Topological Quantum Codes : a model with physical realizability

Beni Yoshida (Physics, MIT)

joint work with Prof. Isaac Chuang
June 8, 2010 @ Obergurgl, Austria

Importance of quantum coding theory

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Protecting a qubit is essential in realizing quantum information theoretical ideas.

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In condensed matter physics,

Several models of correlated spin systems can be considered as quantum codes.

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Quantum code + physical realizability

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"Most" of the models have topological order, and are good quantum codes.

Review of stabilizer codes

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The Hamiltonian

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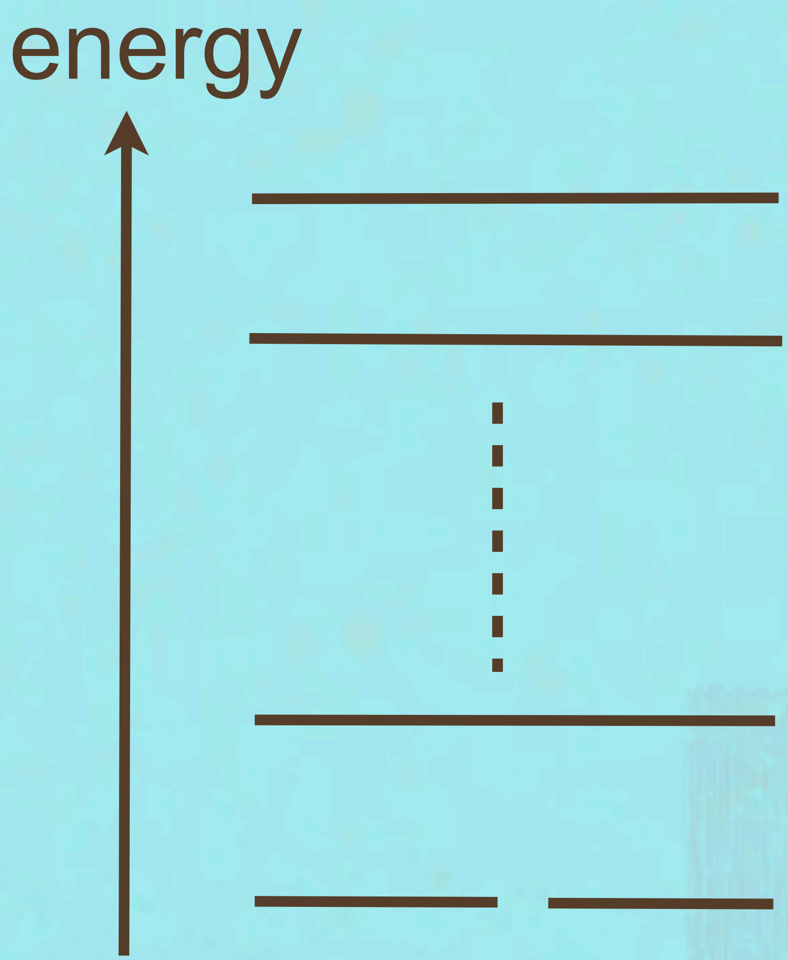

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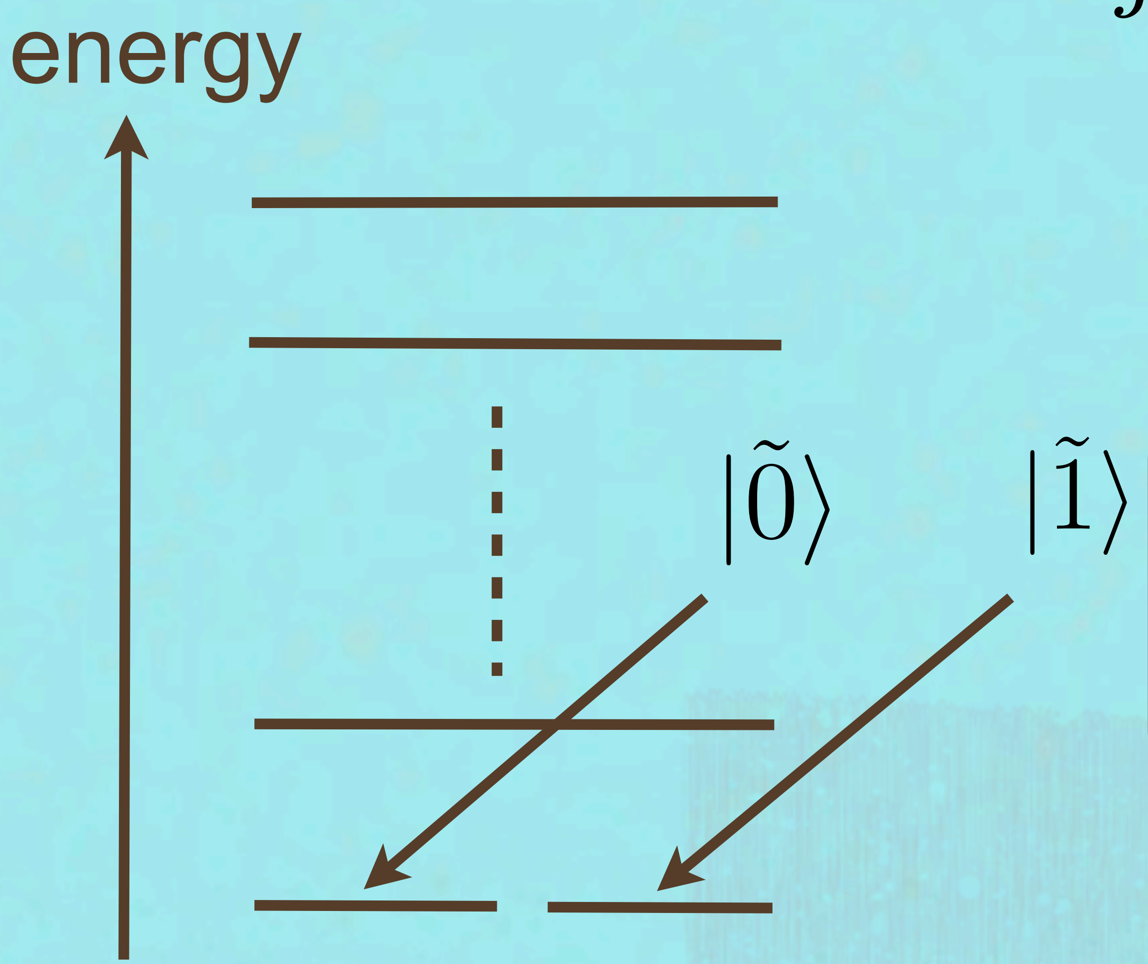
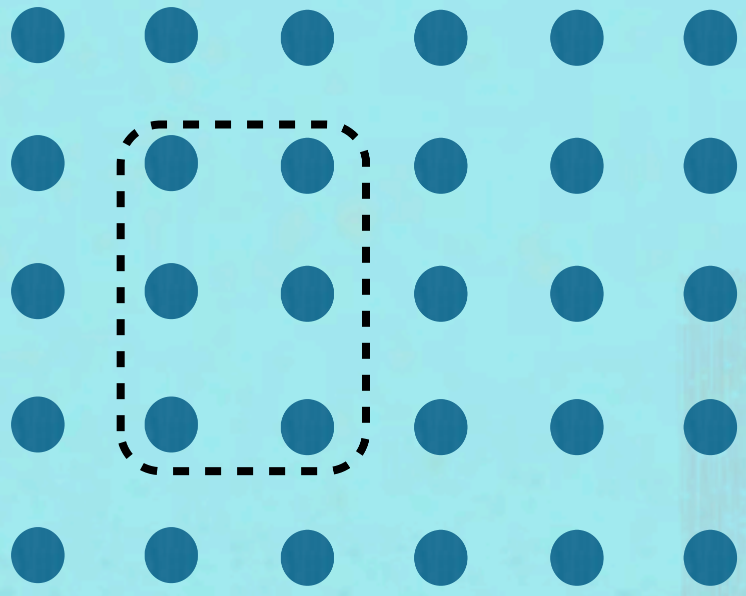


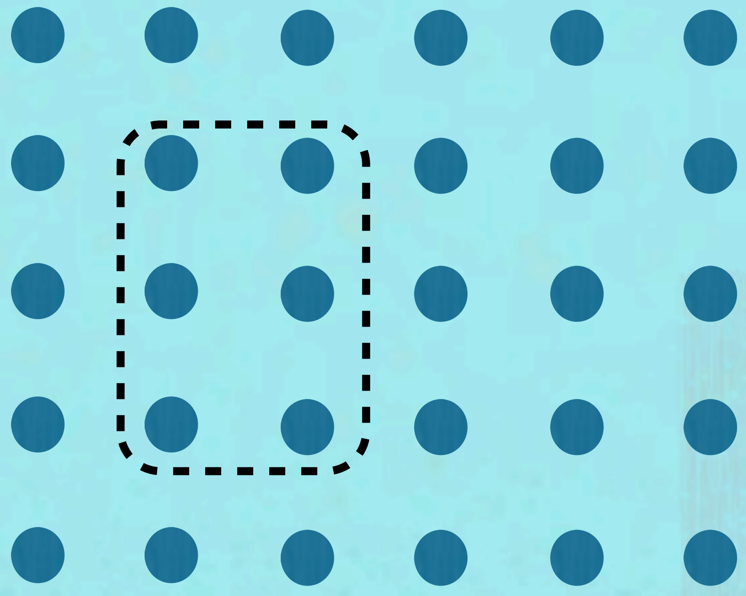
Qubits in the degenerate ground state space

Physically Realizable Code (STS model)



Physically Realizable Code (STS model)

Local interactions



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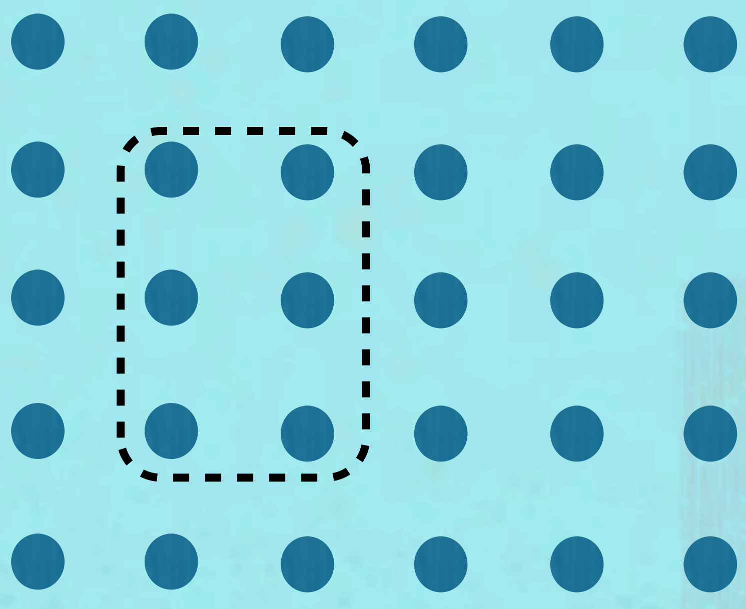
Translation symmetries: the Hamiltonian is invariant under finite translations.



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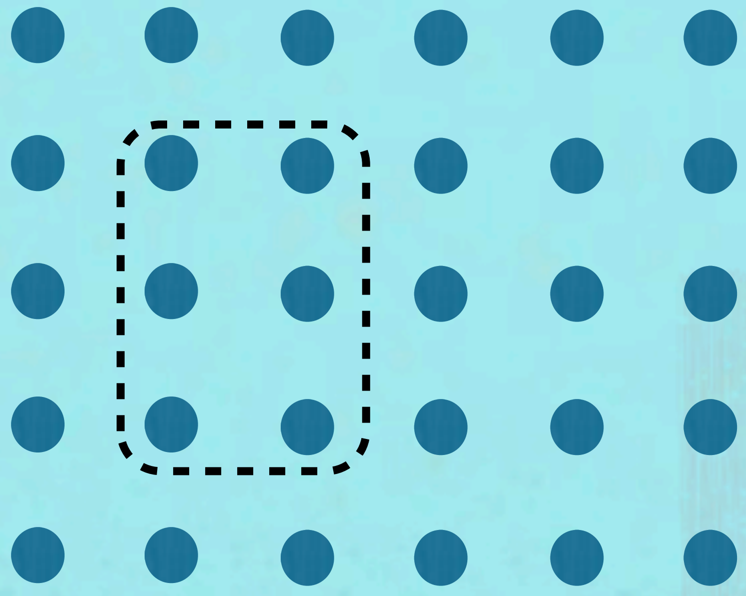

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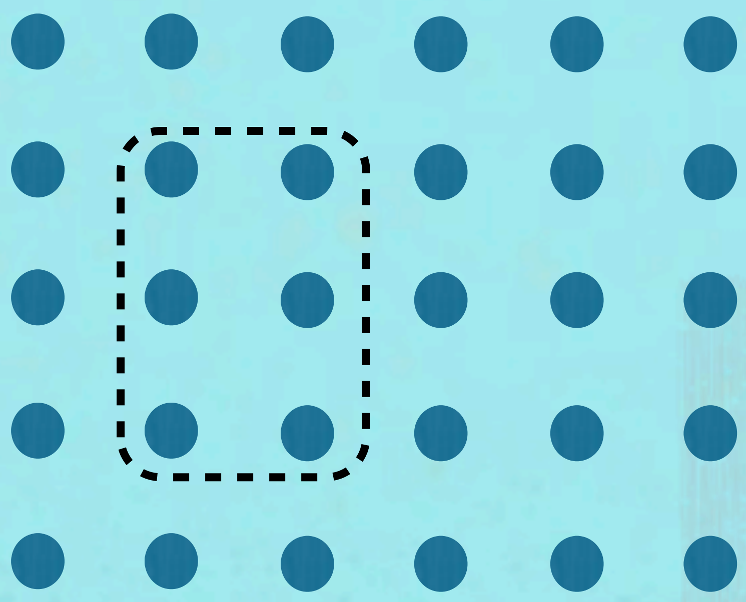

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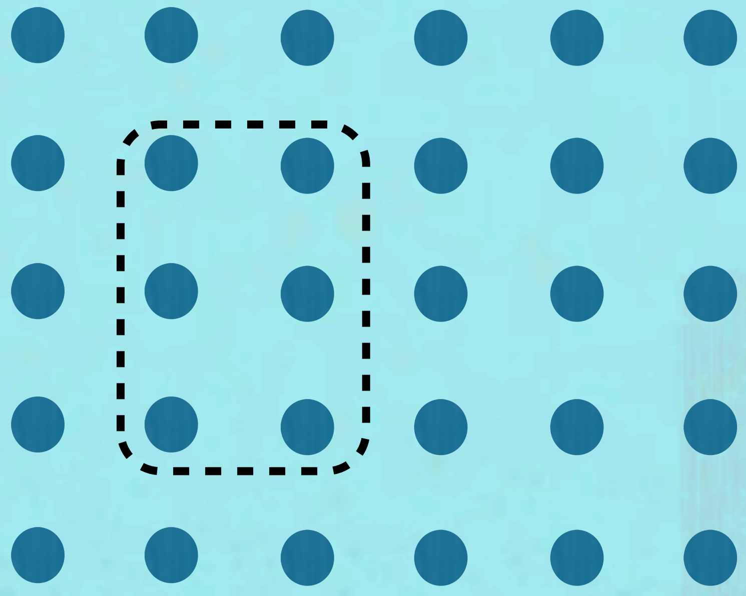

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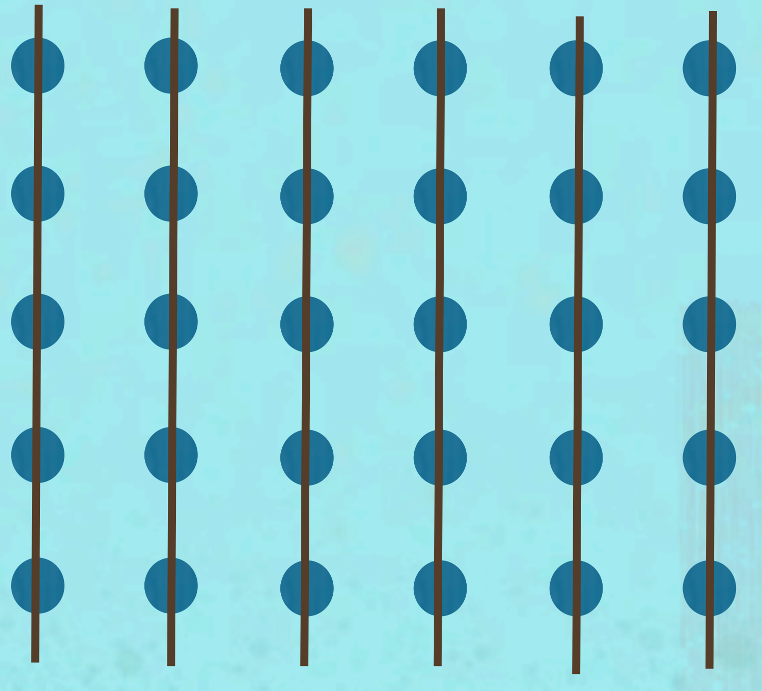

Without scale symmetries, most codes are trivial...

(ex) array of 1D ferromagnet...

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1, Exactly solvable

= logical operators can be easily computable.

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Transform encoded qubits (ground states)

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$$[\ell, S_j] = 0$$

Commute with the Hamiltonian

$$\ell \notin \mathcal{S} = \langle S_1, S_2, \cdots \rangle$$

Not a product of interaction terms

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$$[\ell, S_j] = 0$$ Commute with the Hamiltonian

$$\ell \notin \mathcal{S} = \langle S_1, S_2, \cdots \rangle$$ Not a product of interaction terms

For each encoded qubit...

