{vac}}$$

bandgap

effective mass from periodic crystal potential

Adding spin-orbit interactions...

relativistic correction in vacuum

$$H_{\mathrm{SO}} = \lambda_{\mathrm{vac}}(\nabla V \times \boldsymbol{k}) \cdot \boldsymbol{\sigma}$$

$$\lambda_{\mathrm{vac}} = \hbar^2/4m_0^2c^2 \approx 3.7 \times 10^{-6}\,\mathrm{\AA}^2$$

relativistic correction in a semiconductor

$$H_{\mathrm{SO}} = \lambda_{\mathrm{crystal}}(\nabla V \times \boldsymbol{k}) \cdot \boldsymbol{\sigma}$$

$$\lambda_{\mathrm{crystal}} \approx \hbar^2/4m^* E_g \approx 10^6 \lambda_{\mathrm{vac}}$$

aperiodic part of the total potential:
confinement, impurities, boundaries,
external electric fields,...

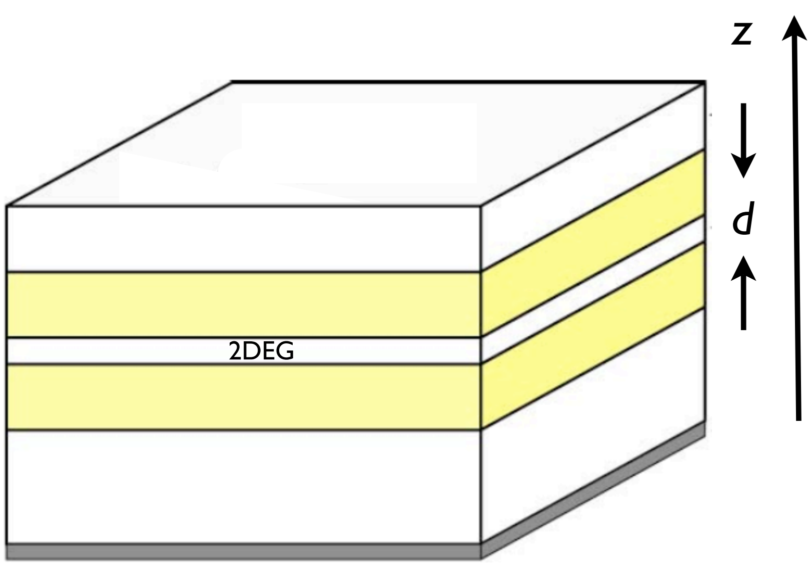
Adding spin-orbit interactions...

relativistic correction in a semiconductor

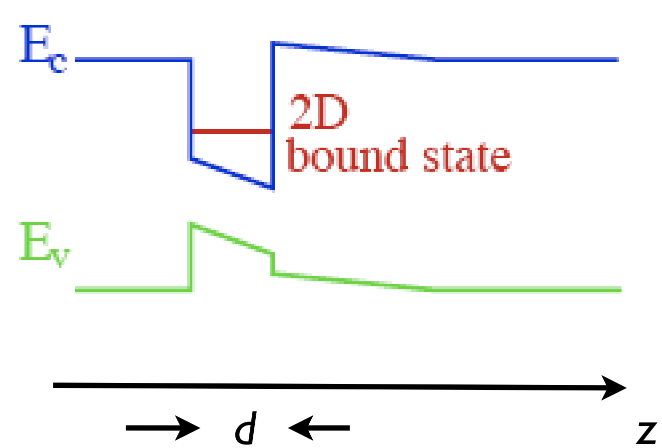
$$H_{\mathrm{SO}} = \lambda_{\mathrm{crystal}}(\nabla V \times \boldsymbol{k}) \cdot \boldsymbol{\sigma}$$

$$\lambda_{\mathrm{crystal}} \approx \hbar^2/4m^* E_g \approx 10^6 \lambda_{\mathrm{vac}}$$

aperiodic part of the total potential:
confinement, impurities, boundaries, external electric fields,...



semiconductor heterostructure



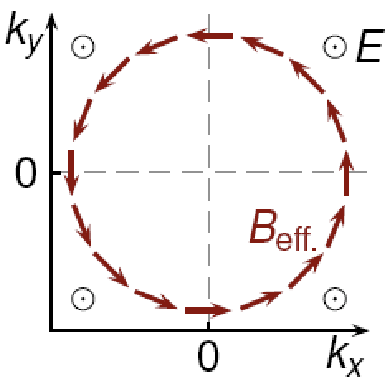
Adding spin-orbit interactions...

relativistic correction in a semiconductor

$$H_{SO} = \lambda_{\rm crystal}(\nabla V \times \boldsymbol{k}) \cdot \boldsymbol{\sigma}$$

$$\lambda_{\rm crystal} \approx \hbar^2/4m^* E_g \approx 10^6 \lambda_{\rm vac}$$

aperiodic part of the total potential:
confinement, impurities, boundaries, external electric fields,...

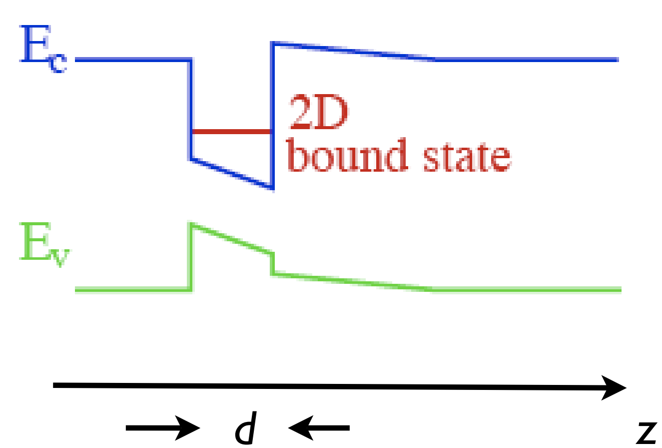


Rashba interaction

E. I. Rashba, Sov. Phys. Solid State **2**, 1109 (1960)

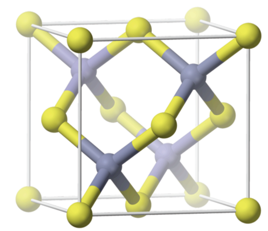
$$H_R = \alpha(k_x \sigma^y - k_y \sigma^x)$$

Spatial asymmetry of band edges mimics an ***E***-field in the *z*-direction



Another type of spin-orbit interaction in 2D semiconductor heterostructures...

zincblende structures:
GaAs, InAs, HgTe,...

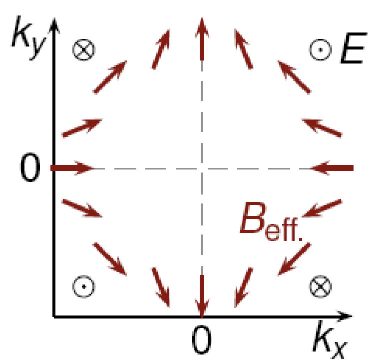


broken lattice
inversion symmetry

Dresselhaus interaction

G. Dresselhaus, Phys. Rev. **100**, 580 (1955)

$$H_D = \beta(k_x\sigma^x - k_y\sigma^y)$$

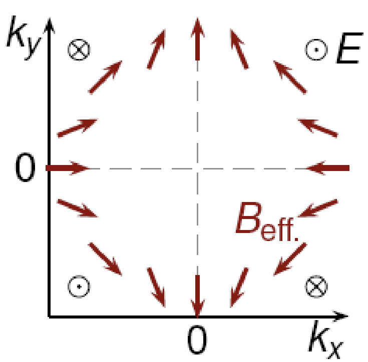


gate controllable

	$\beta k_F/\mathrm{meV}$	$\alpha k_F/\mathrm{meV}$
GaAs/AlGaAs	$0.01-0.1$	$0.01-0.1$
HgTe/CdTe	0.1	30

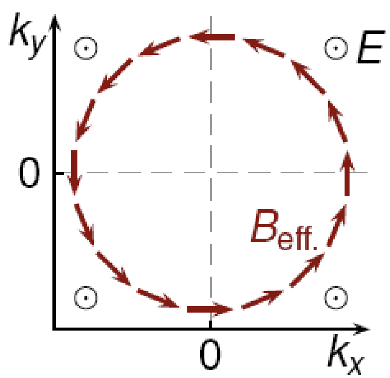
Dresselhaus interaction

$$H_D = \beta(k_x\sigma^x - k_y\sigma^y)$$



Rashba interaction

$$H_R = \alpha(k_x\sigma^y - k_y\sigma^x)$$



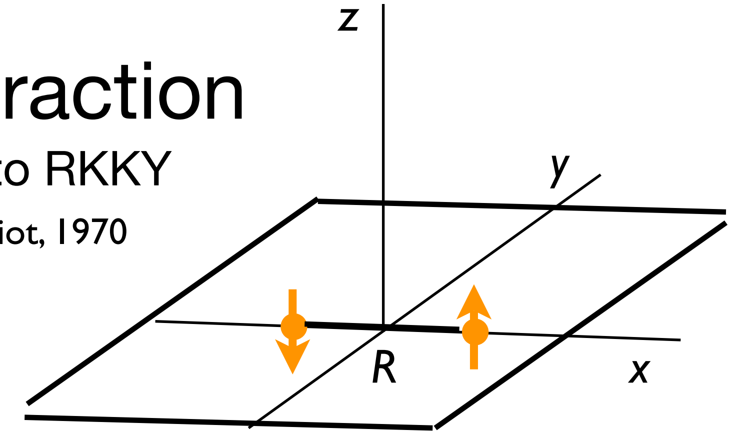
How do Rashba and Dresselhaus spin-orbit interactions influence two-impurity Kondo physics?

