

MINI-COURSES

(Three 1h talks)

Alexander Kleshchev (University of Oregon at Eugene, USA)

Title: Representation theory of symmetric groups and related Hecke algebras

Abstract: We survey some fundamental trends in representation theory of symmetric groups and related objects which became apparent in the last fifteen years. The emphasis is on connections with Lie theory via categorification. We present results on branching rules and crystal graphs, decomposition numbers and canonical bases, graded representation theory, connections with cyclotomic and affine Hecke algebras, Khovanov-Lauda-Rouquier algebras, category \mathcal{O} and \mathcal{W} -algebras.

Ron Donagi (University of Pennsylvania, USA)

Title: Geometric Langlands Conjectures

Abstract: These talks will introduce and explore the Langlands conjectures and especially their geometric versions. These conjectures are at the boundary of algebraic geometry and representation theory. Deep connections to quantum field theory have been unearthed in recent years. The conjectures require existence of a large class of "automorphic objects", or automorphic sheaves in the geometric setting. They are essentially non-abelian: the abelian case, class field theory, is well understood. I will survey much of the background as time permits. In particular, I will state, explain and outline a proof of the classical limit of the geometric Langlands conjecture, which says that the Hitchin integrable system for a simple complex Lie group G is dual to the Hitchin system for the Langlands dual group ${}^L G$. We will explore the two ways that this result relates to the full "quantum" conjecture: it is its classical limit, but may also be equivalent to the full conjecture via a twistor construction. This leads to a physics inspired program to establish the conjecture with the use of non-abelian Hodge theory.

Duiliu Emmanuel Diaconescu (Rutgers University, USA)

Title: ADHM Sheaves, wall-crossing and Local BPS Invariants

Abstract: ADHM invariants are equivariant virtual invariants of moduli spaces of twisted cyclic representations of the ADHM quiver in the abelian category of coherent sheaves on a smooth complex projective curve. This talk will present a generalization of this construction employing a more general stability condition which depends on a real parameter. Two wallcrossing results are proved using Joyce's results on configurations in abelian categories and the theory of generalized Donaldson-Thomas invariants of Joyce and Song. Applications to local Pandharipande-Thomas theory and local Gopakumar-Vafa theory will be presented as well.

TALKS

Andrei Caldararu (University of Wisconsin, Madison, USA)

Title: A conjecture of Duflo and the Ext algebra of branes

Abstract: The Duflo theorem is a statement in Lie theory which allows us to compute the ring structure of the center of the universal enveloping algebra of a finite-dimensional Lie algebra. A categorical version of it was used by Maxim Kontsevich to give a spectacular proof of the so-called "Theorem on complex manifolds," which computes the multiplicative structure of Hochschild cohomology of a complex manifold in terms of the algebra of polyvector fields. In Lie theory there are also more general Duflo-type statements (mostly conjectural), which study the case of a pair (Lie algebra, Lie subalgebra) I will explain how these translate into conjectures about the multiplicative structure of the Ext-algebra of the structure sheaf of a complex submanifold of a complex manifold, and how from this interaction we can hope to gain new insights into both algebraic geometry and Lie theory. (Based on discussions with Damien Callaque).

Alexei Davydov (Macquarie University, Australia)

Title: Witt group of modular categories

Abstract: We describe an abelian group structure on the set of classes of modular categories modulo some equivalence relation. The resulting Witt group of modular categories resembles (and contains an image of) the Witt group of finite abelian groups with quadratic forms. The conjecture of Moore and Seiberg, that all RCFTs come from reductive groups via WZW, coset, and orbifold constructions, can be interpreted as a statement about generators of this Witt group.

Jurgen Fuchs (Karlstad Universitet, Sweden)

Title: Mapping class group representations and sewing constraints in conformal field theory

Abstract: Representations of mapping class groups of surfaces appear in conformal quantum field theory (CFT) through the monodromy properties of conformal blocks. Special elements in the spaces of conformal blocks describe local correlation functions of the CFT; they are singled out by consistency conditions known as sewing constraints. It turns out that such sewing constraints lead naturally to the existence of nontrivial endomorphisms of spaces of conformal blocks, implying the reducibility of the corresponding mapping class group representation. This observation can be used to prove some conjectures about the action of certain automorphisms of affine Lie algebras on conformal blocks that arise in conformal field theories with affine Lie symmetry. I will give an overview over these relationships and describe the relevant tools from conformal and topological field theory.

