Strongly interacting Luttinger liquid state as electronic state inherent in carbon nanotubes

Igor N. Karnaukhov

Institute of Metal Physics of the UAS, Kiev, Ukraine

Quantum Engineering of States and Devices: Theory and Experiments
Obergurgl, Austria, June 7, 2010

Outline

Power-low behavior of electron transport in single-walled carbon nanotubes

Experimental data:

1. Photoemission spectra of carbon nanotubes
2. Differential conductance

Known results – (1+1)D models of strongly interacting many-body systems

1. The Luttinger model
2. The Hubbard model
3. The XXZ spin-1/2 Heisenberg chain or the model of spinless fermions with the density-density interaction

Unknown results – exact solvable (1+1)D model of spinless fermions with hard-core interaction

calculations of the critical exponents

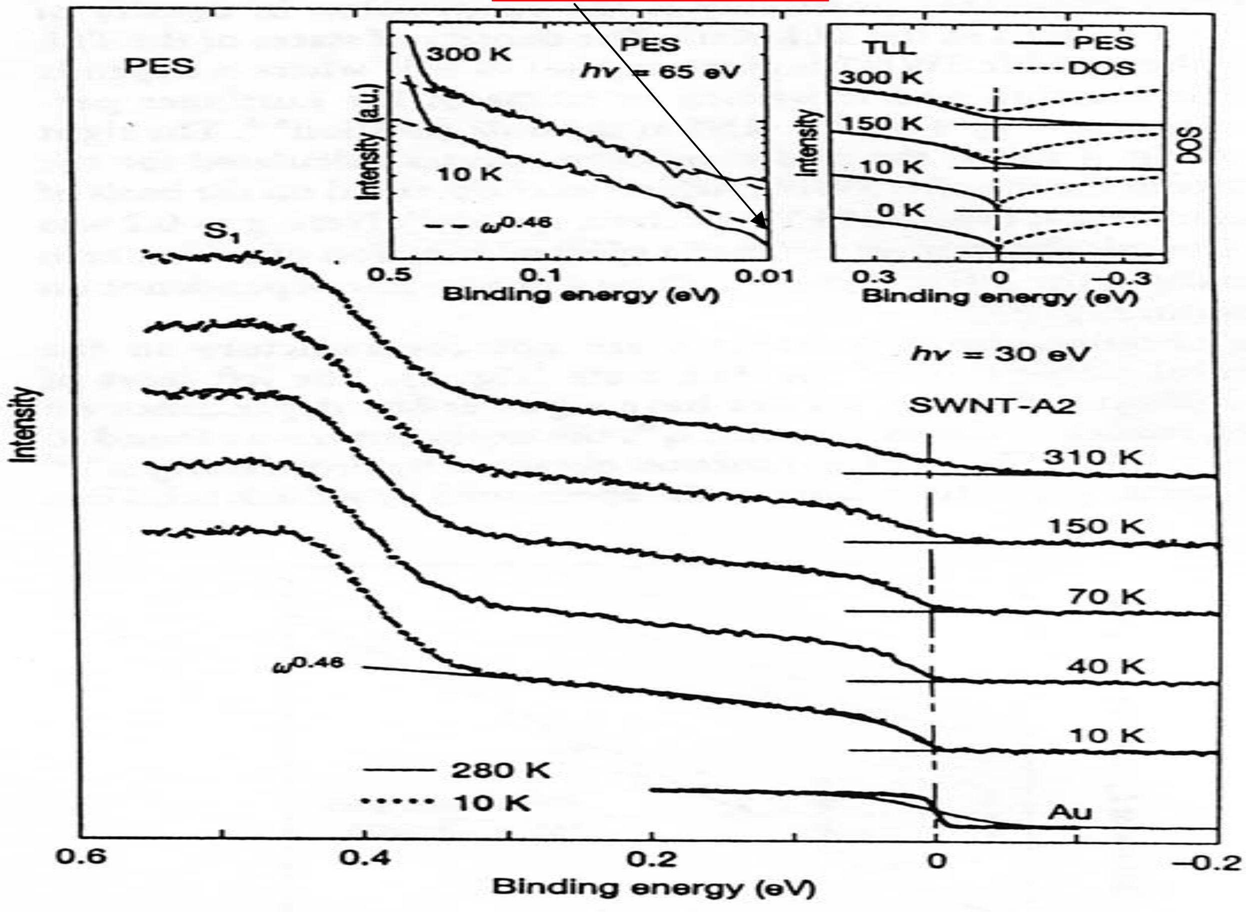
the state of strongly interacting Luttinger liquid state