A pair of anti-commuting logical operators. $\{\ell, r\} = 0$

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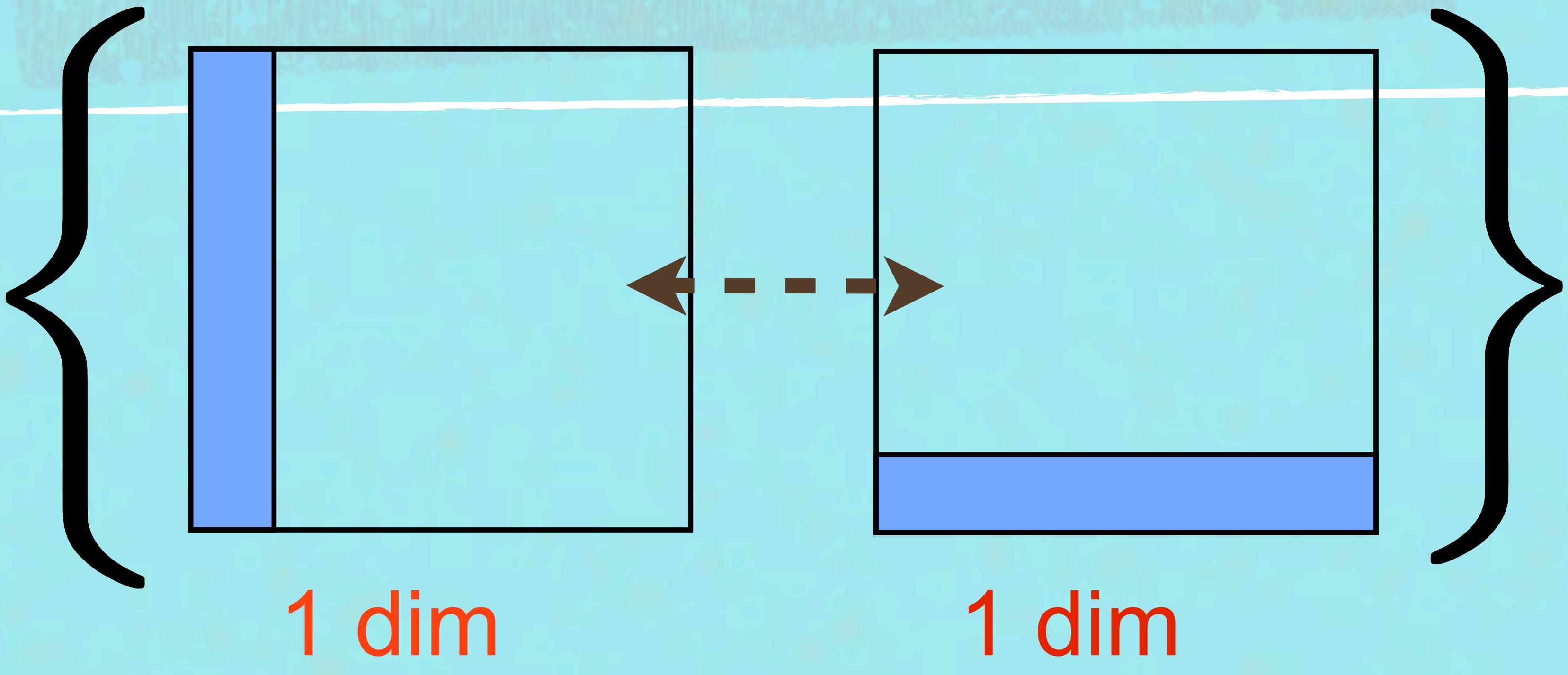

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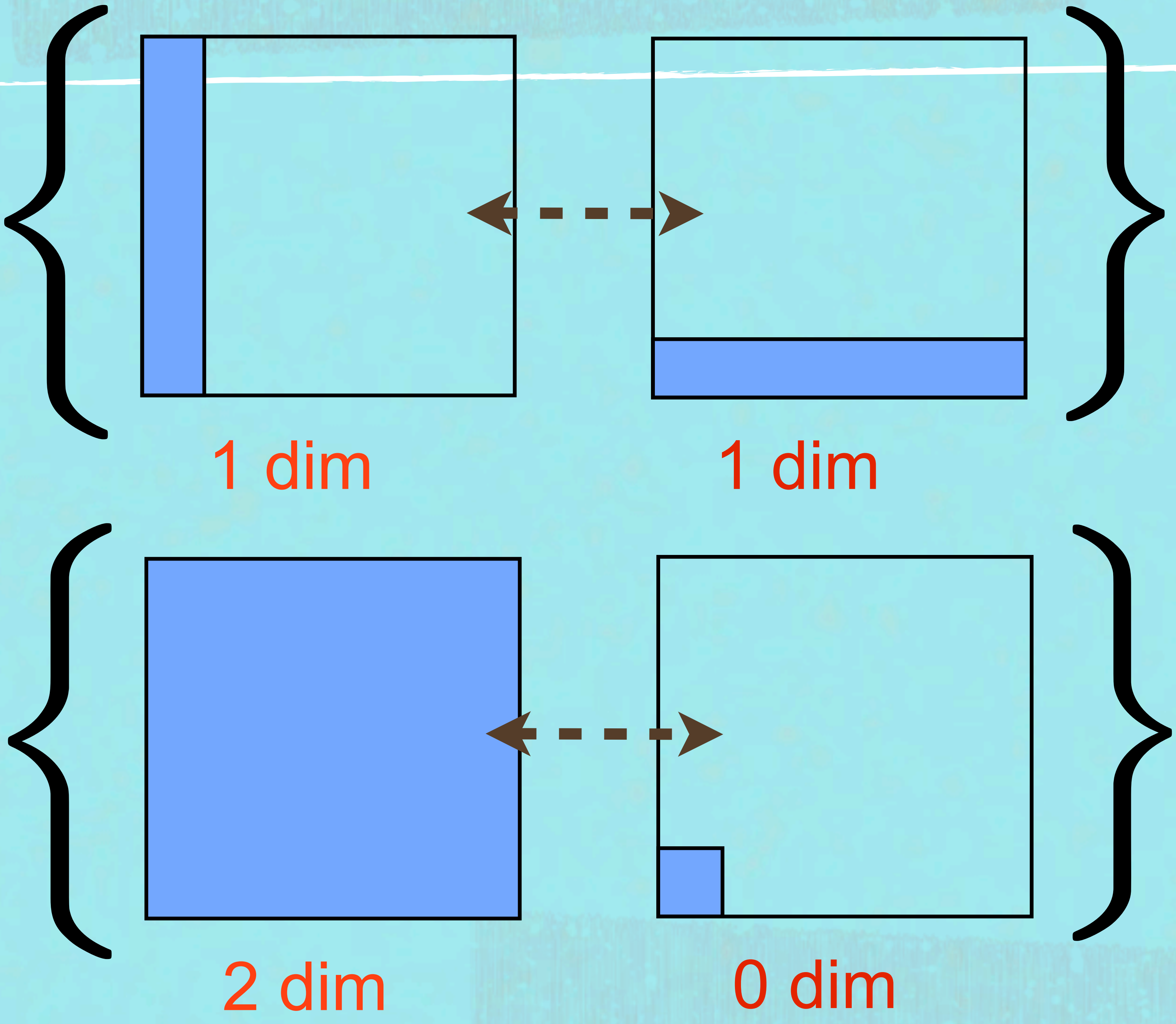
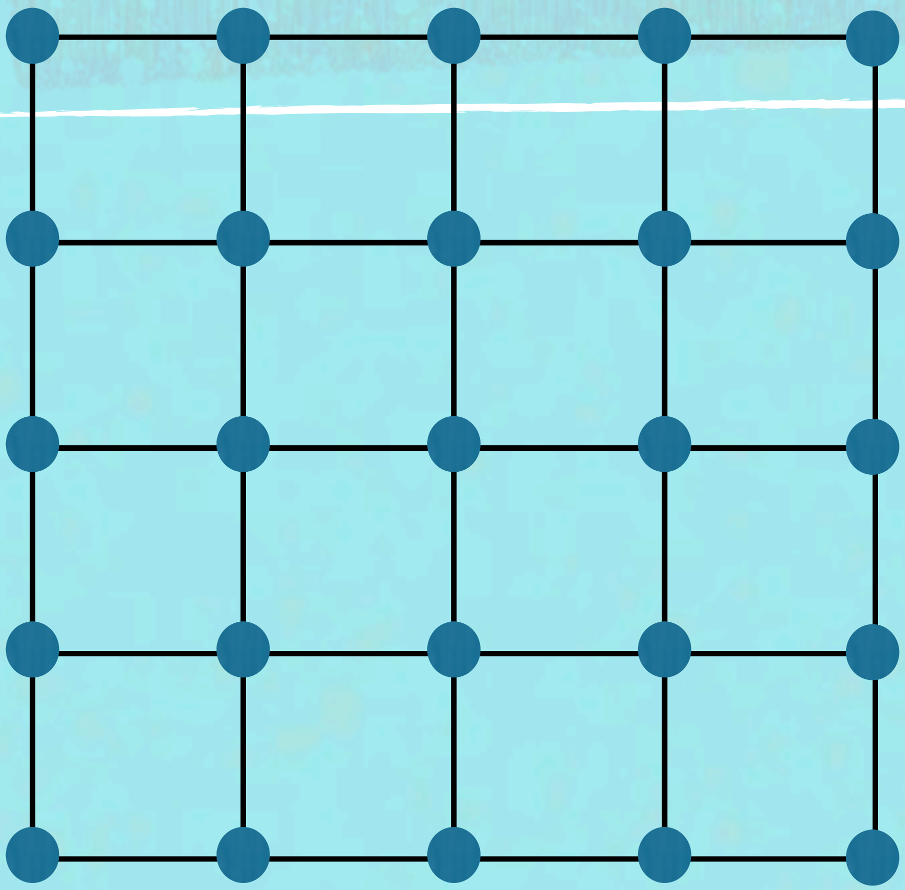


Classical ferromagnet (trivial STS)



Neighboring ZZ interactions

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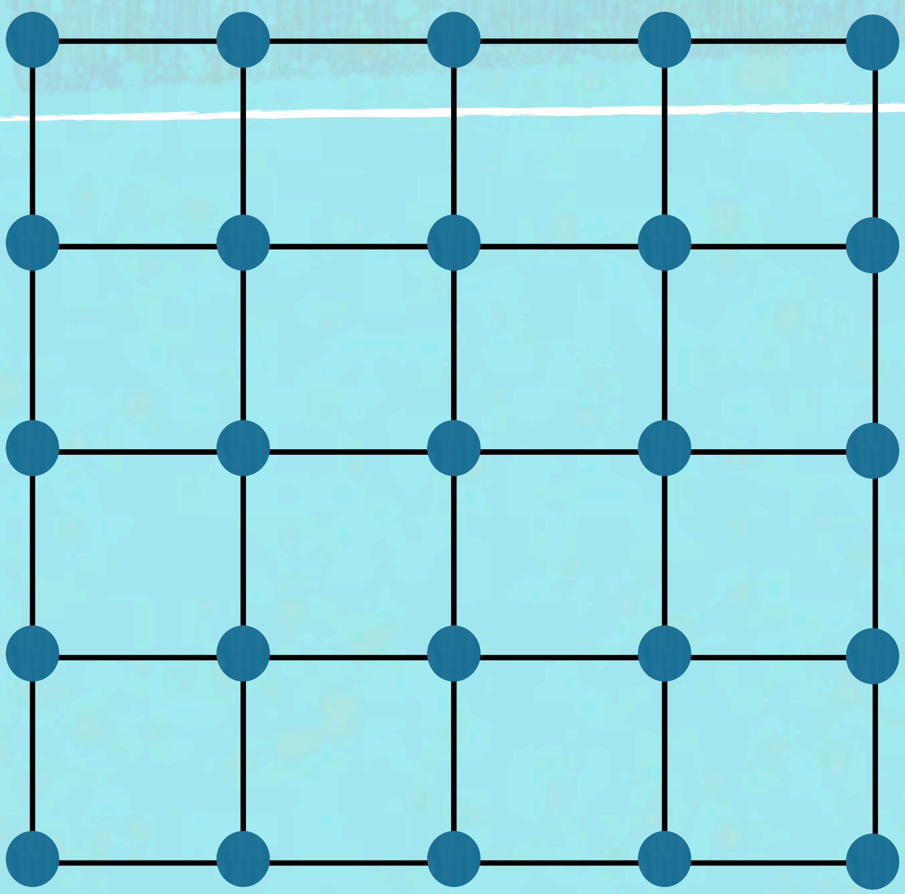

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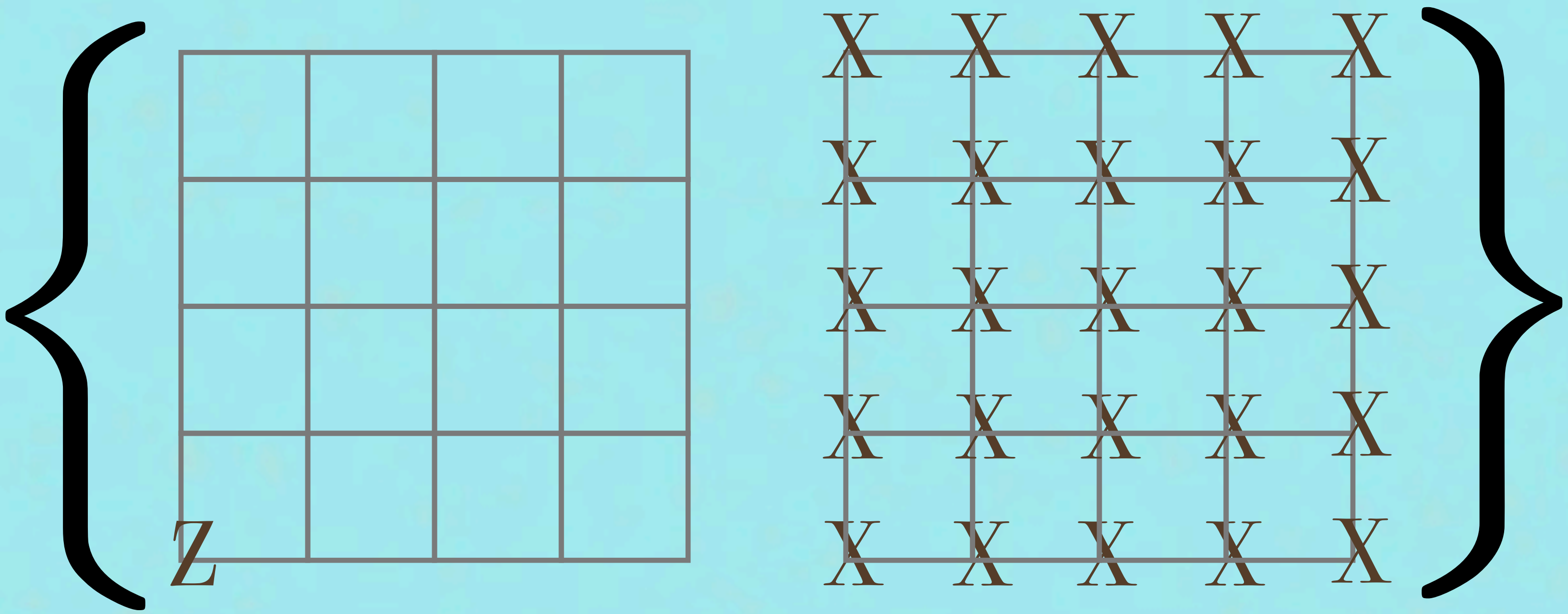

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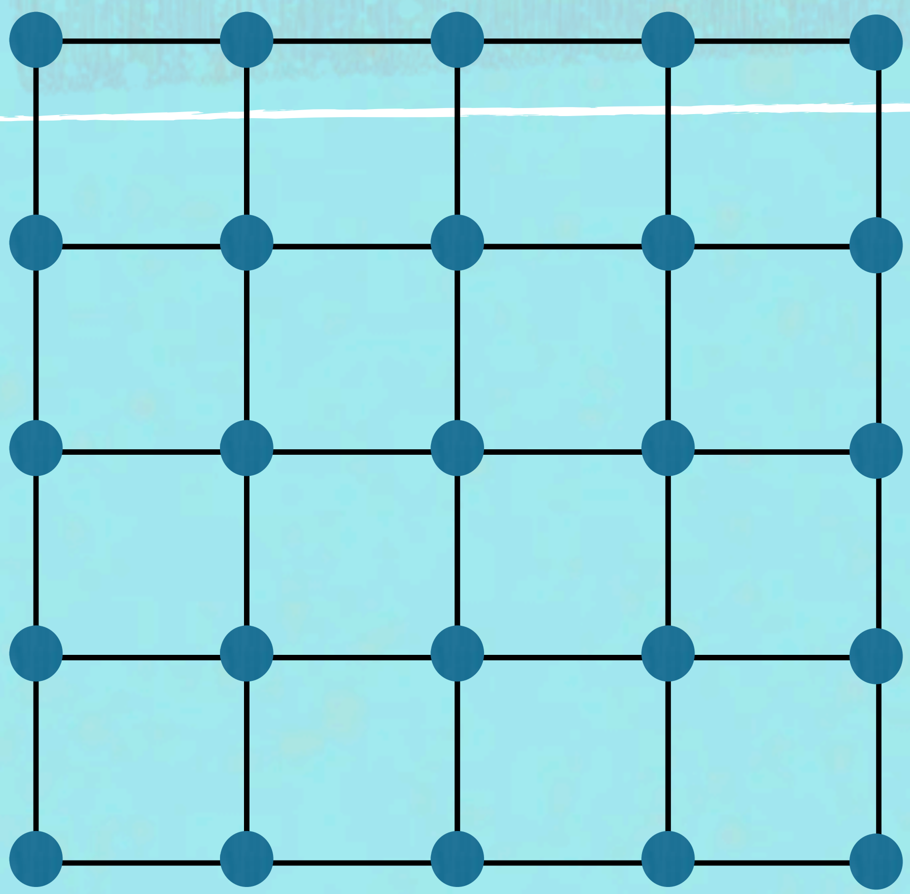

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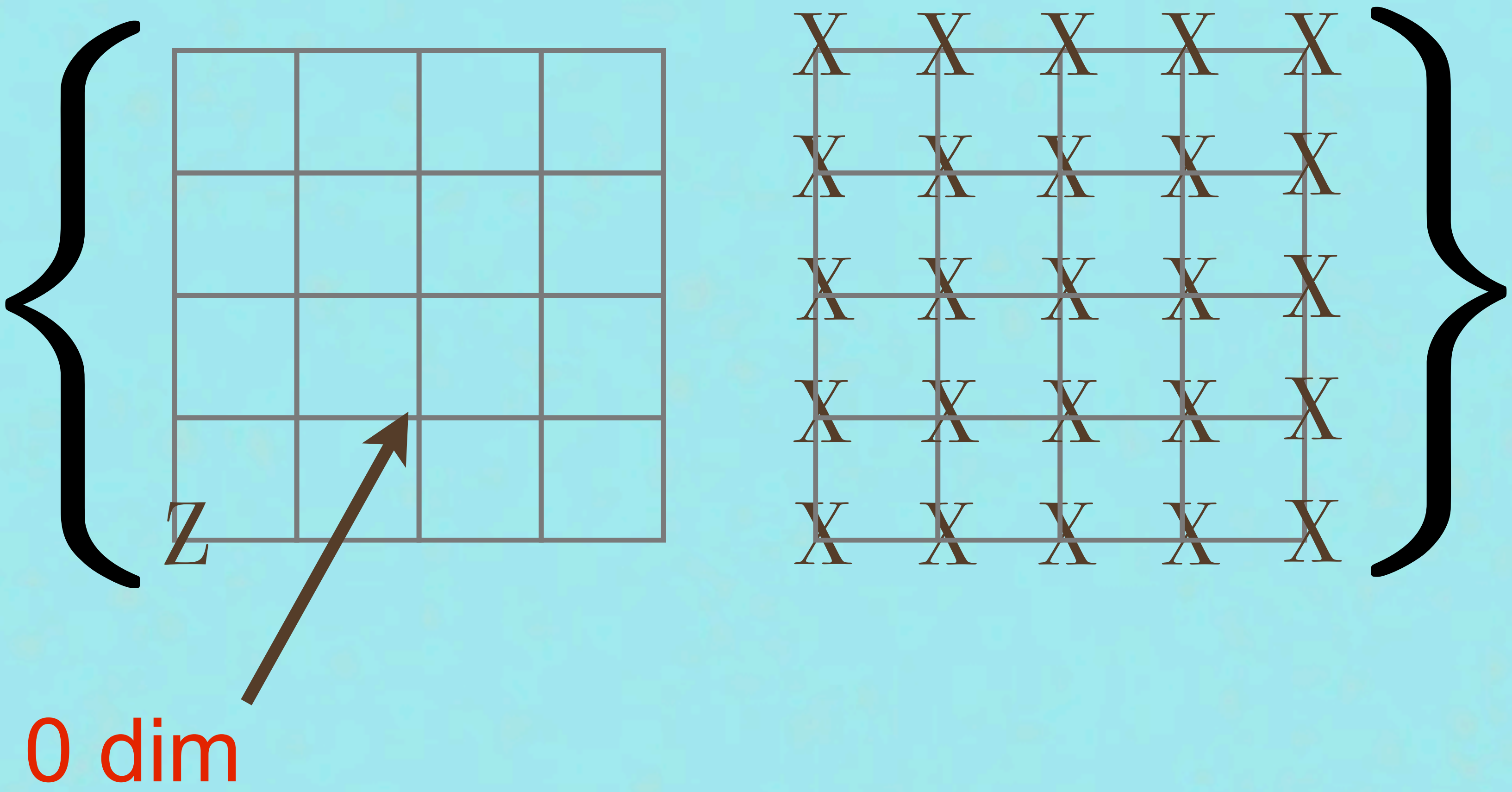