D. F. Mross and H. J., PRB **80**, 155302 (2009)

Spin-orbit effects on the RKKY interaction

from simple extension of standard perturbative approach to RKKY

Blackman and Elliot, 1970



$$H = \frac{\boldsymbol{k}^2}{2m} + \left[\begin{pmatrix} \beta & -\alpha \\ \alpha & -\beta \end{pmatrix} \begin{pmatrix} k_x \\ k_y \end{pmatrix} \right] \cdot \boldsymbol{\sigma}$$

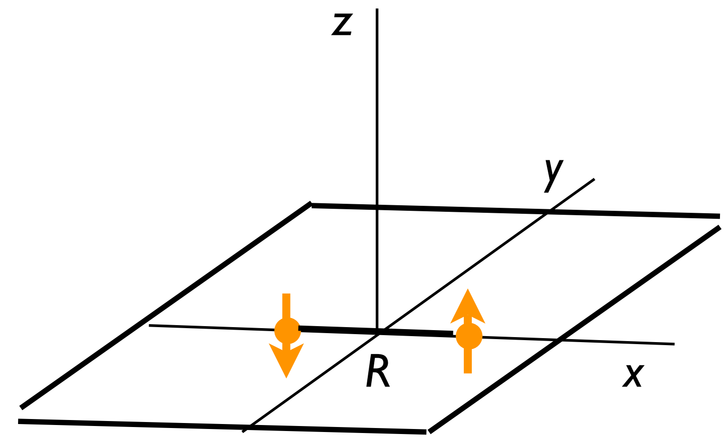
$$G(\boldsymbol{k}, \omega) \equiv (\omega - H(\boldsymbol{k}))^{-1}$$

$$H_{\mathrm{RKKY}} = -\frac{J_1 J_2}{\pi} \mathrm{Im} \int_{-\infty}^{\omega_F} d\omega \ \mathrm{Tr} \left[(\boldsymbol{S}_1 \cdot \boldsymbol{\sigma}) G(R, \omega + i0_+) \times (\boldsymbol{S}_2 \cdot \boldsymbol{\sigma}) G(-R, \omega + i0_+) \right]$$

$$H_{\mathrm{RKKY}}^{\mathrm{SO}} = H_{\mathrm{Heis.}} + H_{\mathrm{Rashba}} + H_{\mathrm{Dress.}} + H_{\mathrm{interf.}}$$

$$\begin{aligned} H_{\mathrm{Heis.}} &= F_0 \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \\ H_{\mathrm{Rashba}} &= \alpha F_1 \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2 \right)^y + \alpha^2 F_2 S_1^y S_2^y \\ H_{\mathrm{Dress.}} &= \beta F_1 \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2 \right)^x + \beta^2 F_2 S_1^x S_2^x \\ H_{\mathrm{interf.}} &= \alpha \beta F_2 \left(S_1^x S_2^y + S_1^y S_2^x \right). \end{aligned}$$

$$H_{\mathrm{RKKY}}^{\mathrm{SO}} = H_{\mathrm{Heis.}} + H_{\mathrm{Rashba}} + H_{\mathrm{Dress.}} + H_{\mathrm{interf.}}$$



$$\begin{aligned}
H_{\mathrm{Heis.}} &= F_0 \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \\
H_{\mathrm{Rashba}} &= \alpha F_1 \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2\right)^y + \alpha^2 F_2 S_1^y S_2^y \\
H_{\mathrm{Dress.}} &= \beta F_1 \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2\right)^x + \beta^2 F_2 S_1^x S_2^x \\
H_{\mathrm{interf.}} &= \alpha\beta F_2 \left(S_1^x S_2^y + S_1^y S_2^x\right).
\end{aligned}$$

$F_i = F_i(\alpha, \beta, R)$ are given by rather complicated integrals *(work in progress).* Analytical expressions in the limit of large distances and a weak *pure* Rashba coupling have been obtained by *H. Imamura et al., PRB 69, 121303(R) (2004).*

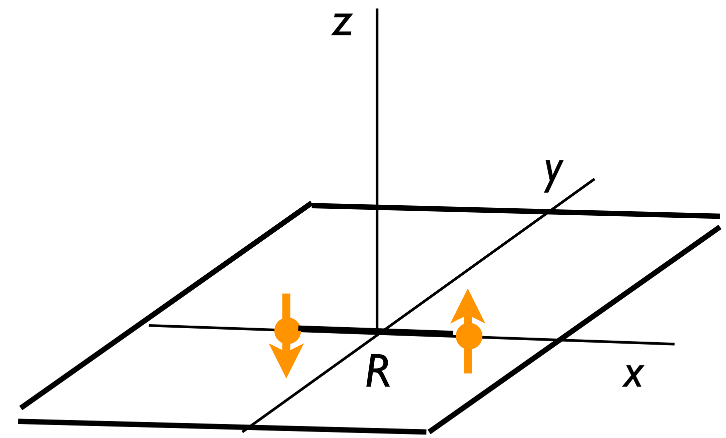
$$F_0(\alpha, 0, R) = -\frac{J^2}{2\pi^2 R^2} \frac{m}{\hbar^2} \sin 2R\sqrt{k_F^2 + \frac{m^2\alpha^2}{\hbar^4}}$$

$k_F R \gg 1$

$\alpha \ll \frac{\hbar^2}{m} k_F$

$$\begin{aligned}
F_1(\alpha, 0, R) &= \frac{F_0(\alpha,0,R)}{\alpha} \sin \frac{2mR\alpha}{\hbar^2} \\
F_2(\alpha, 0, R) &= \frac{F_0(\alpha,0,R)}{\alpha^2} \left(1 - \cos \frac{2mR\alpha}{\hbar^2}\right)
\end{aligned}$$

$$H_{\mathrm{RKKY}}^{\mathrm{SO}} = H_{\mathrm{Heis.}} + H_{\mathrm{Rashba}} + H_{\mathrm{Dress.}} + H_{\mathrm{interf.}}$$



$$\begin{aligned} H_{\mathrm{Heis.}} &= F_0 \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \\ H_{\mathrm{Rashba}} &= \alpha F_1 \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2\right)^y + \alpha^2 F_2 S_1^y S_2^y \\ H_{\mathrm{Dress.}} &= \beta F_1 \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2\right)^x + \beta^2 F_2 S_1^x S_2^x \\ H_{\mathrm{interf.}} &= \alpha\beta F_2 \left(S_1^x S_2^y + S_1^y S_2^x\right). \end{aligned}$$

$F_i = F_i(\alpha, \beta, R)$ are given by rather complicated integrals *(work in progress).* Analytical expressions in the limit of large distances and a weak *pure* Rashba coupling have been obtained by *H. Imamura et al., PRB 69, 121303(R) (2004).*

$$F_0(\alpha, 0, R) = -\frac{J^2}{2\pi^2 R^2} \frac{m}{\hbar^2} \sin 2R\sqrt{k_F^2 + \frac{m^2\alpha^2}{\hbar^4}}$$

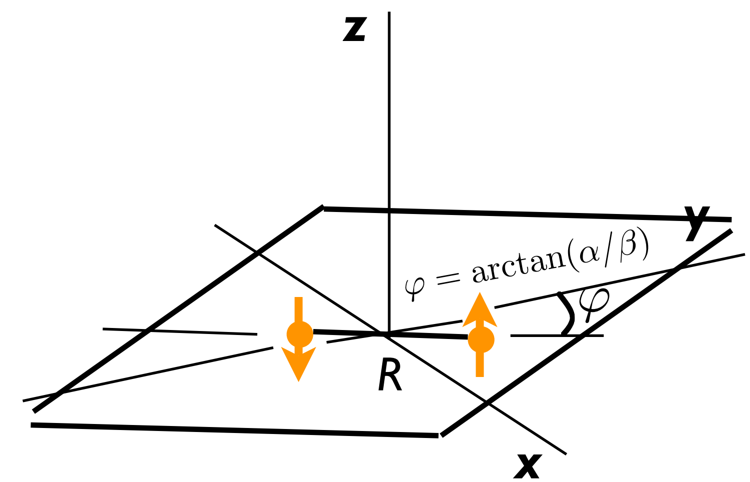
$R \approx$ 10nm

$0.01 < \frac{m\alpha}{\hbar^2} < 0.1$ Å^{-1} via gate control

$k_F \approx 0.02$ Å^{-1}

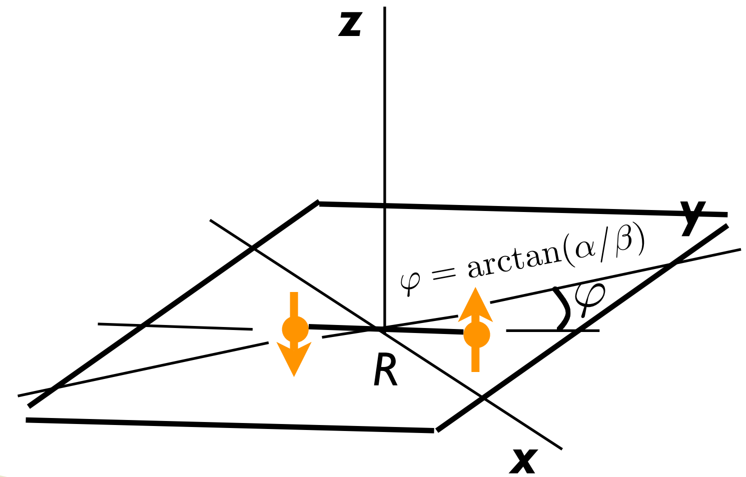
asymmetrically doped InAlAs quantum well