Ishii H., et. al. Direct observation of Tomonaga-Luttiger-liquid state in carbon nanotubes at low temperatures, Nature, 426, 540 (2003);
H.Rauf, et. al. Transition from a Tomonaga-Luttinger Liquid to a Fermi Liquid in Potassium-Intercalated Bundles of Single-Wall Carbon Nanotubes Phys.Rev.Lett.93,096805 (2004).

Photoemission spectra of carbon nanotubes at low T

asymptotic behavior of the spectral function near the Fermi energy

$$\rho(\omega) \approx |\omega|^{\theta} \qquad \theta = 0.46$$



Bockrath M., et. al. Luttinger liquid behavior in carbon nanotubes Nature, 397,598 (1999);
H.W.Ch.Postma, et. al., Electrical transport through carbon nanotube junctions created by mechanical manipulation, Phys.Rev.B 62,R10653 (2000);
A.Bachtold et. al., Suppression of Tunneling into Multiwall Carbon Nanotubes, Phys.Rev.Lett.87, 166801 (2001).

Conductance G $G \propto T^{\theta}$ $eV < k_B T$

Differential conductance $dI/dV \propto V^{\theta}$ $eV > k_B T$

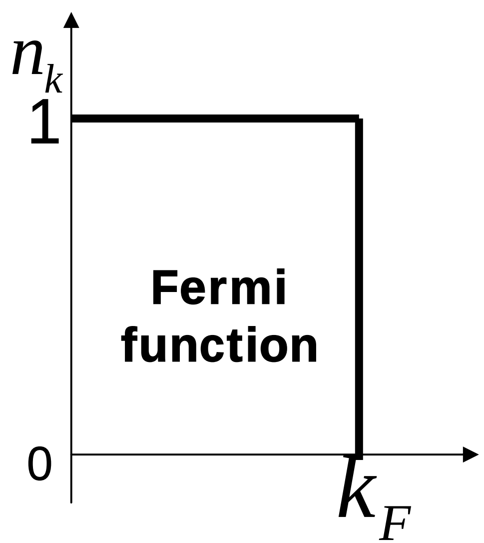
$\theta = 0.43$

the momentum distribution function

$$n_k = \tfrac{1}{2}(1 - const\,|k - k_F|^{\theta}\,\mathrm{sgn}(k - k_F))$$

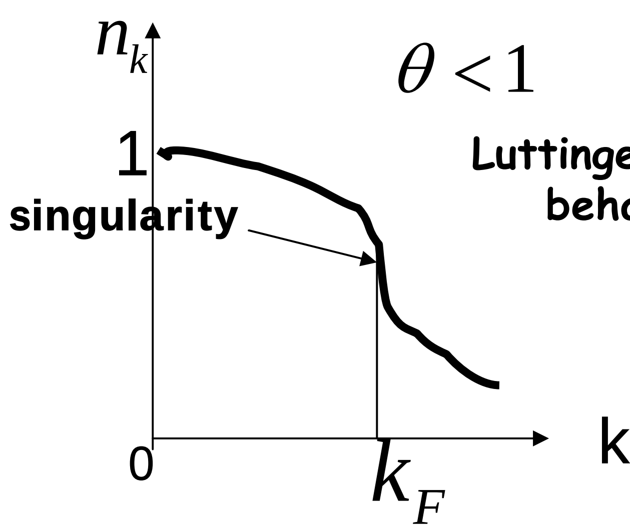
$\theta = 0$

Fermi-liquid behavior



$\theta < 1$

Luttinger liquid behavior



Known results

The Luttinger model for spinless fermions

$$H = \frac{\pi v_F}{L} \sum_{r,k} \rho_r(k)\rho_r(-k) + \frac{\pi v_F}{2L}(N^2 + J^2) +$$

