Dissipative processes for quantum simulation and computation

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Overview

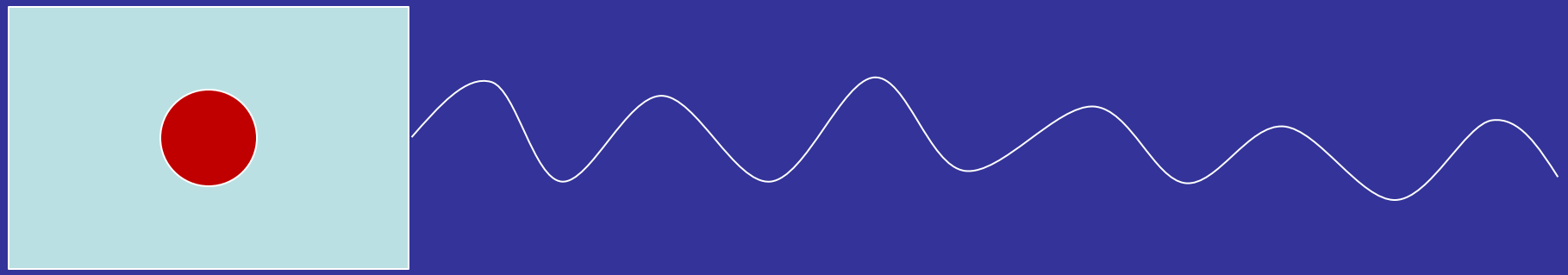
Central theme: are dissipative nonequilibrium processes useful for quantum information tasks?

- Convergence time of non-equilibrium dissipative processes
- Quantum Simulation
  - Preparing ground states of frustration free Hamiltonians
  - Cavity QED as a simulator of quantum field theories
- Quantum Computation:
  - Dissipative universal quantum computation
  - Quantum Metropolis algorithm

Disclaimer

- This is a theory talk; for experiments / experimental proposals related to the talk see
  - Christine Muschik , E. Polzik et al.
  - Dur, Cirac, Rempe et al.
  - Martin Kifner, Hartmann
  - Almut Beige
  - S. Diehl, P. Zoller et al.

Dissipative processes



- Archetypical example: atom in cavity coupled to cavity modes, and photons leaking out of cavity
  - While the whole process is unitary, the quantum state of the atom is described by a mixed state if photons are not measured
- Quantum evolution is described by the Lindblad equation:

$$\begin{aligned}\dot{\rho} &= -i[H,\rho]+\mathcal{L}\rho \\ &\equiv -i[H,\rho]+\sum_{\ell}\kappa_{\ell}\left(2c_{\ell}\rho c_{\ell}^{\dagger}-c_{\ell}^{\dagger}c_{\ell}\rho-\rho c_{\ell}^{\dagger}c_{\ell}\right)\end{aligned}$$

Time-scale of relaxation

- If Lindblad operator is time-independent and ergodic, then the density matrix converges exponentially fast to a fixed point

$$\|\rho(t) - \sigma\|_{\mathrm{tr}}^2 \leq e^{l_1^k t} \chi_k^2(\rho(0), \sigma).$$

  - Asymptotically, convergence time to fixed point if determined by the eigenvalue gap of the Lindblad operator

  - For finite time: the relaxation time is related to the singular value of a symmetrized version of the Lindblad operator

$l_1^k \leq 0$ *refers to the second largest eigenvalue of*

$$\Lambda_k = [\Omega_\sigma^k]^{1/2} \circ \mathcal{L}^* \circ [\Omega_\sigma^k]^{-1/2} + [\Omega_\sigma^k]^{-1/2} \circ \mathcal{L} \circ [\Omega_\sigma^k]^{1/2}.$$

Temme et al., arXiv:1005.2358

Classical analogue of dissipative maps: stochastic maps

- Stochastic processes:

$$\frac{d}{dt}|p\rangle = S|p\rangle$$

- Such processes have been extensively studied in both physics and computer science, and turn to be extremely useful
  - Markov-Chain Monte Carlo methods: obtained a monopoly for the simulation of interacting classical particles
  - Randomized algorithms and mixing times in Computer Science: e.g. finding satisfying assignments in k-SAT (solution of the problem is encoded in the fixed point of the stochastic process)
- Natural question: are quantum dissipative systems equally useful?
  - Can we use it for quantum simulation and for constructing novel quantum algorithms?

