

### THERMO\_PW Tutorial (v.2.0.0)

Andrea Dal Corso (SISSA - Trieste)

### Contents

	0.1 People	5
	0.2 Overview	6
1	Howtos	8
	1.1 How do I make a self-consistent calculation?	8
	1.2 How do I plot the band structure?	9
	1.3 How do I plot the electronic density of states?	10
	1.4 How can I see the crystal structure?	11
	1.5 How can I see the Brillouin zone?	12
	1.6 How can I plot the X-ray powder diffraction spectrum?	13
	1.7 How can I find the space group of my crystal?	14
	1.8 How do I plot the phonon dispersions?	15
	1.9 How do I calculate the vibrational energy, free energy, en-	
	tropy, and heat capacity?	16
	1.10 How do I calculate the atomic B-factor of a solid?	17
	1.11 How do I calculate the elastic constants?	18
	1.12 How do I calculate the Debye temperature?	19
	1.13 How do I calculate the equilibrium structure?	20
	1.14 How do I calculate the equilibrium structure at a given	
	pressure?	21
	1.15 How do I specify the temperature range	22
	1.16 How do I calculate the crystal parameters as a function of	
	temperature?	23
	1.17 How do I calculate the thermal expansion?	24
	1.18How do I calculate the Helmholtz (or Gibbs) free energy as	
	a function of temperature keeping into account the ther-	
	mal expansion?	25
	1.19 How do I calculate the bulk modulus as a function of tem-	• •
	perature?	26
	1.20 How do I calculate the isobaric heat capacity?	27
	1.21 How do I calculate the elastic constants as a function of	00
	temperature?	28
	1.22 How do I calculate the electronic heat capacity of a metal?	29

1.23 How do I calculate the thermal expansion of a metal ac-		
counting for the free energy due to the electronic excita-		
tions?		
1.24 How do I calculate the elastic constants of a metal ac- counting for the free energy due to the electronic excita-		
tions?		
1.25 How do I calculate the temperature dependent elastic con-		
stants of a solid relaxing the internal degree of freedom? . 32		
1.26 How do I calculate the frequency dependent dielectric con-		
stant of a material?		
1.27 How do I calculate the frequency dependent reflectivity of		
a material?		
1.28Which is the meaning of the colors in the electronic bands		
and phonon dispersions plots?		
1.29 How do I specify a custom path for plotting the electronic		
bands and phonon dispersions?		
1.30 How do I specify the plane of <b>k</b> -points when I set the flag		
q2d=.TRUE.?		

### Introduction

This guide gives a brief overview of the THERMO\_PW package. It gives you the minimum information needed to accomplish a given task. It assumes that you have already downloaded and compiled QUANTUM ESPRESSO and THERMO\_PW and you want to calculate some material property but you do not know where to start. The most complete reference for the use of THERMO\_PW is its user's guide. However the capabilities of the code are rapidly expanding and the user's guide is getting long so, if you do not want to read it entirely, here you find where to search the relevant information. There might be some limitations on the type of functionals that you can use or the electronic structure scheme available to compute a given quantity. Please refer to the QUANTUM ESPRESSO documentation for this.

#### 0.1 People

This tutorial has been written by Andrea Dal Corso (SISSA - Trieste).

#### 0.2 Overview

In order to make a calculation with THERMO\_PW you need an input file for the pw.x code of QUANTUM ESPRESSO. This input file requires mainly five information:

- The Bravais lattice.
- The position of the atoms inside the unit cell.
- The type of atoms and the pseudopotentials files that you want to use.
- The cut-off energies.
- The  $\mathbf{k}$ -point mesh used to make integration over the Brillouin zone. The smearing parameter for metals.