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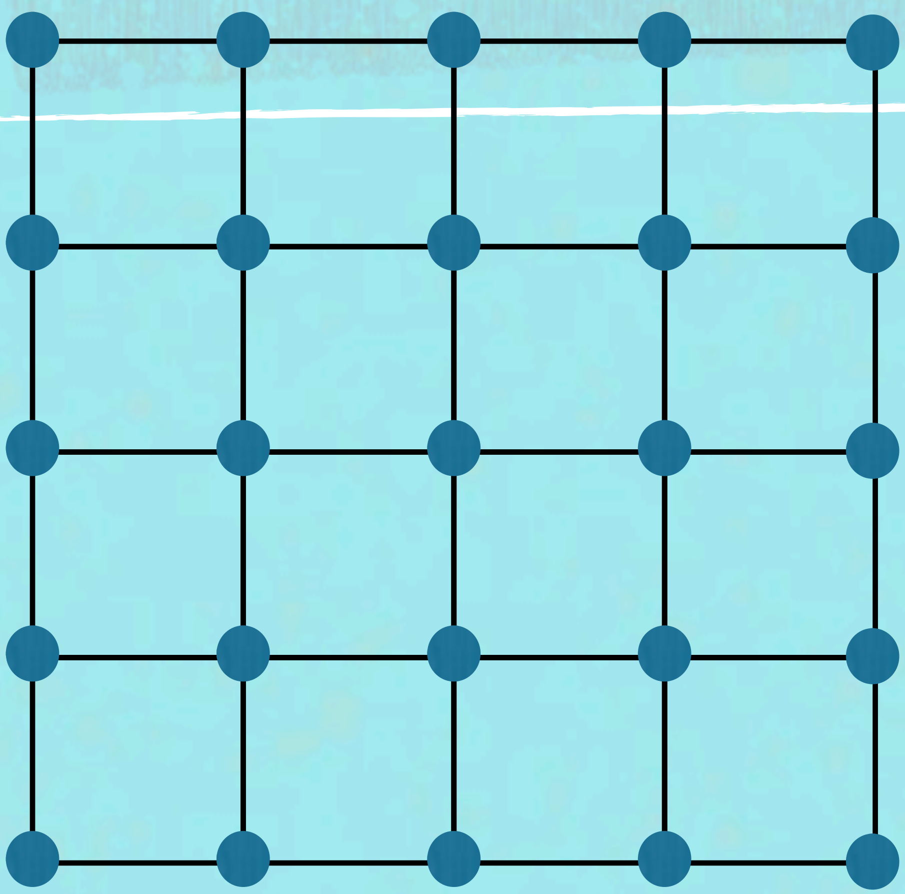

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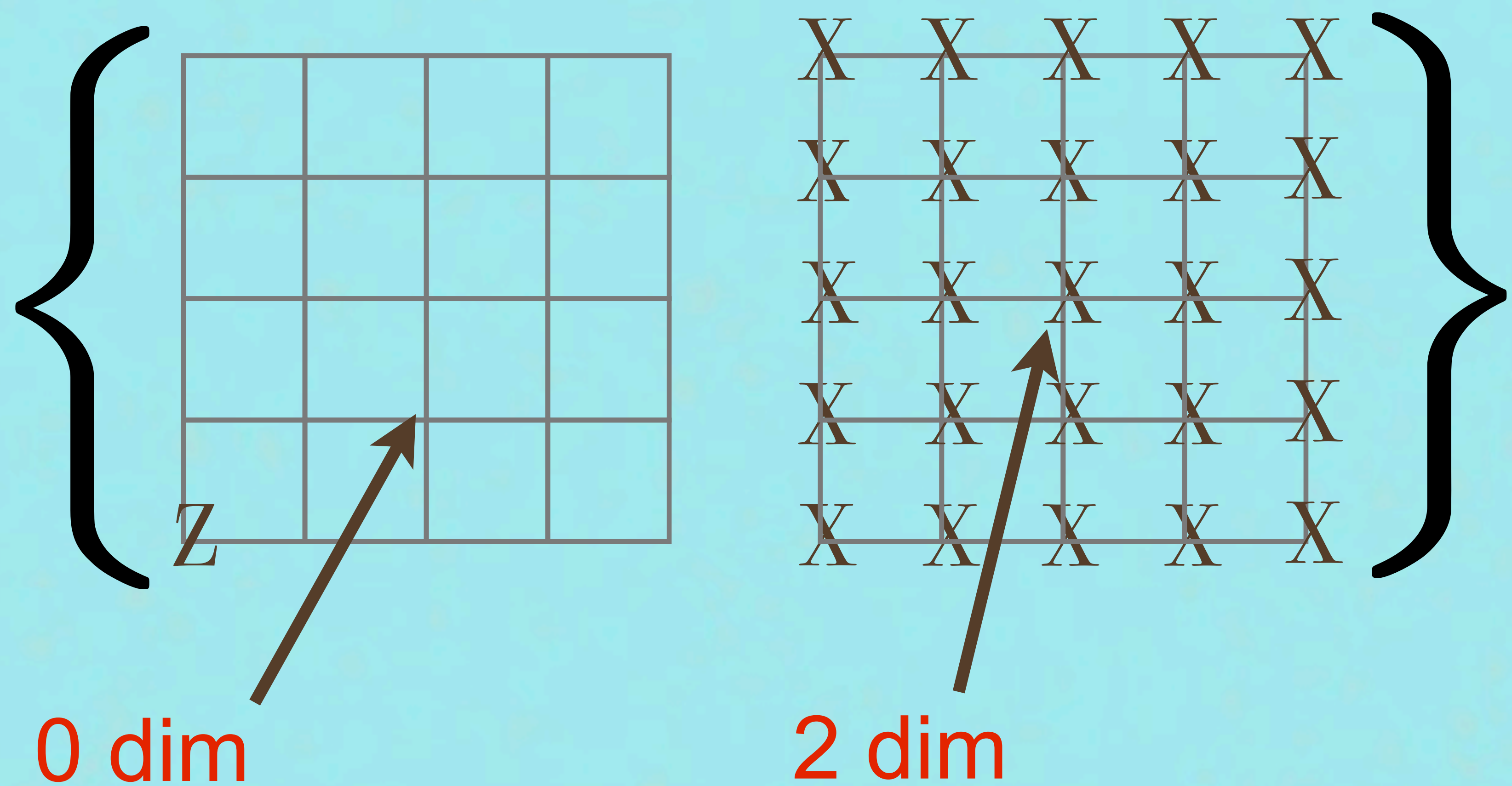

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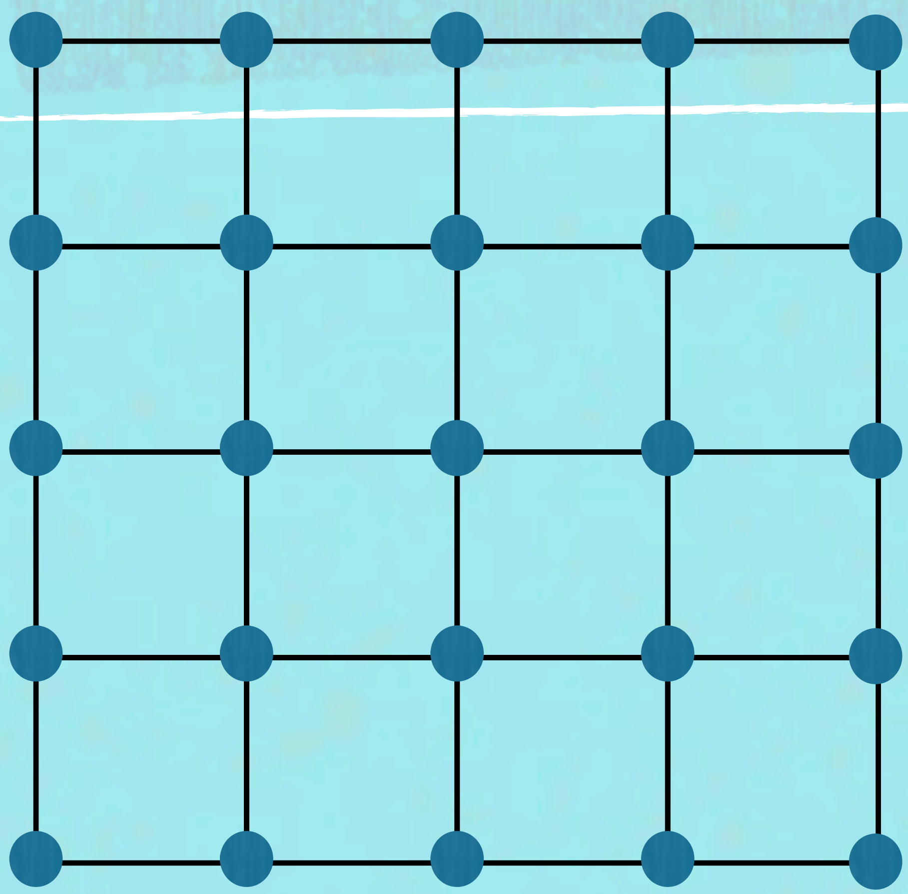

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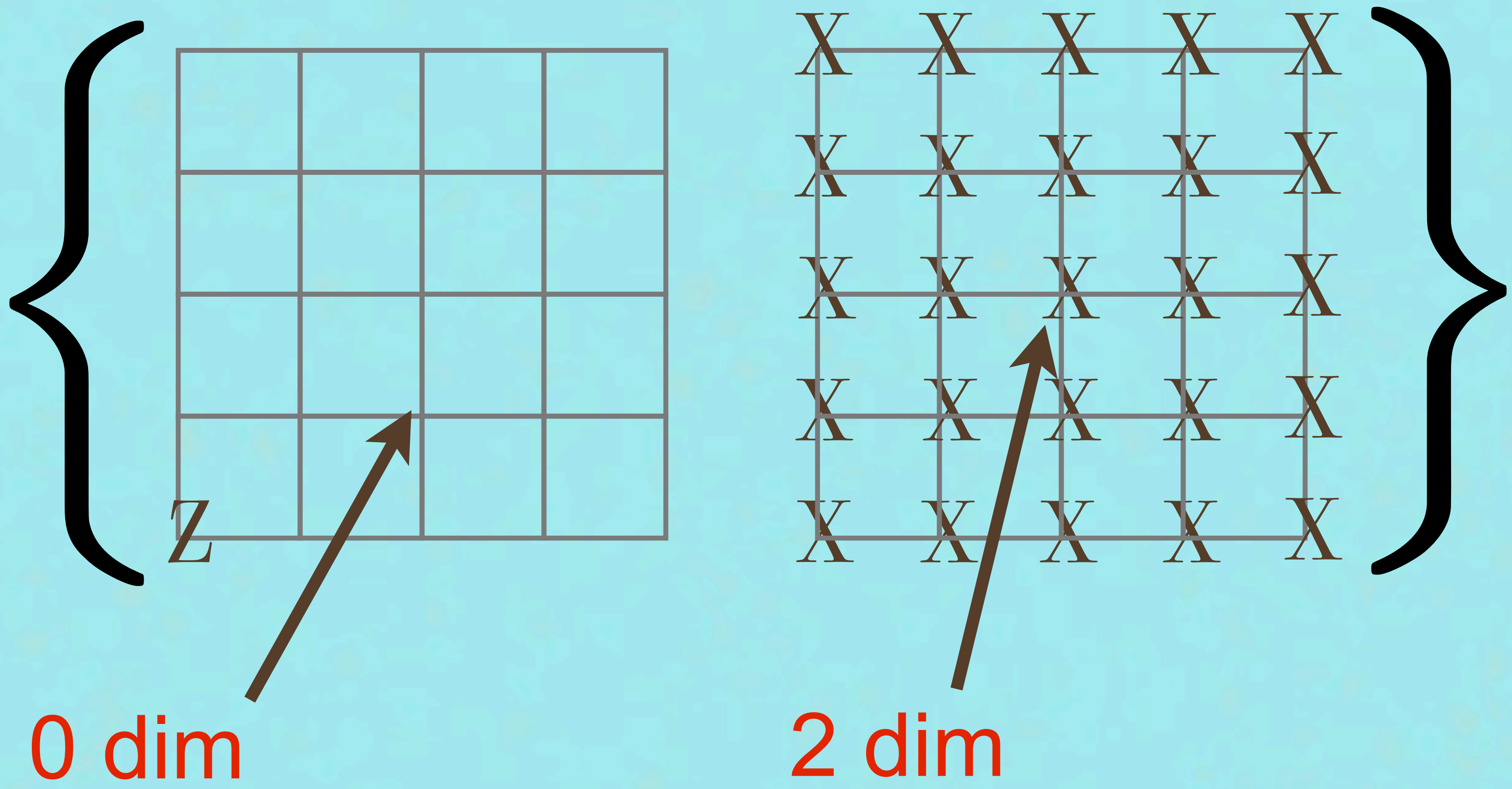
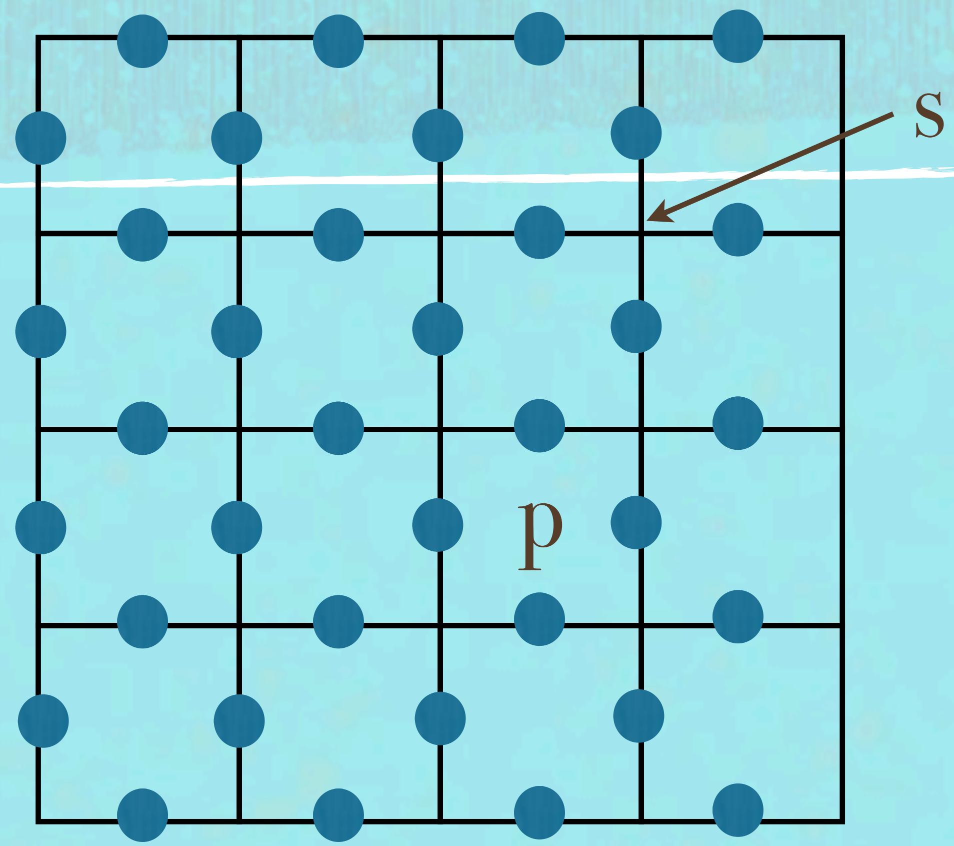


As a quantum code, this is useless...