More useful choice of coordinate system: rotate ***x***, ***y*** by $\pi/2 - \arctan(\alpha/\beta)$ around the ***z***-axis



$$H^{\mathrm{SO}}_{\mathrm{RKKY}} \;=\; K_{\mathrm{H}} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + K_{\mathrm{Ising}} S_1^y S_2^y + K_{\mathrm{DM}} \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2 \right)^y$$

More useful choice of coordinate system: rotate ***x***, ***y*** by $\pi/2 - \arctan(\alpha/\beta)$ around the ***z***-axis

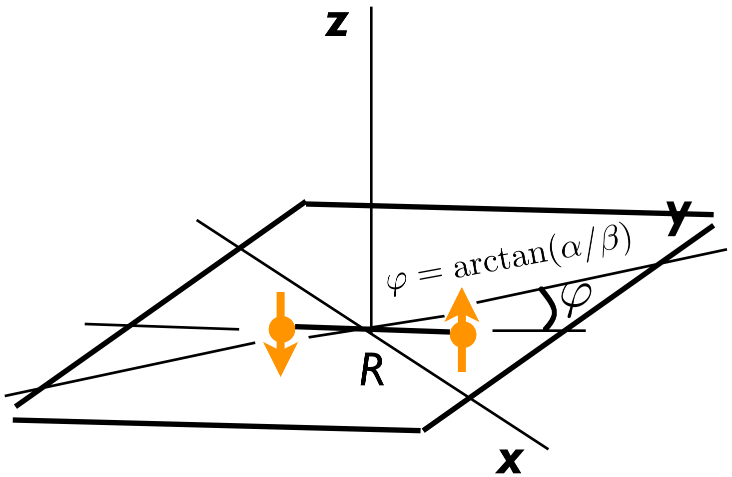


$$H_{\mathrm{RKKY}}^{\mathrm{SO}} = K_{\mathrm{H}} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + K_{\mathrm{Ising}} S_1^y S_2^y + K_{\mathrm{DM}} \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2\right)^y$$

Anisotropies **unwanted** for the CNOT gate

Bad for RKKY-control of two-qubit gating

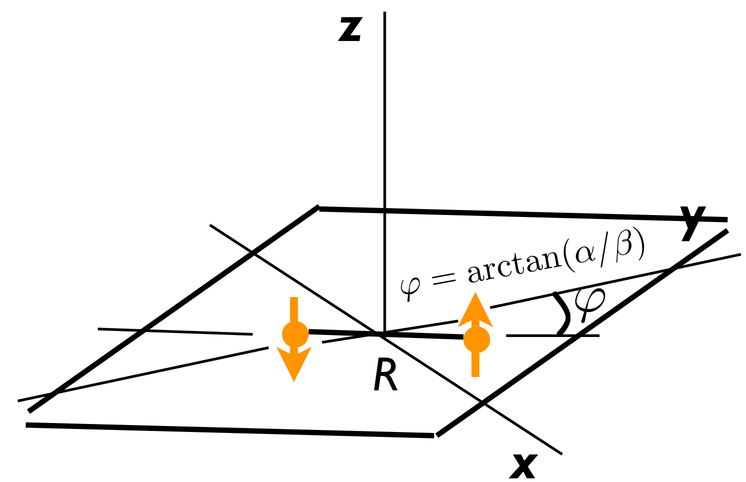
More useful choice of coordinate system: rotate ***x***, ***y*** by $\pi/2 - \arctan(\alpha/\beta)$ around the ***z***-axis



$$H^{\rm SO}_{\rm RKKY} \;=\; K_{\rm H}\boldsymbol{S}_1\cdot\boldsymbol{S}_2 + K_{\rm Ising}S_1^yS_2^y + K_{\rm DM}\left(\boldsymbol{S}_1\times\boldsymbol{S}_2\right)^y$$

SU(2) symmetry recovered when $|\alpha|=|\beta|$!

More useful choice of coordinate system: rotate ***x***, ***y*** by $\pi/2 - \arctan(\alpha/\beta)$ around the ***z***-axis

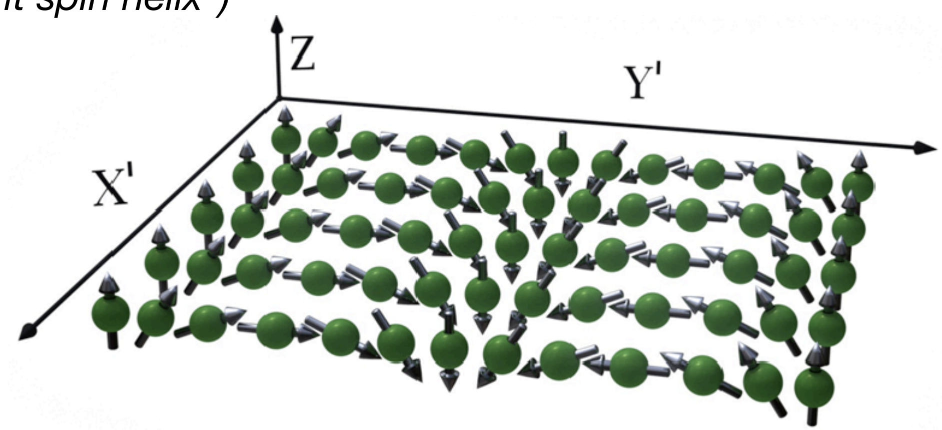


$$H^{\rm SO}_{\rm RKKY} \;=\; K_{\rm H}\boldsymbol{S}_1\cdot\boldsymbol{S}_2 + K_{\rm Ising}S_1^yS_2^y + K_{\rm DM}\left(\boldsymbol{S}_1\times\boldsymbol{S}_2\right)^y$$

SU(2) symmetry recovered when $|\alpha| = |\beta|$!

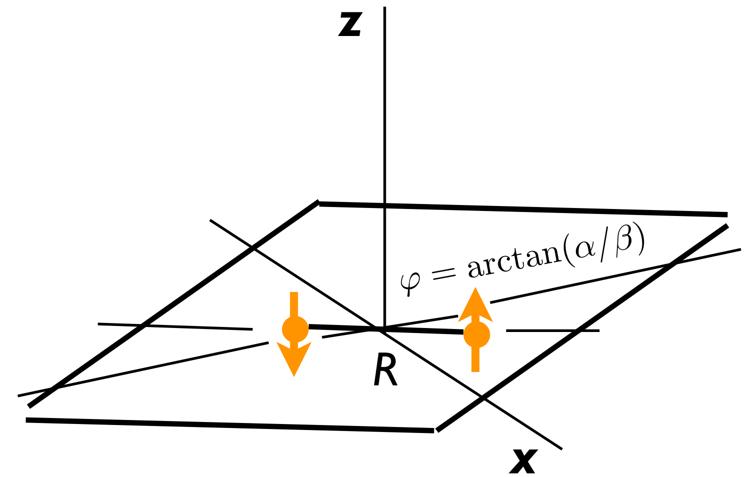
Also predicted and observed in a 2DEG:
conservation of phase and amplitude of
a helical spin structure *("persistent spin helix")*

B. A. Bernevig *et al.*, PRL **97**, 236601 (2006)
J. D. Koralek *et al.*, Nature **458**, 610 (2009)

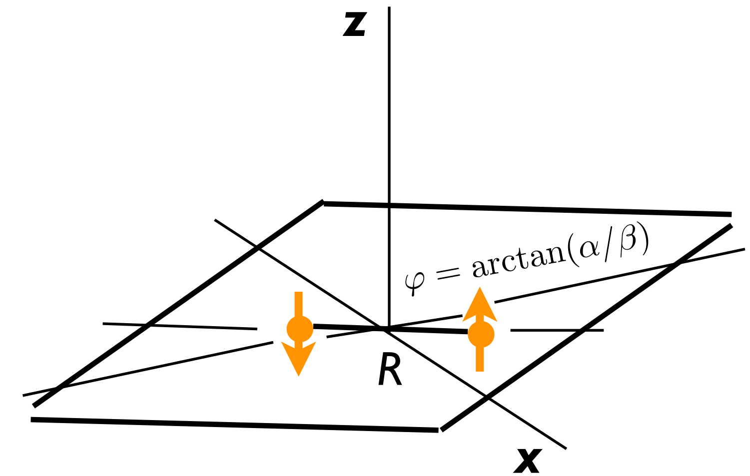


from J. D. Koralek *et al.*, Nature **XX**, YYY (200Z)

More useful choice of coordinate system: rotate ***x***, ***y*** by $\pi/2 - \arctan(\alpha/\beta)$ around the ***z***-axis



$$H_{\mathrm{RKKY}}^{\mathrm{SO}} \;=\; K_{\mathrm{H}}\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + K_{\mathrm{Ising}} S_1^y S_2^y + K_{\mathrm{DM}} \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2\right)^y$$



More useful choice of coordinate system: rotate ***x***, ***y*** by $\pi/2 - \arctan(\alpha/\beta)$ around the ***z***-axis

$$H^{\mathrm{SO}}_{\mathrm{RKKY}} \;=\; K_{\mathrm{H}} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + K_{\mathrm{Ising}} S_1^y S_2^y + K_{\mathrm{DM}} \left(\boldsymbol{S}_1 \times \boldsymbol{S}_2\right)^y$$

$$K^y = K_{\mathrm{H}} + K_{\mathrm{Ising}}$$

$$e^{i\theta} K^{\perp} = K_{\mathrm{H}} + i K_{\mathrm{DM}}$$

$$\boldsymbol{S}_2' = e^{\mathrm{i}\theta S_2^y} \boldsymbol{S}_2 e^{-\mathrm{i}\theta S_2^y}$$

$$H^{\mathrm{SO}}_{\mathrm{RKKY}} = K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2' + (K^y - K^{\perp}) S_1^y S_2'^y .$$

effect of spin-orbit interactions: **twist** and **anisotropy**

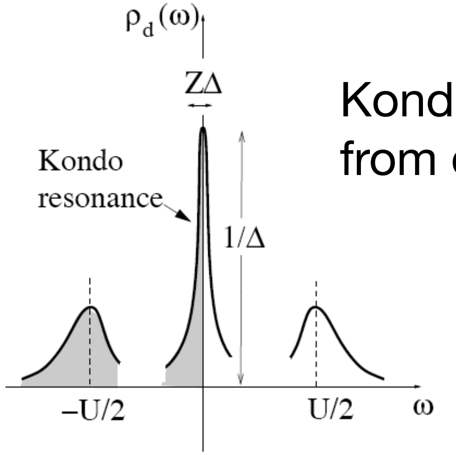
$K^y \neq K^{\perp}$ when Rashba and Dresselhaus are *both* present

By increasing the Kondo scale T_K , the twisted and anisotropic RKKY interaction gets competition from the Kondo effect...