The Bravais lattice is specified by an integer number ibrav and by the crystal parameters celldm (up to six numbers). The ibrav codes and the required crystal parameters are described in the file PW/Doc/INPUT\_PW of the QUANTUM ESPRESSO distribution. In QUAN-TUM ESPRESSO you can use ibrav=0 and give the primitive lattice vectors of the Bravais lattice. Presently THERMO PW needs to know the Bravais lattice number so this form of input is not recommended. If you use it, THERMO PW writes on output ibrav, celldm and the atomic coordinates needed to simulate the same cell and stops. You can just cut and paste these quantities in the input of pw.x or you can set the flag find ibrav=.TRUE. in the THERMO PW input and THERMO PW will make the conversion for you before runnig the job. After setting the correct ibrav and celldm, THERMO\_PW might still tell you that the Bravais lattice is not compatible with the point group. This can happen, for instance, if you have isolated molecules, amorphous solids, defects, or supercells. In these cases you can still continue but symmetry will not be used to reduce the number of components of the physical quantities tensors. In order to use the residual symmetry, you have to use one of the suggested ibrav, adjusting the celldm to the parameters of your cell. For instance if you have a cubic cell, but the symmetry requires a tetragonal lattice, you have to use a tetragonal lattice with celldm(3)=1.0. In rare cases, with lattices such as the face-centered orthorhombic some symmetry operations might be incompatible with the FFT grid found by pw.x. The choice made in QUANTUM ESPRESSO is to discard these symmetries making the lattice incompatible with the point group. In these cases the code needs nr1=nr2=nr3. Set these three parameters in the pw.x input equal to the largest one.

The positions of the atoms inside the unit cell are defined by an integer number nat (the number of atoms) and by nat three-dimensional vectors as explained in the file PW/Doc/INPUT\_PW. You can use several units, give the coordinates in the Cartesian or in the crystal basis or you can give the space group number and the crystal coordinates of the nonequivalent atoms. Note that in centered lattices the crystal basis is the conventional one when using the space group number and the primitive one when not using it. These options are supported by THERMO\_PW. See the pw.x manual for details.

The number of different types of atoms is given by an integer number ntyp and for each atomic type you need to specify a pseudopotential file. Pseudopotential files depend on the exchange and correlation functional and can be found in many different places. There is a pseudopotential page in the QUANTUM ESPRESSO website, or you can consider generating your pseudopotentials with the pslibrary inputs. You can consult the web page

https://dalcorso.github.io/pslibrary/

for more information.

The kinetic energies cut-offs depend on the pseudopotentials and on the accuracy of your calculation. You can find some hints about the required cut-offs inside the pseudopotentials files, but you need to check the convergence of your results with the cut-off energies. Many tests of the kinetic energy cut-offs can be found also at https://www.materials cloud.org/discover/sssp/.

The **k**-point mesh is given by three integer numbers and possible shifts (0 or 1) in the three directions. The convergence of the results with this mesh should be tested. For metals you have also to specify a smearing method (for instance occupations='smearing', smearing='mp') and a value of the smearing parameter (in Ry) (see the variable degauss in the file PW/Doc/INPUT\_PW). Note that the convergence with respect to the **k**-points depends on the value of degauss and must be tested for each degauss.

Once you have an input for pw.x, in order to run THERMO\_PW you have to write a file called thermo\_control that contains a namelist called INPUT\_THERMO. This namelist contains a keyword what that controls the calculation performed by THERMO\_PW. Ideally you need to set only what and call thermo\_pw.x instead of pw.x, giving as input the input prepared for pw.x. In practice each what is controlled by several variables described in the user's guide. These variables have default values that are usually sufficient to give a first estimate of a given quantity but that must be fine tuned to obtain publication quality results.

#### Chapter

### Howtos

#### 1.1 How do I make a self-consistent calculation?

Use what='scf'. See example01. The calculation is equivalent to a call to pw.x and is controlled by its input. In particular in the input of pw.x you can choose a single self-consistent calculation using calculation='scf', an atomic relaxation using calculation='relax', or a cell relaxation using calculation='vc-relax'. The use of calculation='nscf' and calculation ='bands' is not supported by THERMO\_PW and could give unpredictable results. There is no advantage to use THERMO\_PW to do a molecular dynamic calculation.

#### **1.2 How do I plot the band structure?**

Use what='scf\_bands'. See example02. With this option THERMO\_PW calls pw.x twice, making first a self-consistent calculation with the parameters given in the pw.x input and then a band calculation along a path chosen by THERMO\_PW, or along a path given by the user after the INPUT\_THERMO namelist. In this case the path is given as in the pw.x input (see the user's guide for additional details). There are a few parameters that you can give in the INPUT\_THERMO namelist to control the band plot. The most useful are emin\_input and emax\_input that allow you to plot the bands in a given energy range. At the end of the run, the figure of the bands is in a file called by default output\_band.ps, a name that can be changed in the THERMO\_PW input. Check also the option what='scf\_2d\_bands' to plot the surface band structure.