Dissipative processes for simulating strongly correlated quantum spin systems

- Consider a frustration-free Hamiltonian, i.e

$$H = \sum_t P_t \quad ; \quad P_t \geq 0 \quad ; \quad P_t |\chi_0\rangle = 0$$

  - Occurs naturally in condensed matter physics (AKLT, …), quantum complexity/computation theory, BEC, ...
  - Can we find an *efficient* algorithm for finding the ground state?

- This would indeed be possible if we can find a dissipative system for which the fixed point is the state of interest

  - we want the ground state to be the *dark* state of our dissipative process

Dissipative quantum engineering

- Define a dissipative process (master equation) whose fixed point corresponds to ground state of the frustration-free Hamiltonian

$$\frac{d}{dt}\rho = \sum_i L_i \rho L_i^* - \frac{1}{2}\left(\sum_i L_i^* L_i \rho + \sum_i \rho L_i^* L_i \right)$$

  - A sufficient condition for the ground state to be a fixed point:

$$\forall i: \quad L_i \left| \psi_0 \right\rangle = 0$$

  - We want that only ground states are fixed points. This we can do by choosing

$$L_i = U_i P_i$$

where the unitary rotates part of the “bad” subspace into the “good” one. This makes fixed point also unique if the Hamiltonian has a unique ground state

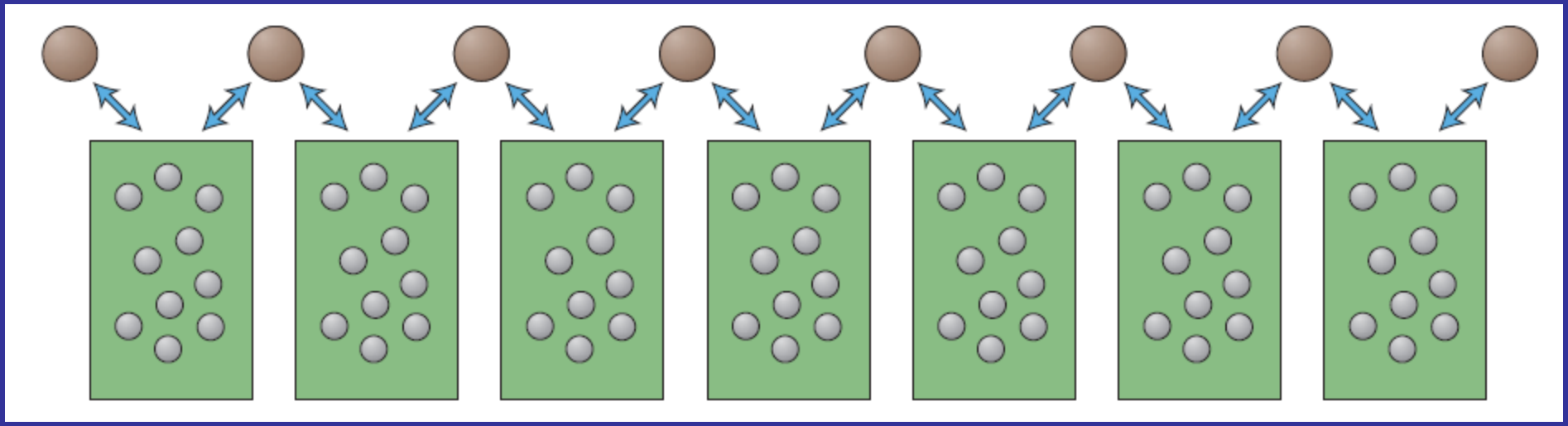
FV, M. Wolf, I. Cirac, Nat. Phys. ’09

Engineering the coupling with the reservoir

- Add qubit ancilla, and couple it via Hamiltonian $H = \Omega(\sigma_- L^\dagger + \sigma_-^\dagger L)$ to the original many-body system. Then simulate dephasing channel $L_a = \sqrt{\Gamma}\sigma_-$ on that ancilla with a strength $\Gamma \gg \Omega$

