Entanglement and Correlations between Disjoint Blocks of 1D Critical Systems

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Outline:

Negativity and correlators between blocks

Entanglement renormalization and dual holographic geometries

Gross features of entanglement measures and correlation functions

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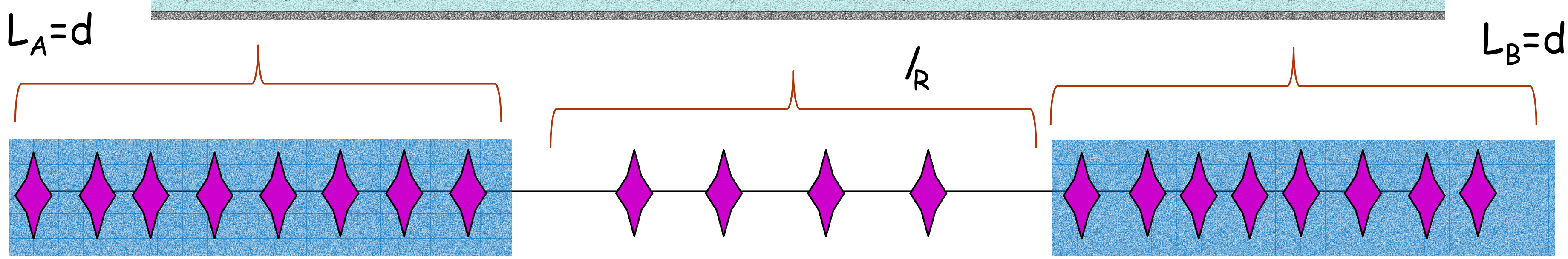
ENTANGLEMENT BETWEEN DISJOINT BLOCKS OF A 1D CRITICAL SPIN CHAIN



A

B

RDM $$\rho_{AB}=\frac{1}{2^{L_A+L_B}}\left[\sum_{even}\langle O_A O_B\rangle O_B^{\dagger} O_A^{\dagger}+\sum_{odd}\langle O_A S O_B\rangle O_B^{\dagger} S O_A^{\dagger}\right]$$

Fagotti & Calabrese arXiv:1003.1110v1

Negativity $$N(\rho_{AB})=\left(\left\|\rho_{AB}^{T_A}\right\|_1-1\right)/2=\sum_k|\lambda_k|-1$$

λ_k eigenvalues of $\langle\rho_{AB}^{T_A}\rangle_{\alpha_A,\beta_B,\gamma_A,\delta_B}=\langle\gamma_A,\beta_B\,|\,\rho_{AB}\,|\,\alpha_A,\delta_B\rangle$

$$N(\rho_{AB})\approx\sum\langle O_A O_B\rangle=\sum\left\langle\phi_A^1(x_A^1)\phi_A^2(x_A^2)\ldots\phi_A^{L_A}(x_A^{L_A})\phi_B^1(x_B^1)\ldots\phi_B^{L_B}(x_B^{L_B})\right\rangle$$

Difficult to evaluate even for Exactly Solvable Models

Operator Product Expansions in CFT

In CFT, OPE reduce n-point functions to 2 point functions

Di Francesco et al, (1997)

$$\left\langle \Phi^1(x^1)\Phi^2(x^2)\Phi^3(x^3)\Phi^4(x^4)\right\rangle = \left\langle \sum_k C_{12k}\Phi_k(x_2)\sum_k C_{34n}\Phi_n(x_4)\right\rangle =$$

$$\sum_{k,n} C_{12k}C_{34n}\left\langle \Phi_k(x_2)\Phi_n(x_4)\right\rangle = \sum_n C_{12n}C_{34n}\left\langle \Phi_n(x_2)\Phi_n(x_4)\right\rangle$$

OPE $$\Phi^i(x)\Phi^j(y) \equiv \sum_k c_{ijk}\,|x-y|^{\Delta_k-\Delta_i-\Delta_j}\,\Phi_k(y)$$

Valid for |x-y|« ξ macroscopic preferred length scale in the theory

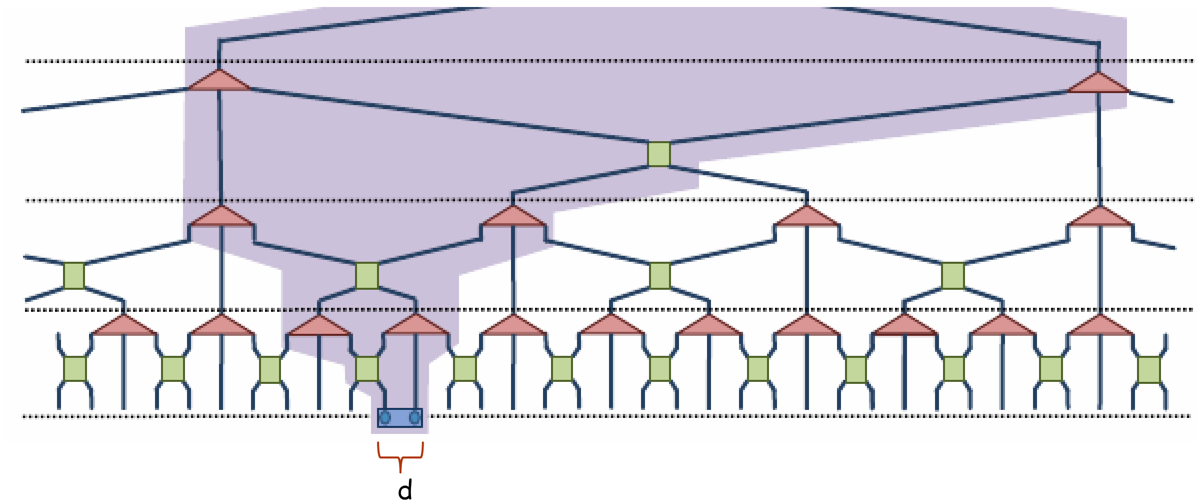
In a complete scale invariant theory any ξ diverges to infinity and OPE is exact