### **1.3 How do I plot the electronic density of states?**

Use what='scf\_dos'. See example18. With this option THERMO\_PW calls pw.x twice, making first a self-consistent calculation followed by a non self-consistent calculation on a uniform **k**-point mesh. This mesh can be specified in the THERMO\_PW input (if none is given THERMO\_PW uses the default values). At the end of the run, the figure of the density of states is in a file called by default output\_eldos.ps, a name that can be changed in the THERMO\_PW input.

#### **1.4 How can I see the crystal structure?**

THERMO\_PW is not a crystal structure viewer, but you can use the XCrySDen code, which reads the pw.x input, to see the crystal structure. If you use what='plot\_bz', THERMO\_PW produces a .xsf file with the input structure that can be read by XCrySDen. This could be useful when you give the space group number and the nonequivalent atomic positions since this input is presently not readable by XCrySDen. The generated .xsf file contains all the symmetry equivalent atomic positions. For the same purpose you could also use the output of pw.x.

#### **1.5 How can I see the Brillouin zone?**

Use what='plot\_bz'. See example12. With this option THERMO\_PW does not call pw.x, it just produces a script for the asymptote code with the instructions to plot the Brillouin zone and the standard path (or the path that you have given in the THERMO\_PW input).

#### **1.6 How can I plot the X-ray powder diffrac**tion spectrum?

Use what='plot\_bz' to see the spectrum corresponding to the geometry given in the pw.x input. You can also see the spectrum corresponding to a relaxed structure using for instance what='scf', asking for an atomic (cell) relaxation in the pw.x input and using lxrdp=.TRUE. variable in the THERMO\_PW input. The X-ray powder diffraction spectrum is shown in a file called by default output\_xrdp.ps, a name that can be changed in the THERMO\_PW input. The scattering angles and intensities are also written in a file called by default output\_xrdp.dat, which can also be changed in the THERMO\_PW input.

# 1.7 How can I find the space group of my crystal?

Use what='plot\_bz' and look at the output. The space group is identified. In the case you have a structure with ibrav=0 and the primitive lattice vectors use the option find\_ibrav=.TRUE. in the THERMO\_PW input (see the THERMO\_PW user's guide in the subsection *Coordinates and structure*). This option has presently some limitations. It does not work for noncollinear magnetic system, or for supercells, or when the Bravais lattice and the point group are incompatible.

#### **1.8 How do I plot the phonon dispersions?**

Use what='scf\_disp'. See example04. In this case you have to prepare an input for the ph.x code that must be called ph\_control. The required information in this input is the **q**-point mesh on which the dynamical matrices are computed and the name of the files where the dynamical matrices are written. See the ph.x guide if you need information on this point. At the end of the run, the phonon dispersions are found in a file called by default output\_disp.gl.ps, a name that can be changed in the THERMO\_PW input. The vibrational density of states is found in a file called by default output\_dos.gl.ps, which can also be changed in the THERMO\_PW input.

# **1.9 How do I calculate the vibrational energy,** free energy, entropy, and heat capacity?

Use what='scf\_disp'. See example04. These quantities are calculated after the phonon dispersion for the default temperature range (1 K - 800 K) or for the range given in the THERMO\_PW input. The figure of these quantities is in the file output\_therm.gl.ps. Note that they are calculated at the geometry given in the input of pw.x.

#### **1.10 How do I calculate the atomic B-factor of a solid?**

Use what='scf\_disp' as in example04 and add the flag with\_eigen=.TRUE.. These quantities are calculated after the phonon dispersions for the default temperature range (1 K - 800 K) or for the range given in the THERMO\_PW input. The figure of these quantities is in the file output\_therm.gl\_dw.p Note that they are calculated at the geometry given in the input of pw.x.

#### **1.11 How do I calculate the elastic constants?**

Use what='scf\_elastic\_constants'. See example13. The elastic constants appear in the output of THERMO\_PW and also in a file called by default output\_el\_cons.dat, a name that can be changed in the THERMO\_PW input. This file can be read by THERMO\_PW for the options that require the knowledge of the elastic constants.

#### **1.12 How do I calculate the Debye tempera**ture?

Use what='scf\_elastic\_constants'. See example13. The Debye temperature appears in the output of THERMO\_PW. A file called output\_therm\_debye.gl.ps contains plots of the vibrational energy, free energy, entropy, and heat capacity computed within the Debye model.

