SEVENTH FRAMEWORK PROGRAMME

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Annex I - "Description of Work"

Project acronym: QuaDynEvoPro

Project full title: Quasistatic and Dynamic Evolution Problems in Plasticity and Fracture

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Principal Investigator: Gianni Dal Maso

Host Institution: Scuola Internazionale Superiore di Studi Avanzati

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QuaDynEvoPro

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Principal Investigator:
Gianni Dal Maso

Host Institution:
Scuola Internazionale Superiore di Studi Avanzati (SISSA)

Proposal Title:
Quasistatic and Dynamic Evolution Problems in Plasticity and Fracture

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QuaDynEvoPro

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Summary

This research project deals with *nonlinear evolution problems* that arise in the study of the *inelastic behaviour* of *solids*, in particular in *plasticity* and *fracture*. The project will focus on selected problems, grouped into three main topics, namely:

1. *Plasticity with hardening and softening,*
2. *Quasistatic crack growth,*
3. *Dynamic fracture mechanics.*

The analysis of the models of these mechanical problems leads to deep mathematical questions originated by two common features: the energies are *not convex* and the solutions exhibit *discontinuities* both with respect to space and time. In addition, plasticity problems often lead to *concentration* of the strains, whose mathematical description requires *singular measures.* Most of these problems have a *variational structure* and are governed by *partial differential equations*. Therefore, the construction of consistent models and their analysis need advanced mathematical tools from the *calculus of variations,* from *measure theory* and *geometric measure theory,* and also from the theory of *nonlinear elliptic* and *parabolic partial differential equations.* The models of dynamic crack growth considered in the project also need results from the theory of *linear hyperbolic equations.*

Our goal is to develop new mathematical tools in these areas for the study of the selected problems. Quasistatic evolution problems in plasticity with hardening and softening will be studied through a *vanishing viscosity* approach, that has been successfully used by the P.I. in the study of the Cam-Clay model in soil mechanics. Quasistatic models of crack growth will be developed under different assumptions on the elastic response of the material and on the mechanisms of crack formation. For the problem of crack growth in the dynamic regime our aim is to develop a model that predicts the *crack path* as well as the *time evolution* of the crack along its path, taking into account all inertial effects.

Contents

Section 1: The Principal Investigator

a. Scientific Leadership Profile

The research area of Gianni Dal Maso is the Calculus of Variations. Most of the results of the first part of his career are devoted to Γ-convergence (54 publications). He is author of the first book on this subject: An introduction to Γ-convergence, *Birkhäuser*, Boston, 1993, cited 595 times according to MathSciNet.

Content and impact of major scientific contributions. Only the major contributions before 2001 are considered here. See "Top 10 publications" in part c for the last ten years.

Manuscripta Math. **30** (1980), 387-416, studies the lower semicontinuous extension to $BV(\Omega)$ of the integral $\int_\Omega f(x,u(x),\nabla u(x))dx$. The techniques introduced here are a basic tool for many other lower semicontinuity and relaxation results on $BV(\Omega)$ (citations: ISI 74, MathSciNet 41).

J. Anal. Math. **37** (1980), 145-185, with Buttazzo, contains a compactness result for a wide class of integral functionals. It is the first paper where the integral representation of the Γ-limit is obtained without assuming convexity or equi-Lipschitz continuity (citations: ISI 52, MathSciNet 18).

Appl. Math. Optim. **15** (1987), 15-63, with Mosco, is the first self-contained presentation of the notion of relaxed Dirichlet problems, which represents the completion, under Γ-convergence, of the Dirichlet problems in perforated domains (citations: ISI 88, MathSciNet 39). These results have also been used in *Appl. Math. Optim.* **23** (1991), 17-49, with Buttazzo, which contains a relaxed formulation of some shape optimization problems for elliptic equations with Dirichlet boundary conditions (citations: ISI 53, MathSciNet 38).

Nonlinear Anal. **18** (1992), 481-496, with Acerbi, Chiadò Piat, and Percivale, solves the homogenization problem for elliptic equations with Neumann boundary conditions in an arbitrary periodic connected domain with holes (citations: ISI 70, MathSciNet 46).

Acta Math. **168** (1992), 89-151, with Morel and Solimini, proves the solvability of the Mumford-Shah problem in dimension two and derives some properties of the solutions (citations: ISI 65, MathSciNet 40).