In a theory in which scale invariance is broken by a macroscopic preferred length scale, correlators should depend on

$$\exp\left(-\frac{|x-y|}{\xi}\right)$$

MERA

Casual Cones in MERA



a) Operators always become more local

b) General result

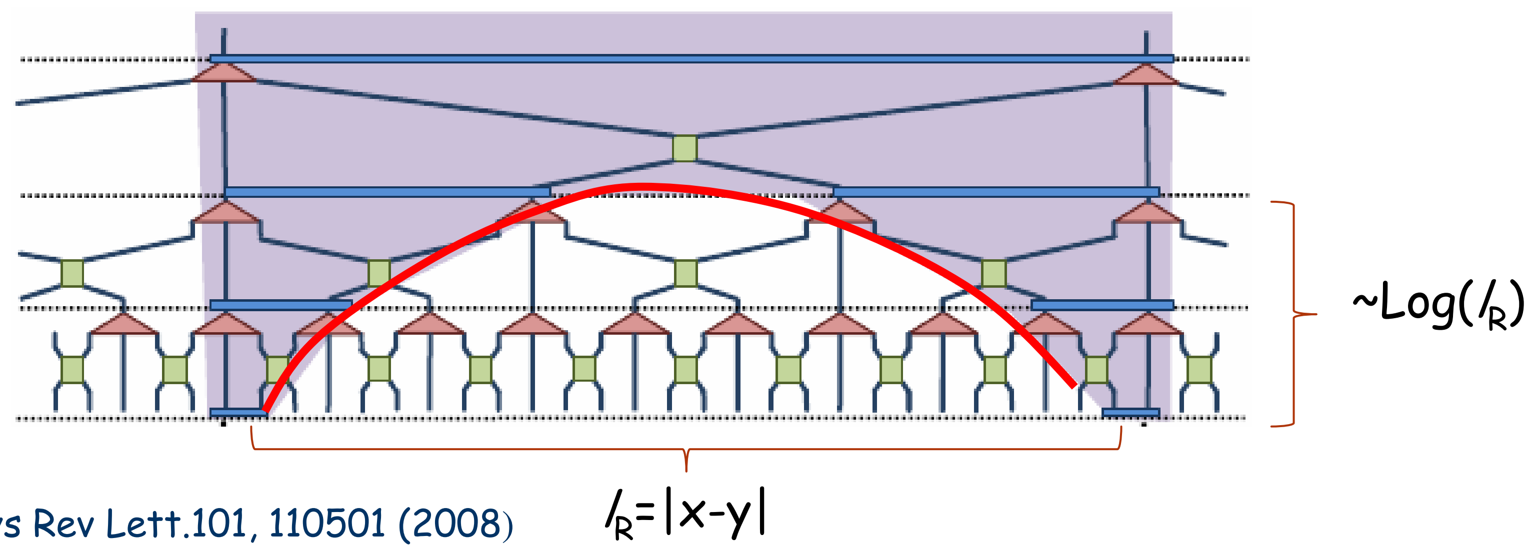
$$d_{\tau+1} < d_{\tau}$$

i.e

Casual Cones of large Blocks tend to shrink after ~ Log (d) RG steps

Vidal, Phys Rev Lett. 99, 220405 (2007)

Correlators in MERA

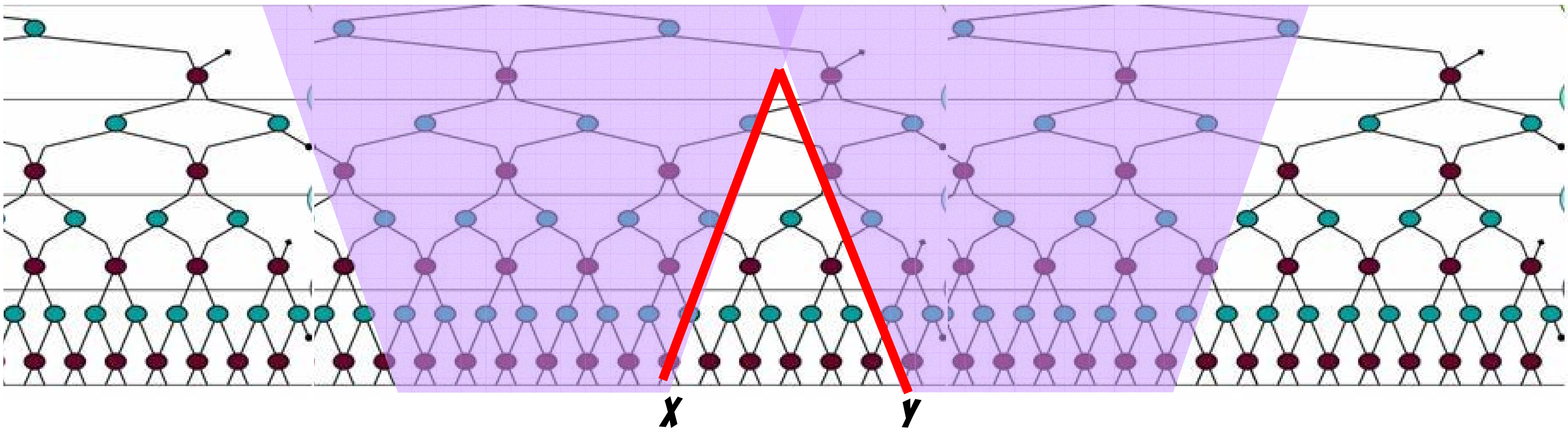


Vidal, Phys Rev Lett.101, 110501 (2008)