- Usual adiabatic elimination (i.e. second order perturbation theory) yields desired master equation

- If Hamiltonian was local, so are Lindblad terms



- What about the efficiency?
  - Convergence time is related to the gap of the Lindblad equation
  - Gap of Lindblad is not directly related to gap of Hamiltonians
    - E.g. spin glass: cst gap for Hamiltonian, exp. Small gap for Lindbladian
  - efficient for matrix product states
    - E.g GHZ, W, AKLT - states
  - efficient for all stabilizer states, e.g. toric code (note: different scaling for cluster state and toric code states with topological order)
    - Log(N) vs N Log(N) scaling for topological order
  - Coherent versions of classical spin Hamiltonians (e.g. 2-D Ising)
    - Exhibit dissipatively driven quantum phase transitions!

The Feynman-Kitaev Hamiltonian

- The most famous frustration-free Hamiltonian in quantum information theory is the Feynman-Kitaev Hamiltonian, which maps a quantum circuit to a Hamiltonian

  - Encodes every different quantum gate term in the quantum circuit into a local term

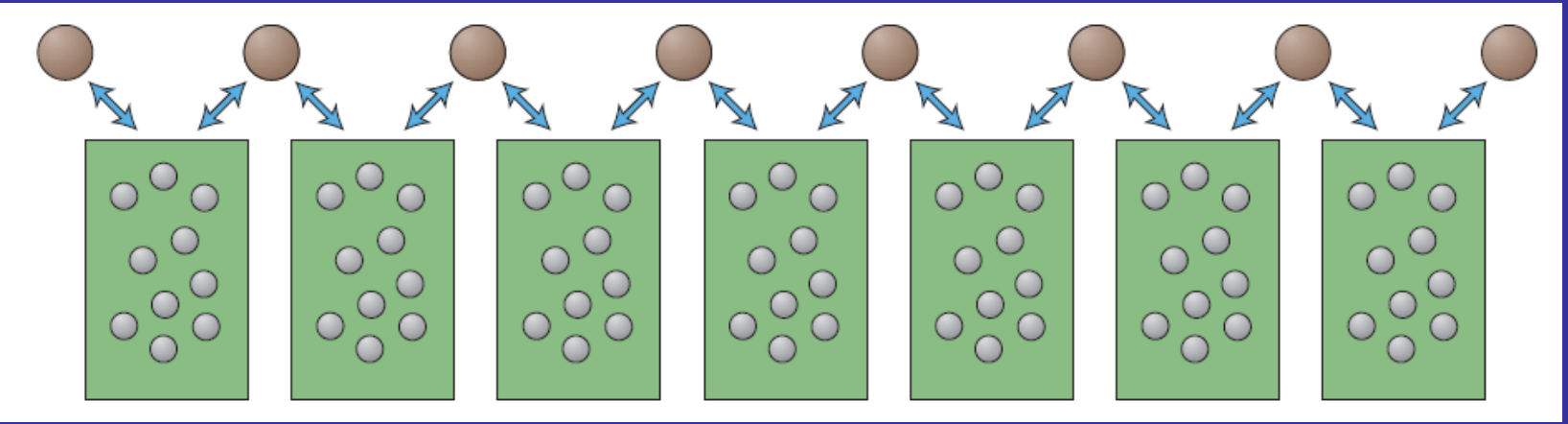
$$H = H_0 + \sum_{t=0}^{T-1} H_t + H_f$$

$$H_0 = \left(\sum_{k=anc} |1\rangle_k \langle 1| \right) \otimes |0\rangle\langle 0| \qquad ; \qquad \mathrm{H_f} = |1\rangle_{res} \langle 1| \otimes |T\rangle\langle \mathrm{T}|$$

$$H_t = U_t \otimes |t+1\rangle\langle t| + U_t^{-1} \otimes |t\rangle\langle t+1|$$