#### **1.13 How do I calculate the equilibrium struc**ture?

Use what='mur lc'. See example05 for the cubic case and refer to the user's guide for anisotropic solids. The crystal parameters are written in the THERMO PW output file. Note that the structure is searched interpolating with a polynomial or with the Murnaghan equation the energy calculated for several geometries close to the geometry given in the input of pw.x so the closer this structure to the actual equilibrium structure the better the fit and the closer the value found by THERMO\_PW to the real minimum. You can check on the file output\_mur.ps (when lmur=.true.) or output\_energy.ps (when lmur=.false.) if the minimum is within the range of calculated structures. If it is not, the calculated minimum is probably inaccurate and it is better to repeat the calculation starting from it. Note also that almost all options can be specified using what='mur\_lc\_...' instead of what='scf\_...'. In this case the calculations are carried out at the equilibrium geometry instead of the geometry given in the pw.x input. Setting a finite pressure in the THERMO\_PW input, the equilibrium geometry is the one at the given pressure (see below) and the calculations are carried out at this geometry.

#### **1.14 How do I calculate the equilibrium struc**ture at a given pressure?

Use what='mur\_lc' and specify pressure=value in kbar in the THERMO\_PW input. Note that in the input of pw.x you should give a geometry which is as close as possible to the equilibrium value found at the given pressure (see above).

#### 1.15 How do I specify the temperature range

See the subsection *Temperature and Pressure* in the THERMO\_PW user's guide.

#### **1.16 How do I calculate the crystal parameters as a function of temperature?**

Use what='mur\_lc\_t'. See example09. Note that for this option you need to give also the ph.x input. For anisotropic solids using lmurn=.TRUE. you calculate the volume as a function of temperature varying celldm(1) but all the other crystal parameters are kept constant, while using lmurn=.FALSE. you can calculate all the crystal parameters as a function of temperature. The crystal parameters are plotted as a function of temperature in the standard range (T = 1 K - T =800 K) or in the range requested in input, in the file output\_anharm.ps, a name that can be changed in the THERMO\_PW input. Presently no temperature dependence is calculated for the atomic coordinates, so the present approach is applicable to solids in which equilibrium atomic positions are fixed by symmetry, while it is an approximation in the other cases.

#### 1.17 How do I calculate the thermal expansion?

Use what='mur\_lc\_t'. See example09. The components of the thermal expansion tensor are shown as a function of temperature in the file output\_anharm.ps, a name that can be changed in the THERMO\_PW input.

#### 1.18 How do I calculate the Helmholtz (or Gibbs) free energy as a function of temperature keeping into account the thermal expansion?

Use what='mur\_lc\_t'. The Helmholtz (or Gibbs at finite pressure) free energy is shown as a function of temperature in the file output\_anharm.ps. Note that the absolute value of this energy depends on the pseudopotentials as the total energy. You can however compare the free energies for different crystal structures and predict if a phase transition occurs and at which temperature (also as a function of pressure).

# **1.19 How do I calculate the bulk modulus as a function of temperature?**

Use what='mur\_lc\_t' and the option lmurn=.TRUE.. This approach is rigorously valid only for cubic solids, for anisotropic solid it is an approximation in which only celldm(1) is changed while the other crystal parameters are kept constant. For the general case, see the elastic constants as a function of temperature. This calculation gives also the bulk modulus as a function of temperature.

#### **1.20 How do I calculate the isobaric heat capacity?**

Use what='mur\_lc\_t' and the option lmurn=.TRUE.. This approach is rigorously valid only for cubic solids, for anisotropic solid it is an approximation in which only celldm(1) is changed while the other crystal parameters are kept constant. For the general case, see the elastic constants as a function of temperature. This calculation gives also the isobaric heat capacity as a function of temperature.

# **1.21 How do I calculate the elastic constants as a function of temperature?**

There are two ways. Both of them are a two step calculation. The fastest, but less accurate, method uses the "quasi-static" approximation. First use what='elastic\_constants\_geo'. This option computes the elastic constants at T = 0 K for all the geometries used by what='mur\_lc\_t' and saves them in the directory elastic\_constants. In the second step, using the same input, run again THERMO\_PW with what='mur\_lc\_t' after copying in your working directory the directory elastic\_constants obtained in the previous step. Elastic constants are read only when lmurn=.FALSE.. The "quasi-static" approximation means that the code interpolates the T = 0 K elastic constants found in the first step at the geometry that minimizes the Helmholtz (or Gibbs at finite pressure) free energy at temperature T.