MERA

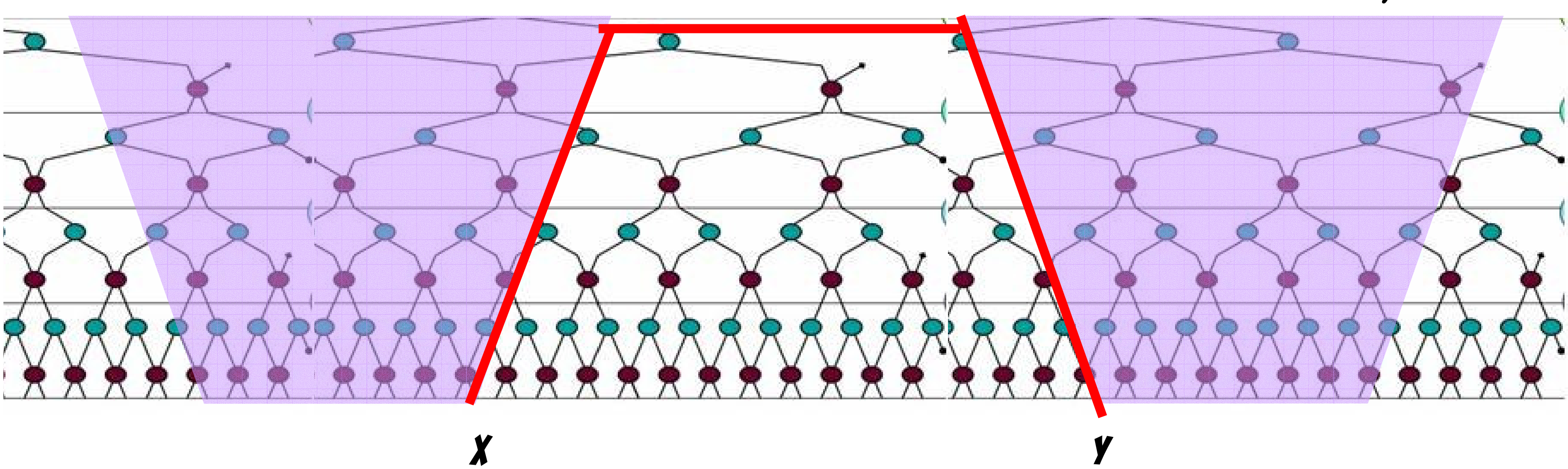
CC overlap before shrinking

$|x-y|<d$



CC DO NOT overlap before shrinking

$|x-y|>d$



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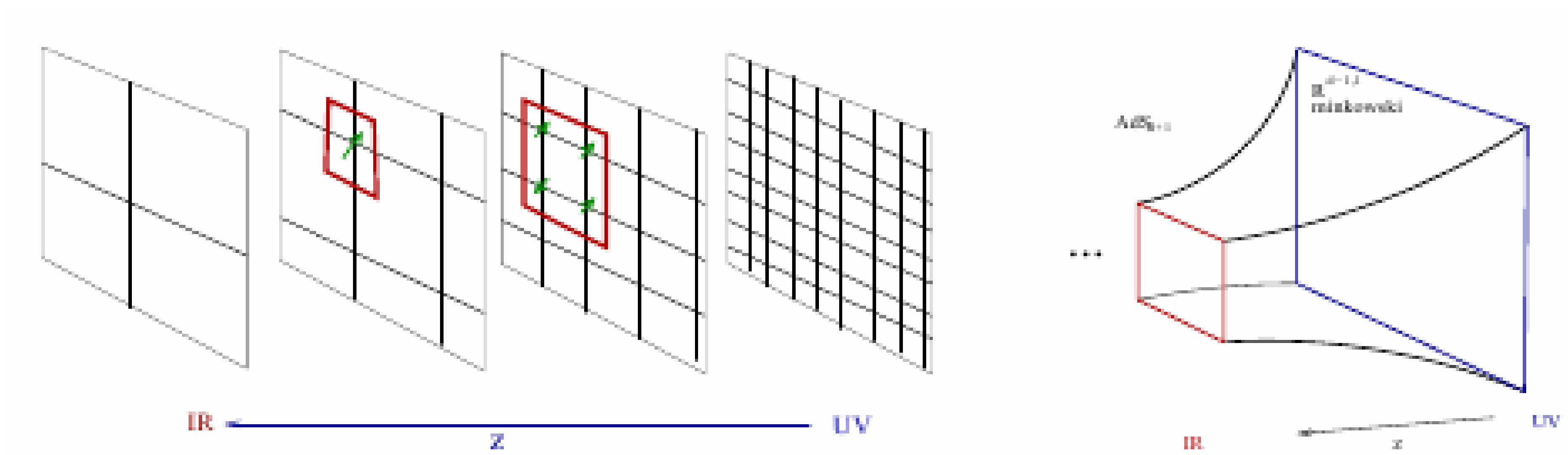
Entanglement Renormalization and Holography.