The Toric code (non-trivial STS)



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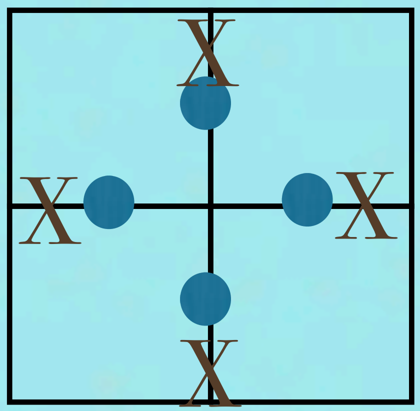
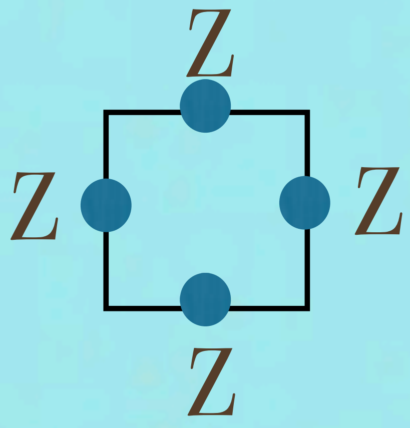
A_s 

B_p 

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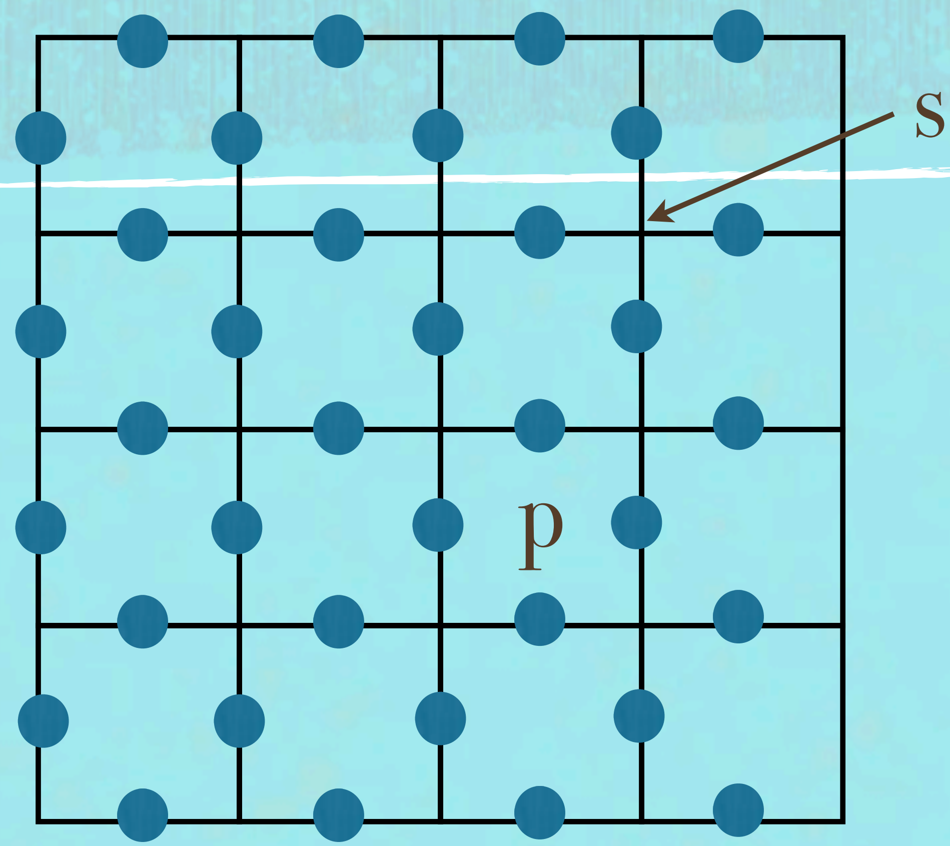

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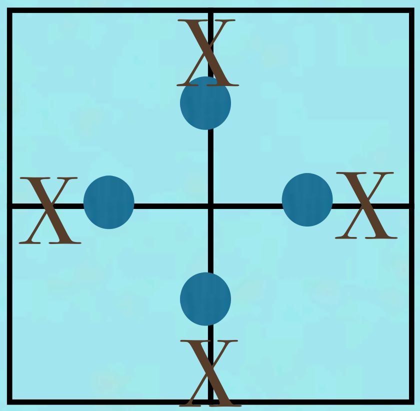

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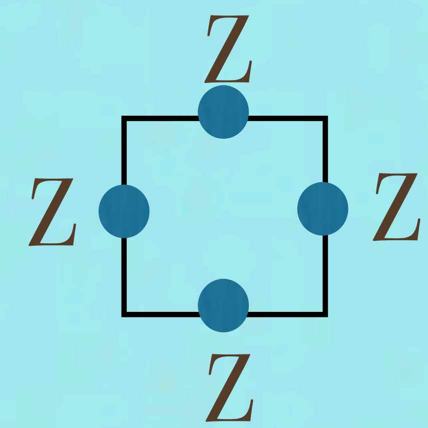

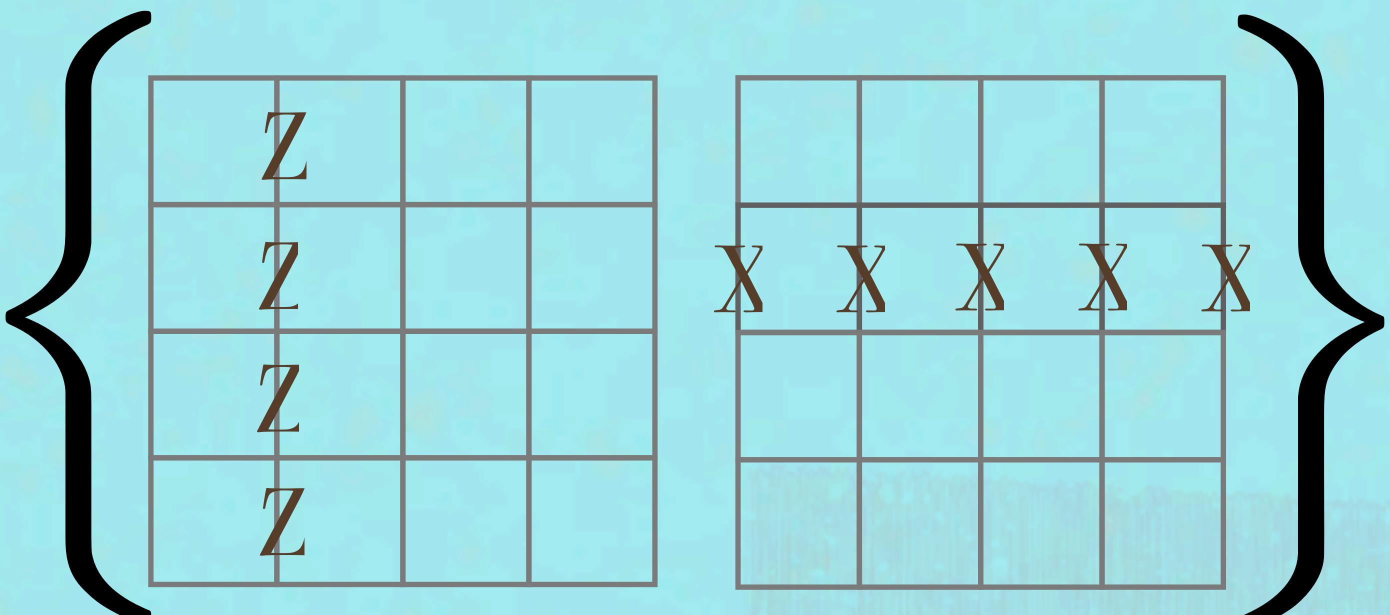

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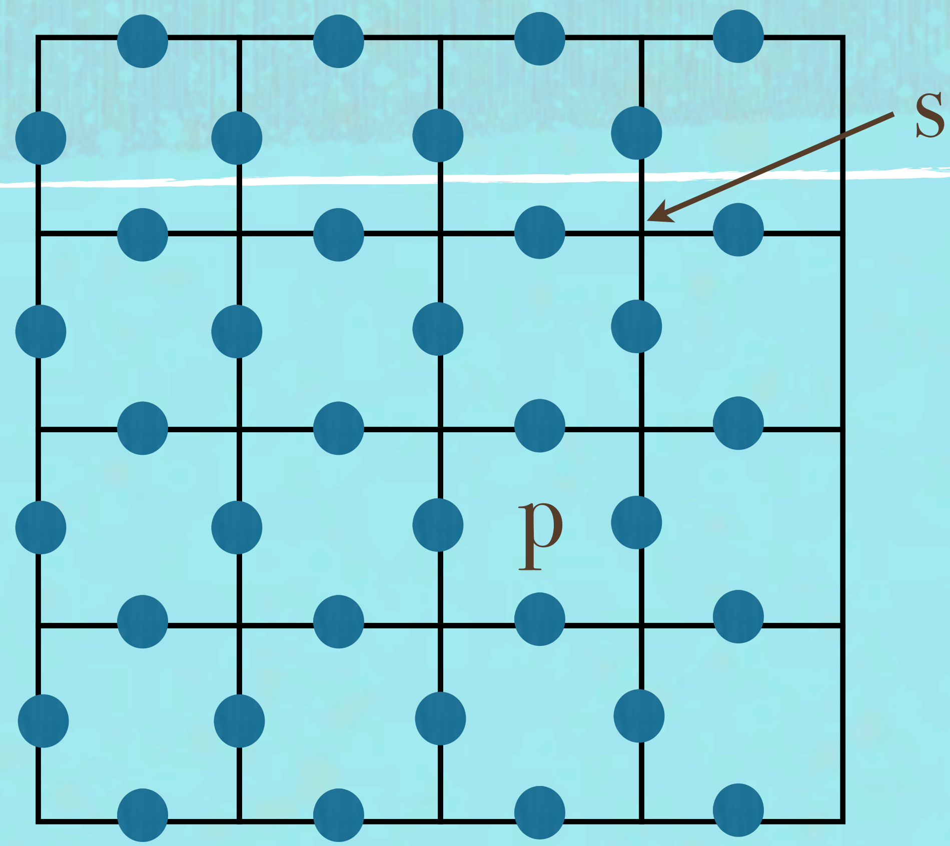

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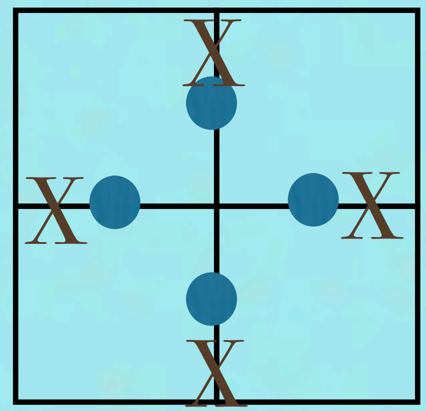

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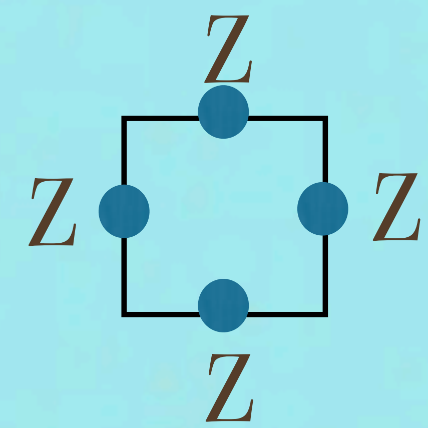

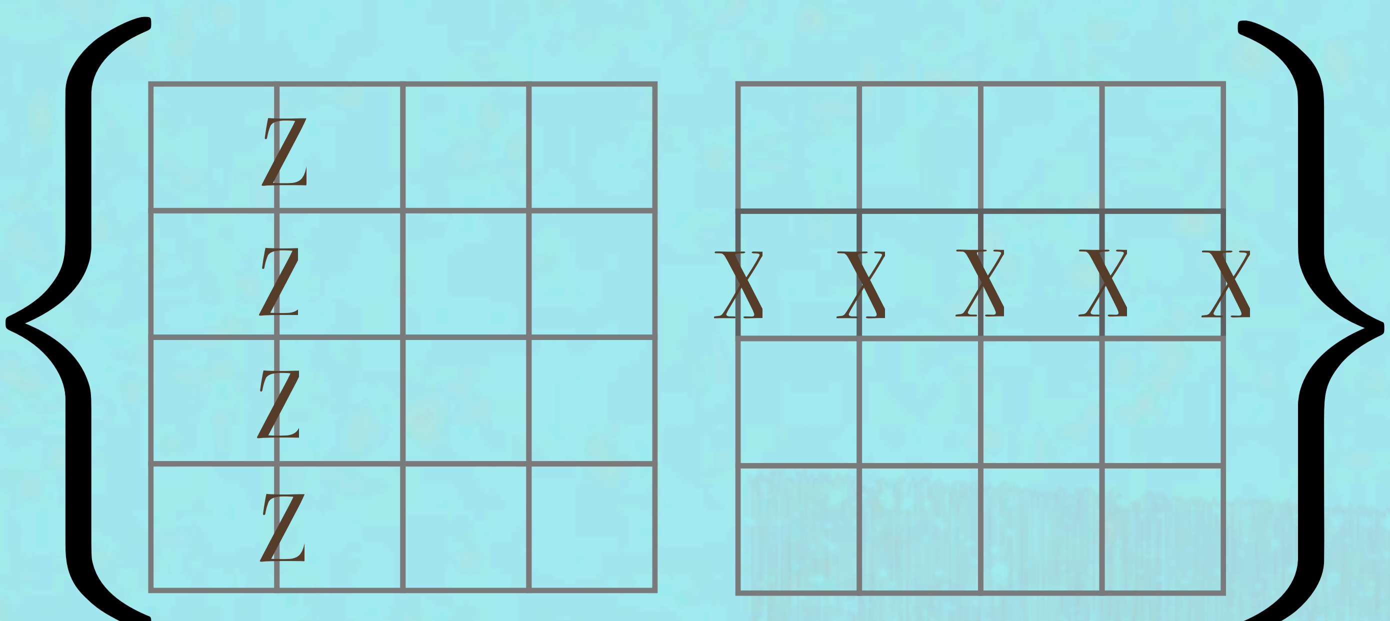

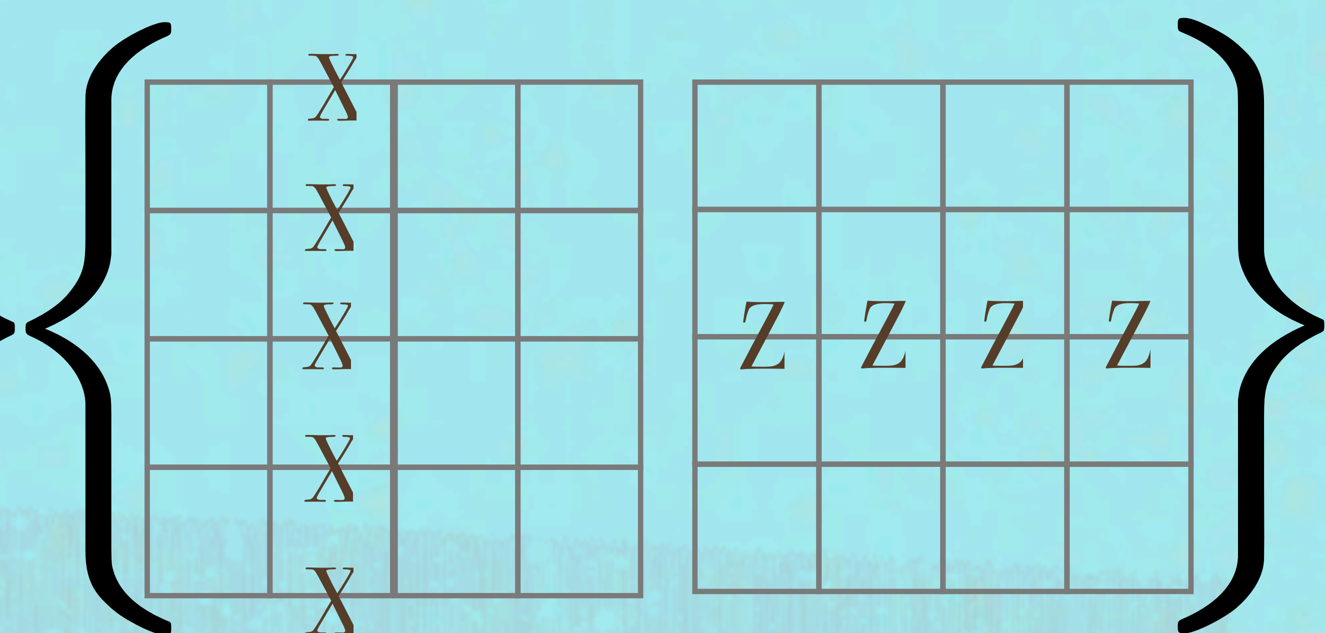

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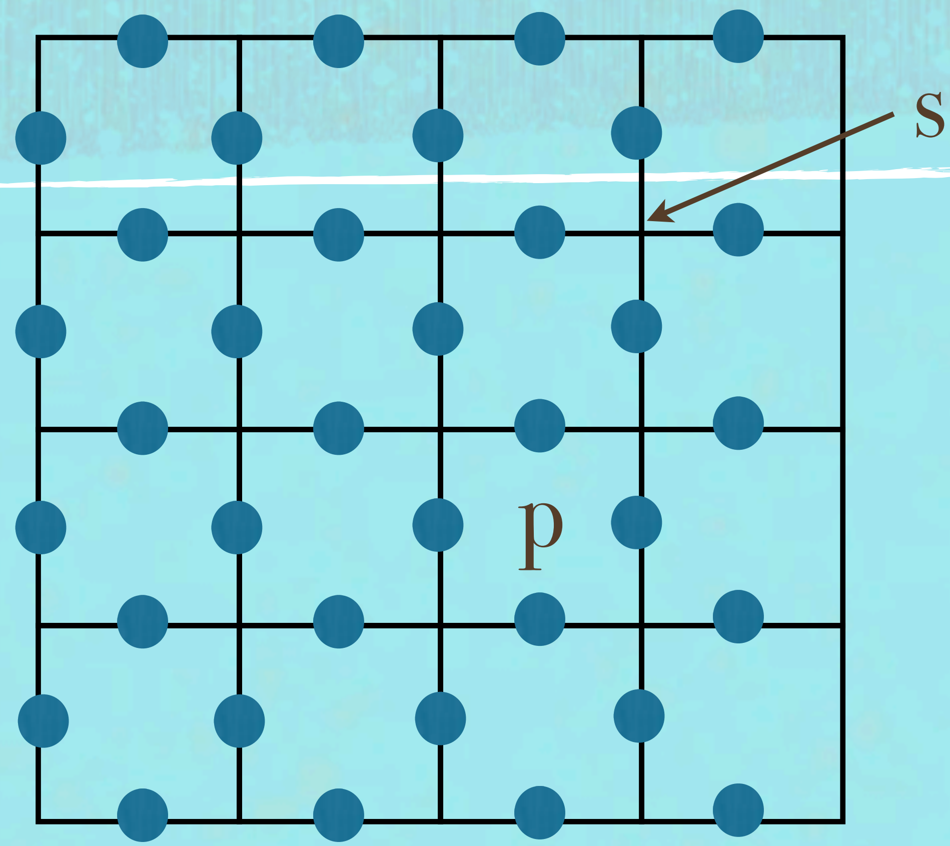

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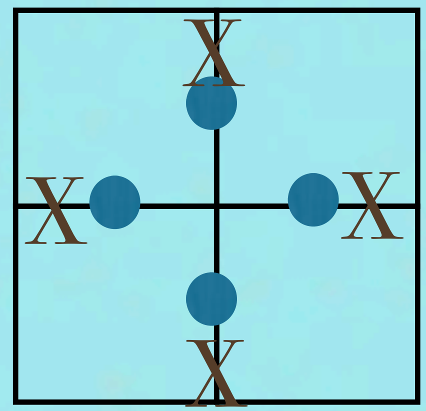

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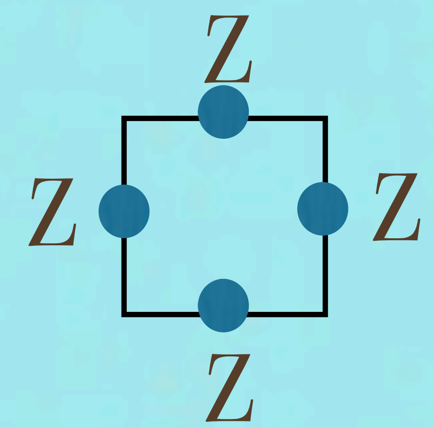