Elizabeth Gasparim (University of Edinburgh, UK)

Title: Level-rank Duality

Abstract: I will report on some new isomorphisms of moduli spaces and discuss equivalence of partition functions for supersymmetric gauge theories over certain local surfaces and Calabi-Yau threefolds.

Jonas Hartwig (IME-USP, São Paulo, Brazil)

Title: Twisted generalized Weyl algebras and Serre-type relations.

Abstract: The twisted generalized Weyl algebras (TGWAs) constitute a large class of algebras which include many interesting examples such as quantized Weyl algebras, Mickelsson step algebras, Gelfand-Zetlin algebras etc. We define the notion of a locally finite TGWA and show that one can associate a generalized Cartan matrix to any such algebra so that corresponding Serre-type relations hold. All examples studied so far, including the above mentioned, are "trivial" in the sense that they are of type A_1^n . We give a new construction which associates a locally finite TGWA to any symmetric generalized Cartan matrix. We conjecture that the corresponding quantum Serre relations give, together with other relations, a complete presentation of these algebras, and prove it in type A_2 . This gives a new link between TGWAs and (quantized) enveloping algebras of Lie algebras.

A. Amar Henni (SISSA, Italy)

Title: Monads for torsion-free sheaves on multi-blow-ups of the complex projective plane

Abstract: We construct monads for framed torsion-free sheaves with fixed Chern character on the multi-blow-ups of the complex projective plane. Using these monads we prove that the moduli space of such sheaves is a smooth algebraic variety. Moreover we construct monads for families of such sheaves parametrized by a reduced noetherian scheme S of finite type. A universal monad on the moduli space is introduced and used to prove that the moduli space is fine.

Eduardo Hoefel (UFPR, Brazil)

Title: OCHA via resolutions of operads.

Abstract: We will show that the OCHA operad is a resolution of the suboperad of top dimensional homology classes of the swiss-cheese operad. However, the resolution is not a "resolution of operads" but only a "resolution of modules over the L_∞ operad". The OCHA operad appears naturally in the first row of the E_1 term of the Spectral Sequence of the real Fulton-MacPherson compactification of the configuration space of points in the closed disc. That spectral sequence does not collapse at E_2 in general, nonetheless the resolution can be obtained by restricting attention to its first row.

Ludmil Katzarkov (University of Miami, USA)

Title: Homological Mirror Symmetry and Algebraic cycles

Abstract: In this talk we will look at some classical questions computing Griffiths groups and Bloch Conjecture from the point of view of Homological Mirror Symmetry.

Takashi Kimura (Boston University, USA)

Title: An equivariant Riemann-Hurwitz formula for families

Abstract: We present an equivariant Riemann-Hurwitz formula for families of complex curves with an effective action of a finite group. This formula has coefficients in the representation ring of the group. In the special case where the base of the family is a point and the group action is forgotten, this formula reduces to the ordinary Riemann-Hurwitz formula which relates the genus of a complex curve to the genus of the ramified cover. We describe the significance of this formula for equivariant cohomological field theory and orbifold Gromov-Witten theory. We also discuss some possible generalizations.

Johan Martens (UNICAMP, Campinas, Brazil)

Title: Moduli of parabolic Higgs bundles and Atiyah algebroids

Abstract: We will discuss the holomorphic Poisson geometry of the moduli space of (non-strongly) parabolic Higgs bundles over a Riemann surface with marked points in terms of Lie algebroids. Using the Hitchin system in this setting we will show how this provides a global analogue of the Grothendieck-Springer resolution. This is joint work with M. Logares.

Andrei Mironov (ITEP, Russia)

Title: Kontsevich-Hurwitz Generating Functions: Integrability and Matrix Models

Abstract: We discuss various properties of the Kontsevich-Hurwitz partition function which counts the weighted numbers of ramified coverings over the Riemann sphere and is, simultaneously, a generating function of Hodge integrals. Special emphasis is given to integrability and to matrix model representations.

Alexei Morozov (ITEP, Russia)

Title: Basics of Nonlinear Algebra

Abstract: Concise introduction to a relatively new subject of non-linear algebra: a part of algebraic geometry, which is a literal extension of text-book linear algebra to the case of non-linear equations and maps. This powerful science is based on the notions of discriminant (hyperdeterminant) and resultant, which today can be effectively studied both analytically and by modern computer facilities. We shall mostly consider various representations of resultants of non-linear maps. First steps will be described in direction of Mandelbrot-set theory, which is direct extension of the eigenvalue problem from linear algebra, and is related through renormalization group ideas to the theory of phase transitions and dualities.