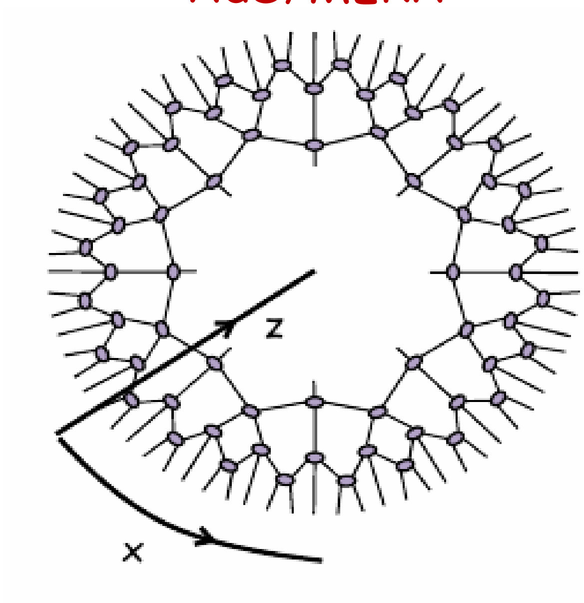
MERA Tensor Network implements a discrete version of
Anti de Sitter (AdS) Space



Emergence of the gravity dual picture is intimately related to the quantum entanglement of degrees of freedom in the corresponding conventional quantum system.

Swingle arXiv: 0905.1317,
McGreevy arXiv: 0909.0518
Van Raamsdonk arXiv: 1005.3035, arXiv: 0907.2939

AdS/MERA

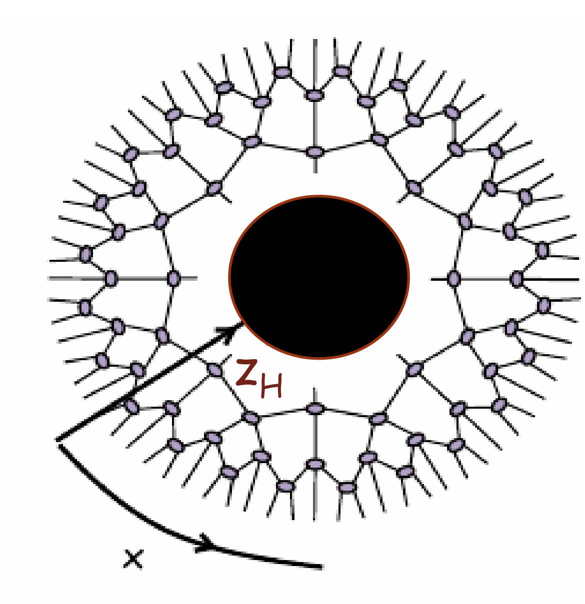


Anti de Sitter Space AdS

$$ds^2 = \kappa^2 \left(\frac{dz^2 + dx^2}{z^2} \right)$$

Finite size of blocks leads to relevant deformations that break scale invariance in the IR.

AdS Black Hole/MERA



Only one nontrivial solution to Einstein equations of this form is AdS Black Hole:

$$ds_{BH}^2 = \frac{\kappa^2}{z^2} \left(\frac{dz^2}{f(z)} + dx^2 \right) \qquad f(z) = 1 - \left(\frac{z}{z_H} \right)^2$$

Asymptotically AdS as $z \to 0$. (UV)

Horizon at $z_H = d$ (IR) i.e $T_H \sim 1/z_H$

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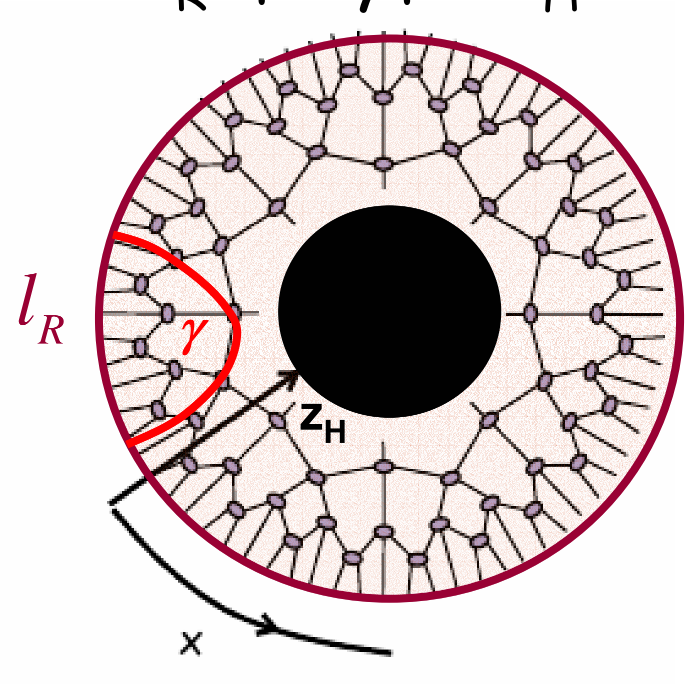
HOLOGRAPHIC COMPUTATION OF 2 POINT FUNCTIONS IN AdS BH (I)

$l_R = |x-y| \ll z_H$



$$L_\gamma \approx \kappa Log\left(\frac{2l_R}{z_\varepsilon}\right)^2$$

$$S_{l_R} = \frac{L_\gamma}{4G_N} = \frac{c}{3} Log\left(\frac{l_R}{z_\varepsilon}\right) + c_1^{'}$$

$$\langle \phi(x)\phi(y) \rangle \sim e^{(-m_\phi L_\gamma)} \approx \frac{1}{|x-y|^{2\Delta_\phi}}$$

