Braverman Shiraishi

Macdonald polynomials type A_{N-1}

symmetric $y_1, \dots, y_N$ $\mathbb{Q}(t,q)$

eigenfunctions of $D_N P_\lambda =$

$$= \left(\sum_{i=1}^{N} q^{\lambda_i} t^{N-i} \right) P_\lambda$$

$$D_N = \sum_{i=1}^{N} \prod_{j \neq i} \frac{t y_i - y_j}{y_i - y_j} T_{q, y_i}$$

$$T_{q,y_i} f(y_1 \dots y_n) = f(y_1 \dots q y_i, \dots y_n)$$

b) $P_\lambda = m_\lambda + \sum_{\mu < \lambda} u_{\mu\lambda} m_\mu$

$N=1$, $SL(2)$ $\quad y_1 = z$, $y_2 = z^{-1}$

Rogers : $P_0 = 1$, $P_1 = z + z^{-1}$, $P_2 = z^2 + z^{-2} + \frac{(1+q)(1-t)}{1-qt}$

Pieri rule: $(z+z^{-1})P_n = P_{n+1} + \frac{(1-q^n)(1-t^2q^{n-1})}{(1-tq^n)(1-tq^{n-1})}P_{n-1}$

Laumon moduli spaces

$\alpha = \sum d_i \alpha_i$, $d_i \in \mathbb{N}$ $\quad \mathcal{Q}^\alpha$ = moduli

of flags $0 \subset W_1 \subset W_2 \subset \dots \subset W_N = \mathcal{O}_C^N$ $\quad C = \mathbb{P}^1$

$\operatorname{rk} W_i = i$, $\deg W_i = -d_i$

Q^{α} smooth projective, connected,

$$\dim = 2|\alpha| + \frac{N(N-1)}{2}$$
$$2\sum d_i$$

$$\mathring{Q}^{\alpha} : \mathcal{W}_i \text{ vector subbundles}$$
$$= \mathrm{Maps}^{\alpha}(C, \mathcal{B}_N)$$

SL_2 : $\mathcal{W}_1 \subset \mathcal{O}_C^2$, $\mathcal{W}_1 \cong \mathcal{O}(-d_1)$

$$Q^d = \mathbb{P}\Gamma(C, \mathcal{O}^2(d)) = \mathbb{P}^{2d+1}$$

$$W_i \subset \mathcal{O}_C^N \implies \Lambda^i W_i \subset \Lambda^i \mathcal{O}_C^N$$

$$\Lambda^i W_i \cong \mathcal{O}(-d_i)$$

$$\phi : Q^\alpha \longrightarrow \prod_{i=1}^{N-1} \mathbb{P}\Gamma(C, \Lambda^i \mathcal{O}_C^N(d_i))$$

image : Drinfeld comp. of $\mathrm{Maps}^\alpha(C, \mathcal{B}_N)$

$$\lambda = \sum_{i=1}^{N} l_i \omega_i \implies \mathcal{O}_\lambda = \phi^* \boxtimes_{i=1}^{N-1} \mathcal{O}(l_i) \text{ on } Q^\alpha$$

Local version $Z^\alpha \subset Q^\alpha$:

loc. closed

a) $\mathcal{W}_i$ are vect. subbundles near $\infty \in C \simeq \mathbb{P}^1$

b) $\mathcal{W}_\bullet|_\infty$ is the standard flag in $\mathbb{C}^N = \mathcal{W}_n|_\infty$

closure of $\mathrm{BasedMaps}^\alpha(C, \infty; \mathcal{B}_N, F_0)$

smooth, connected, dim $= 2|\alpha|$

Symmetries: $\overset{q}{\mathbb{C}^*}$ - loop rotations

$\mathbb{C}^* \times T$ acts on $\qquad \rotatebox{90}{=}\ \mathrm{Aut}(\mathbb{C};\infty,0)$

$Q^\alpha, \mathcal{O}_\lambda$

T: Cartan torus of SL_N

$\overset{y_1}{z_1}, \dots, \overset{y_{N-1}}{z_{N-1}}, \overset{y_N}{z_N} := (z_1 \cdots z_{N-1})^{-1}$

$w_i = z_1 \cdots z_i$

$H^\alpha_\lambda(q,t,z)$: Laurent poly : f-n on $\mathbb{C}^* \times T$

generating f-n : $[H^\cdot(Q^\alpha, \Omega^\cdot_{Q^\alpha} \otimes \mathcal{O}_\lambda)] =$