  - This Hamiltonian is frustration-free!
  - Ground state of this Hamiltonian contains the history of the quantum computation -> basis for adiabatic quantum computation

Dissipative quantum computer



- Only ingredient needed is a time-independent purely dissipative Lindblad evolution

$$\frac{d}{dt}\rho = \sum_i L_i \rho L_i^* - \frac{1}{2}\left(\sum_i L_i^* L_i \rho + \sum_i \rho L_i^* L_i \right)$$

  - the gap of Liouvillian is poly(1/T): efficient!
  - gap independent of actual quantum computation done

- Intrinsic robustness: if errors are made, this does not matter too much because system is evolving to “dark state” anyway
- Defies most of DiVincenzo’s criteria for building a quantum computer:
  - No initialization, no unitaries, time-independent
  - Conceptually a step further than the one-way quantum computer

Dissipative Quantum Algorithms

- Can we go further, and develop new quantum algorithms that are based on constructing fixed points of Lindblad evolutions?
- Obvious candidate: simulation of finite-T quantum many body systems
- Once we have quantum computers, their main application will probably be quantum simulation
  - Quantum chemistry: finding ground states
  - Condensed Matter Physics: determine phase diagram of e.g. Hubbard model as a function of T,U,J
  - Quantum Chromodynamics: simulate quark-gluon plasma, determine masses of elementary particles

Preparing Gibbs states

- We could just simulate complete unitary dynamics of system + heat bath, wait for long enough, and then trace out heat bath

- Problem:
  - Coupling between system+bath has to be very weak for this to work, and for many interesting systems of interest, the relaxation time is up to 10 orders of magnitude slower than time scale governing microscopic dynamics
  - Even for classical interacting systems, this approach does not work for this reason (e.g polymers or binary mixtures)

- Solution:
  - For classical case: Metropolis / Monte Carlo algorithm

    "We devised a general method to calculate the properties of any substance comprising individual molecules with classical statistics."
  - Can we device a quantum version of that algorithm?

The Metropolis algorithm

- Let's for simplicity consider a classical Ising spin system with Hamiltonian

$$\mathcal{H} = \sum_{ij} J_{ij} s_i s_j + \sum_i K_i s_i$$

- Instead of simulating the action of a heat bath (which would require much more resources and would be very slow), Metropolis et al. set up a stochastic map that flips spins from one configuration i to another one j in a controlled random way:

$$|\uparrow\uparrow\downarrow\uparrow\downarrow \ldots\rangle \rightarrow |\uparrow\downarrow\downarrow\uparrow\downarrow \ldots\rangle$$

$$p_{i\rightarrow j} = \min\left(1, \exp\left(-\beta(E_j - E_i)\right)\right)$$

  - The stationary state of this stochastic map is the Gibbs state with

$$p_i = \frac{\exp(-\beta E_i)}{Z}$$

Quantum Metropolis Sampling

- A Quantum Metropolis sampling algorithm should allow for the preparation of static ground and thermal states of generic quantum Hamiltonians
    - Cfr. universal quantum simulator of Lloyd which only allows the simulation of dynamics generated by a local Hamiltonian. The main properties of interest in quantum chemistry and theory of strongly correlated quantum systems are however mainly in static properties!
    - Technical problem: construct Lindblad operator for which fixed point is the Gibb's state of a given Hamiltonian
    - Can of course also be used for preparing ground states

K. Temme, K. Vollbrecht, T. Osborne, D. Poulin, F. Verstraete '09

Quantum Metropolis Sampling

- Basic idea: use quantum phase estimation to sample in the eigenbasis of your Hamiltonian
  - Sampling is random walk algorithm on the eigenstates of your Hamiltonian; a move is accepted or rejected according to the energy difference
  - Convergence to Gibb's state is guaranteed if the probabilities of those moves obey detailed balance
  - this automatically solves the infamous sign problem!
- Central problem for quantum sampling:
  - How can we reject a move: prepare an eigenstate, move to another one, measure its energy, and go back to the original one?
  - Is seemingly in violation with the no-cloning theorem
  - Central trick: do not measure the full energy, but do the measurement in superposition such that reveals ***only 1 bit of information***: accept or reject