...what happens at the TIKM critical point?

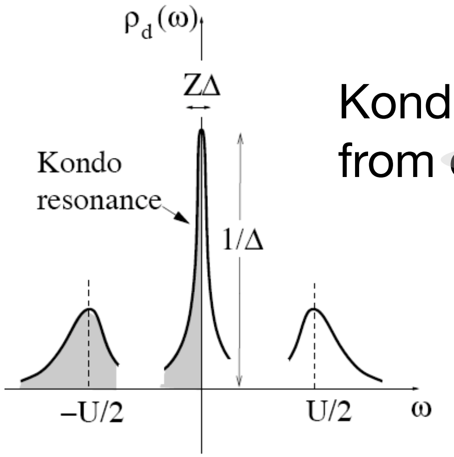
Effect from spin-orbit interactions on single-impurity Kondo effect?

Effect from spin-orbit interactions on single-impurity Kondo effect?

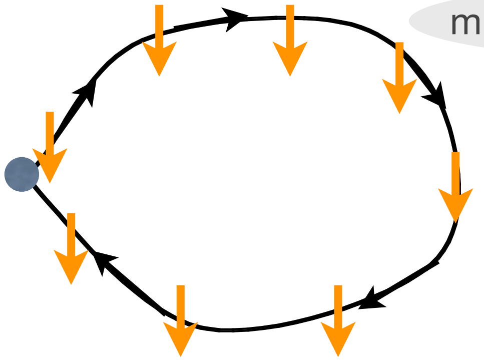


Kondo resonance at the Fermi level
from divergence in the electron self-energy

Effect from spin-orbit interactions on single-impurity Kondo effect?

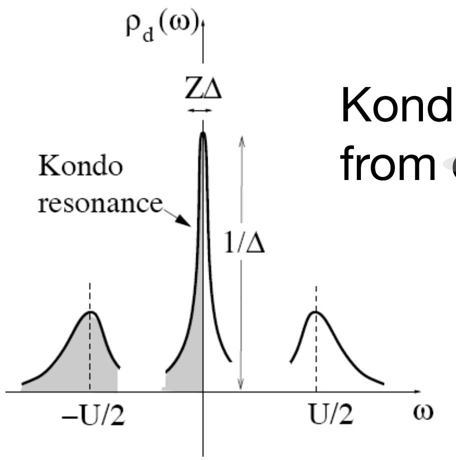


Kondo resonance at the Fermi level
from divergence in the electron self-energy

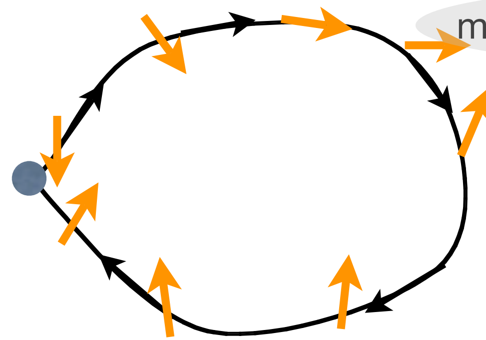


multiple electron-impurity scattering

Effect from spin-orbit interactions on single-impurity Kondo effect?



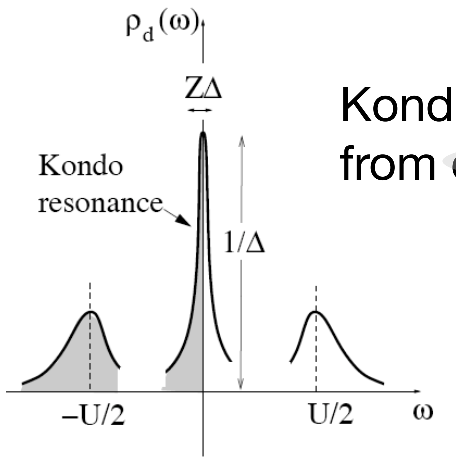
Kondo resonance at the Fermi level
from divergence in the electron self-energy



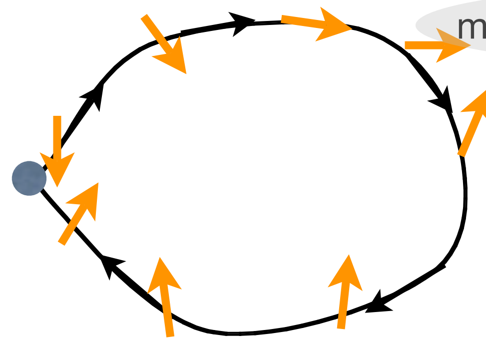
multiple electron-impurity scattering

non-conservation of electron spin in the presence of spin-orbit interactions

Effect from spin-orbit interactions on single-impurity Kondo effect?



Kondo resonance at the Fermi level
from divergence in the electron self-energy



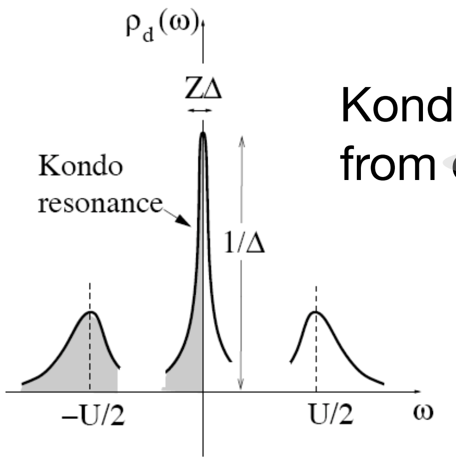
multiple electron-impurity scattering

non-conservation of electron spin in the presence of spin-orbit interactions!

But,... experiments show that the Kondo effect is insensitive to spin-orbit scattering...

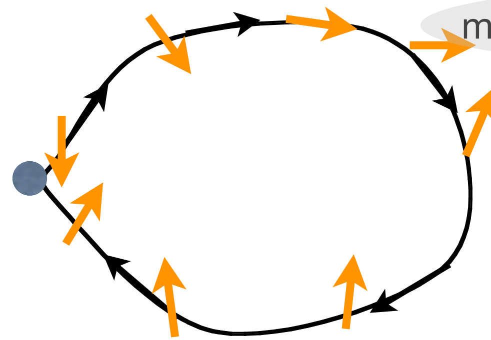
G. Bergmann, PRL 57, 1460 (1986)

Effect from spin-orbit interactions on single-impurity Kondo effect?

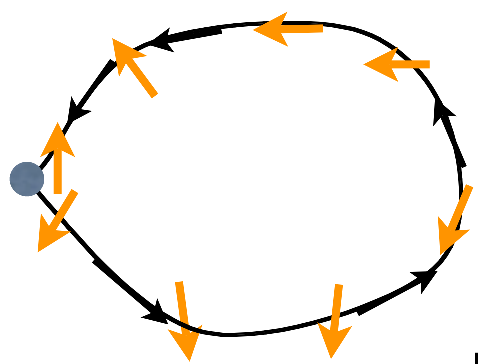


Kondo resonance at the Fermi level
from divergence in the electron self-energy

multiple electron-impurity scattering

\+ 

non-conservation of electron spin in the presence of spin-orbit interactions!

\- 

But,... experiments show that the Kondo effect is insensitive to spin-orbit scattering...