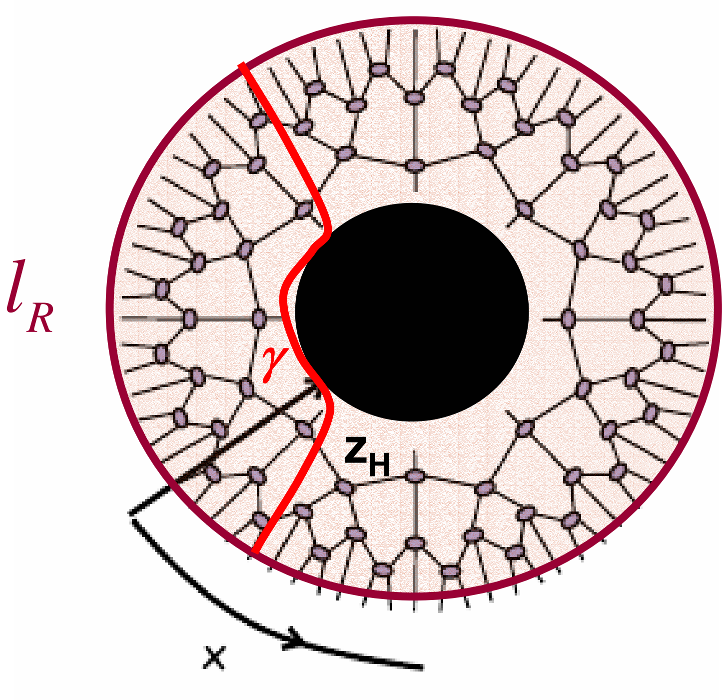
$$c = \frac{3\kappa}{2G_N}$$

Ryu & Takanayagi, PRL 96, 181602 (2006)

$$\Delta_\phi \approx m_\phi \kappa \gg 1$$

Holographic computation of 2 point functions in AdS BH (II)

$l_R = |x-y| > z_H$



$$L_\gamma = 2\kappa\pi \frac{l_R}{z_H}$$

$$S_{l_R} = \frac{L_\gamma}{4G_N} = \frac{\pi c}{3} \frac{l_R}{z_H}$$

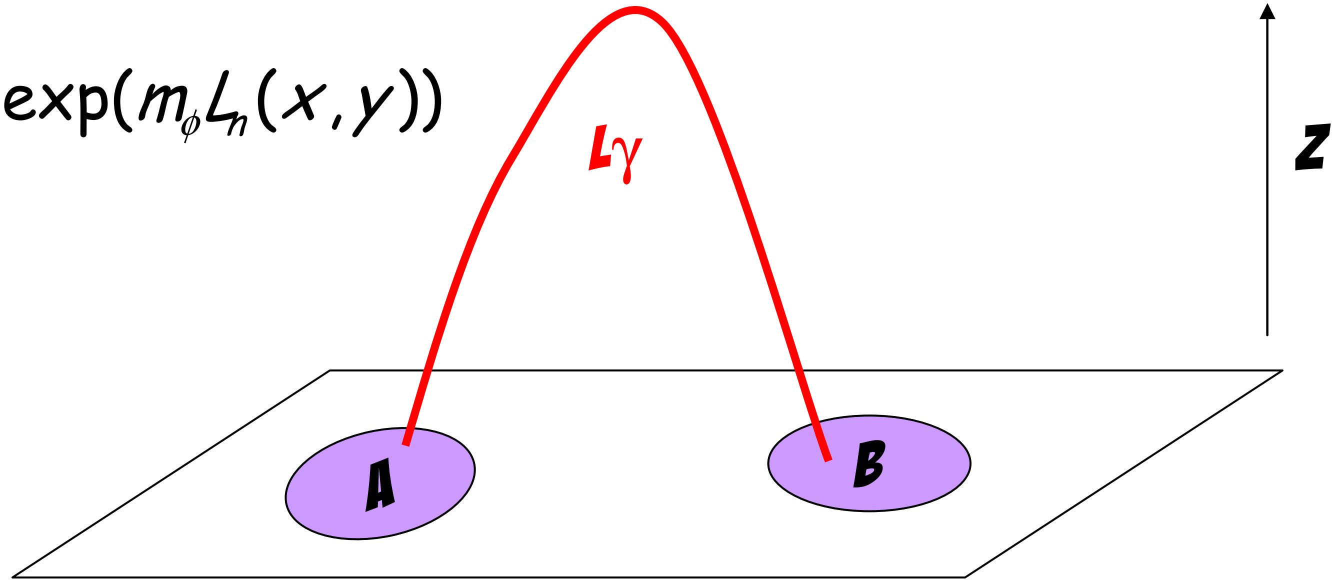
$$\langle \phi(x)\phi(y) \rangle \sim e^{(-m_\phi L_\gamma)} = \exp\left(-2\pi\Delta_\phi \frac{l_R}{z_H} \right)$$

$$c = \frac{3\kappa}{2G_N}$$

Ryu & Takanayagi, PRL 96, 181602 (2006)