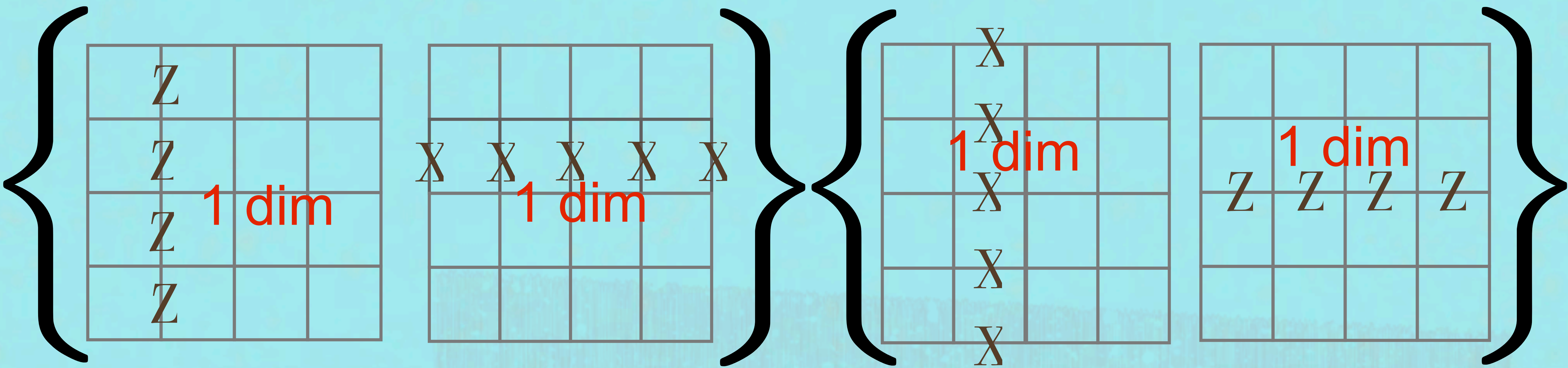

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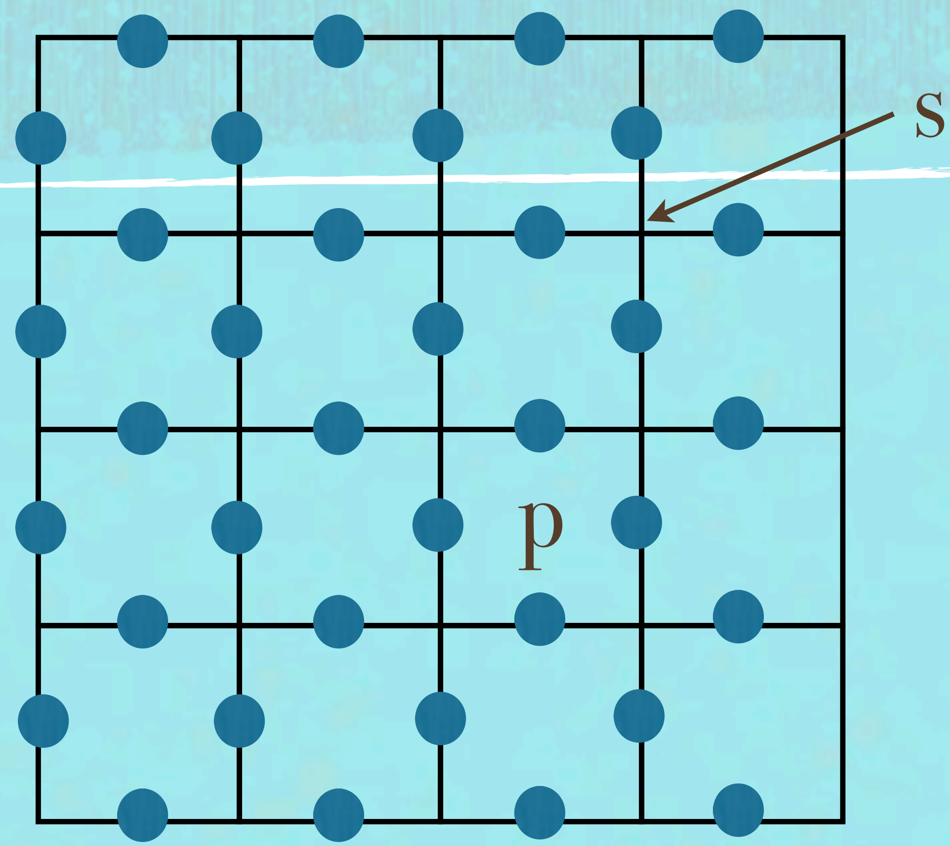

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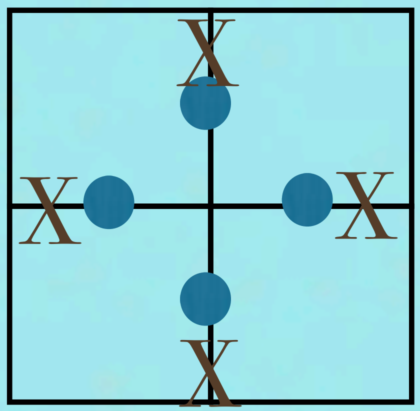

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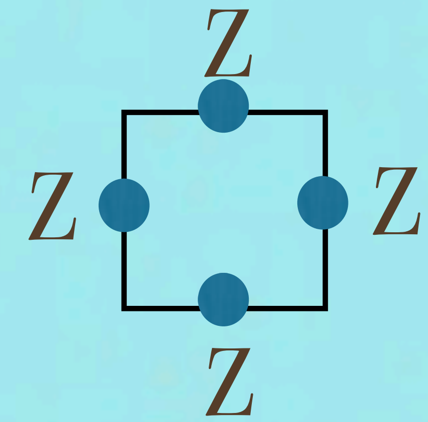

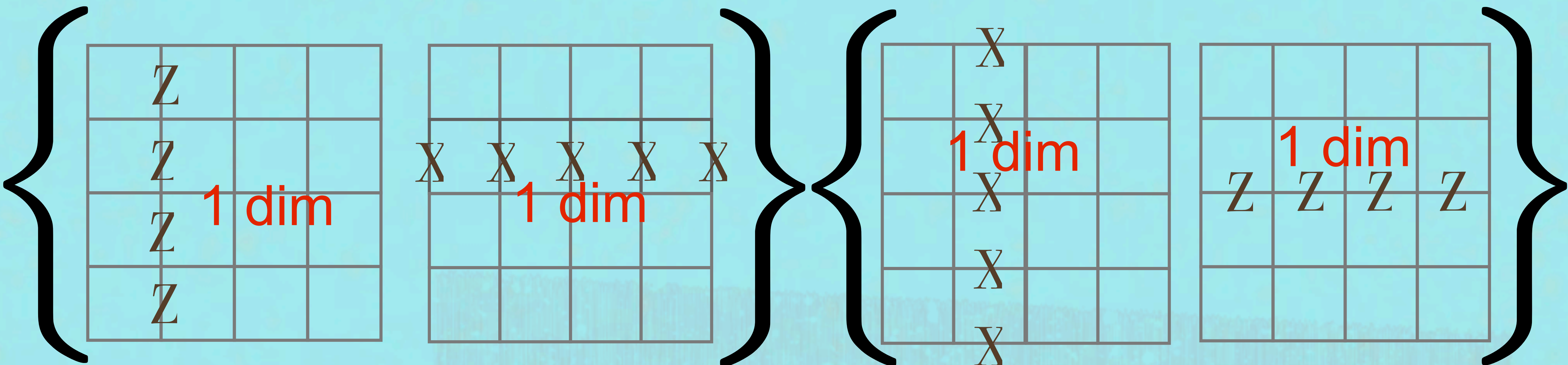

Very good code with topological order

Properties of STS model

1, Exactly solvable

= logical operators can be easily computable.

2, Topological deformation of logical operators

3, Non-trivial STS always has topological order

Equivalence of logical operators

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So many equivalent representations...

Topological deformation of logical operators

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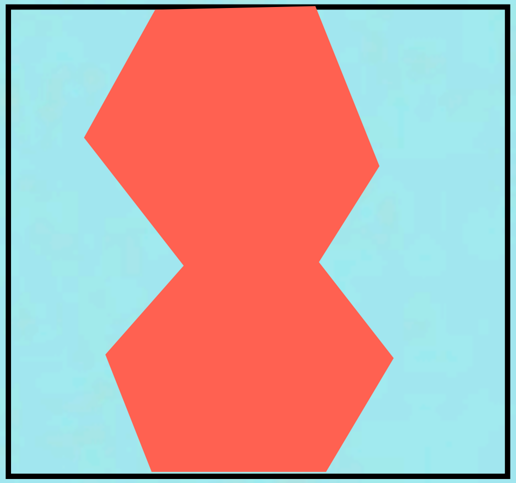

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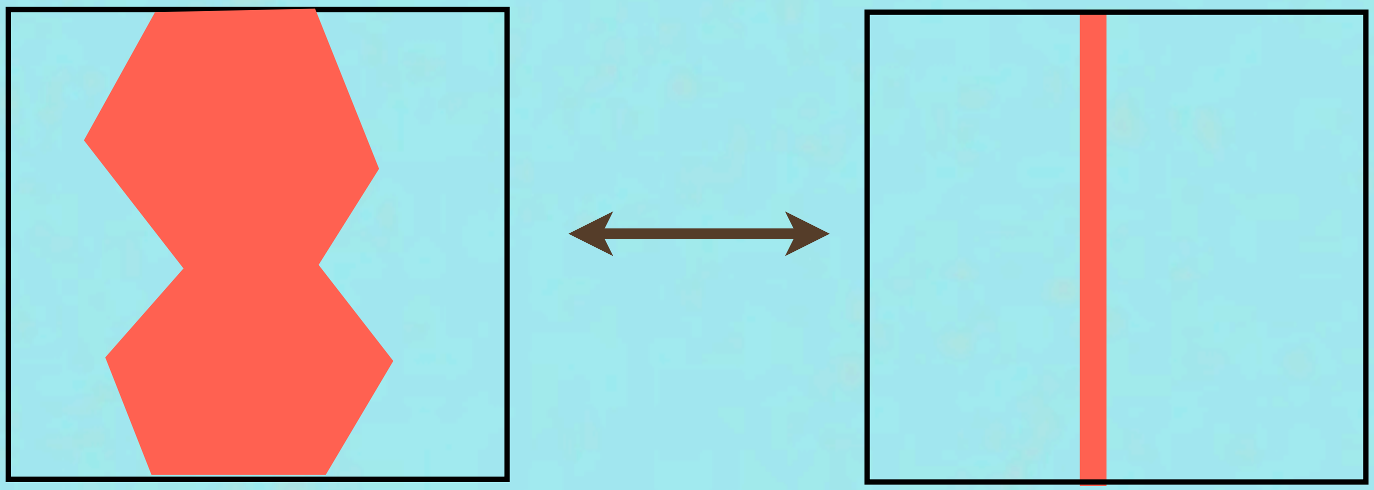

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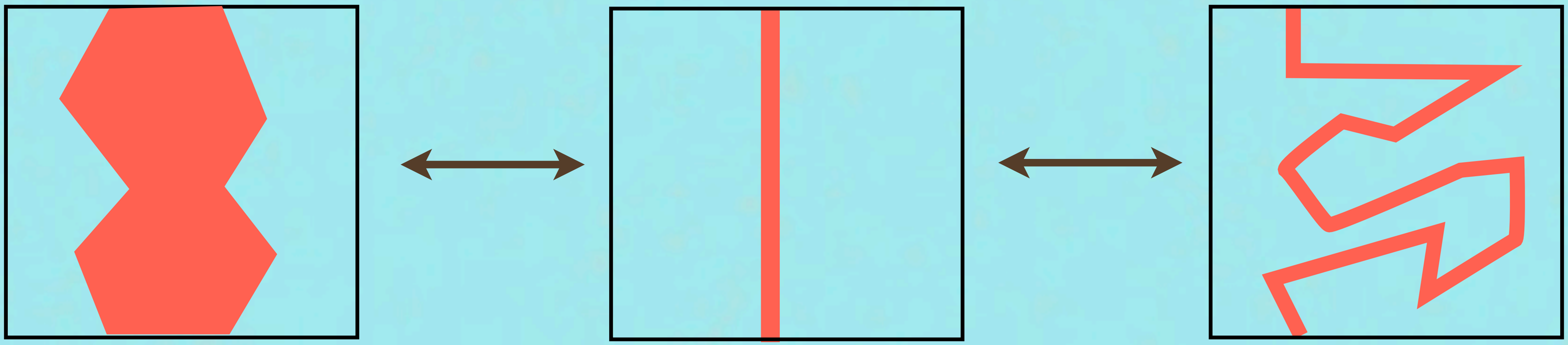

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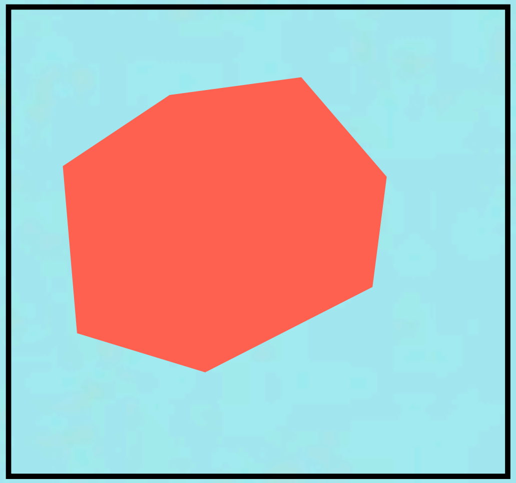

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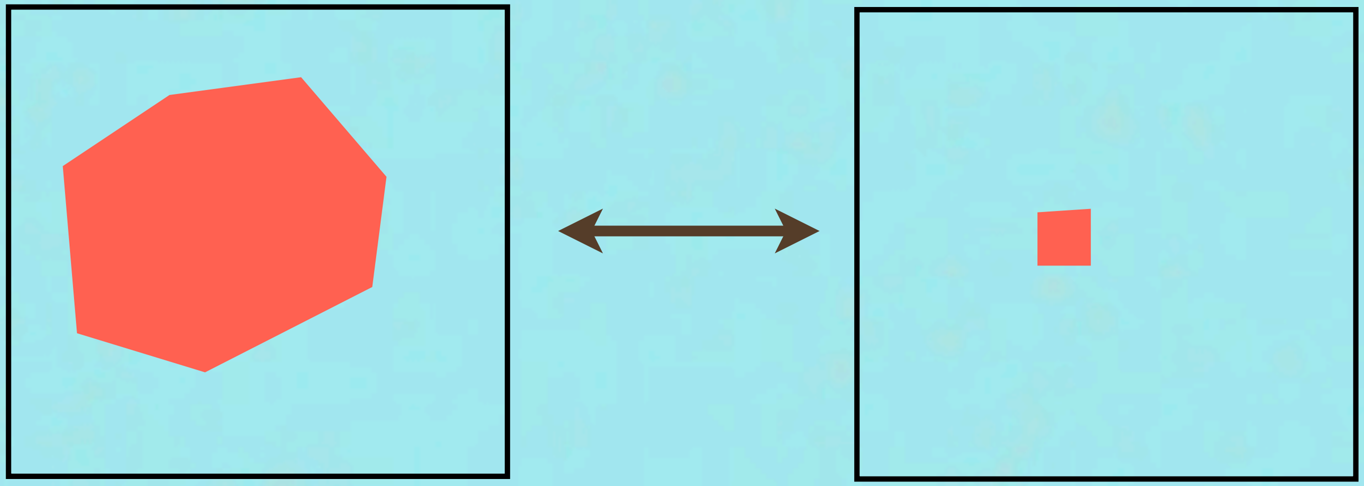

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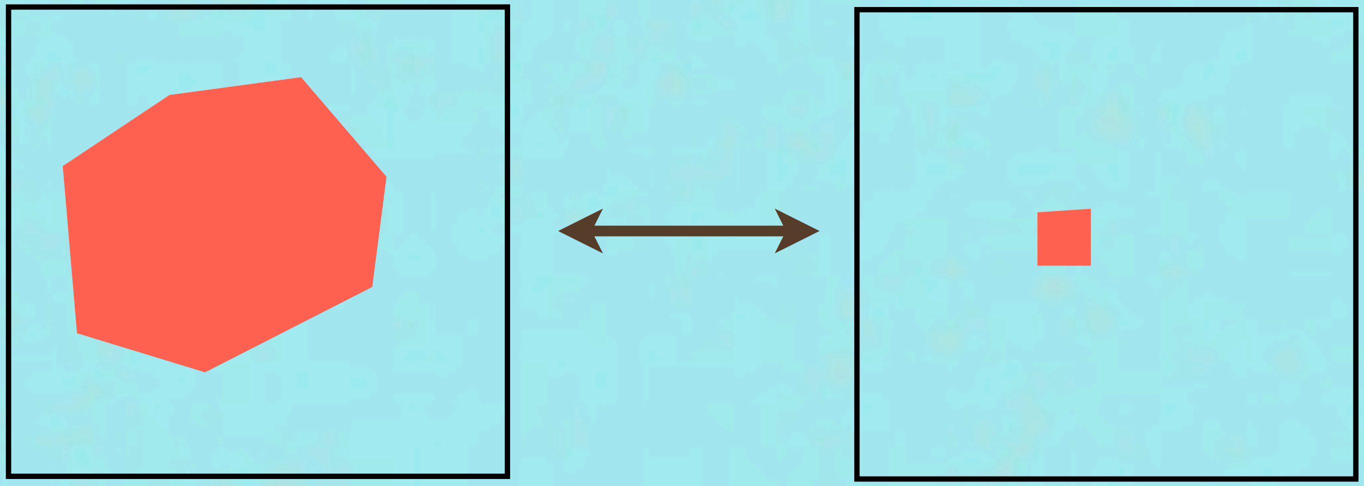

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Anyons

Topological entropy

Stability against quantum fluctuation

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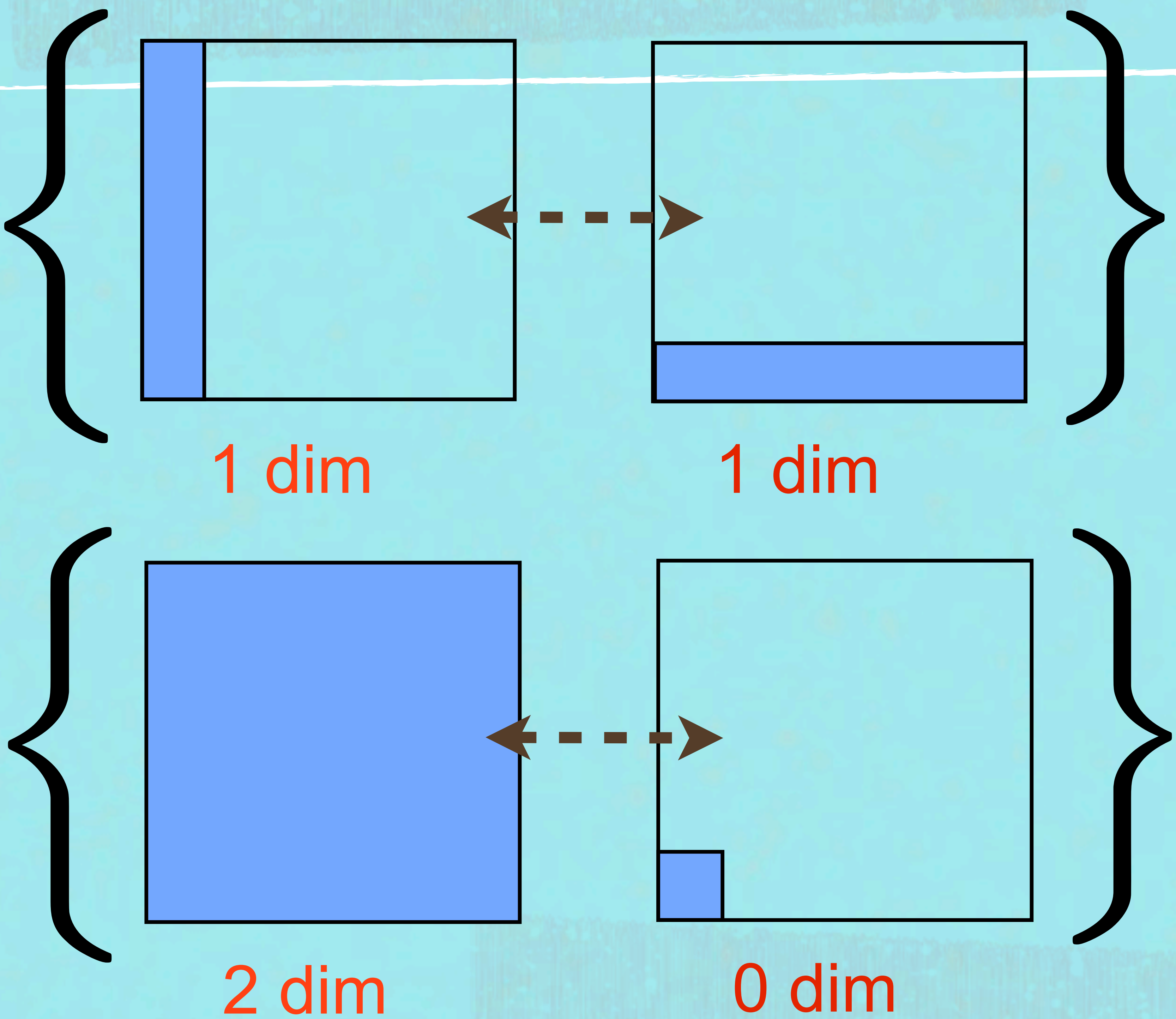
Exact solvability: logical operators in STS