The second method uses the "quasi-harmonic" approximation and requires many phonon calculations at many geometries. First use what= 'elastic\_constants\_geo' and set use\_free\_energy= .TRUE.. This option computes the temperature dependent elastic constants taking as unperturbed geometries those used by what='mur\_lc\_t' and saves them in the directory anhar\_files. In the second step, using the same input, run again THERMO\_PW with what= 'mur\_lc\_t' after copying in your working directory the directory anhar\_files obtained in the previous step. Elastic constants are read only when lmurn= .FALSE.. The "quasi-harmonic" approximation means that the code interpolates the temperature dependent elastic constants found in the first step as second derivatives of the Helmholtz free energy (corrected to give the stress-strain elastic constants), at the geometry that minimizes the Helmholtz free energy.

The plot of the elastic constants will be found in a file whose default name is output\_anhar.el\_cons.ps. Note that when the elastic constant are available in a file, with the option lmurn=.FALSE. the thermodynamic properties are calculated also for non-cubic solids. The main approximation of the present implementation is that the atomic coordinates are relaxed only at T = 0 K minimizing the energy and the free energy is not calculated as a function of the atomic positions. Please be careful to use the same tmin, tmax, and deltat in these calculations. Only a tmax larger in the what='elastic\_constants\_geo' calculation than in what='mur\_lc\_t' is allowed.

### **1.22** How do I calculate the electronic heat capacity of a metal?

Use what='scf\_dos'. See example18. In the metallic case in addition to a plot of the density of states this option produces also a plot of the electronic energy, free energy, entropy, heat capacity, and chemical potential in the standard temperature range (T = 1 K - T = 800 K) or in the range requested in input. These quantities are found in the file called by default output\_eltherm.ps, a name that can be changed in the THERMO\_PW input. Please be careful with the value of deltae or the electronic thermodynamic properties could be wrong at low temperatures.

#### 1.23 How do I calculate the thermal expansion of a metal accounting for the free energy due to the electronic excitations?

This is a two step calculation. First use what='mur\_lc' and the flag lel\_free\_energy=.TRUE. to write on files the electronic thermodynamic properties for each geometry used by what='mur\_lc'. Then copy the files with the electronic thermodynamic properties in the therm\_files directory and run THERMO\_PW with what='mur\_lc\_t' and the flag lel\_free\_energy=.TRUE.. The electronic free energy is read from files and added to the vibrational free energy before computing the equilibrium crystal structure. See example24 for this case.

#### **1.24** How do I calculate the elastic constants of a metal accounting for the free energy due to the electronic excitations?

This is a four step calculation. First use what='mur\_lc' and the flag lel\_ free\_energy=.TRUE. as in the previous point in order to get the electronic thermodynamic properties for all the unperturbed geometries. Then use what='elastic\_constants\_geo' with the flag lel\_free \_energy=.TRUE. and the flag use\_free\_energy=.FALSE. to write on file the electronic thermodynamic properties for all strained geometries. Then use what='elastic\_constants\_geo' with lel\_free \_energy=.TRUE. and use\_free\_energy=.TRUE. after copying the electronic thermodynamic properties obtained at previous point in the therm\_files directory, so the free energy=.TRUE. and use\_free\_energy=.TRUE. TRUE. and use\_free\_energy=.TRUE. after copying the electronic contribution. Finally use what='mur\_lc\_t' with lel\_free\_energy=.TRUE. and use\_free\_energy=.TRUE. after copying the electronic stants obtained at previous point in the elastic constants has also the electronic contribution. Finally use what='mur\_lc\_t' with lel\_free\_energy=.TRUE. and use\_free\_energy=.TRUE. after copying the elastic constants obtained at previous step in the anhar\_files directory and the electronic thermodynamic properties obtained at previous step in the first step in the therm\_files directory.

#### 1.25 How do I calculate the temperature dependent elastic constants of a solid relaxing the internal degree of freedom?

This is possible only within the ZSISA approximation. This means that for each value of the crystal parameters the ions are relaxed (with frozen\_ions=.FALSE.) to their T = 0 K equilibrium positions. The phonons (and the free energy) are computed only in this configuration. For solids in which internal relaxations are described by one degree of freedom, it is possible to compute the phonons (and the free energy) for several values of this parameter. The free energy as a function of this parameter is then minimized at each strain and each temperature (See the parameter stype in what='scf\_elastic\_constants').

#### **1.26 How do I calculate the frequency dependent dielectric constant of a material?**

Use what='scf\_ph'. See example16 and example20. Please note that this calculation is made by the extended version of the phonon code available in THERMO\_PW, so you have to provide specific flags to the phonon input. See the user's guide for details. The dielectric constant is in the file output\_epsilon.ps. Note that for metals you need to specify a finite wave-vector **q** and you cannot plot the dielectric constant when **q** is the  $\Gamma$  point. See example17 and example21 for this case.