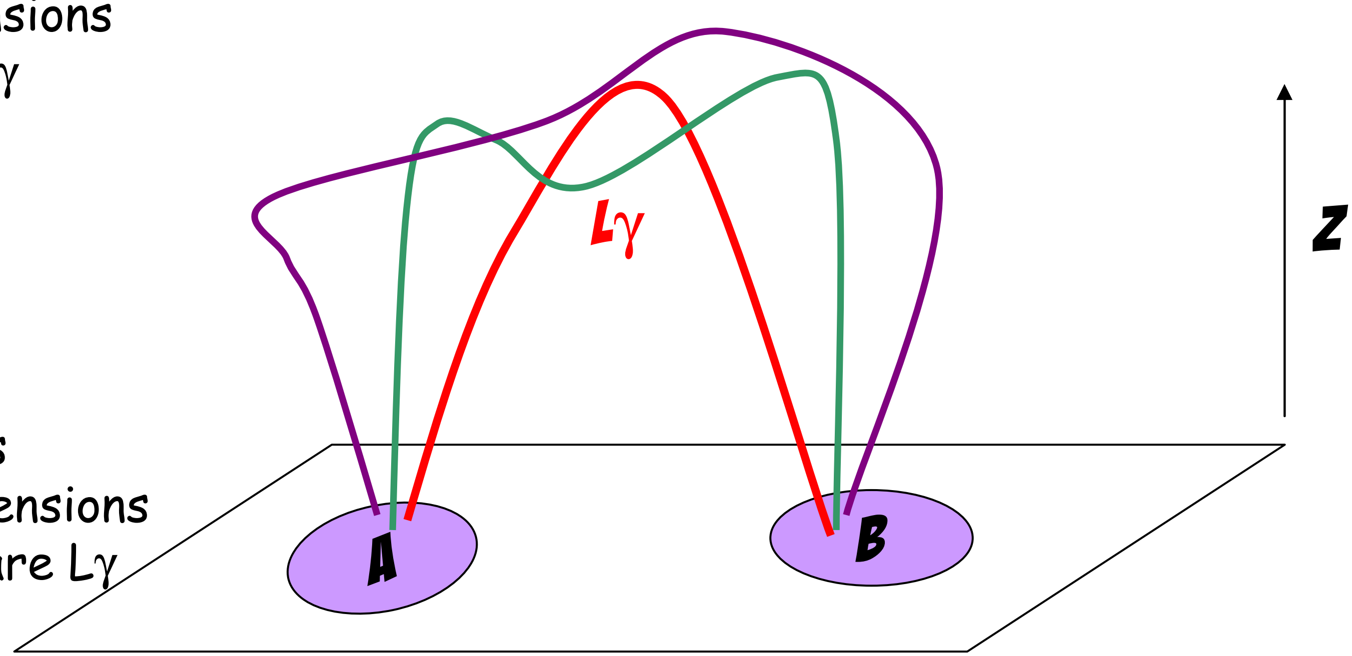
$$\Delta_\phi \approx m_\phi \kappa \gg 1$$

$$\langle \phi(x)\phi(y) \rangle \sim \sum_n \exp(m_\phi L_n(x,y))$$



$$\Delta_\phi \approx m_\phi \kappa \gg 1$$

For massive particles
i.e large scaling dimensions
main contribution is Lγ



$$\Delta_\phi \chi \approx m_\phi \kappa$$

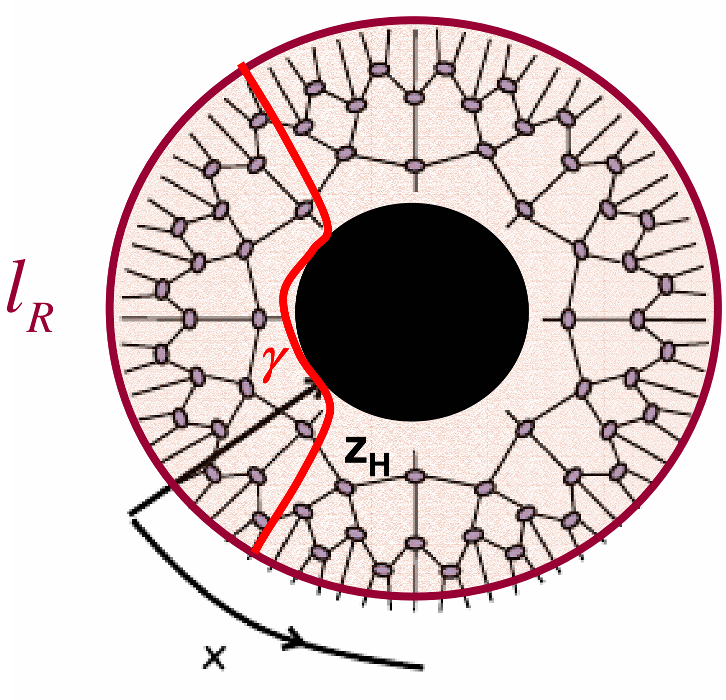
For lighter particles
i.e small scaling dimensions
main contributions are Lγ
and other paths

Holographic computation of 2 point functions in AdS BH (III)