Adriano Moura (UNICAMP, Brazil)

Title: On minimal affinizations of quantum affine algebras

Abstract: Let \mathfrak{g} be a finite-dimensional simple Lie algebra over the field of complex numbers. Consider its loop algebra \mathfrak{g} and the corresponding quantized enveloping algebras $U_q(\mathfrak{g})$ and $U_q(\tilde{\mathfrak{g}})$. It is known that, unless \mathfrak{g} is of type A, there is no quantum group analogue of the evaluation maps $\tilde{\mathfrak{g}} \rightarrow \mathfrak{g}$. In particular, the concept of evaluation representations is not available in the context of the quantum affine algebra $U_q(\tilde{\mathfrak{g}})$ in general. Chari and Pressley introduced and studied the concept of minimal affinizations which, in some sense, place the role of evaluation modules. We plan to discuss some new results concerning their characters.

Hossein Movasati (IMPA, Brazil)

Title: Moduli of polarized Hodge structures arising from mirror Calabi-Yau threefolds

Abstract: The moduli of polarized Hodge structures was introduced by P. Griffiths around 1970 in similarity with Poincaré upper half plane and Siegel domain and it is nowadays known as Griffiths or period domain. In this talk I explain the construction of a new period domain which lives on Griffiths domain and I reinterpret many classical facts, such as Griffiths transversality, in the context of this new domain. I mainly focus

on Hodge structures with Hodge numbers $h^{30}=h^{21}=h^{12}=h^{03}=1$. This class of Hodge structures appear in mathematical physics. The corresponding varieties are mirror dual to quintic Calabi-Yau threefolds. The final objective in this case is to construct the theory of quasi automorphic functions in such a way that it includes the generating series for counting virtual rational curves on generic quintic hypersurfaces introduced by physicists Candelas, de la Ossa, Green and Parkes in 1991.

Eric Ragoucy (LAPTH-CNRS, France)

Title: Nested Bethe ansatz for spin chains

Abstract: We present in a unified way the nested Bethe ansatz for spin chains based on $gl(n)$, $gl(m|n)$ and their deformations. We perform the ansatz for closed and open spin chains. In the case of open spin chains, we use diagonal boundary conditions. In all cases, we deduce a recursion formula, and a trace formula for Bethe vectors.

Henrique N. Sá Earp (Unicamp, Campinas, Brazil)

Title: Towards the moduli space of G2–instantons over Kovalev manifolds

Abstract: Compact G2–manifolds have drawn attention from physicists over this decade as plausible models for the 7 extra space-time dimensions predicted by M-theory. In this context, physics as we know it would assume the form of a gauge theory, which can be rather rigorously formulated in close analogy with the typical 4–dimensional case. The exceptional geometry expressed by the G2–structure induces a natural notion of G2–instantons, indeed critical points of the Yang-Mills functional, and one can meaningfully expect their moduli space to reveal interesting new invariants.

In 2002 A. Kovalev constructed original families of smooth compact G2–manifolds M by the ‘twisted’ gluing of products $W_i^3 \times S^1$, $i = 1, 2$, where each W_i^3 is an asymptotically cylindrical Calabi-Yau 3–fold. This opened a road map towards obtaining concrete moduli spaces of G2–instantons associated to holomorphic bundles $E \rightarrow M$, parameterised by certain ‘asymptotic stability’ constraints.

I will briefly introduce this construction and the relevant notions of higher-dimensional gauge theory, then explain how the first milestone in this long-term programme - namely the existence of ‘half’ of the first G2–instanton yet - has been reached in my thesis, following the analytical methods of S. Donaldson, C. Simpson et al.

Alexander Schmitt (Freie Universität Berlin, Germany)

Title: Decorated Principal Bundles

Abstract: Given a reductive linear algebraic group G (say, over the complex numbers) and a representation r of G on the finite dimensional \mathbf{C} -vector space V , we get G -actions on the affine space V and the projective space $\mathbf{P}(V)$. Geometric Invariant Theory grants the existence of the categorical quotient $V//G$ as affine algebraic variety and gives an open subset $\mathbf{P}(V)^{ss}$ of $\mathbf{P}(V)$ of semistable points, such that the categorical quotient $\mathbf{P}(V)^{ss}//G$ exists as projective variety. Now, suppose we are also given a polarized projective base manifold $(X, \mathcal{O}_X(I))$ and a line bundle L on X .