$$\frac{1}{L} \sum_{k} [g_2(k)\rho_+(k)\rho_-(-k) + \frac{g_4(k)}{2} \sum_{r} \rho_r(k)\rho_r(-k)]$$

r=+,- describes 1D right- and left-moving fermions, $N, J = N_+ \pm N_-$

The Hamiltonian is diagonalized as

$$H = \sum_{r,k} \frac{\pi v(k)}{L} \rho_r(k)\rho_r(-k) + \frac{\pi}{2L}(v_N N^2 + v_J J^2)$$

with the velocities

$$v(k) = \sqrt{[v_F + \frac{g_4(k)}{2\pi}]^2 - [\frac{g_2(k)}{2\pi}]^2},$$

$$v_N = v_F + [g_4(0) + g_2(0)]/(2\pi), v_J = v_F + [g_4(0) - g_2(0)]/(2\pi)$$

and stiffness constant

$$K(k) = \sqrt{\frac{2\pi v_F + g_4(k) - g_2(k)}{2\pi v_F + g_4(k) + g_2(k)}}$$

the momentum distribution function

$$n_k = \tfrac{1}{2}(1 - const\,|k - k_F|^{\theta}\,\mathrm{sgn}(k - k_F))$$

$$\theta = \tfrac{1}{2}(K_\rho + 1/K_\rho - 2), K_\rho = K(0) = \sqrt{\frac{2\pi v_F + g_4(0) - g_2(0)}{2\pi v_F + g_4(0) + g_2(0)}}$$

$$g_2(k) << 2\pi v_F, g_4(k) << 2\pi v_F, K_\rho \approx 1 - g_2(0)/(2\pi v_F)$$

$K_\rho = 1$ corresponds to free spinless fermions

the case $K_\rho < 1$ characterizes repulsion interaction between fermions,

$K_\rho < 1$ attractive interaction

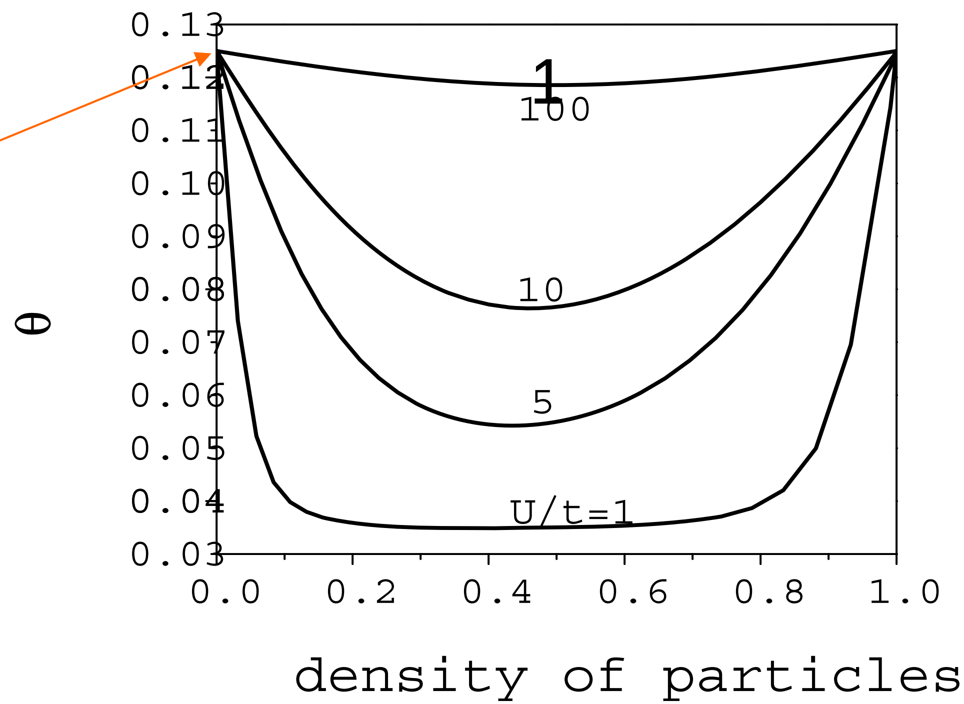
$\theta \approx [\frac{g_2(0)}{2\pi v_F}]^2 << 1$ in the Luttinger model