Arch. Ration. Mech. Anal. **122** (1993), 183-195, with Buttazzo, proves the existence of solutions of some shape optimization problems with prescribed volume, when a suitable monotonicity condition is satisfied (citations: ISI 52, MathSciNet 36).

Arch. Ration. Mech. Anal. **139** (1997), 201-238, with L. Ambrosio and A. Coscia, studies the fine properties of *BD* functions, whose symmetric gradient is a measure (citations: ISI 46, MathSciNet 38).

J. Math. Pures Appl. **74** (1995), 483-548, with Le Floch and Murat, studies the stability properties of a suitable definition of the product $g(u)u'$ when $g\colon \mathbb{R}^n \to \mathbb{R}^n$ is locally bounded and $u\colon (a,b) \to \mathbb{R}^n$ has bounded variation. These results have been widely applied to nonlinear hyperbolic systems (citations: ISI 163, MathSciNet 110).

Arch. Ration. Mech. Anal. **146** (1999), 23-58, with Braides and Garroni, shows that an elasticity model allowing for softening and fracture in dimension one can be obtained as limit of discrete models subject to nearest-neighbour nonlinear interactions (citations: ISI 57, MathSciNet 32).

International recognition. The scientific prizes are mentioned in the CV in part b. As for the publications, Gianni Dal Maso has written 117 articles in peer-reviewed scientific journals, 11 articles in conference proceedings, 5 chapters in collective volumes and 3 chapters in encyclopedias. According to MathSciNet his publications have been cited 2359 times by 1141 authors. According to ISI Web of Knowledge 87 publications of Gianni Dal Maso have been cited 1884 times (h-index 25, average citations per item 21.66).

Efforts and ability to inspire younger researchers. Gianni Dal Maso has been advisor of 30 SISSA students. One (Bellettini) is full professor in Italy, one (Notarantonio) is professor in Mexico, 9 (Defranceschi, Balzano, Chiadò Piat, Vitali, De Cicco, Amar, Coscia, Garroni, Morini) are associate professors in Italy, 8 (Paderni, Malusa, Toader, Leone, Mora, Negri, Giacomini, Ponsiglione) are assistant professors in Italy, 7 (Zanini, Barchiesi, Cagnetti, Scardia, Fiaschi, Lazzaroni, Solombrino, Morandotti) are post-docs (three in Italy, the others in Portugal, Netherlands, Germany, Austria, and USA), one (Cortesani) is global head of tranches market making in a bank in the USA, one (Demyanov) is quantitative analyst in a bank in England, and one (Dall'Aglio) is teacher in Italy.

b. Curriculum Vitae

Gianni Dal Maso, born in Vicenza, Italy, in 1954.

Professional Preparation
1973-1977: Student of the undergraduate courses in Mathematics at the University of Pisa, Italy, and student of the Scuola Normale Superiore in Pisa.
1977: Degree in Mathematics with honors at the University of Pisa. Thesis: Γ-limits of set functions. Advisor: Ennio De Giorgi.
1977: "Diploma" in Mathematics of the Scuola Normale Superiore, Pisa.
1977-1981: Post-graduate Research Fellowship in Mathematics "Perfezionamento", Scuola Normale Superiore, Pisa.

Appointments
1982-1985: Assistant Professor of Mathematical Analysis, Faculty of Engineering, University of Udine, Italy.
1985-1987: Associate Professor of Mathematical Analysis, SISSA, Trieste, Italy.
1987-present: Full Professor of Calculus of Variations, SISSA, Trieste, Italy.
1993-1998 and 2001-2010: Head of the Sector of Functional Analysis and Applications, SISSA, Trieste, Italy.
2010-present: Deputy Director, SISSA, Trieste, Italy.

Scientific Councils
1990-1996: Member of the Scientific Council of the Centro Internazionale Matematico Estivo (CIME).
1994-present: Member of the Scientific Council of the Italian Mathematical Union (UMI).

International Prizes
1982: Stampacchia Prize, awarded by the Scuola Normale Superiore, for an article on obstacle problems (ex aequo with M. Aizenman, H. W. Alt, L.A. Caffarelli, A. Friedman, B. Simon).
1991: Caccioppoli Prize, awarded by the Italian Mathematical Union.
1996: Medaglia dei XL per la Matematica, awarded by the Accademia Nazionale delle Scienze detta dei XL.
2003: Prize of the Minister for the Cultural Heritage for Mathematics and Mechanics, awarded by the Accademia Nazionale dei Lincei.
2005: Prize Luigi and Wanda Amerio, awarded by the Istituto Lombardo Accademia di Scienze e Lettere.