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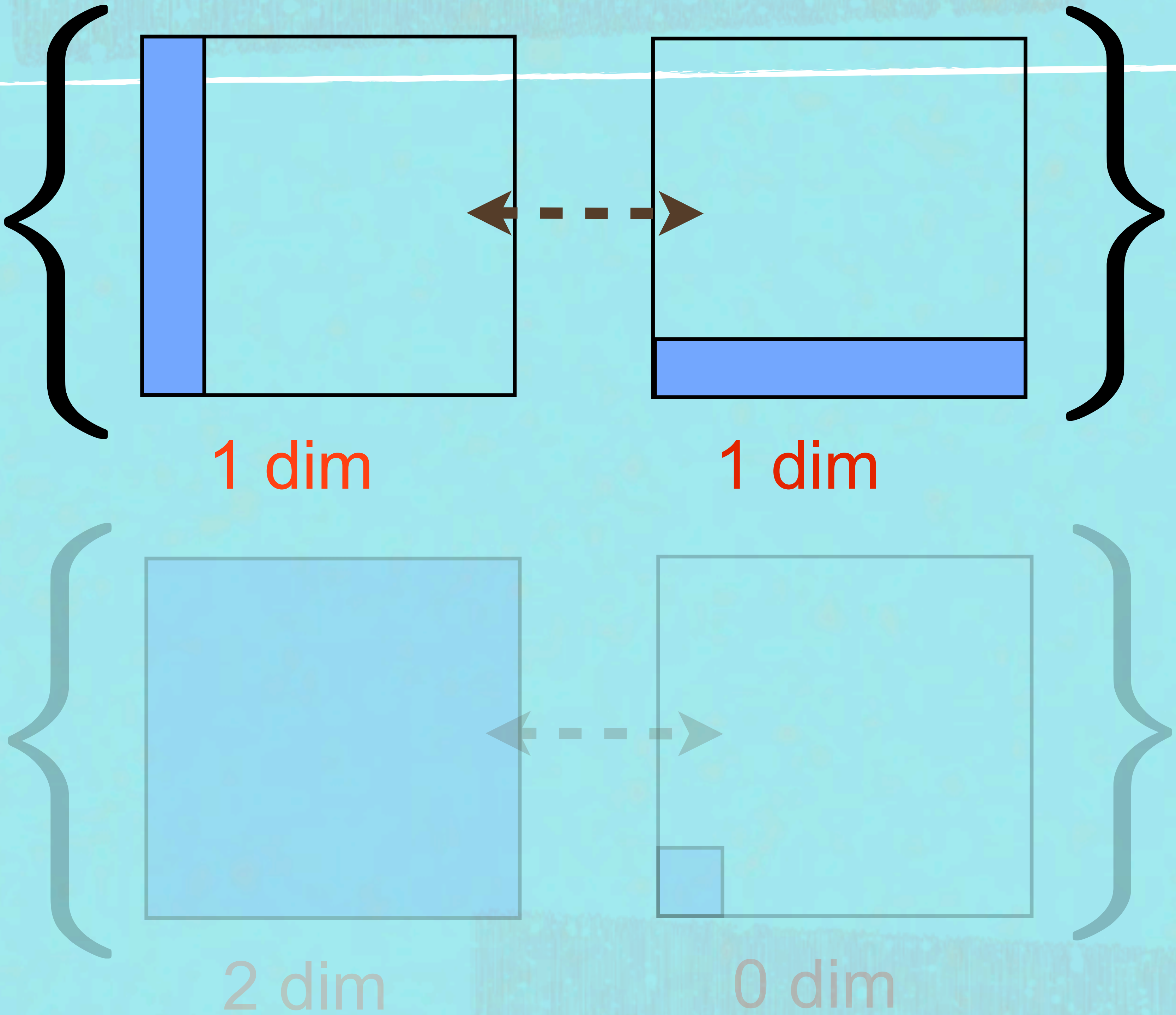

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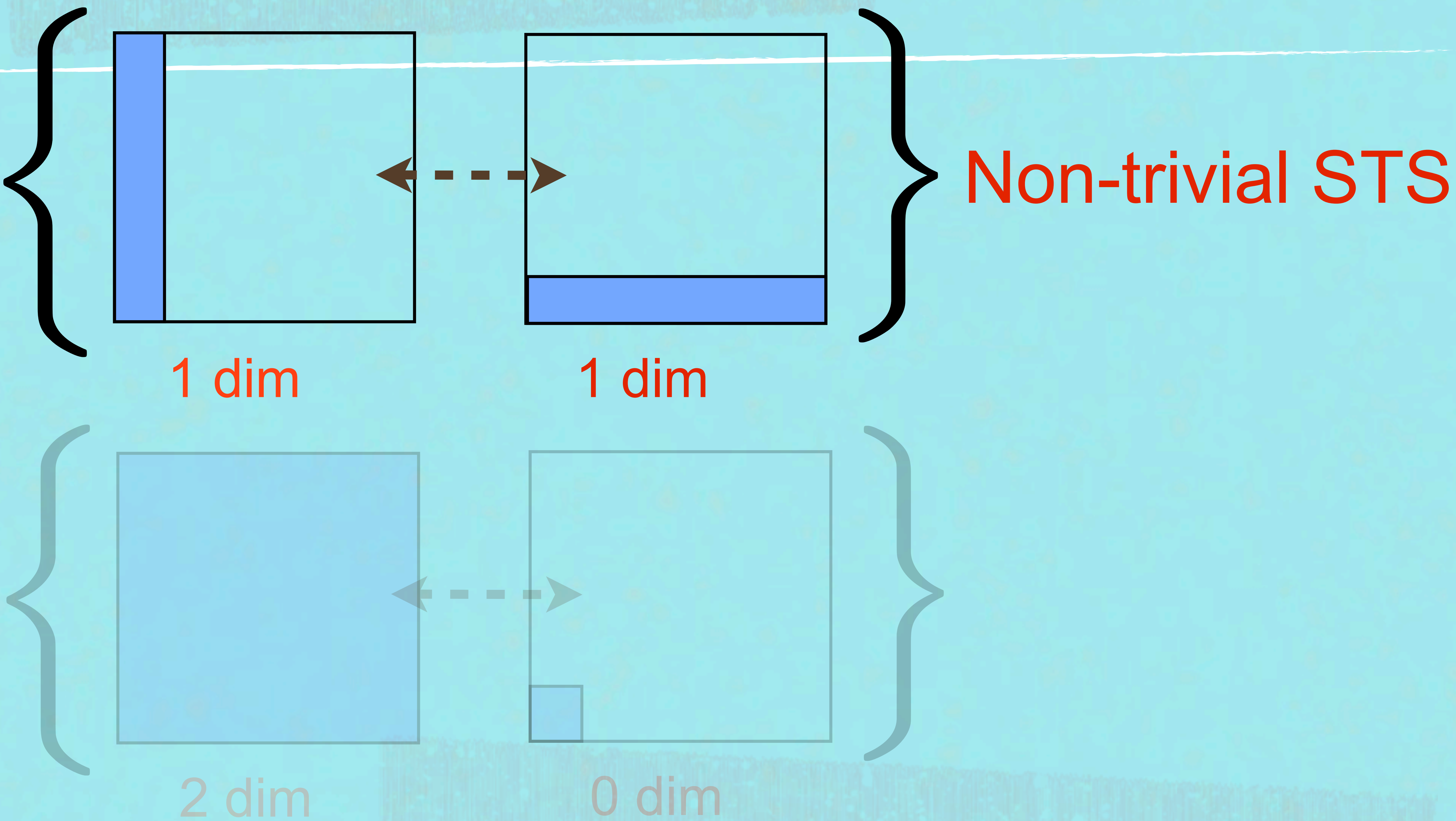

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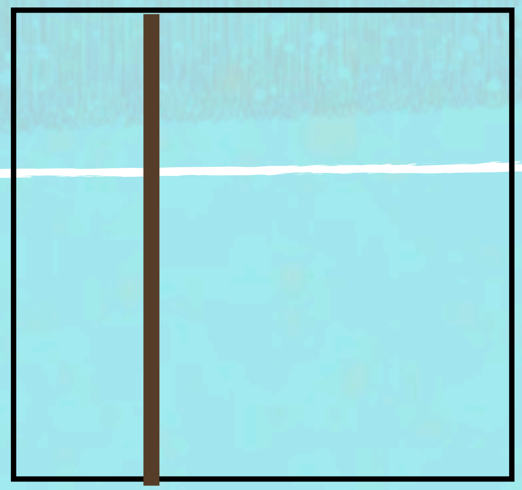

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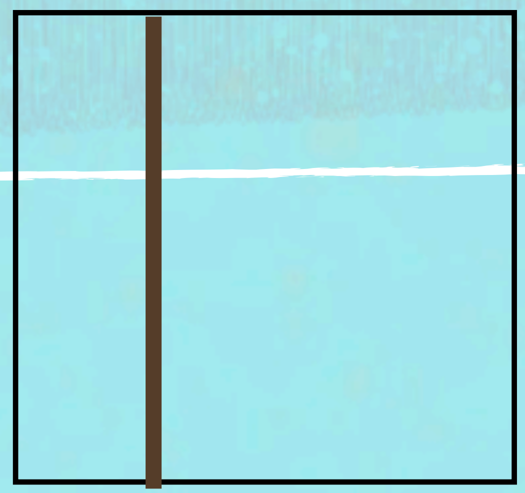
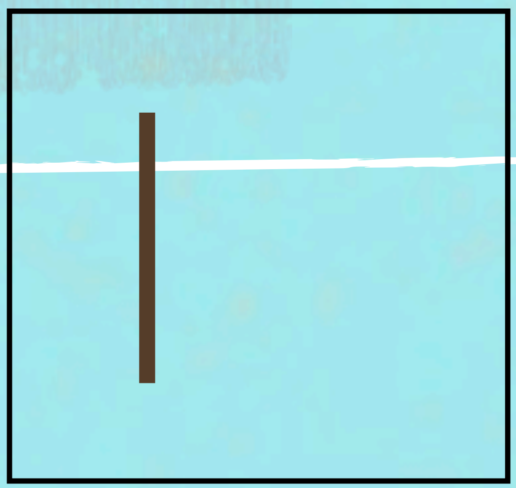