Hydrodynamic regime

$l_R = |x-y| \gg z_H$

$\Delta_\phi \chi \sim m_\phi \kappa$



$$L_\gamma = 2\kappa\pi \frac{l_R}{z_H}$$

$$\langle \phi(x)\phi(y) \rangle \sim \exp\left(-2\pi\Delta_\phi \frac{l_R}{\xi} \right)$$

$$\xi = (1/\chi) z_H$$

Son & Starinets, Ann.Rev.Nucl.Part.Sci.57:95-118,2007

Entropy generating process: Shear Viscosity

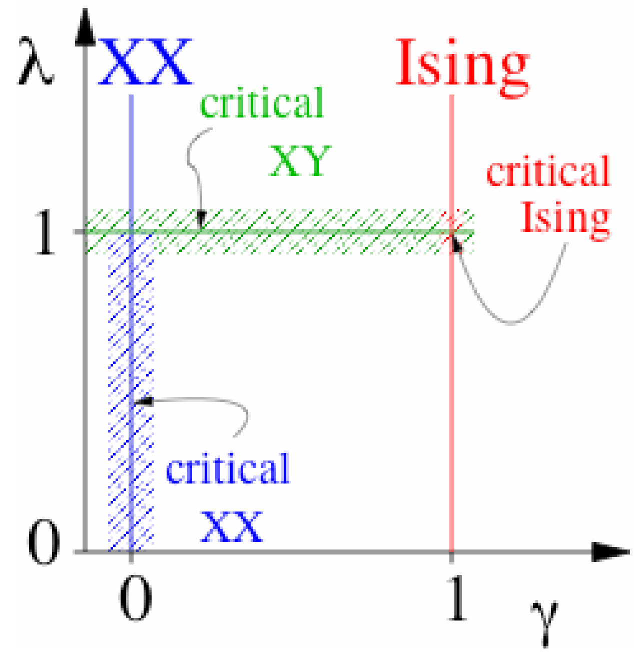
$$\eta = \Phi(z_H) s \sim \xi \varepsilon$$

Negativity between disjoint blocks

Prediction: Our choice of the partition dynamically generates a mass gap

$$N_{AB} \sim \exp\left(-\alpha \frac{l_R}{z_H}\right)$$

$$\alpha = \frac{3 v_F \varepsilon}{c} \Delta_\phi$$



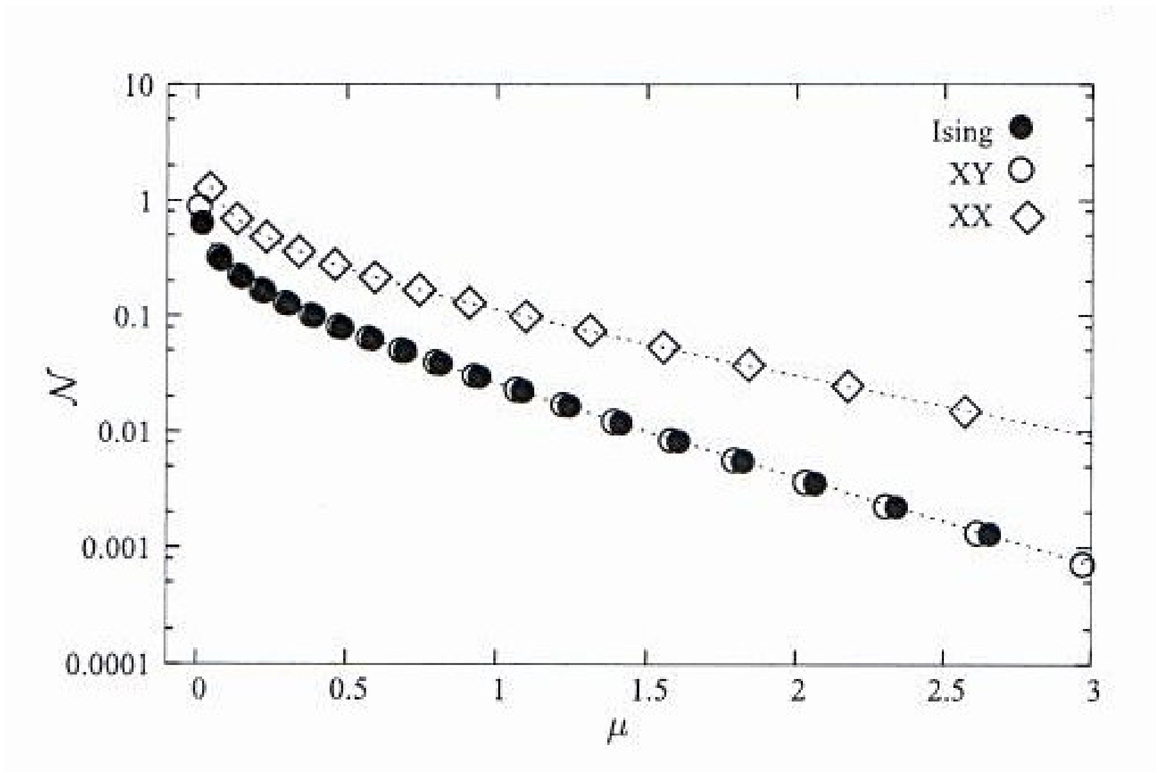
$$H = -\sum_i \frac{(1+\gamma)}{2} \sigma_i^x \sigma_{i+1}^x + \frac{(1-\gamma)}{2} \sigma_i^y \sigma_{i+1}^y - \lambda \sigma_i^z$$