We study the problem of classifying pairs (E, s) which consist of a principal G -bundle E on X and a section s of the vector bundle with fiber V that is associated to E via r . Gauge theory gives a notion of semistability for such pairs.

Recently, the speaker formulated an algebraic version of this semistability condition and proved the existence of quasi-projective moduli spaces for the semistable objects. They are equipped with a projective morphism to an affine variety, the generalized Hitchin map. This result may be considered as a version of Geometric Invariant Theory relative to the base variety X . Examples include Bradlow pairs, Higgs bundles, and ADHM sheaves.

Eric Sharpe (Virginia Tech, USA)

Title: A-twisted Landau-Ginzburg models, gerbes, and Kuznetsov's homological projective duality

Abstract: In this talk we will summarize several recent developments related to Landau-Ginzburg models. We will begin by describing how one A-twists a Landau-Ginzburg model in physics, and how a physical process known as *renormalization group flow* sometimes identifies correlation functions in such A-twisted Landau-Ginzburg models with ordinary Gromov-Witten invariants. Then, after briefly reviewing how one associates a CFT to a gerbe and associated technical issues, we will outline some applications of CFT's of gerbes, including Landau-Ginzburg (Toda) mirrors to gerbes on projective spaces, and a physical realization of Kuznetsov's homological projective duality, which provides examples of LG's and GLSM's describing non-birational spaces on the same GLSM Kahler moduli space, as well as examples in which the geometry arises via nonperturbative effects, rather than the usual complete intersection story.

Francesco Toppan (CBPF, Rio da Janeiro, Brazil)

Title: Supersymmetric Extensions of the Hopf maps.

Abstract: The supersymmetric extensions of the Hopf maps will be discussed. Non-linear representations of supersymmetry are induced and 1D supersymmetric invariant sigma models whose bosonic target coordinates parameterize the Hopf spheres are constructed. For the first Hopf map we obtain four $N=4$, 1D sigma models and their mutual relations: they are given by

- (I) the (4,4) linear "root" supermultiplet (supersymmetric extension of \mathbf{R}^4),
- (II) the (3,4,1) linear supermultiplet (supersymmetric extension of \mathbf{R}^3),
- (III) the (3,4,1) non-linear supermultiplet living on S^3 ,
- (IV) the (2,4,2) non-linear supermultiplet living on S^2 .

The (I) \rightarrow (II) map is the supersymmetric extension of the $\mathbf{R}^4 \rightarrow \mathbf{R}^3$ bilinear map, while the (II) \rightarrow (IV) map is the supersymmetric extension of the $S^3 \rightarrow S^2$ Hopf fibration. The restrictions on the S^3 , S^2 spheres are expressed in terms of the stereographic projections.

In a related result, the extension of the Schur lemma to minimal linear supermultiplets up to $N=8$ is presented.

Bernardo Uribe (Universidad de los Andes, Colombia)

Title: Chern-Weil homomorphism in twisted equivariant cohomology

Abstract: I will describe how the equivariant De Rham theorem is generalized to the twisted case and I will recall the relation between the twisted equivariant cohomology and the equivariant cohomology of Courant algebroids. In the case that the group is not

compact I will describe a relation between actions of groups on exact Courant algebroids and cohomology classes of a model of equivariant cohomology defined by Getzler.

Dmitri Vassilevich (UFABC, Brazil)

Title: Diffeomorphisms in noncommutative gravity

Abstract: After a short discussion of various proposals for realization of diffeomorphisms on noncommutative spaces I introduce a covariant star product and use it to construct a fully diffeomorphism invariant noncommutative gravities in two dimensions (which is the first noncommutative gravity on the market which is invariant under general coordinate transformations). One model in the family appears to be intergrable. I conclude with discussing properties of the geometries which arise in this context.

Katrin Wendland (Universitat Augsburg, Germany)

Title: How to attach gauge theory data to elliptic fibrations

Abstract: We investigate how to attach gauge theory data to certain elliptically fibered Calabi-Yau threefolds by means of string-string duality. We discuss various intrinsic geometric consequences of the conjectured duality, some of which can be proved within geometry.