The Hubbard model

$$H = -t\sum_{j\sigma}(c^{+}_{j\sigma}c_{j+1\sigma} + c^{+}_{j+1\sigma}c_{j\sigma}) + U\sum_{j} n_{j\uparrow}n_{j\downarrow}$$

t is the hopping integral, U is the constant of the on-site repulsive (U>0) interaction

$\theta \leq 1/8$



Θ<0.46 that is realized in carbon nanotubes

$\theta = \frac{1}{4}(K_\rho + 1/K_\rho - 2)$ for the Hubbard model

Spin-1/2 XXZ Heisenberg chain

$$H = -2\sum_j (S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + J\sum_j S_j^z S_{j+1}^z$$

the Jordan-Wigner transformation

$$S_j^+ = c_j \exp(-i\pi \sum_{i>j} n_i),$$

$$S_j^z = 1/2 - n_j ; n_j = c_j^+ c_j$$

or model of spinless fermions

$$H = -\sum_j (c_j^+ c_{j+1} + c_{j+1}^+ c_j) + J\sum_j n_j n_{j+1}$$

the hopping integral is equal to unity, J is the constant of the density-density interaction

$K_\rho = \zeta(\Lambda)^2$ is the universal Luttinger parameter, ζ is the dressed charge

$$\theta = \tfrac{1}{2}(K_\rho + 1/K_\rho - 2)$$

Unknown results

The exact solution of the model Hamiltonian with hard-core interaction

The Hamiltonian of the chain of spinless fermions with density-density interactions

$$H = -\sum_j (c_j^+ c_{j+1} + c_{j+1}^+ c_j) + J_1 \sum_j n_j n_{j+1} + J_2 \sum_j n_j n_{j+2} + \ldots J_l \sum_j n_j n_{j+l}$$

here $J_1 > J_2 \ldots > J_l$

The case $J_1 \to \infty, J_2 \to \infty, \in J_{l-1} \to \infty, J_l$ is reduced to the Hamiltonian with hard-core interaction, where hard-core radius is equal to l/2

The model has exact solution for arbitrary values of the hard-core radius l/2 and coupling constant J_l

$$\theta = \tfrac{1}{2}(K_\rho + 1/K_\rho - 2)$$

$K_\rho = \zeta(\Lambda)^2$ **is the universal Luttinger parameter, ζ is the dressed charge**

large value

small value

the Bete equations have the following form

$$\left[\frac{\sinh\frac{1}{2}(\lambda_j+i\eta)}{\sinh\frac{1}{2}(\lambda_j-i\eta)}\right]^{(L-lN)} = \exp(-ilP)\prod_{i=1}^{N}\frac{\sinh\frac{1}{2}(\lambda_j-\lambda_i+2i\eta)}{\sinh\frac{1}{2}(\lambda_j-\lambda_i-2i\eta)}$$

$$J_l = \cos\eta, \exp(ik_j) = \frac{\sinh\frac{1}{2}(\lambda_j+i\eta)}{\sinh\frac{1}{2}(\lambda_j-i\eta)}, P = \sum_{j=1}^{N} k_j$$

k_j, λ_j are the momenta and charge rapidities of spinless fermions

integral equation for the distribution function ρ(λ)

$$\rho(\lambda) + \int_{-\Lambda}^{\Lambda} d\lambda' K_2(\lambda-\lambda')\rho(\lambda') = (1-\mathrm{n}\,l)K_1(\lambda)$$

