Towards electron-electron entanglement in Penning traps

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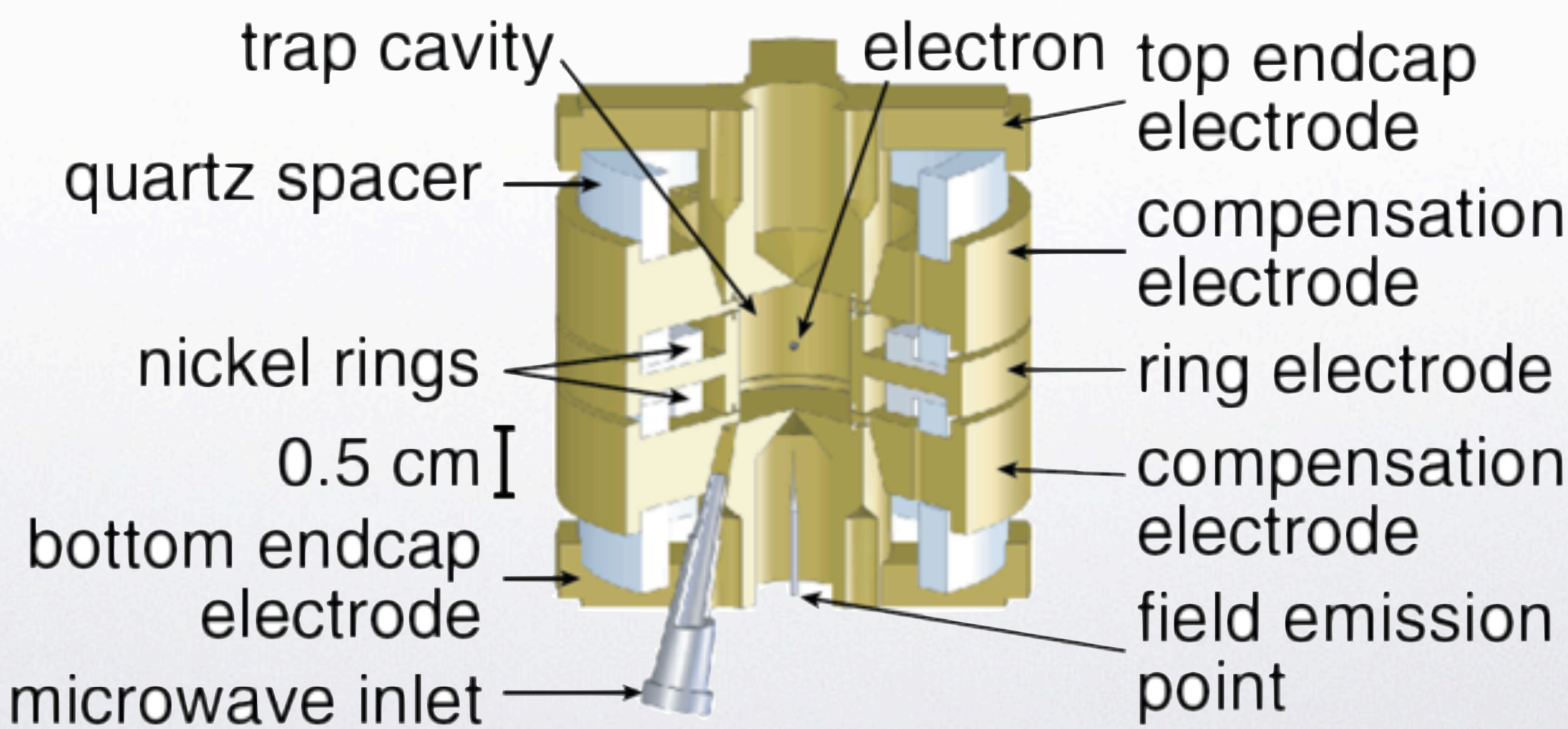
Outline

- Introduction: electrons in Penning traps
- Motivation
- Our proposal: entangling gate
- Applications
- Conclusions

Introduction: Penning traps

- Dynamical trapping of charged particles
- Constant magnetic field + quadrupole electric potential