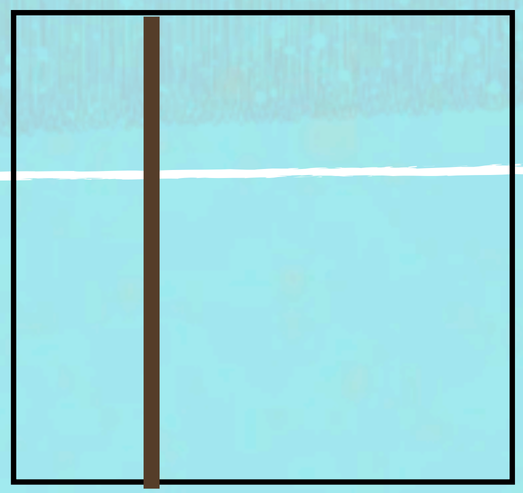
Logical operators



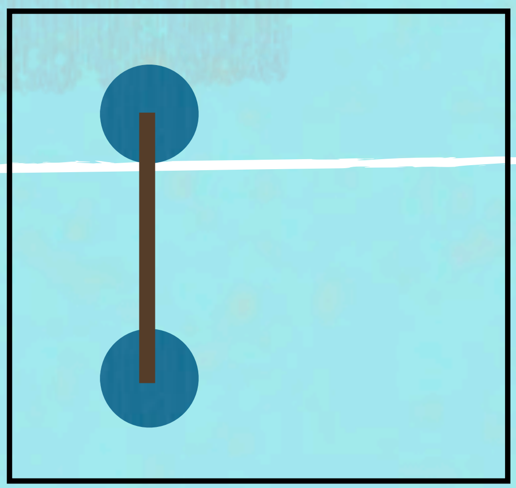
Segment of
logical operators

Anyons in non-trivial STS

Anyons



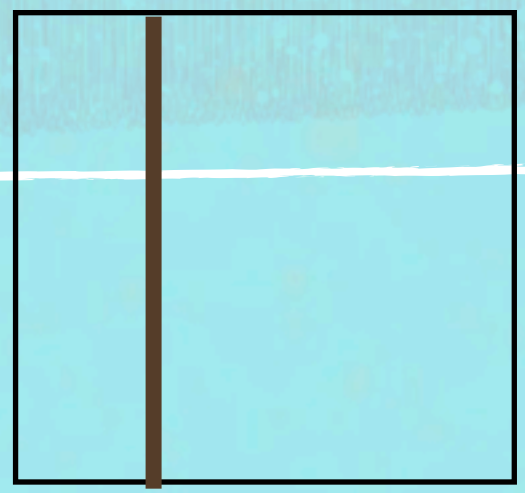
Logical operators



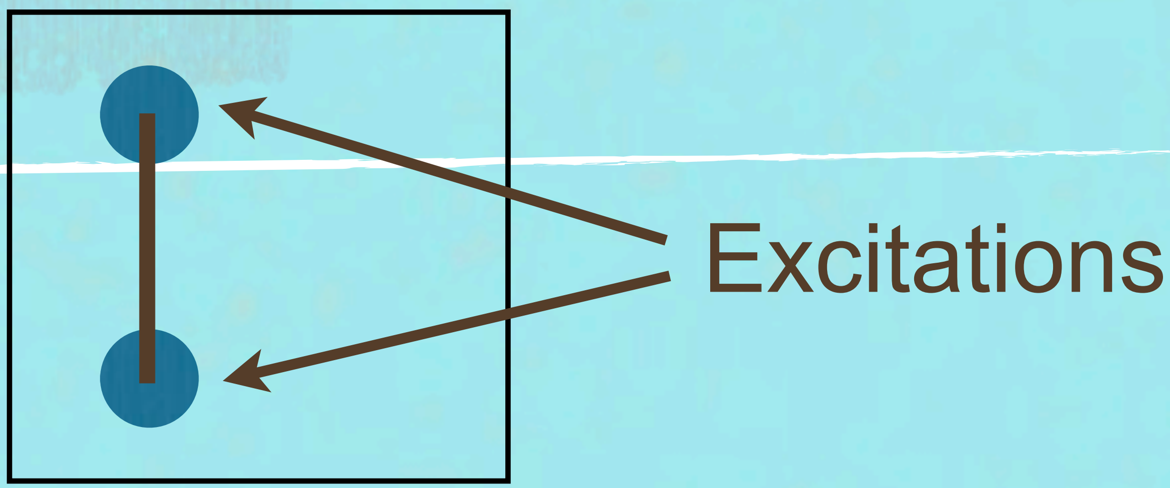
Segment of
logical operators

Anyons in non-trivial STS

Anyons



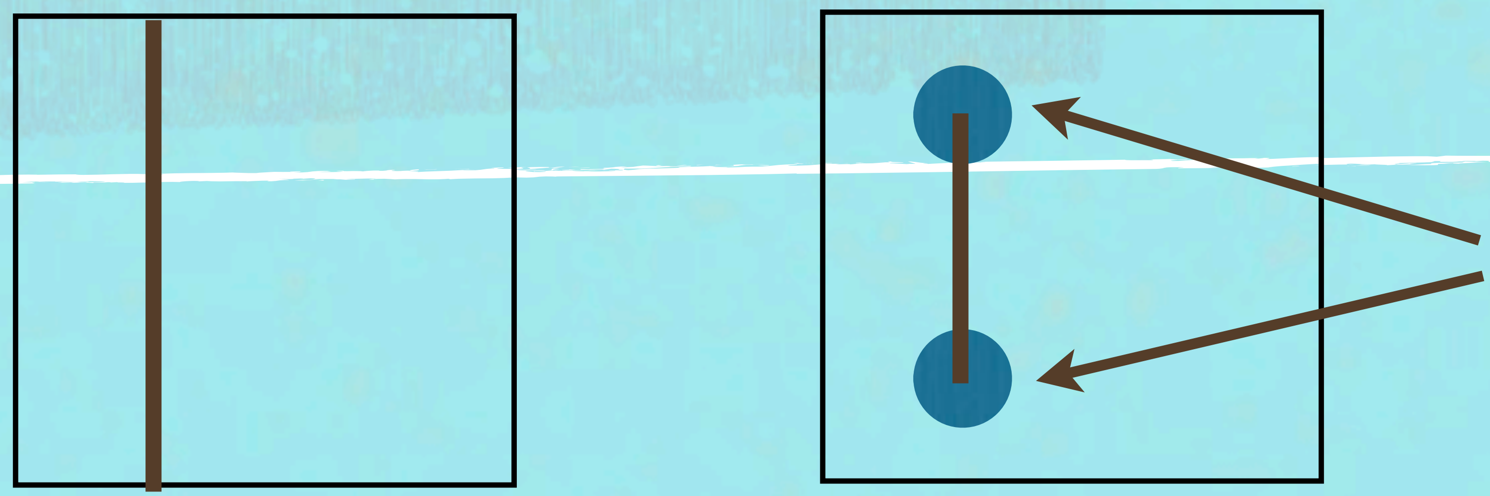
Logical operators



Segment of
logical operators

Anyons in non-trivial STS

<u>Anyons</u>



Excitations

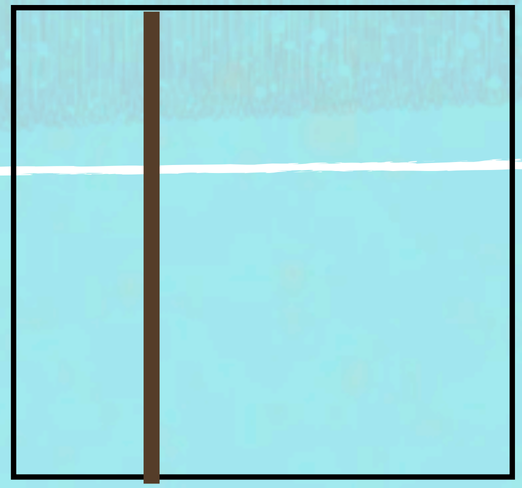
= anyons

Logical operators

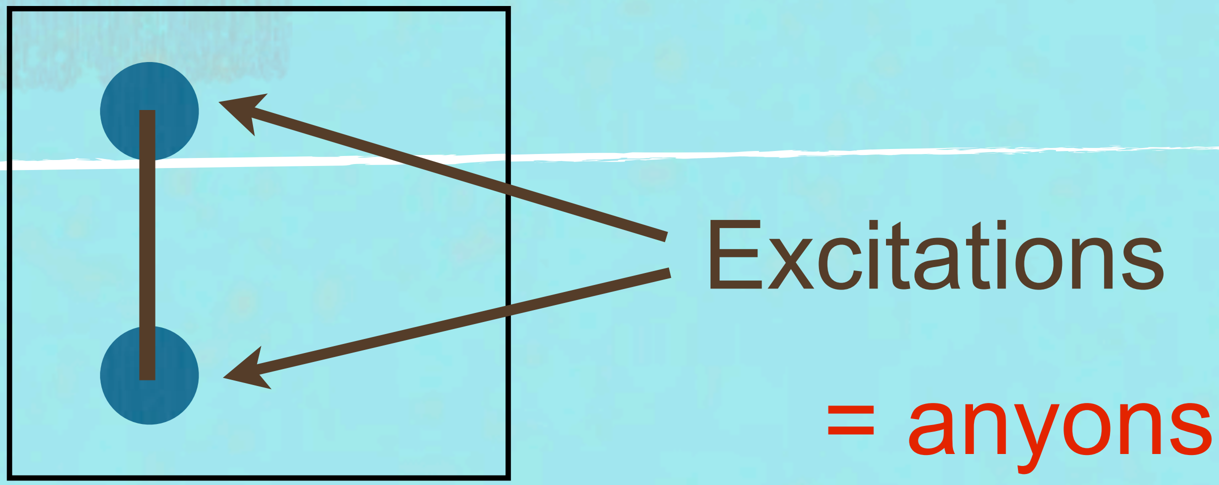
Segment of
logical operators

Anyons in non-trivial STS

Anyons



Logical operators

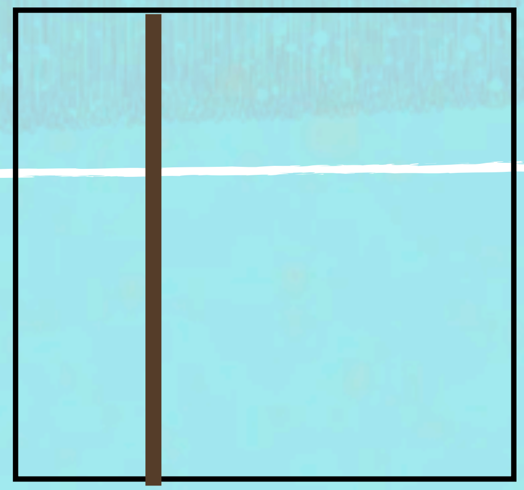


Segment of logical operators

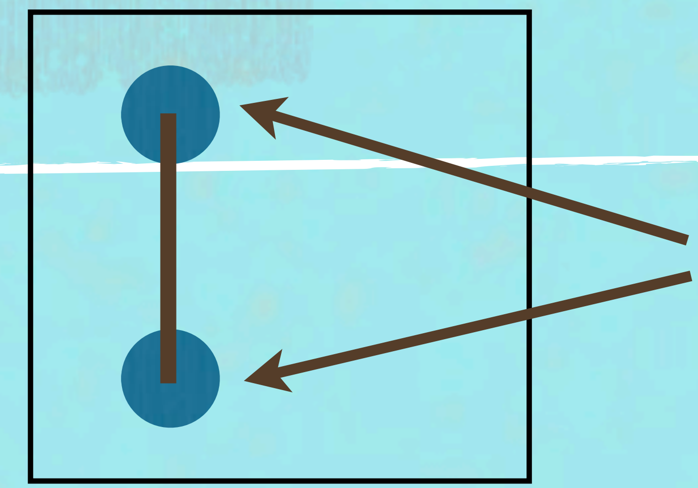
Moving anyons

Anyons in non-trivial STS

Anyons



Logical operators

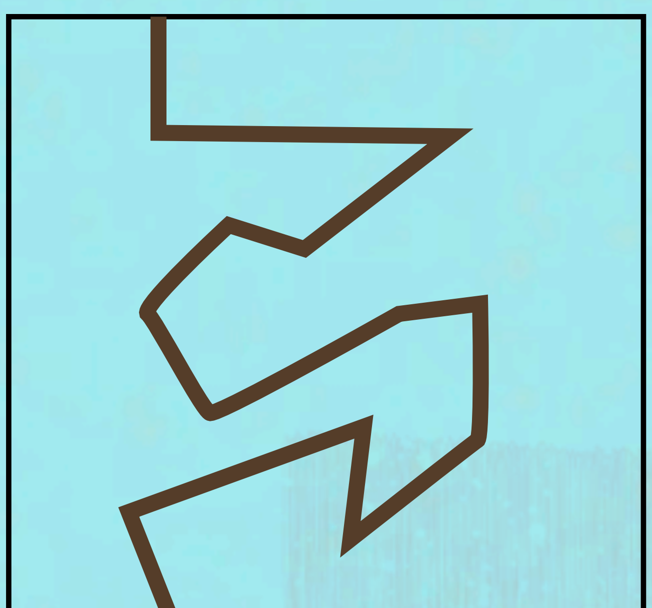


Excitations

= anyons

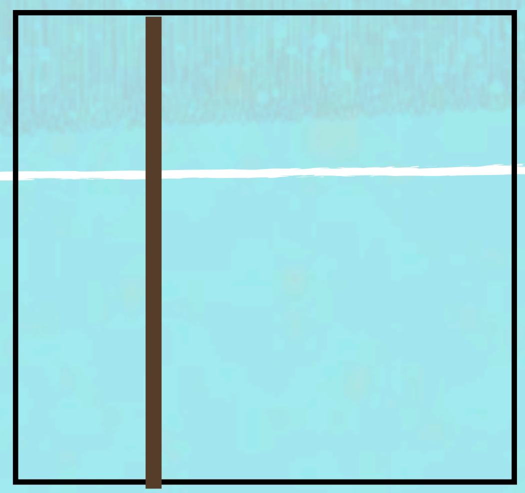
Segment of logical operators

Moving anyons

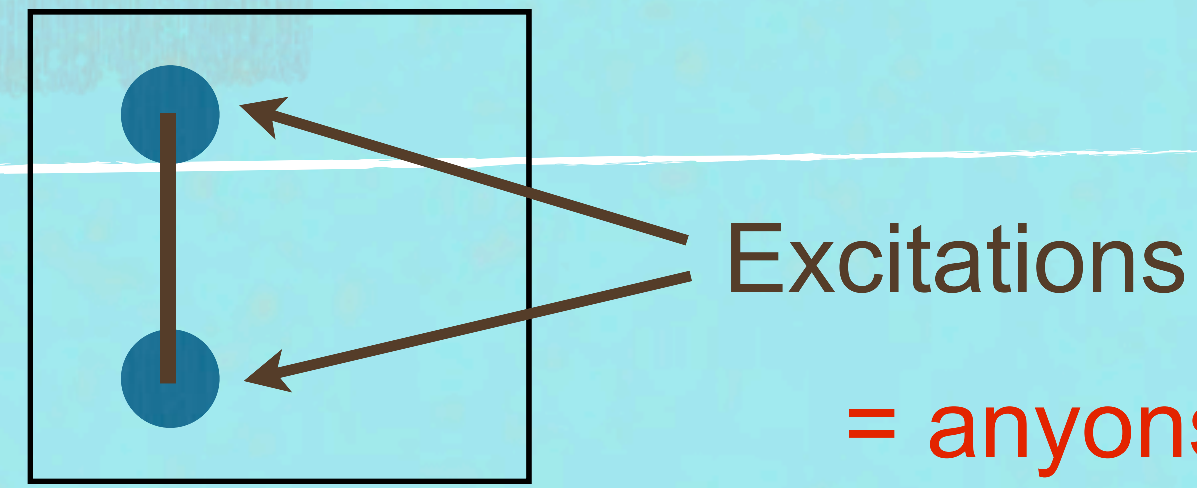


Anyons in non-trivial STS

<u>Anyons</u>



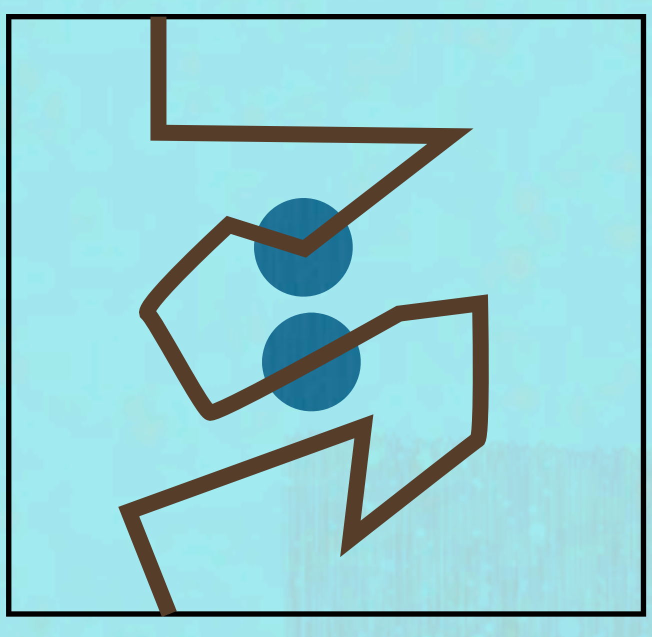
Logical operators



= anyons

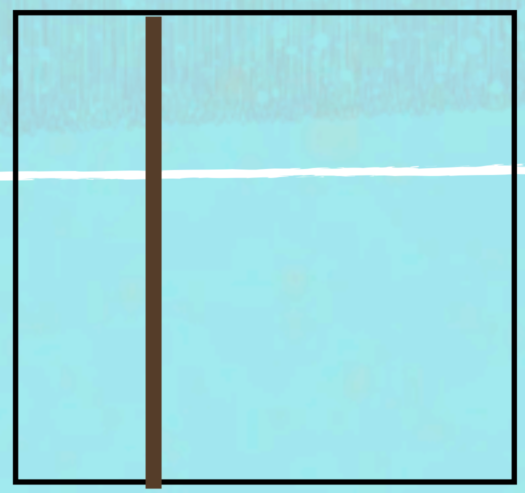
Segment of logical operators

<u>Moving anyons</u>

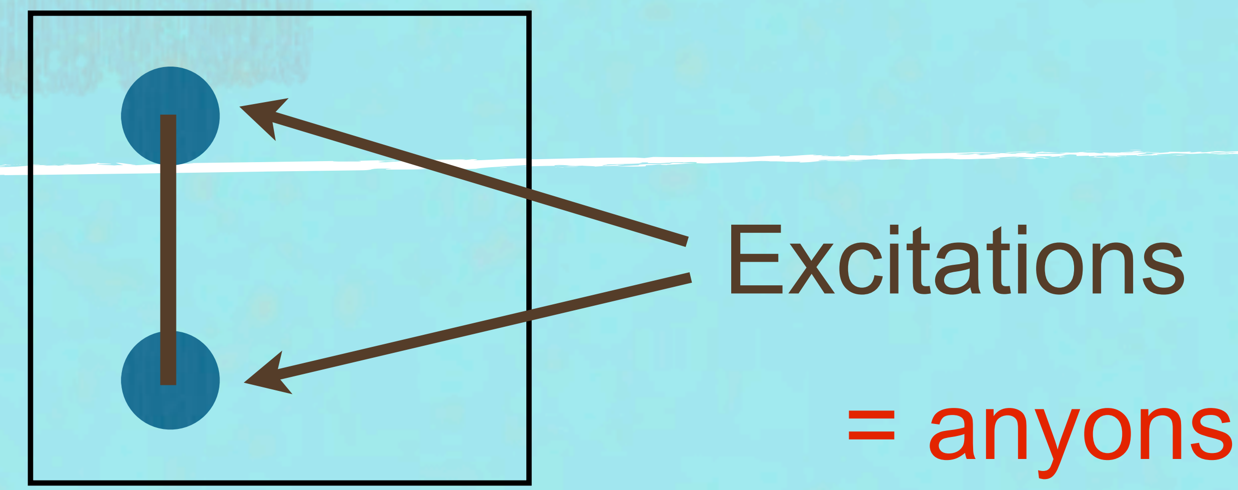


Anyons in non-trivial STS

Anyons



Logical operators

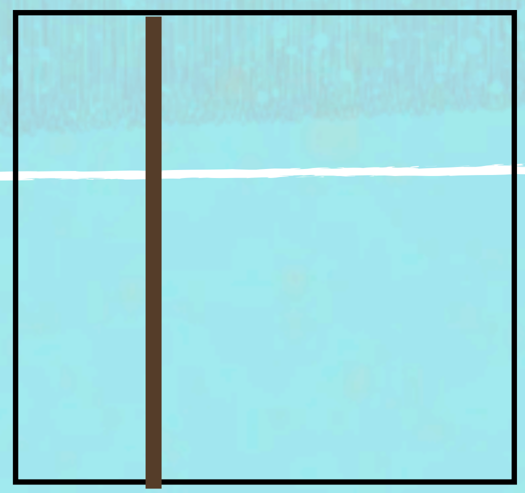


Segment of logical operators

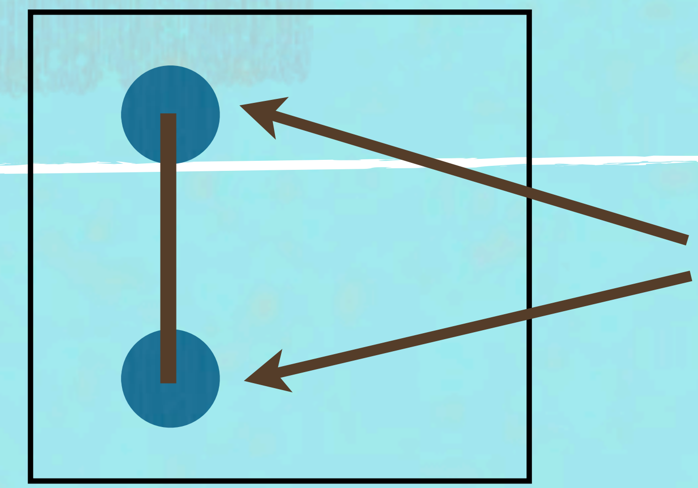
Moving anyons

Anyons in non-trivial STS

Anyons



Logical operators

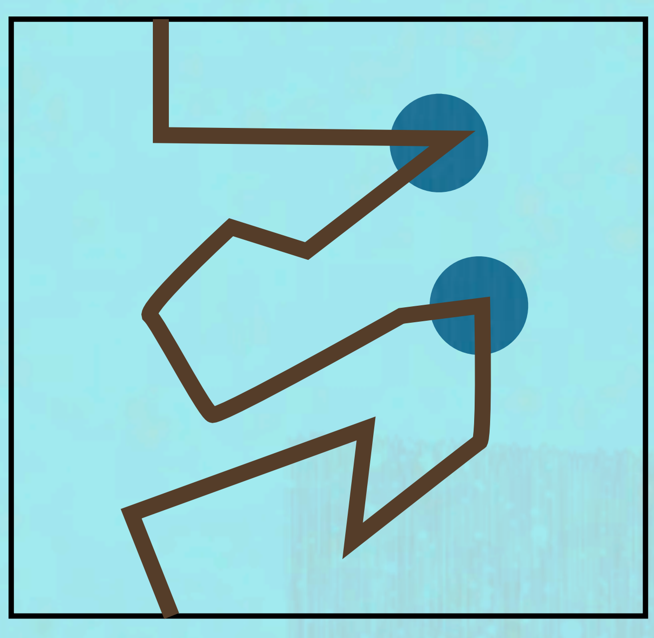


Excitations

= anyons

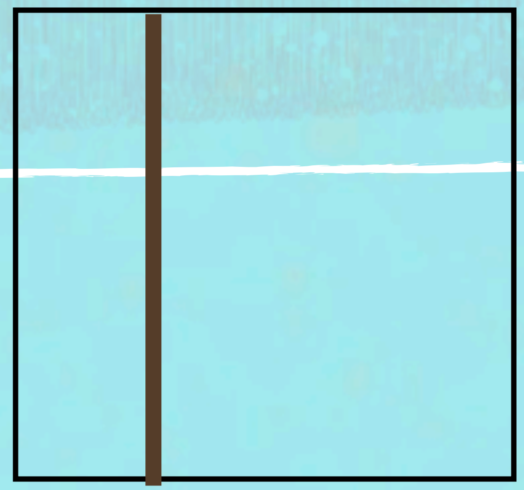
Segment of logical operators

Moving anyons

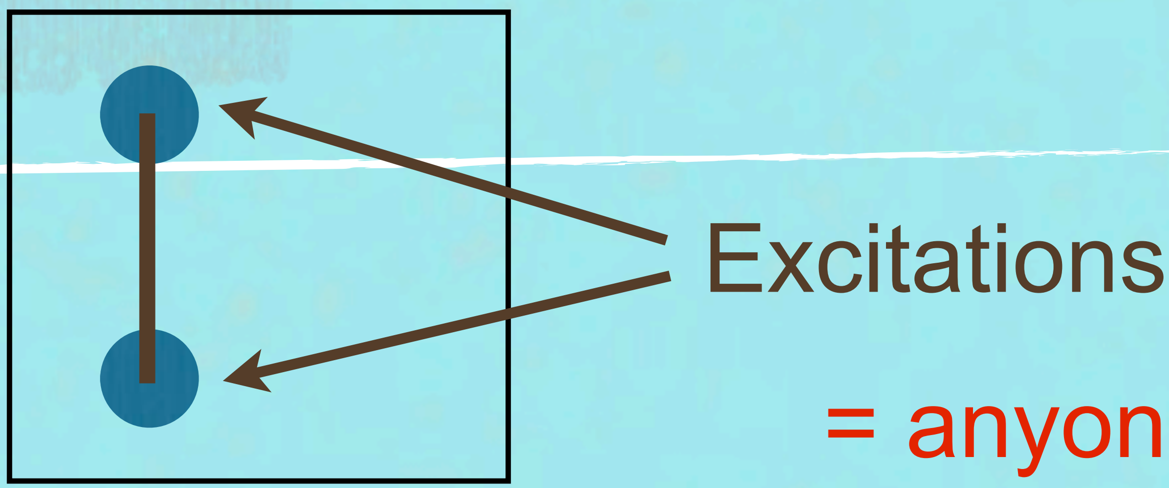


Anyons in non-trivial STS

Anyons

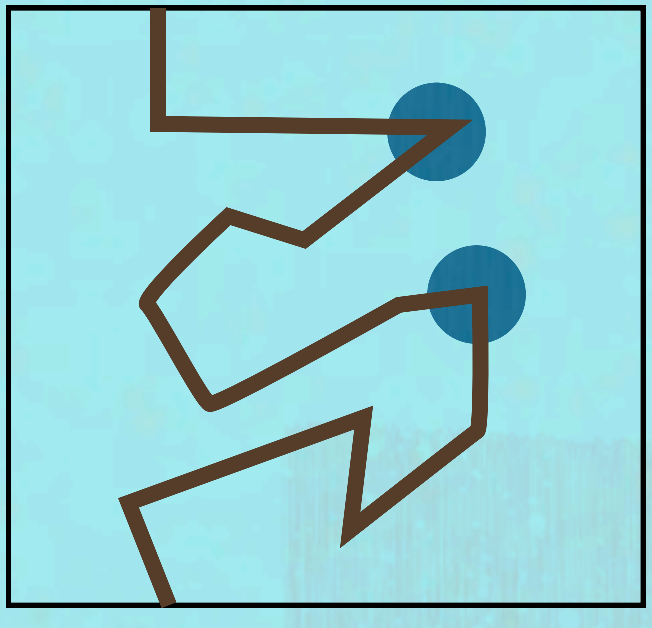


Logical operators



Segment of logical operators

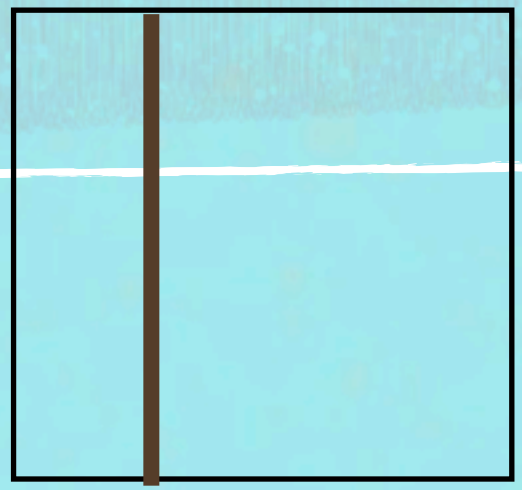
Moving anyons



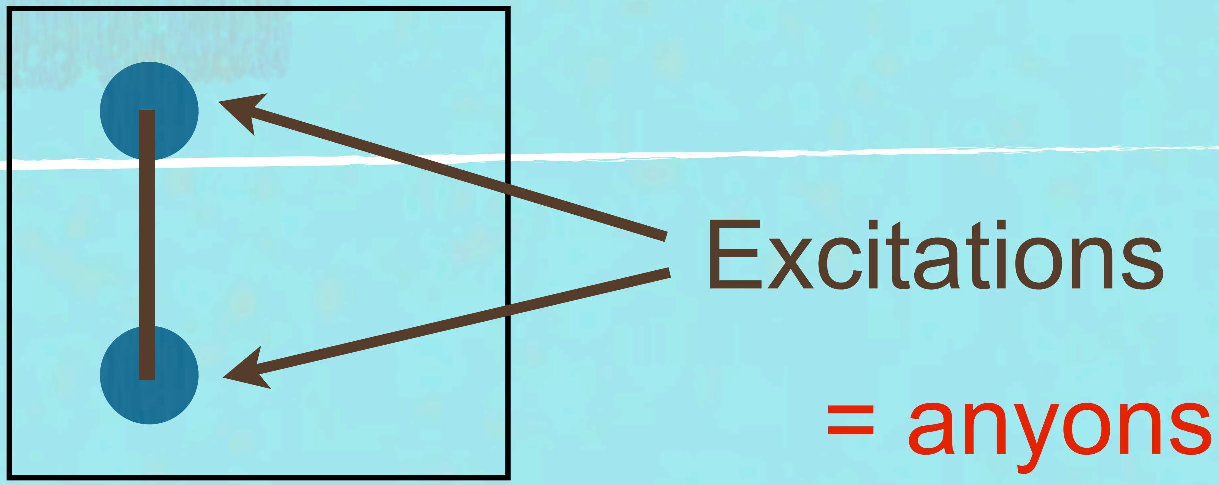
Braiding of anyons

Anyons in non-trivial STS

Anyons

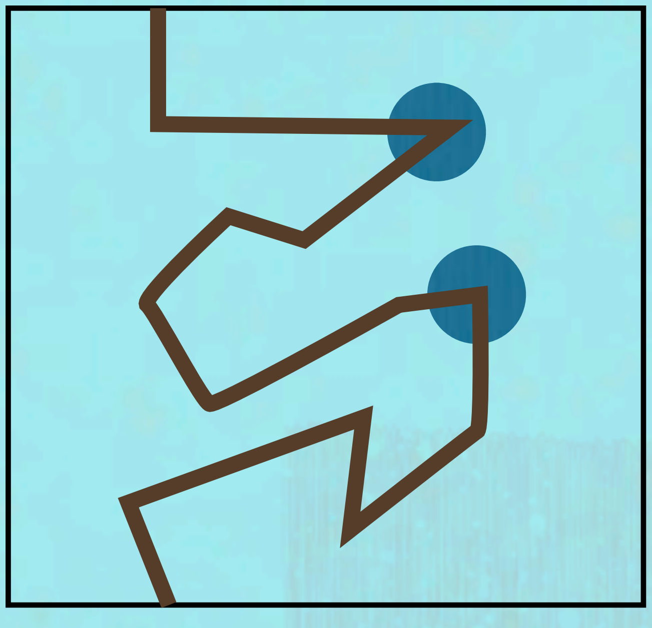


Logical operators



Segment of logical operators

Moving anyons

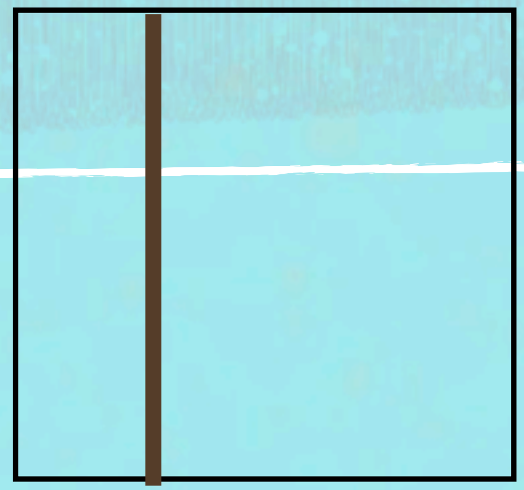


Braiding of anyons

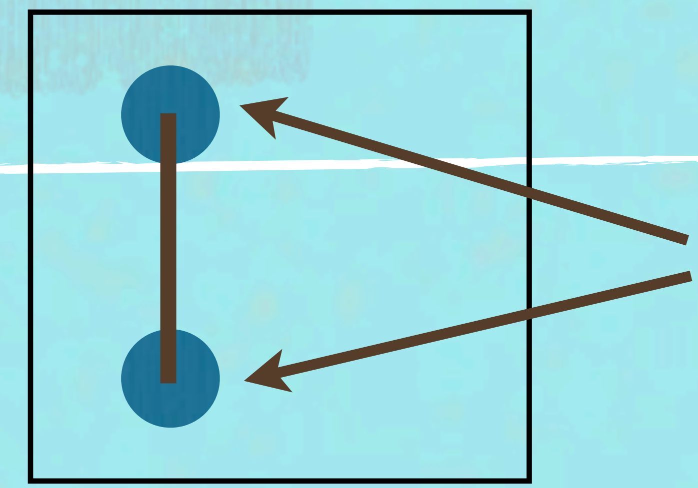
Phase factor "-1"

Anyons in non-trivial STS

Anyons



Logical operators

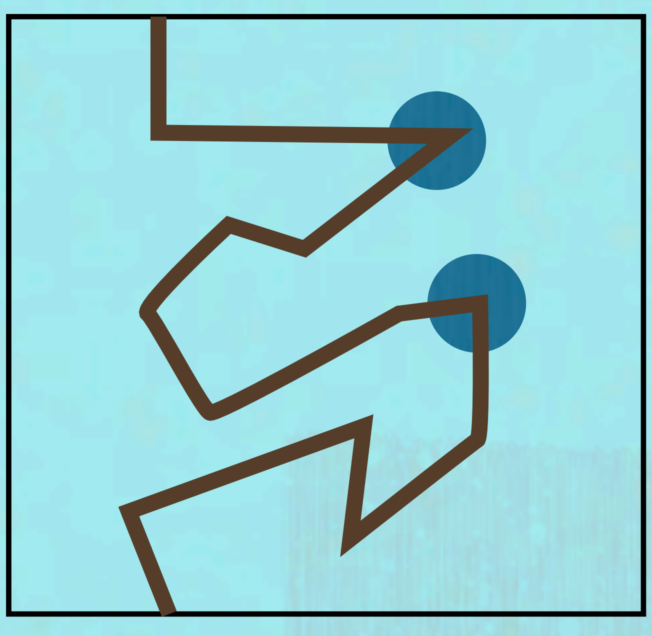


Segment of logical operators

Excitations

= anyons

Moving anyons



Braiding of anyons

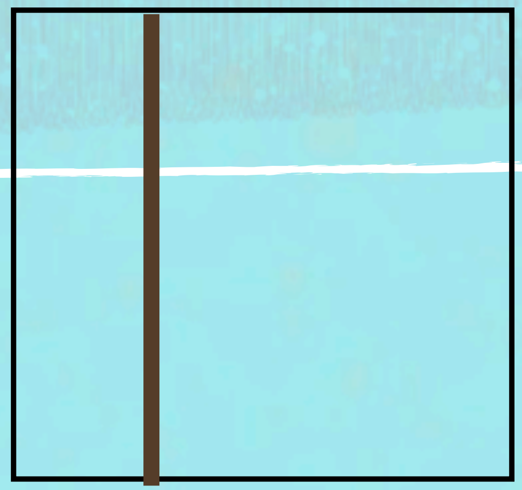
Phase factor “-1”

↑