with the kernel being for $J_l < 1$

$$K_n(\lambda) = \frac{1}{2\pi}\frac{\sin(n\eta)}{\cosh\lambda - \cos(n\eta)}.$$

n is the density of particles

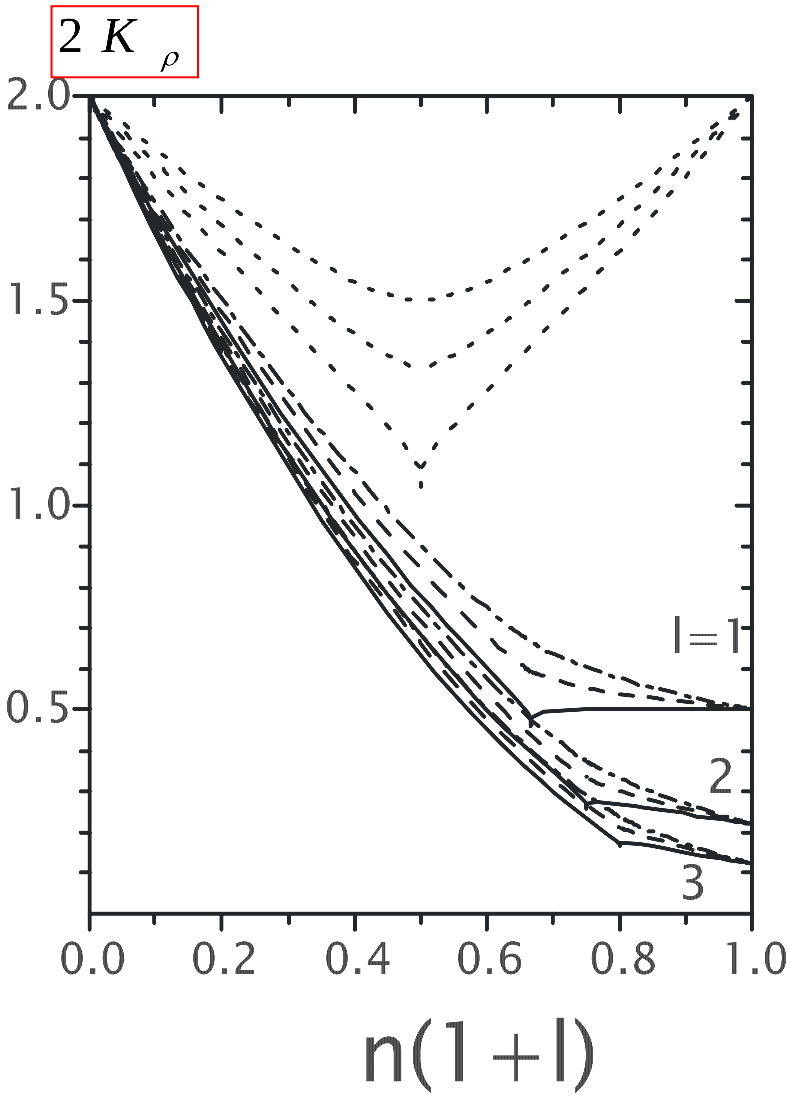
$$n = \int_{-\Lambda}^{\Lambda} \rho(\lambda)d\lambda$$