Ph.D. Theses
Gianni Dal Maso has been advisor of the Ph.D. thesis of 30 SISSA students (Defranceschi, Balzano, Paderni, Chiadò Piat, De Cicco, Vitali, Amar, Bellettini, Coscia, Notarantonio, Garroni, Malusa, Cortesani, Toader, Dall'Aglio, Leone, Mora, Morini, Negri, Giacomini, Ponsiglione, Zanini, Barchiesi, Cagnetti, Scardia, Demyanov, Fiaschi, Lazzaroni, Solombrino, Morandotti). At present he is advisor of 4 SISSA students (Agostiniani, Racca, Iurlano, Scala).

Jury member abroad
1984: Jury member (Rapporteur) of the "Thèse de Doctorat d'Etat ès Sciences Mathématiques" on "Comportement limite de solutions d'inéquations variationnelles associées à une suite de contraintes de type obstacle" by Colette Picard, Université Paris Sud, France.
1992: Jury member (Fakultetsopponent) of the "Doctoral Thesis" on "G-convergence and Homogenization of Sequences of Linear and Nonlinear Partial Differential Operators" by Nils Svanstedt, Department of Applied Mathematics, Lule University of Technology, Sweden.
1996: Jury member (Rapporteur) of the "Thèse de Doctorat" on "Homogénéisation de problèmes de Dirichlet non linéaires dans des ouverts perforés" by Juan Casado-Diaz, Université Pierre et Marie Curie, Paris VI, France.

1998: Jury member (Rapporteur) of the “Habilitation à Diriger des Recherches” on “Effets de la geometrie dans quelques problèmes d’homogénéisation” by Marc Briane, Université Pierre et Marie Curie, Paris VI, France.
2002: Jury member (Examinateur) of the “Habilitation à Diriger des Recherches” on “Problèmes variationnels aux frontières ou discontinuités libres. Applications au traitement d’images, à l’optimisation de formes, à la mécanique de la rupture” by Antonin Chambolle, Université Paris IX Dauphine, France.
2008: Jury member (Rapporteur) of the “Habilitation à Diriger des Recherches” on “Questions variationnelles autour d’un problème de restauration d’images” by Simon Masnou, Université Pierre et Marie Curie, Paris VI, France.

Visiting Professor
1989, March: Université de Metz.
1990, November: IMA, University of Minnesota, Minneapolis.
1991, April: Université de Toulon et du Var.
1991, June: Université de Metz.
1994, April: Carnegie Mellon University, Pittsburgh.
1998, January: Academia Sinica, Taipei.
1998, October-November: Université Pierre et Marie Curie, Paris VI.
1999, October-November: Newton Institute, Cambridge.
2000, May: Paris IX Dauphine.
2001, January : Paris IX Dauphine.
2002, October: Carnegie Mellon University, Pittsburgh.
2004, May: Keio University, Yokohama.
2004, May: Caltech, Pasadena.
2005, October: Carnegie Mellon University, Pittsburgh.
2007, May: Carnegie Mellon University, Pittsburgh.
2008, April: Carnegie Mellon University, Pittsburgh.
2008, October: Max-Planck Institute, Leipzig.
2009, April: Worcester Polytechnic Institute.
2009, September-October: Carnegie Mellon University, Pittsburgh.
2010, May: Carnegie Mellon University, Pittsburgh, and Worcester Polytechnic Institute.
2011, May: Carnegie Mellon University, Pittsburgh.

Funding ID
Gianni Dal Maso is the Scientific Coordinator of the National Research Project on “Variational problems with multiple scales” in the PRIN program funded by the Italian Ministry for Education, University, and Research (181.929,00 € for 46 participants for 2 years). There is and there will be no funding overlap with the ERC grant requested and any other source of funding for the same activities and costs that are foreseen in this project.

c. 10-Year-Track-Record

10 publications

1. Dal Maso G., Toader R.: A model for the quasi-static growth of brittle fractures: existence and approximation results. *Arch. Ration. Mech. Anal.* **162** (2002), 101-135 (citations: ISI 65, MathSciNet 50).