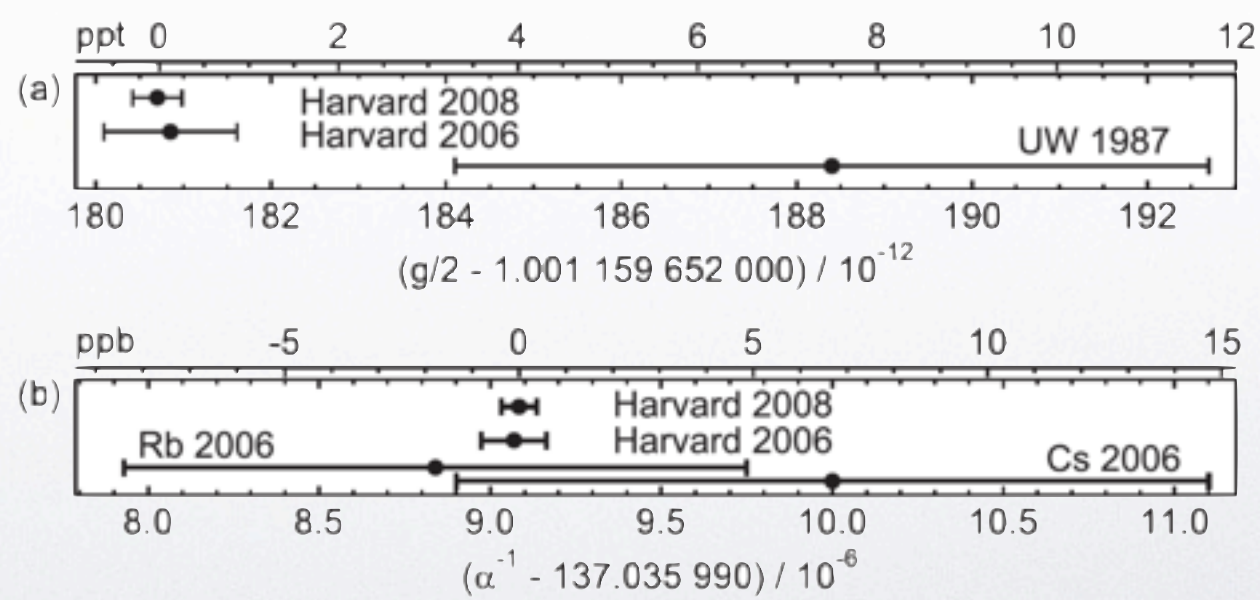


Introduction: Penning traps

- One-electron experiments
- Highest precision in g factor and α

$$\vec{\mu} = -\frac{g}{2}\mu_B \frac{\vec{S}}{\hbar/2}$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$$



Introduction: Penning traps

- Experiments for *g* factor measurements:
- Cylindrical traps (Gabrielse lab, 1987-today)

PRL **100,** 120801 (2008) PHYSICAL REVIEW LETTERS week ending 28 MARCH 2008

New Measurement of the Electron Magnetic Moment and the Fine Structure Constant

D. Hanneke, S. Fogwell, and G. Gabrielse*

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(Received 4 January 2008; published 26 March 2008)

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- Initial experiments: Mainz ('06,'07), Ulm ('08,'10)