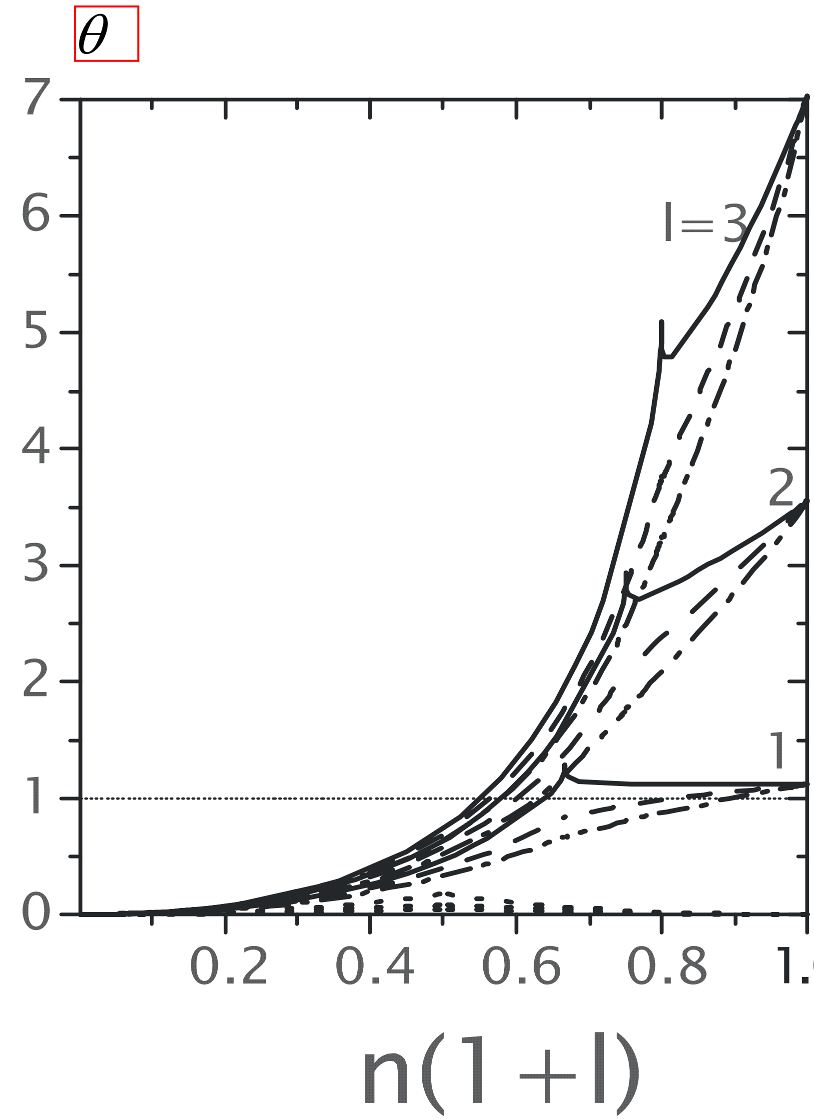
integral equation for the dressed function ζ(λ)

$$\zeta(\lambda) + \int_{-\Lambda}^{\Lambda} d\lambda' K_2(\lambda-\lambda')\zeta(\lambda') = 1-\mathrm{n}\,l$$

Critical exponents for $J_l < 1, J_l = \cos\eta$

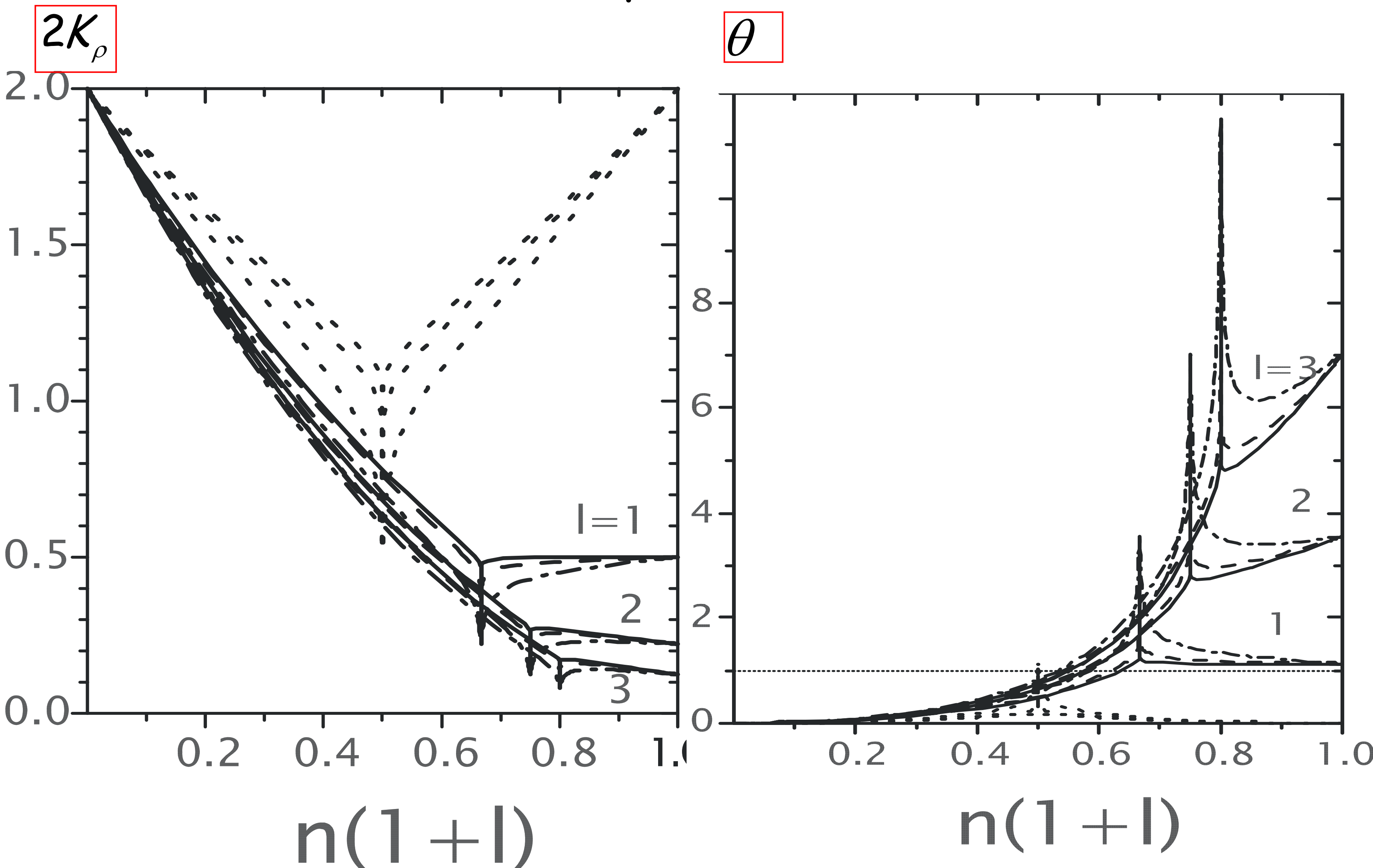
as a function of the density of fermions for η =0.1(solid lines),η=π/4 (dashed); η=π /3 (dashed with points) and *l* =0,1,2,3. Result for *l* =0 is plotted (dotted line) for comparision