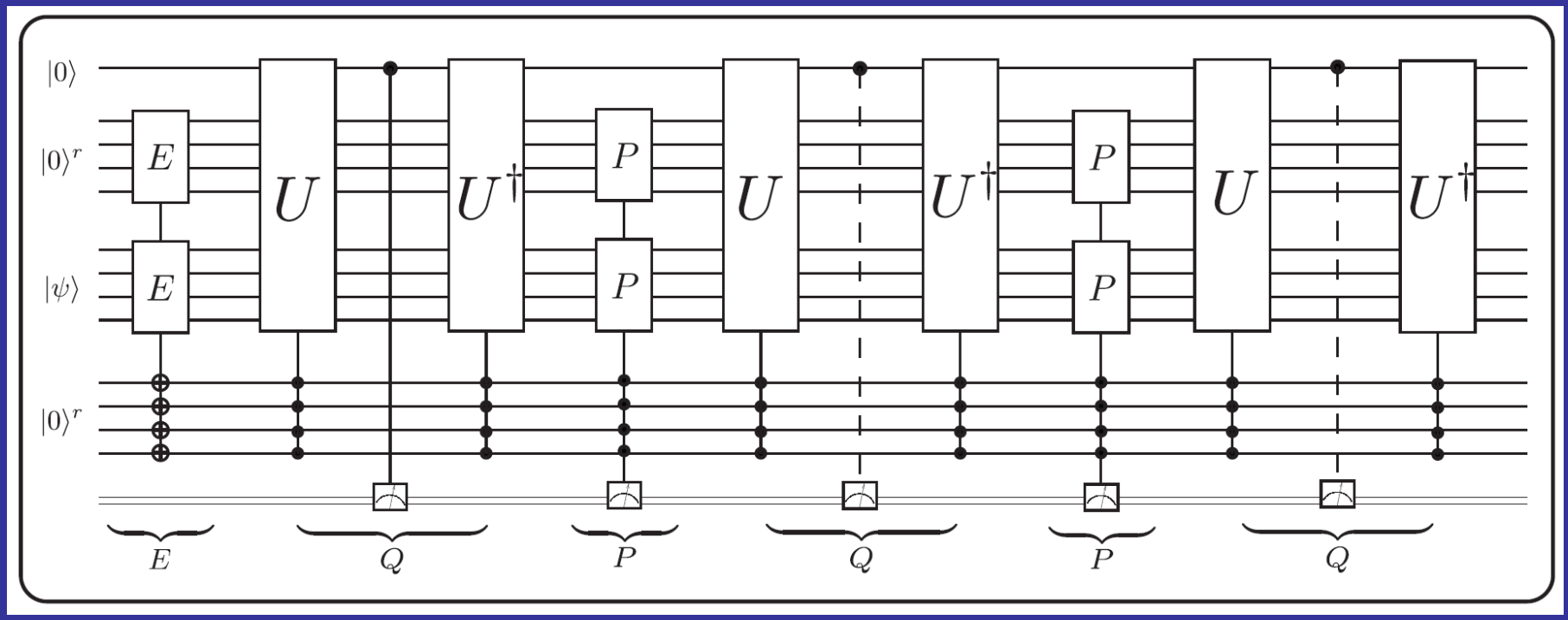
Undoing a binary quantum measurement

- Basic theorem in linear algebra: given 2 projectors P and Q, then there is a basis in which they can be written as

$$P_1 = \begin{pmatrix} \mathbb{I}_p & 0_{n-p,p} \\ 0_{p,n-p} & 0_{n-p,n-p} \end{pmatrix}$$