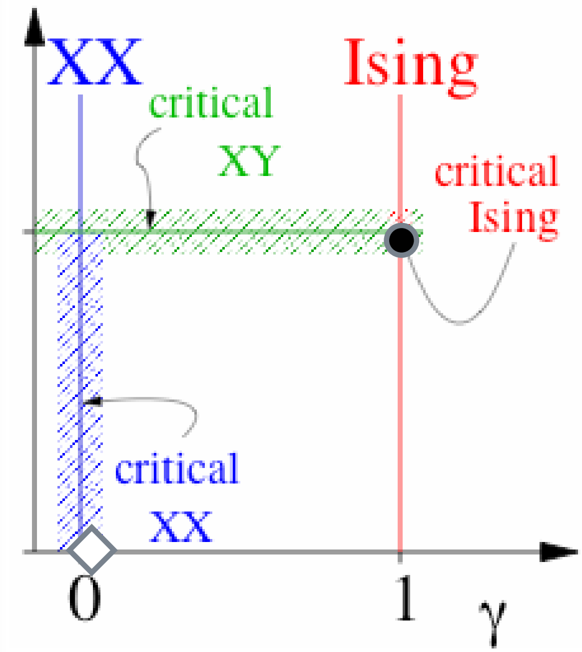
$$\alpha_{Is} = 2\alpha_{XX}$$

$$\alpha_{Is} = 1,9$$

$$\alpha_{XX} = 0,95$$

Negativity between disjoint blocks: DMRG results





$\alpha_{Is} \sim 1.7$

$\alpha_{XX} \sim 0.99$

Data fitted to the functional ansatz

$$N_{AB}(\mu) \sim \mu^{-h} \exp(-\alpha\mu)$$

$$\mu \equiv \frac{l_R}{d}$$

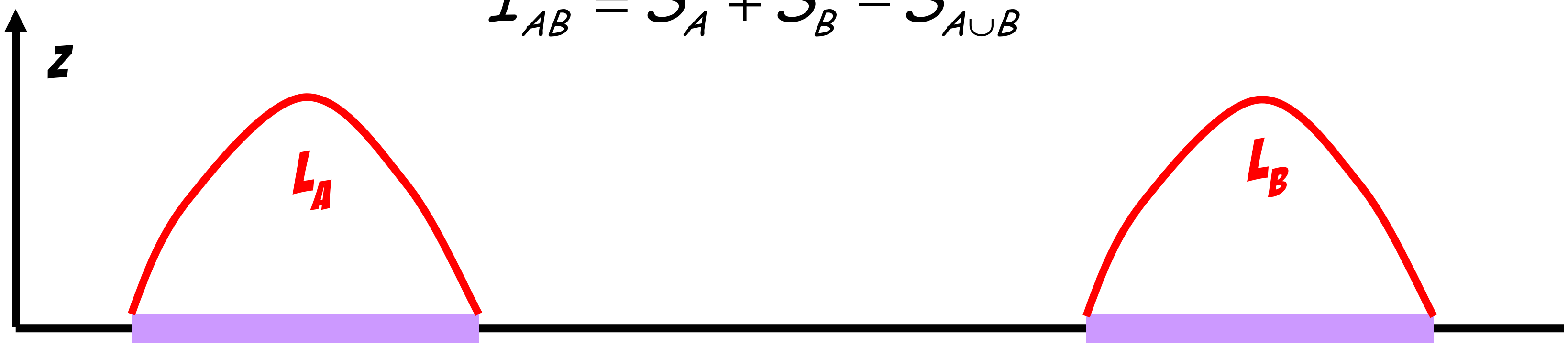
Wichterich et al, PRA 80, 010304 (2009)

Marcovitch et al, PRA 80, 012325 (2009)

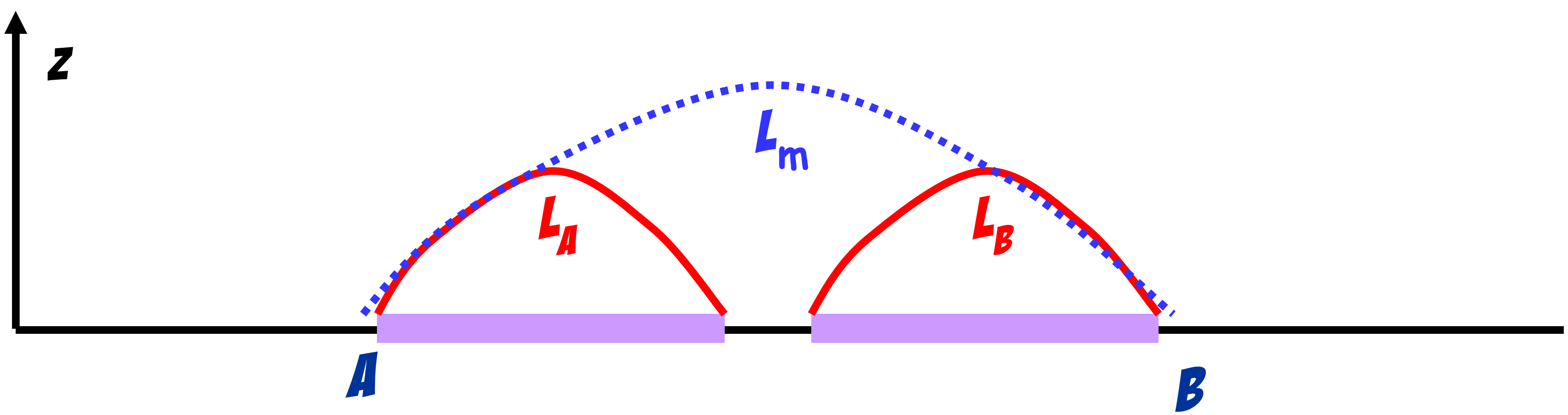
Ryu-Takanayagi formula predicts a similar "phase transition" in mutual information

Headrick arXiv: 1006.0047

$$I_{AB} = S_A + S_B - S_{A\cup B}$$



$$\min L_{A\cup B} \sim L_A + L_B \Rightarrow S_{A\cup B} \sim S_A + S_B \Rightarrow I_{AB} \sim 0$$



$$\min L_{A\cup B} \sim L_m \Rightarrow I_{AB} > 0$$

THANK YOU!