G. Bergmann, PRL **57**, 1460 (1986)

...protected by time-reversal invariance

Y. Meir and N.S. Wingreen, PRB **50**, 4947 (1994)

= 0

Analysis of 2D single-impurity Kondo + Rashba model

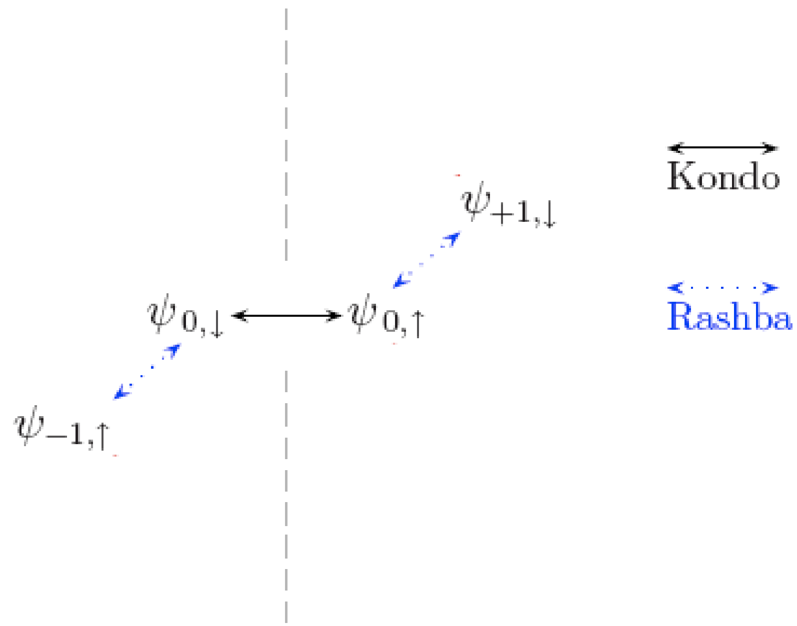
J. Malecki, J. Stat. Phys. **129**, 741 (2007)

Pseudospin basis (mixing spin and angular momentum)

$$a_{k\pm\uparrow} = \frac{1}{\sqrt{2}}(\psi_{k0\uparrow} \mp i\psi_{k+1\downarrow})$$
$$a_{k\pm\downarrow} = \frac{1}{\sqrt{2}}(\psi_{k0\downarrow} \mp i\psi_{k-1\uparrow})$$

two-channel anisotropic Kondo model;
the weakly coupled channel drops out
at low temperatures

$$H = v_F \int dk\, k\,(a^\dagger_{k\mu} a_{k\mu} + \tilde{a}^\dagger_{k\mu}\tilde{a}_{k\mu}) + J^{\rm eff}\frac{V k_F^0}{2\pi}\int dk\, dk'\, \boldsymbol{S}\cdot a^\dagger_{k\mu}\boldsymbol{\sigma}_{\mu\nu}\, a_{k'\nu}$$



usual 1D low-energy Kondo model with rescaled coupling $J^{\rm eff} = J\sqrt{1 + m\alpha^2/2\epsilon_F}$

Extension to Kondo + Dresselhaus *(only)* is straightforward...

Pseudospin basis (mixing spin and angular momentum)

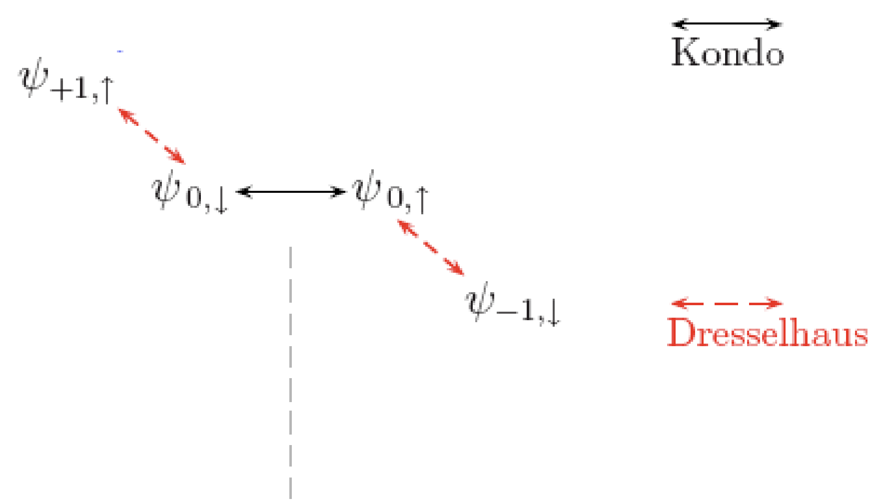
$$a_{k\pm\uparrow} = \frac{1}{\sqrt{2}}(\psi_{k0\uparrow} \mp i\psi_{k\mathbf{-1}\downarrow})$$

$$a_{k\pm\downarrow} = \frac{1}{\sqrt{2}}(\psi_{k0\downarrow} \mp i\psi_{k\mathbf{+1}\uparrow})$$

two-channel anisotropic Kondo model;
the weakly coupled channel drops out
at low temperatures

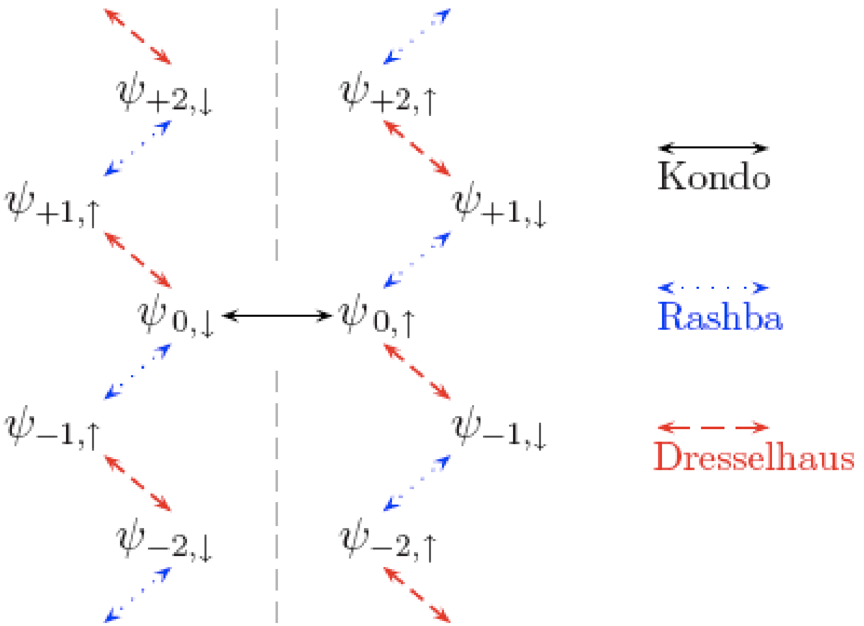
$$\begin{aligned} H &= v_F \int dk\, k\, (a^\dagger_{k\mu} a_{k\mu} + \tilde{a}^\dagger_{k\mu}\tilde{a}_{k\mu}) \\ &+ J^{\text{eff}} \frac{V k_F^0}{2\pi} \int dk\, dk' \boldsymbol{S} \cdot a^\dagger_{k\mu} \boldsymbol{\sigma}_{\mu\nu}\, a_{k'\nu} \end{aligned}$$

usual 1D low-energy Kondo model with
rescaled coupling $J^{\text{eff}} = J\sqrt{1 + m\beta^2/2\epsilon_F}$



Rashba + Dresselhaus couple an infinite number of orbital angular modes to the impurity

work in progress...



Summarizing spin-orbit effects:

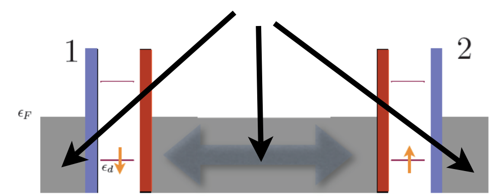
rescaled coupling

Kondo exchange: $H^{\mathrm{SO}}_{\text{el-imp}} = J\boldsymbol{S}\cdot\boldsymbol{\sigma}$

RKKY: $H^{\mathrm{SO}}_{\mathrm{RKKY}} = K^{\perp}\boldsymbol{S}_1\cdot\boldsymbol{S}_2' + (K^y - K^{\perp})S_1^y S_2'^y.$

twist anisotropy

TIKM with spin-orbit interactions



$$H^{\mathrm{SO}}_{\mathrm{TIKM}} = H_{\mathrm{kin}} + J_1\boldsymbol{S}_1\cdot\boldsymbol{\sigma}_1 + J_2\boldsymbol{S}_2'\cdot\boldsymbol{\sigma}_2 + K^{\perp}\boldsymbol{S}_1\cdot\boldsymbol{S}_2' + (K^y - K^{\perp})S_1^y S_2'^y$$

Spin-orbit effects:

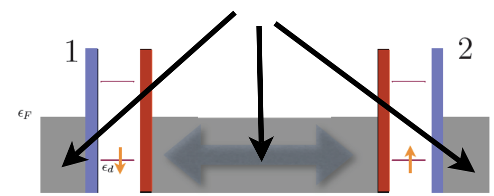
rescaled coupling

Kondo exchange: $H^{\text{SO}}_{\text{el-imp}} = J\boldsymbol{S}\cdot\boldsymbol{\sigma}$

RKKY: $H^{\text{SO}}_{\text{RKKY}} = K^{\perp}\boldsymbol{S}_1\cdot\boldsymbol{S}'_2 + (K^y - K^{\perp})S^y_1 S'^y_2$.

twist   anisotropy

TIKM with spin-orbit interactions



$$H^{\text{SO}}_{\text{TIKM}} = \underbrace{H_{\text{kin}} + J_1\boldsymbol{S}_1\cdot\boldsymbol{\sigma}_1 + J_2\boldsymbol{S}'_2\cdot\boldsymbol{\sigma}_2 + K^{\perp}\boldsymbol{S}_1\cdot\boldsymbol{S}'_2}_{} + (K^y - K^{\perp})S^y_1 S'^y_2$$

with Rashba ***or*** Dresselhaus

Spin-orbit effects:

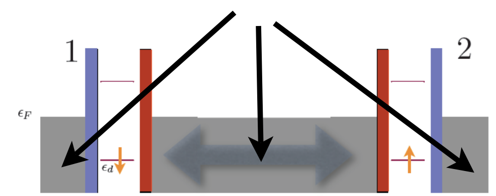
rescaled coupling

Kondo exchange: $H^{\mathrm{SO}}_{\text{el-imp}} = J\boldsymbol{S}\cdot\boldsymbol{\sigma}$

RKKY: $H^{\mathrm{SO}}_{\mathrm{RKKY}} = K^{\perp}\boldsymbol{S}_1\cdot\boldsymbol{S}'_2 + (K^y - K^{\perp})S^y_1 S'^y_2$.

twist anisotropy

TIKM with spin-orbit interactions



$$H^{\mathrm{SO}}_{\mathrm{TIKM}} = \underbrace{H_{\mathrm{kin}} + J_1\boldsymbol{S}_1\cdot\boldsymbol{\sigma}_1 + J_2\boldsymbol{S}'_2\cdot\boldsymbol{\sigma}_2 + K^{\perp}\boldsymbol{S}_1\cdot\boldsymbol{S}'_2 + (K^y - K^{\perp})S^y_1 S'^y_2}$$

with Rashba ***and*** Dresselhaus in the central reservoir /
Rashba ***or*** Dresselhaus in the external leads

$$H^{\mathrm{SO}}_{\mathrm{TIKM}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}'_2 \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}'_2 + (K^y - K^{\perp}) S^y_1 S'^y_2$$

Critical behavior?

$$H_{\mathrm{TIKM}}^{\mathrm{SO}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}'_2 \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}'_2$$

Critical behavior?