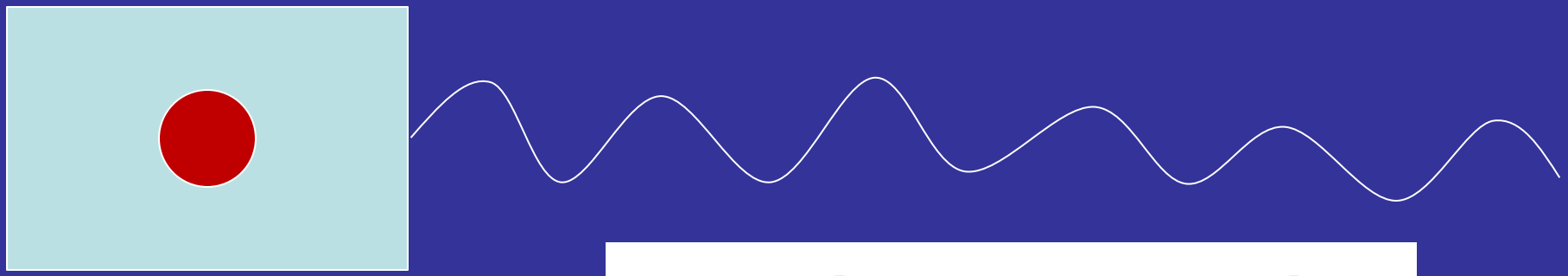
$$Q_1 = \begin{pmatrix} D_p & \sqrt{D_p(\mathbb{I}_p - D_p)} & 0 & 0 \\ \sqrt{D_p(\mathbb{I}_p - D_p)} & \mathbb{I}_p - D_p & 0 & 0 \\ 0 & 0 & \mathbb{I}_{q-p} & 0 \\ 0 & 0 & 0 & 0_{n-(q+p),n-(q+p)} \end{pmatrix}.$$

- Suppose P was the basis for which the original state was an eigenstate with eigenvalue 1, and Q is the basis in which we measure
  - then due to the simple structure of the matrices, recursive measurements of the type, (P,I-P), (Q,I-Q), (P,I-P), … will take you back to the original state with probability exponentially close to 1
  - This is how the reject step can be implemented!


$|0\rangle$
$|0\rangle^r$
$|\psi\rangle$
$|0\rangle^r$
E
E
U
$U^\dagger$
P
P
U
$U^\dagger$
P
P
U
$U^\dagger$
E
Q
P
Q
P
Q

Using dissipation / nonequilibrium processes to simulate quantum field theories

- Actually, dissipative processes can also be used directly for quantum simulation:
  - The state of the photons leaking out of the cavity is a nontrivial quantum many-body state



$$|\chi\rangle = \mathrm{Tr}_{aux}\left[\mathcal{P}e^{\int_0^L dx\left[Q(x)\otimes\mathbb{1}+R(x)\otimes\hat{\psi}^\dagger(x)\right]}\right]|\Omega\rangle$$

  - Atom-cavity coupling can be engineered in such a way that the quantum state of the photons leaking out of cavity is equivalent to the vacuum state of an interacting quantum field theory (such as e.g. Lieb-Liniger model)

FV, I. Cirac, PRL ‘10; T. Osborne, J. Eisert, FV ‘10

- This is a manifestation of *the holographic principle*: description of 0-dimensional nonequilibrium / dissipative dynamics is equivalent to the description of all static ground state properties of a 1+1 dimensional quantum field theory

  – Temporal correlation functions for cavity are in 1 to 1 correspondence with spatial correlation functions of vacuum

- This seems to be a generic feature: ground states of 2+1 dimensional theories obey same physics as nonequilibrium 1+1 dimensional theories (in Minkowski space)

- This property forms core of the success of real-space renormalization group methods (DMRG, MPS, PEPS, …)

  – Allows to simulate quantum field theories variationally without putting them on a lattice

Conclusion

- Quantum dissipation can be very useful for quantum simulation and quantum computation
  - Natural way of simulating frustration-free Hamiltonians
  - Allows for universal quantum computation

- Novel quantum algorithms can be constructed using dissipative ingredients: Quantum Metropolis Sampling, …

- Non-equilibrium 0-dimensional dissipative processes are in many ways equivalent to static 1+1 dimensional quantum field theories: holographic principle