$= \sum_{i,j} (-1)^{i+j} t^j [H^i(Q^\alpha, \Omega^j \otimes \mathcal{O}_\lambda)]$

Global Theorem: a) Let λ be non dominant

Then for given $j,k \in \mathbb{N}$, and $\alpha \gg 0$

$$H^k(Q_\alpha, \Omega^j \otimes \mathcal{O}_\lambda) = 0$$

b) $\forall \lambda \ \exists \lim_{\alpha\to\infty} H^\alpha_\lambda(q,t,z) =: H_\lambda(q,t,z)$

$$\lambda \not> 0 \Rightarrow H_\lambda = 0$$

c) $\lambda = 0$ $\quad H_0(q,t,z) =$ $\quad (p;q)_n = (1-p)(1-pq)\cdots(1-pq^{n-1})$

$$\frac{(1+t)(1+t+t^2)\cdots(1+t+\ldots+t^{N-1})}{(1-t^{N-1})^2(1-t^{N-2})^4\cdots(1-t^2)^{2N-4}\quad(1-t^N)(1-t)^{N-2}}$$

d) $\lambda = \sum l_i \omega_i, \; l_i \in \mathbb{N}$

$$H_\lambda = H_0 P_\lambda \cdot \prod_{1 \le i \le j \le N-1} \frac{(t^{j-i+1};q)_{l_i+\ldots+l_j}}{(t^{j-i}q;q)_{l_i+\ldots+l_j}}$$

Local Theorem about Z^α

$$H_n = H_0 \cdot P_n \cdot \frac{(1-t)(1-tq)\cdots(1-tq^{n-1})}{(1-q)\cdots\cdots(1-q^n)}$$

$\alpha = \sum d_i a_i$

$d_i \to \infty$

$Q^d = \mathbb{P}^{2d+1}$ $\left[H^{\cdot}\left(\Omega^{\cdot}_{\mathbb{P}^{2d+1}}(\lambda)\right)\right]$ $H_0 = (1-t)^{-1}$

$$H_1 = \frac{Z + Z^{-1}}{1-q}$$

$$H_2 = \frac{(1-tq)}{(1-q)(1-q^2)}\left(Z^2 + Z^{-2}\right) + \frac{(1-t)(1+q)}{(1-q)(1-q^2)}$$

$$J_\alpha(q,t,z) = [H^\cdot(Z^\alpha, \Omega^\cdot_{Z^\alpha})]$$

$$J(q,t,z,x) = \sum_{\alpha \in \mathbb{N}^{n-1}} x^\alpha J_\alpha(q,t,z)$$

$$J = \prod x_i^{\log \omega_i / \log q} \quad x = (x_1, \ldots, x_{n-1})$$

Macdonald-Baker-Akhiezer

$$T_{i,q^{\pm 1}} F(q,t,z,x) \simeq F(q,t,z, x_1, \ldots, x_{i-2}, q^{\mp 1} x_{i-1}, q^{\pm 1} x_i \ldots)$$

a) $D\,J = z_1 + \dots + z_{N-1} + (z_1 \cdots z_{N-1})^{-1}\, J$

$$D = \sum_{i \geq 1}^{N} \prod_{j < i} \frac{1 - q^{-1} t^{i-j-1} x_1 \cdots x_{i-1}}{1 - t^{i-j} x_j \cdots x_{i-1}} \prod_{j < i}^{N} \cdots$$

b) $$\lim_{\alpha \to \infty} J_\alpha(q, t, z) = \prod_{1 < i < j < N} \frac{\left(q t \frac{z_i}{z_j} ; q\right)_\infty}{\left(q \frac{z_i}{z_j} ; q\right)_\infty} \times \cdots$$

$$\left(z^\alpha\right)^{T \times \eta^*}$$

$Z^\alpha = \mathbb{Q}^\alpha \qquad (\mathbb{Q}^\alpha)^T \times \mathbb{C}^* \quad , \quad \mathbb{C}, 0, \infty$

$W_1 = \mathcal{O}(\mathbb{B} \cdot 0 + C_1 \cdot \infty)$ locally $\mathbb{Q}^\alpha \approx Z^\beta \times Z^\gamma \times \mathbb{B}$

$W_2 =$

around

a fixed point

$\beta + \gamma = \alpha$

$\mathbb{Q}^N = \langle e_1, -, e_N \rangle$