$K^{\perp} = K^{y},\ \theta$ arbitrary

rotate also the spins of the electrons which couple to $\boldsymbol{S}'_2$

$$\psi_2 \to \psi'_2 = e^{-i\theta\tau^y/2}\psi_2$$

$$H_{\mathrm{TIKM}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S'}_2 \cdot \boldsymbol{\sigma}'_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}'_2$$

The model represented in a twisted spin basis = the ordinary TIKM

→ **same critical behavior for all** θ

$$H^{\mathrm{SO}}_{\mathrm{TIKM}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}_2 \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + (K^y - K^{\perp}) S_1^y S_2^y$$

Critical behavior?

$$K^{\perp} \neq K^y, \;\; \theta = 0$$

$$SU(2) \longrightarrow U(1)$$

Kondo exchange anisotropies do not produce any RG-relevant or new correction-to-scaling operator

I. Affleck *et al.*, PRB **52**, 9528 (1995)

↓ ?

same critical behavior for all $K^{\perp} \neq K^y$

$$H^{\rm SO}_{\rm TIKM} = H_{\rm kin} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}_2 \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + (K^y - K^{\perp}) S_1^y S_2^y$$

Critical behavior?

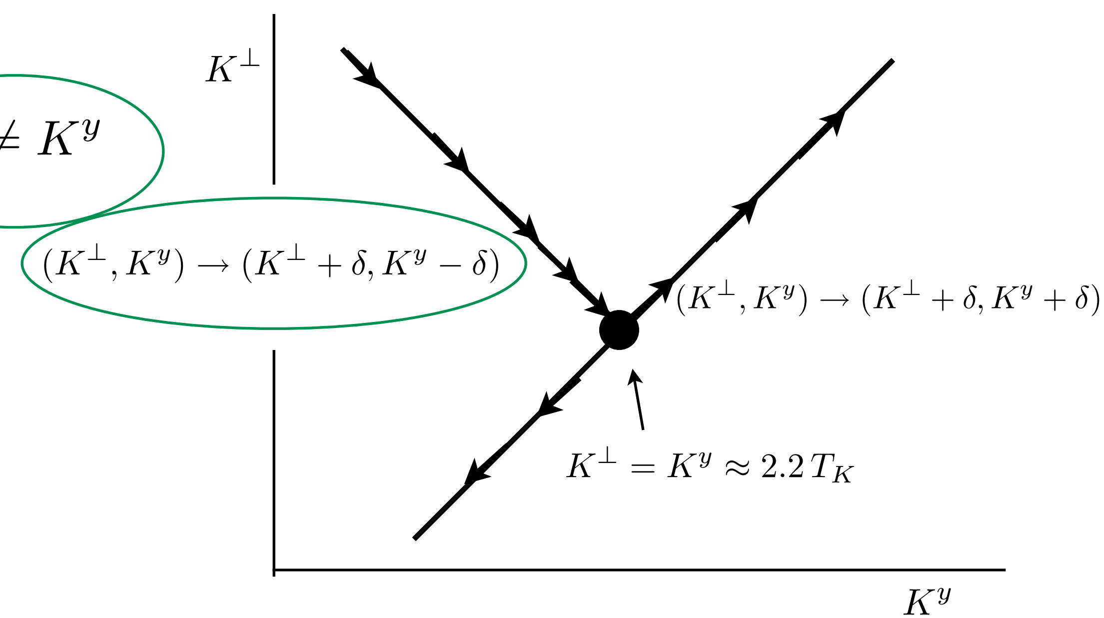
$K^{\perp} \neq K^y, \ \theta = 0$

$$SU(2) \rightarrow U(1)$$

Kondo exchange anisotropies do not produce any RG-relevant or new correction-to-scaling operator

I. Affleck *et al.*, PRB **52**, 9528 (1995)

same critical behavior for all $K^{\perp} \neq K^y$

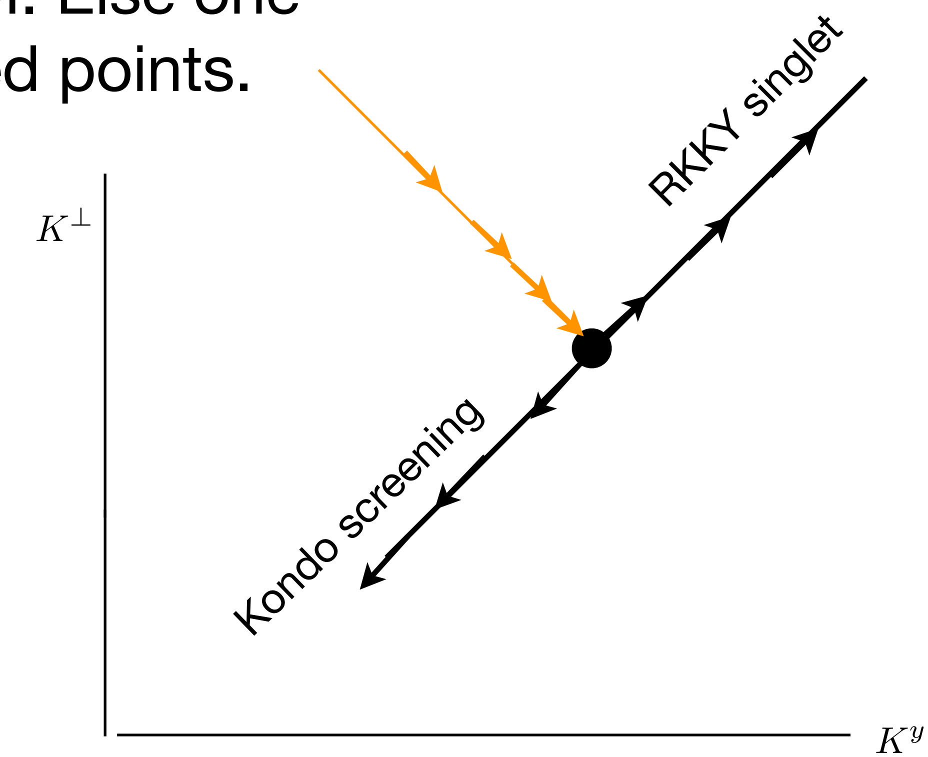


$$H^{\mathrm{SO}}_{\mathrm{TIKM}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}'_2 \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}'_2 + (K^y - K^{\perp}) S^y_1 S'^y_2$$

Critical behavior?

$K^{\perp} \neq K^y, \ \theta$ arbitrary

With fine-tuned $K^y, K^{\perp}$ the critical behavior is the *same* as for the ordinary TIKM. Else one flows towards one of the stable fixed points.

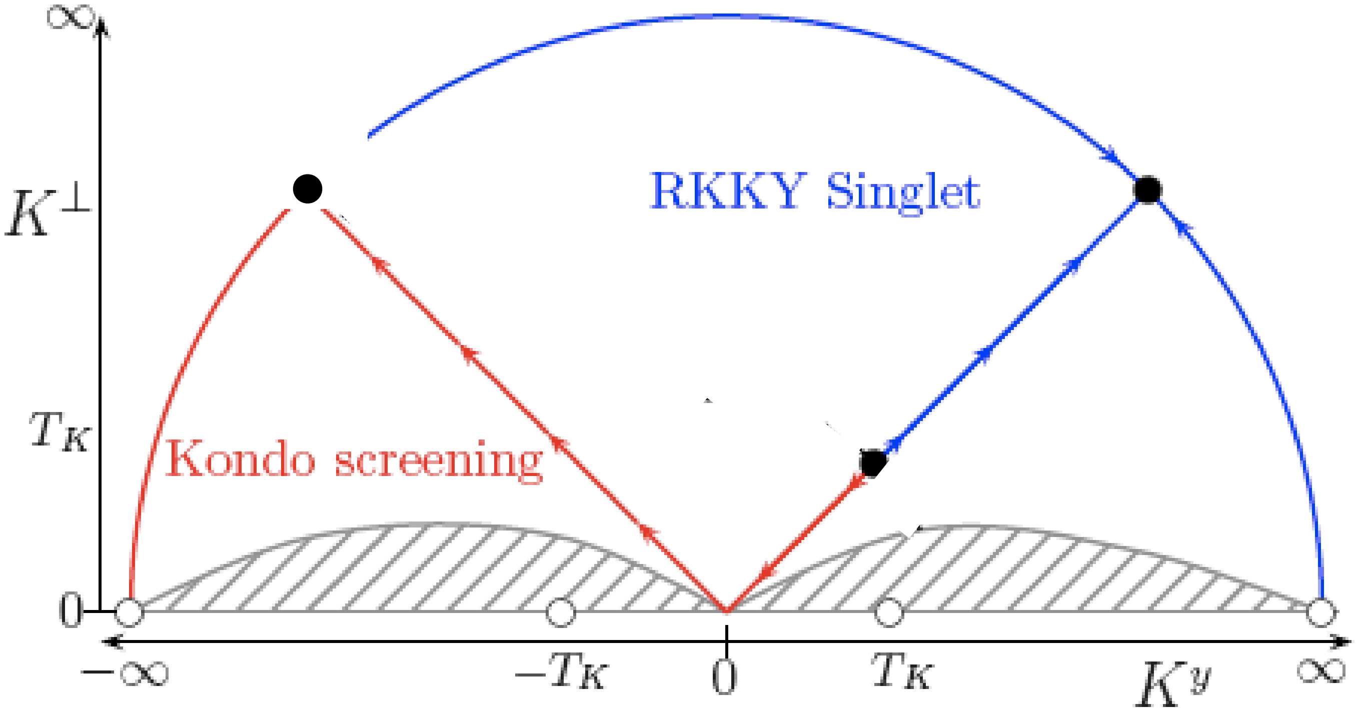


$$H_{\mathrm{TIKM}}^{\mathrm{SO}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}_2' \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2' + (K^y - K^{\perp}) S_1^y S_2'^y$$

Global RG flow?