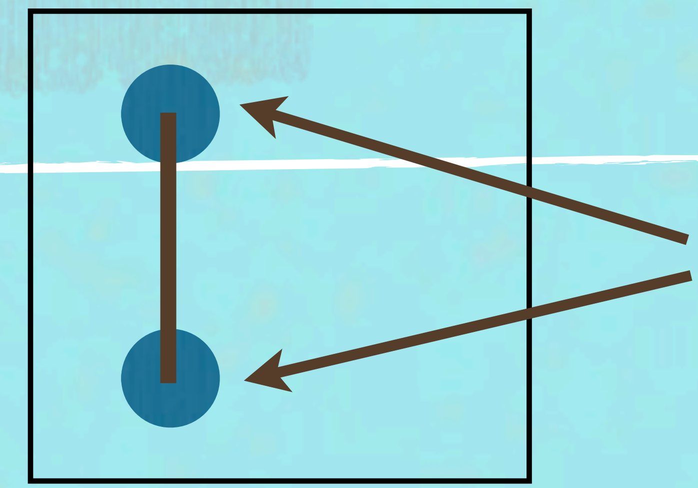
Anti-commutation between logical operators

Anyons in non-trivial STS

Anyons



Logical operators

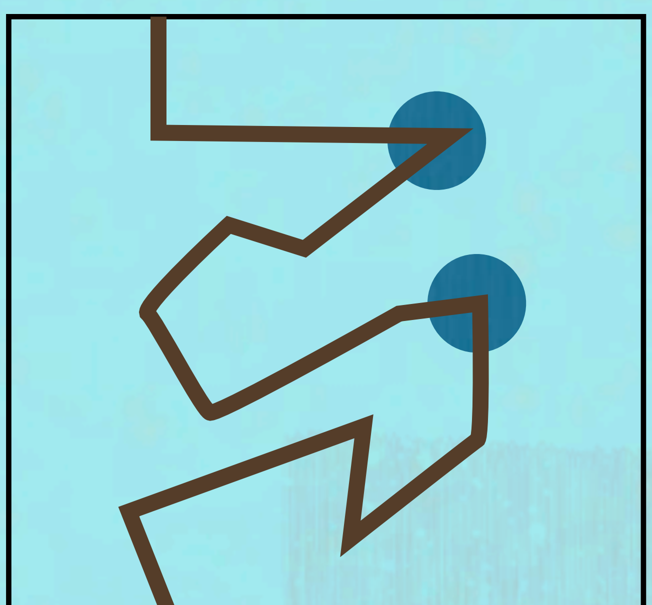


Excitations

= anyons

Segment of logical operators

Moving anyons



Braiding of anyons

Phase factor "-1"

↑

Anti-commutation between logical operators

Non-trivial STS always has topological order.

Conclusion

Stabilizer code

Conclusion

Stabilizer code ⟵ Physical realizability

Conclusion

Stabilizer code ⟵ Physical realizability

Future work

Conclusion

Stabilizer code ⟵ Physical realizability

Future work

Subsystem code

Conclusion

Stabilizer code ⟵ Physical realizability

Future work

Subsystem code ⟵ Physical realizability

Conclusion

Stabilizer code ⟵ Physical realizability

Future work

Subsystem code ⟵ Physical realizability

Complexity of Hamiltonian

Conclusion

Stabilizer code ⟵ Physical realizability

Future work

Subsystem code ⟵ Physical realizability

Complexity of Hamiltonian ⟵ Physical realizability

Conclusion

Stabilizer code ⟵ Physical realizability

Future work

Subsystem code ⟵ Physical realizability

Complexity of Hamiltonian ⟵ Physical realizability

Entanglement entropy

Conclusion

Stabilizer code ⟵ Physical realizability

Future work

Subsystem code ⟵ Physical realizability

Complexity of Hamiltonian ⟵ Physical realizability

Entanglement entropy ⟵ Physical realizability

Conclusion

Stabilizer code ⟵ Physical realizability

Future work

Subsystem code ⟵ Physical realizability

Complexity of Hamiltonian ⟵ Physical realizability

Entanglement entropy ⟵ Physical realizability

Entanglement entropy → Area law

Conclusion

Stabilizer code ← Physical realizability

Future work

Subsystem code ← Physical realizability

Complexity of Hamiltonian ← Physical realizability

Entanglement entropy ← Physical realizability

→ Area law

QI theory

Conclusion

Stabilizer code ⟵ Physical realizability

Future work

Subsystem code ⟵ Physical realizability

Complexity of Hamiltonian ⟵ Physical realizability

Entanglement entropy ⟵ Physical realizability

Entanglement entropy → Area law

QI theory ⟵ Physical realizability

Back up: Example of non-trivial STS

```
    Z
X   X   X
    Z
```

on a square lattice.

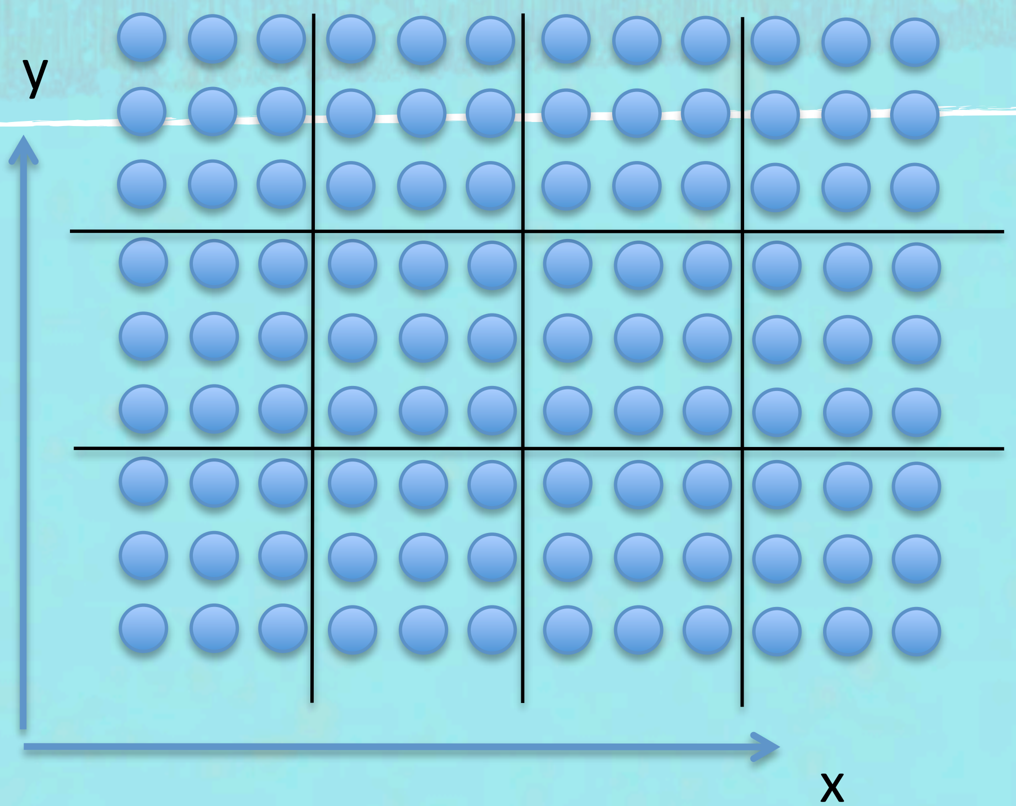
4 encoded qubits

Logical operators

ZZIZZIZZIZZI

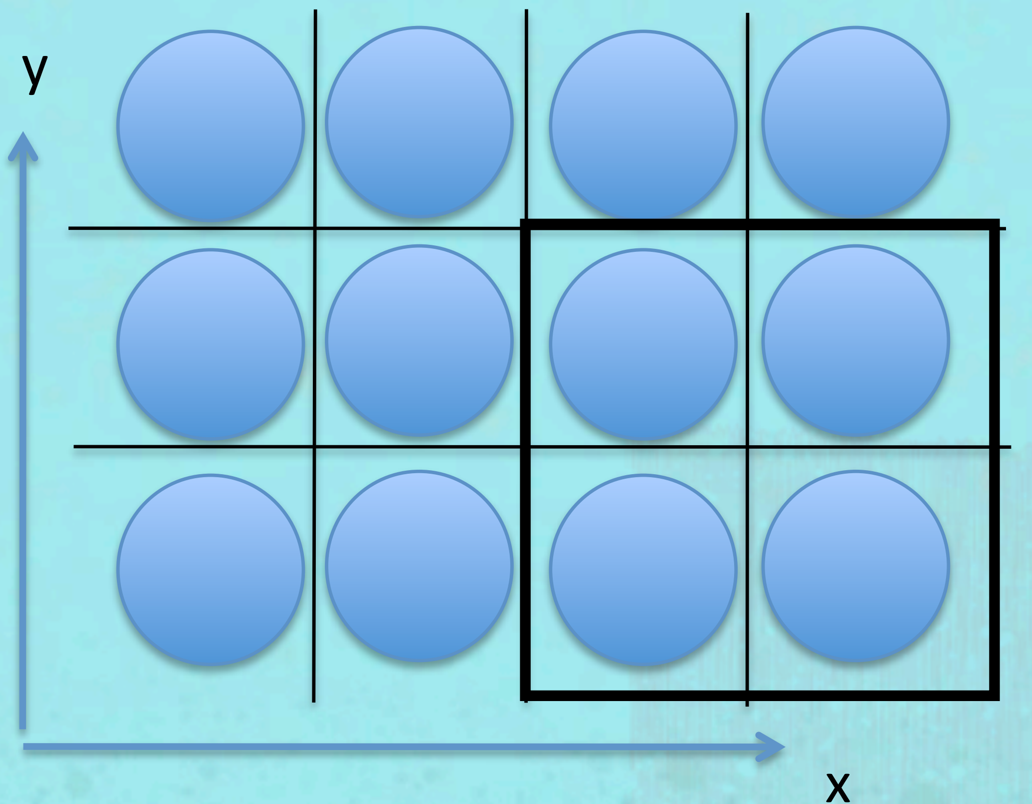
X
I
X
I
X
I
X
I

Back up: more precise def of STS model



Translation symmetry with qubits

coarse-grain



Scale symmetry : with respect to a coarse-grained lattice.