# **1.27** How do I calculate the frequency dependent reflectivity of a material?

The calculation of this quantity is implemented only for isotropic (cubic) solids. It is plotted together with the frequency dependent dielectric constant in insulating cubic solids when  $\mathbf{q}$  is the  $\Gamma$  point. In this case the code plots also the absorption coefficient. Both quantities can be plotted as a function of the frequency or of the wavelength.

# 1.28 Which is the meaning of the colors in the electronic bands and phonon dispersions plots?

Different colors correspond to different irreducible representations of the point co-group of the **k** or **q** wavevector. To see the correspondence color-representation see the point\_groups.pdf manual. The point group used for each **k** or **q** point is written in the THERMO\_PW output and also in the plot if you set enhance\_plot=.TRUE.. In the output you can also find, close to each band energy or phonon frequency value, the name of the irreducible representation. Relevant character tables are given in the point\_groups.pdf manual, in the THERMO\_PW output, or by the crystal\_point\_group.x tool.

#### **1.29 How do I specify a custom path for plotting the electronic bands and phonon dispersions?**

You have to give an explicit path after the INPUT\_THERMO namelist (see what='scf\_bands' and what='scf\_2d\_bands'). There are two possibilities to specify a path in the Brillouin zone. You can use explicit coordinates of the first and last points of each line (in cartesian or crystal coordinates) and a weight that indicates the number of points in the line, or you can use labels to indicate the first and/or the last points. The available labels depend on the Brillouin zone and are illustrated in the file Doc/brillouin\_zones.pdf in the main QE documentation. Labels for points given by explicit coordinates can be added after the weights. Usually, the same label that indicates the coordinates of the point is written in the dispersion plot, but it is also possible to override this label, adding a letter after the weight as for the points given by explicit coordinates. Note that labels are given without quotes, while the additional letter after the weight must be written between single quotes. An example of a path for the fcc lattice:

7 gG 20 X 20 W 20 'gS1' 1.0 1.0 0.0 20 'X' gG 0 gG 20 L 0

This path has five lines, the first from  $\Gamma$  to X with 20 points, the second from X to W (that in the plot will have the label  $\Sigma_1$ ) with 20 points, the third from W to the point of coordinates (1.0, 1.0, 0.0) (that in the plot will have the label X), the fourth from (1.0, 1.0, 0.0) to  $\Gamma$  with 20 points and a final line from  $\Gamma$  to L with 20 points. Until version 1.8.1 labels are read as three characters, so it is important not to write the weights too close to the labels. Grüneisen parameters are not analytic at the  $\Gamma$  point so to obtain a correct plot, the  $\Gamma$  label in the middle of a plot must be repeated twice, as in the example.

#### 1.30 How do I specify the plane of k-points when I set the flag q2d=.TRUE.?

In this case you must give three **k** points:  $\mathbf{k}_1$ ,  $\mathbf{k}_2$ , and  $\mathbf{k}_3$  after the INPUT\_THERMO namelist.  $\mathbf{k}_1$  is the origin and the two vectors  $\mathbf{a}_1 = \mathbf{k}_2 - \mathbf{k}_1$  and  $\mathbf{a}_2 = \mathbf{k}_3 - \mathbf{k}_1$  determine the plane

$$\mathbf{k} = \mathbf{k}_1 + \lambda \mathbf{a}_1 + \mu \mathbf{a}_2, \tag{1.1}$$

where  $0 \le \lambda \le 1$ ,  $0 \le \mu \le 1$ . The interval of  $\lambda$  is divided into  $n_1$  points, while  $\mu$  is divided  $n_2$  points, where  $n_1$  and  $n_2$  are the weights of the points  $k_2$  and  $k_3$ . An example of the input for determining the plane is: 3

k\_{x,1} k\_{y,1} k\_{z,1} n0 k\_{x,2} k\_{y,2} k\_{z,2} n1 k\_{x,3} k\_{y,3} k\_{z,3} n2

where the **k** points are given in cartesian coordinates in units of  $\frac{2\pi}{a}$  (*a* is the lattice constant celldm(1) or in crystal coordinates using q\_in\_cryst\_coord=.TRUE. in the INPUT\_THERMO namelist.  $n_0$  is not used and can be set to any integer value.