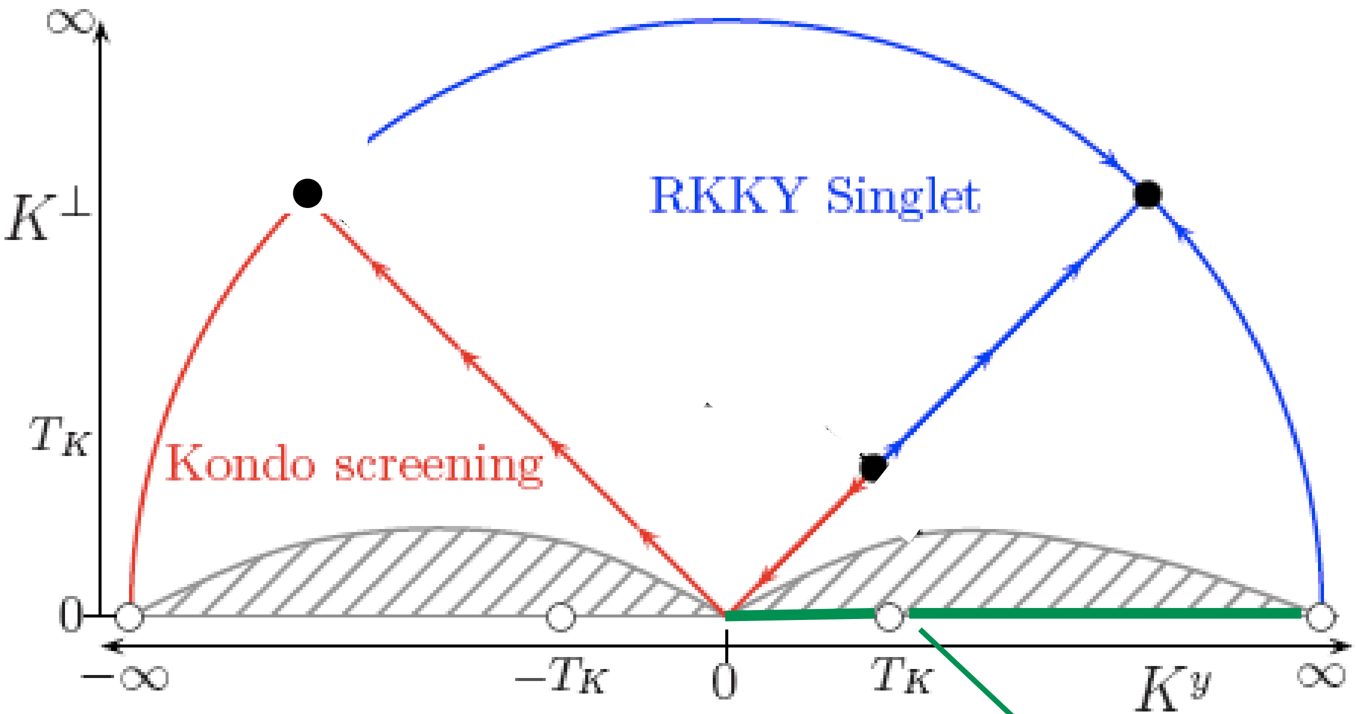
$$H^{\mathrm{SO}}_{\mathrm{TIKM}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}'_2 \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}'_2 + (K^y - K^{\perp}) S^y_1 S'^y_2$$

Global RG flow?



$$H^{\mathrm{SO}}_{\mathrm{TIKM}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}'_2 \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}'_2 + (K^y - K^{\perp}) S^y_1 S'^y_2$$

Global RG flow?

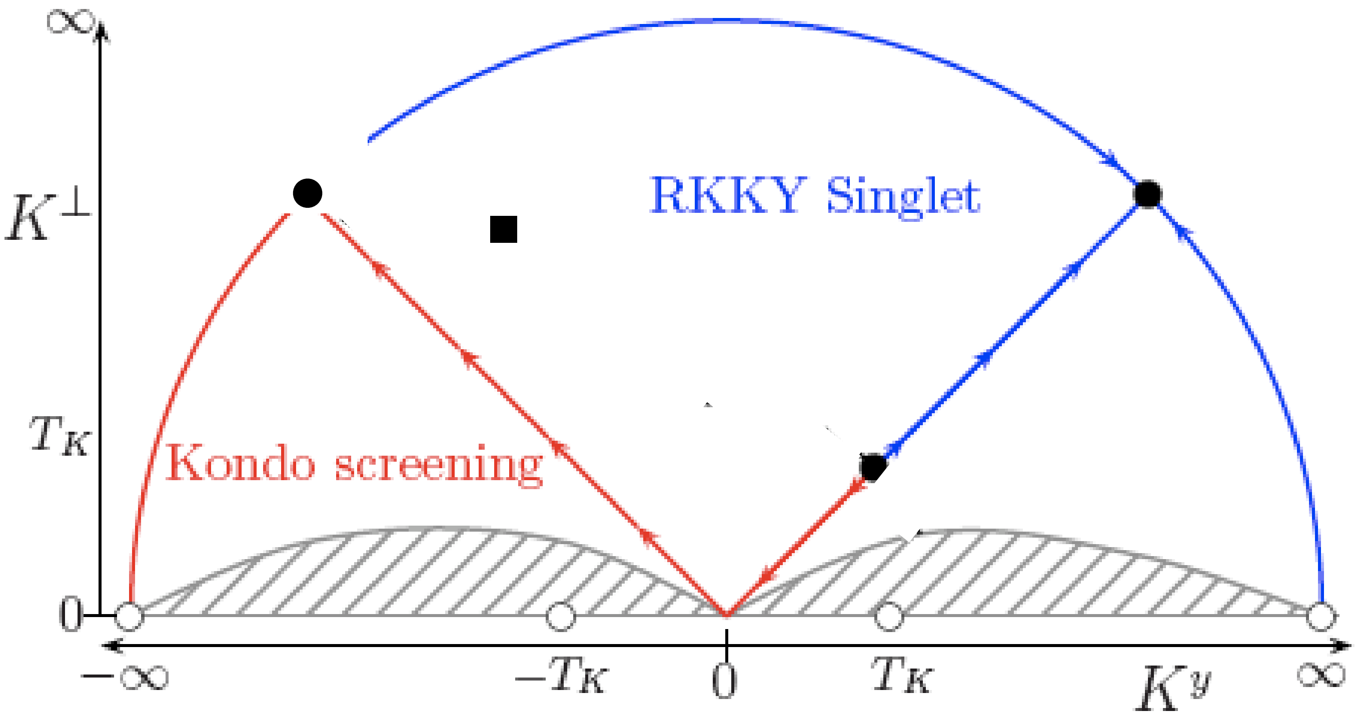


Ising-coupled impurities, quantum phase transition at $K^y \approx T_K$ with different behavior.

N. Andrei *et al.*, PRB **60**, 5125(R) (1999)

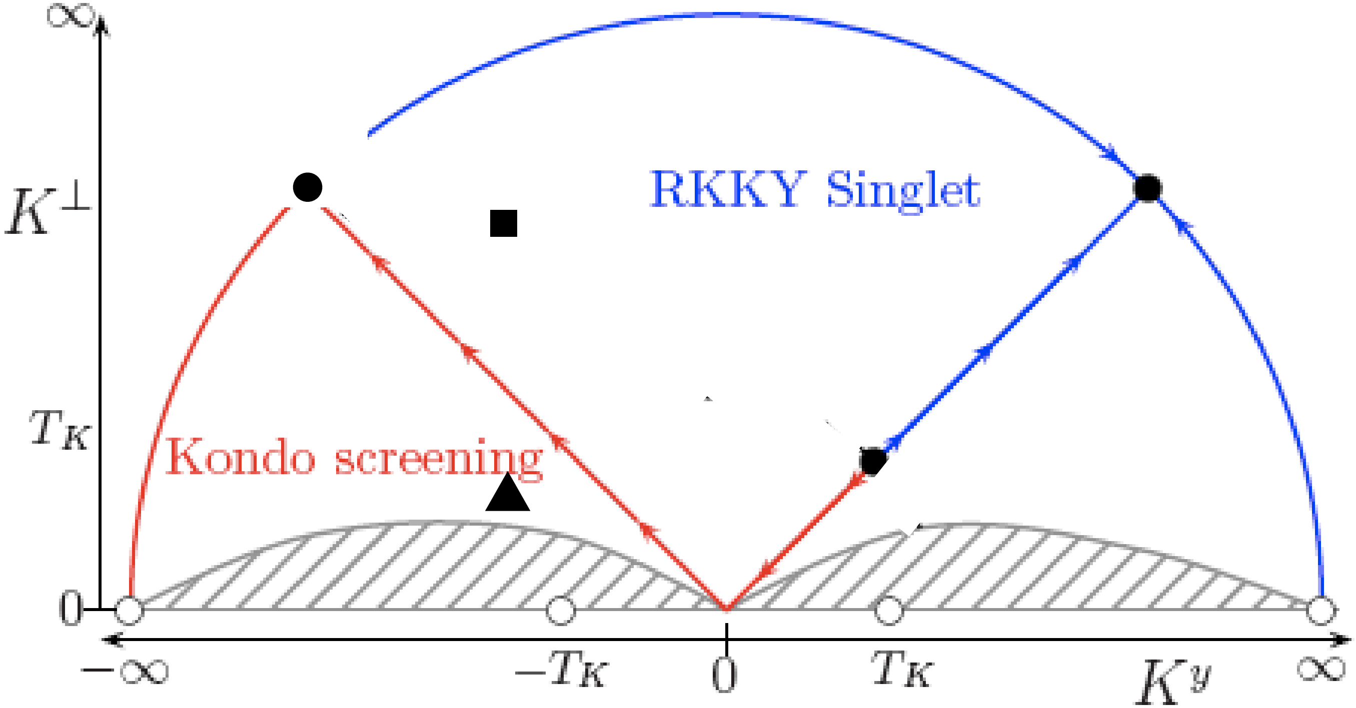
$$H_{\mathrm{TIKM}}^{\mathrm{SO}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}_2' \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2' + (K^y - K^{\perp}) S_1^y S_2'^y$$