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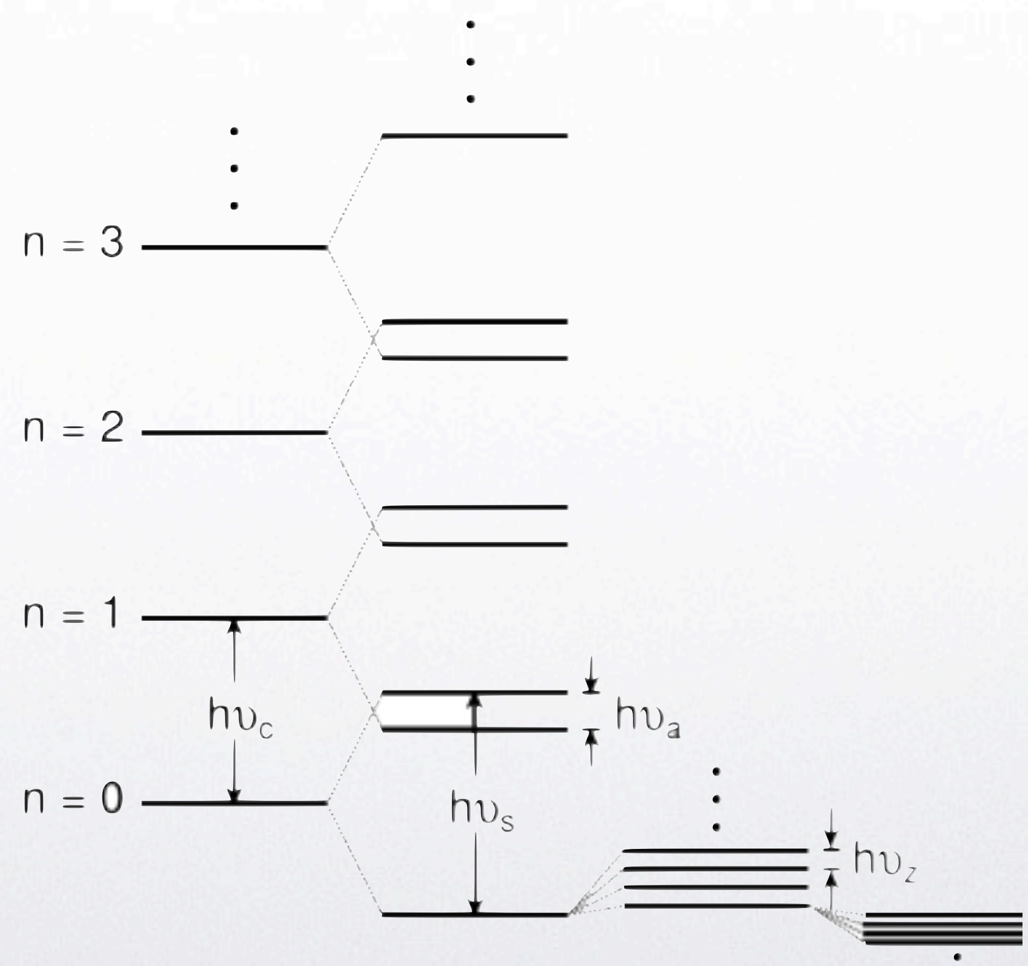
Introduction: Penning traps

- Quantized energy levels



Motivation

- Frequency hierarchy ⟷ Clean system

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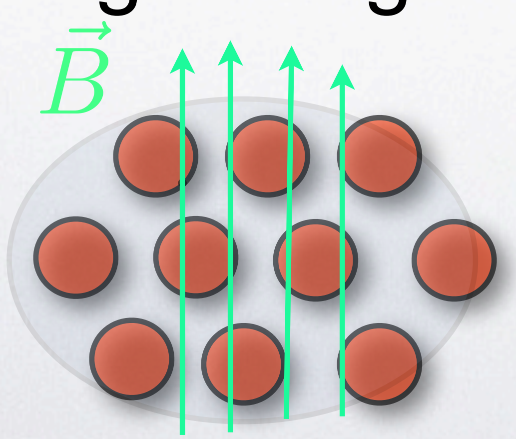
Good for quantum coherence

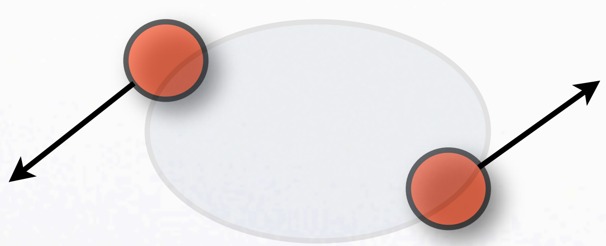
Our proposal

- A method of entangling two electrons in a Penning trap (as a *proof of principle* experiment)

- Long term goal: Quantum simulators/metrology



Two-electron system



- Dynamical equilibrium positions
- Normal modes
- *cm* mode → Two-qubit gate
- Decoherence

Two-electron system

$$H = \sum_{i=1,2} \{ \frac{\omega_s}{2}\sigma_z^{(i)} + \frac{[\vec{p}_i + e\vec{A}(x_i)]^2}{2m} + V_q + V_{12} \}$$

$$V_q(x_i) = \sum_{i=1,2} \frac{m\omega_z^2}{2}[z_i^2 - (x_i^2 + y_i^2)/2], \;\; V_{12}(x_i) = \frac{e^2}{4\pi\epsilon_0|\vec{x}_1 - \vec{x}_2|}$$