Critical exponents for $J_l > 1, J_l = \cosh\eta$

as a function of the electron density for η =0.1 (solid lines), 0.5 (dashed); 1 (dashed with points) and l=0,1,2,3, result for l=0 is plotted (dotted line) for comparision



The leading asymptotics of the density-density correlation

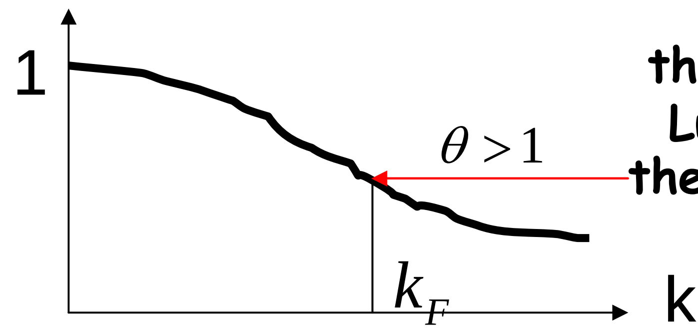
$$< n(x)n(0) > \approx n^2 + A_0 x^{-2} + A_1^{(1)} x^{-2K_\rho} \cos(k_F x)$$

small value

The long-distance asymptotic of the one-particle correlation function

$$< c^+(x)c(0) > \approx B_1 x^{-\theta-1} \cos(k_F x)$$

$$n_k \approx n_{k_F} - const\,|k-k_F|^\theta \,\mathrm{sgn}(k-k_F)$$



the state of strongly interacting Luttinger liquid state

the singularity vanishes

the state of strongly interacting Luttinger liquid state is characterized by anomalously strong density-density correlations in the many-body system and a disappearing residual Fermi surface

Conclusions

The photoemission experiments, obtained on the organic conductors, show much larger value of θ =1.25 than in carbon nanotubes where θ=0.46. Such large values of the critical exponent θ are explained in the framework of state strongly interacting Luttinger liquid, that is realized in 1D systems with a hard-core repulsive interaction.

- I hope, that I answered on the question What is the interaction that leads to large values of the critical exponents ?

- And to propose the family of the integrable (1+1)D models that describe the behavior of strongly interacting Luttinger liquid state

- Open question about the nature (realization) of a hard-core in carbon nanotubes

Thank you for attention