Global RG flow?



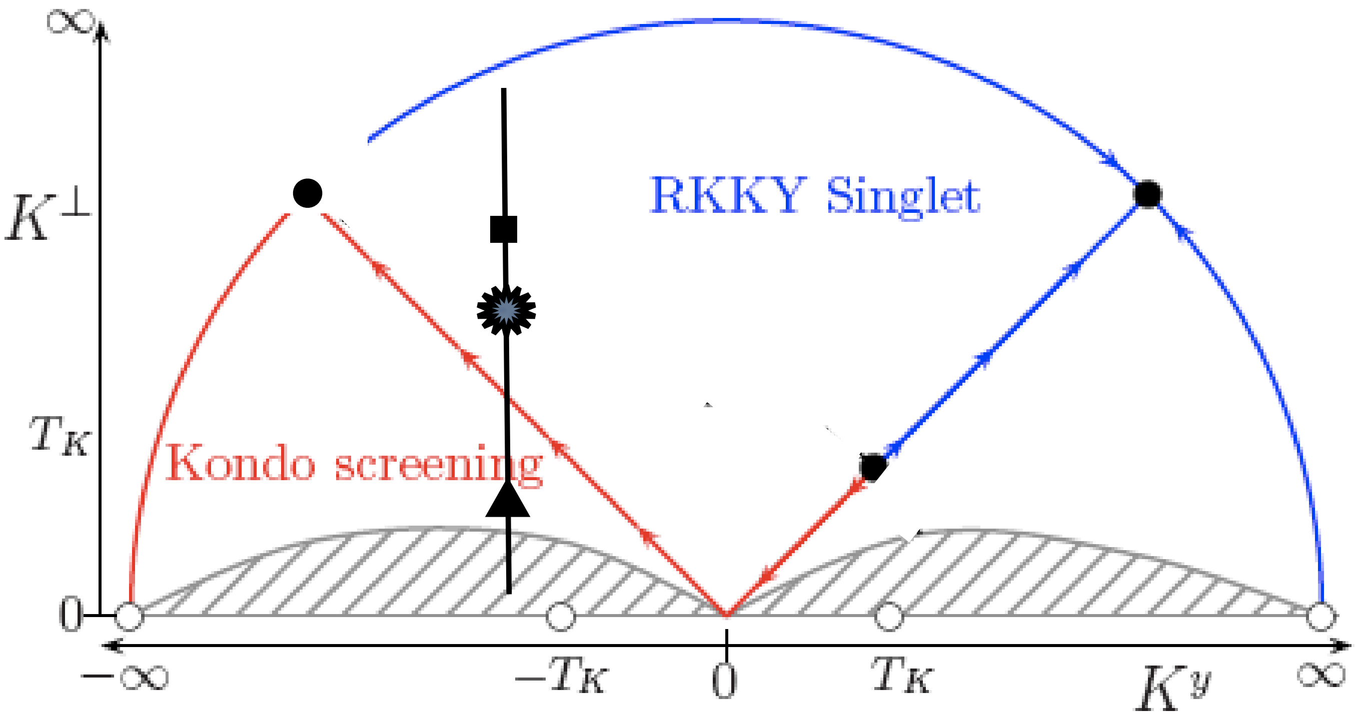
$$H^{\mathrm{SO}}_{\mathrm{TIKM}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}'_2 \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}'_2 + (K^y - K^{\perp}) S^y_1 S'^y_2$$

Global RG flow?



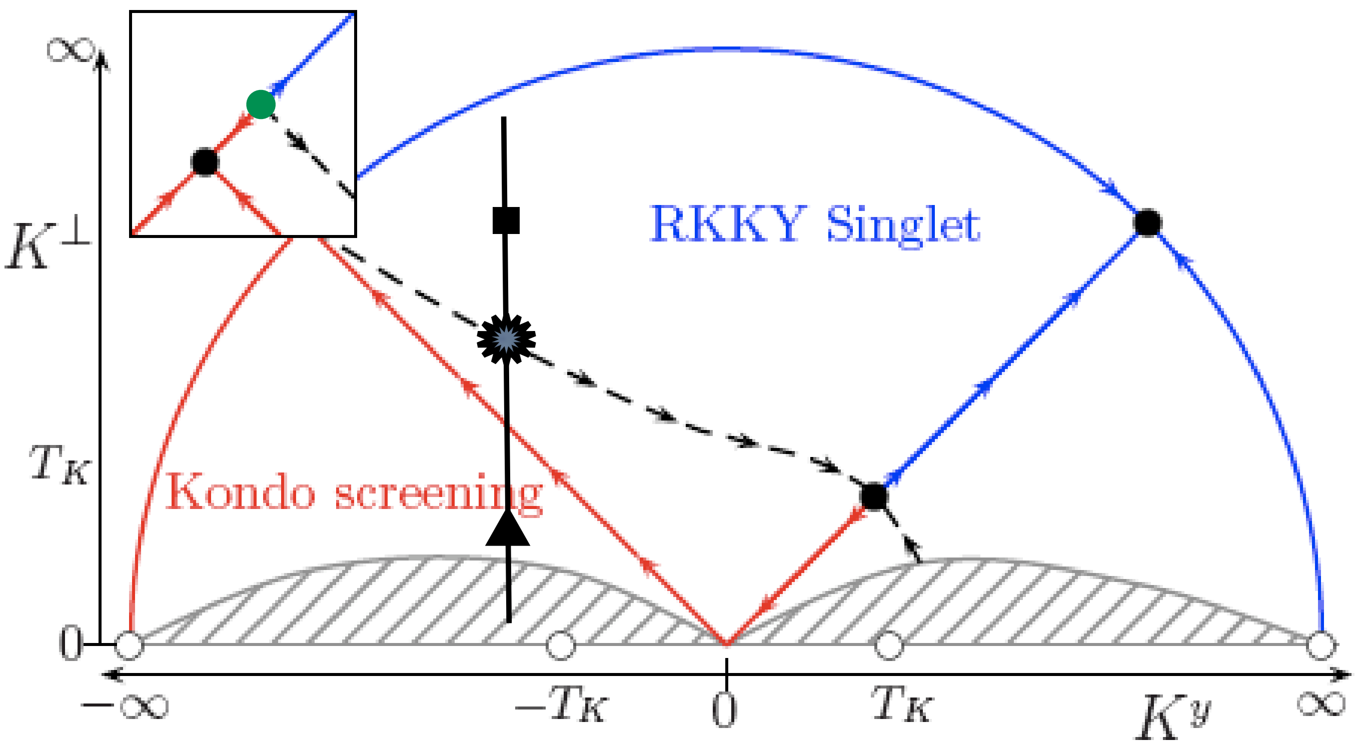
$$H_{\text{TIKM}}^{\text{SO}} = H_{\text{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}'_2 \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}'_2 + (K^y - K^{\perp}) S_1^y S_2'^y$$

Global RG flow?



$$H^{\mathrm{SO}}_{\mathrm{TIKM}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}'_2 \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}'_2 + (K^y - K^{\perp}) S^y_1 S'^y_2$$

Global RG flow?



Summary

2D two-impurity Kondo model with *spin-orbit interactions*

$$H_{\mathrm{TIKM}}^{\mathrm{SO}} = H_{\mathrm{kin}} + J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}_2' \cdot \boldsymbol{\sigma}_2 + K^{\perp} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2' + (K^y - K^{\perp}) S_1^y S_2'^y$$

- the RKKY interaction gets “twisted”, with an Ising anisotropy
- SU(2) invariance recovered when |Rashba| = |Dresselhaus|

  good for RKKY-controlled two-qubit gates
- “fine-tuning” of $K^y, K^{\perp}$ ⟶ same quantum critical behavior as with no spin-orbit interactions
- possible new unstable fixed point for $(K^y, K^{\perp}) \to (K_0^y, \infty)$

Numerics on amplitudes, crossover effects from higher-order tunneling, Rashba *and* Dresselhaus in the leads, global RG flow,... *work in progress*

...spin-orbit interactions *in* the Kondo dots (multi-channel two-impurity Kondo model...?)