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- Interaction Hamiltonian:

$$H_I = \frac{g}{2}\mu_B\beta_2 \sum_{i=1,2} [(z_i^2 - \rho_i^2/2)\sigma_z^{(i)} - z_i\vec{\rho}_i \cdot \vec{\sigma}_i]$$

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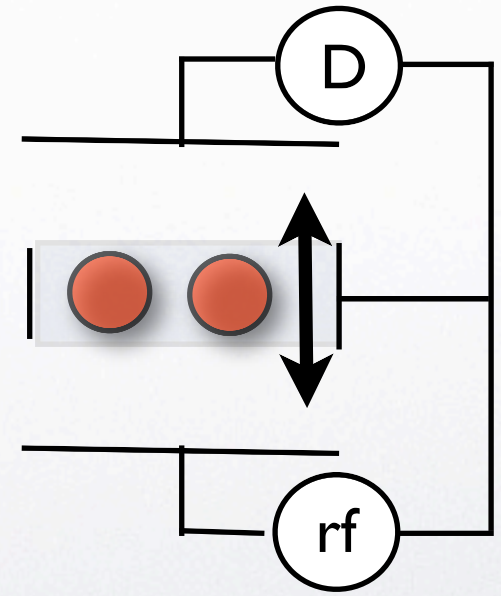
Gate term

Two-electron gate

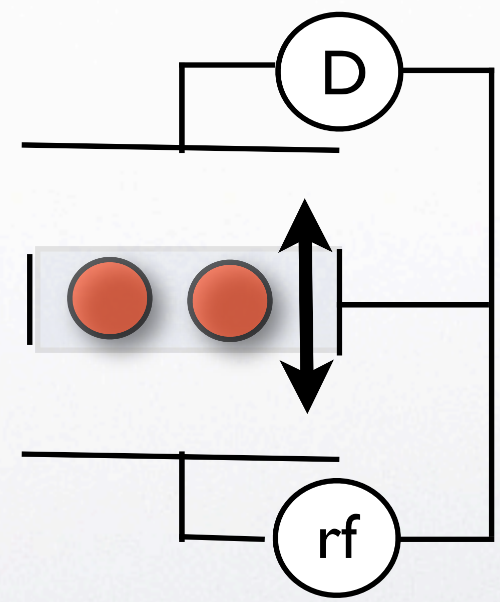
$$-z_i \vec{\rho}_i \cdot \vec{\sigma}_i \qquad z_{cm} = z_0 \cos[(\omega_a + \Delta)t]$$



Two-electron gate

$$-z_i \vec{\rho}_i \cdot \vec{\sigma}_i \qquad z_{cm} = z_0 \cos[(\omega_a + \Delta)t]$$

- Axial driving + *cm,c*



$$\omega_a = \omega_s - \omega_{cm,c}$$

Two-electron gate

- Rotating-wave approximation retains anomaly transition terms

$$H_I^{\text{gate}} = \hbar\Omega \sum_{i=1,2} (\sigma_i^+ a_{cm,c} e^{-i\Delta t} + \sigma_i^- a_{cm,c}^\dagger e^{i\Delta t})$$

$$\Omega = \frac{g}{2} \frac{\mu_B \beta_2 z_0}{\sqrt{4m\hbar}[\omega_c^2 - 2\omega_z^2]^{1/4}}$$

Applications

$$H_I^{\text{gate}} = \hbar\Omega \sum_{i=1,2} (\sigma_i^+ a_{cm,c} e^{-i\Delta t} + \sigma_i^- a_{cm,c}^\dagger e^{i\Delta t})$$

- When $\Delta \gg \Omega$ adiabatically eliminate *cm,c*

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- When $\Delta \gg \Omega$ adiabatically eliminate *cm,c*

$$H_I^{\rm off} = \frac{\Omega^2}{\Delta} \sum_{i,j} [-(n+1)\sigma_i^+ \sigma_j^- + n\sigma_i^- \sigma_j^+]$$

Two qubit gate (quantum computing)

Applications

- Tests of quantum mechanics with entangled elementary particles: e^-, e^+
- Possibility of quantum metrology protocols
  g, α
- Possibility of two-qubit gates

Conclusions

- Penning traps: possibility to perform high-precision measurements (low decoherence)
- Here: how to entangle two electrons
- Many interesting applications: quantum metrology, two qubit gates, decoherence analysis