This paper develops the first complete mathematical analysis of a continuous-time formulation of a model of quasistatic crack growth proposed by Francfort and Marigo and based on Griffith's criterion. It studies only the case of antiplane linear elasticity in dimension two.

2. Dal Maso G., Toader R.: A model for the quasi-static growth of brittle fractures based on local minimization. *Math. Models Methods Appl. Sci.* **12** (2002), 1773-1800 (citations: ISI 15, MathSciNet 10).

This paper provides the first mathematical model of quasistatic crack growth where the global minimality condition considered by Francfort and Marigo is replaced by a sort of local minimality condition, which is closer to the original formulation of Griffith's criterion.

3. Alberti G., Bouchitté G., Dal Maso G.: The calibration method for the Mumford-Shah functional and free-discontinuity problems. *Calc. Var. Partial Differential Equations* **16** (2003), 299-333 (citations: ISI 5, MathSciNet 8).

This paper introduces a calibration method for the Mumford-Shah functional, which allows us to prove the minimality of some solutions of the corresponding Euler equation.

4. Dal Maso G., Murat F.: Asymptotic behaviour and correctors for linear Dirichlet problems with simultaneously varying operators and domains. *Ann. Inst. H. Poincaré Anal. Non Linéaire* **21** (2004), 445-486 (citations: ISI 11, MathSciNet 15).

This paper is the most general homogenization result for linear Dirichlet problems for elliptic equations in domains with many small holes.

5. Dal Maso G., Francfort G.A., Toader R.: Quasistatic crack growth in nonlinear elasticity. *Arch. Ration. Mech. Anal.* **176** (2005), 165-225 (citations: ISI 66, MathSciNet 50).

This paper studies a model of quasistatic crack growth for elastic brittle materials in arbitrary space dimension, assuming a quasiconvex bulk energy with polynomial growth.

6. Dal Maso G., DeSimone A., Mora M.G.: Quasistatic evolution problems for linearly elastic - perfectly plastic materials. *Arch. Ration. Mech. Anal.* **180** (2006), 237-291 (citations: ISI 28, MathSciNet 21).

This paper studies the quasistatic evolution of elastoplastic materials by adapting the variational framework of rate independent processes to suitable spaces of measures.

7. Dal Maso G., DeSimone A., Mora M.G., Morini M.: A vanishing viscosity approach to quasistatic evolution in plasticity with softening. *Arch. Ration. Mech. Anal.* **189** (2008), 469-544 (citations: ISI 8, MathSciNet 7).

This paper studies a model problem of small strain associative plasticity with softening, without regularizing terms, by considering the limit of the solutions to viscoplastic evolution problems in suitable spaces of Young measures.

8. Dal Maso G., Zeppieri C.I.: Homogenization of fiber reinforced brittle materials: the intermediate case. *Adv. Calc. Var.* **3** (2010), 345-370. (citations: ISI 0, MathSciNet 0).

This paper studies the macroscopic effect of a large number of brittle microcracks separated by unbreakable fibres and provides the first rigorous example of cohesive zone model obtained through a homogenization process.

9. Dal Maso G., Toader R.: Quasistatic crack growth in elasto-plastic materials: the two-dimensional case. *Arch. Ration. Mech. Anal.* **196** (2010), 867-906 (citations: ISI 0, MathSciNet 0).

This paper studies a model of quasistatic crack growth in linearly elastic - perfectly plastic materials in dimension two.

10. Dal Maso G., Lazzaroni G.: Quasistatic crack growth in finite elasticity with non-interpenetration. *Ann. Inst. H. Poincaré Anal. Non Linéaire* **27** (2010), 257-290 (citations: ISI 0, MathSciNet 0).

This paper studies a model of quasistatic growth of brittle cracks in the framework of finite elasticity with noninterpenetration.

Chapters in collective volumes

Dal Maso G.: Calcolo delle variazioni. *In "Storia della Scienza", directed by S. Petruccioli, Vol. IX*, 293-301, *Istituto della Enciclopedia Italiana, Roma*, 2003.

Dal Maso G., Francfort G.A., Toader R.: Quasi-static crack evolution in brittle fracture: the case of bounded solutions. *Calculus of Variations. Topics from the Mathematical Heritage of Ennio De Giorgi*, 245-266, *Quaderni di Matematica, Dip. di Matematica, Seconda Università di Napoli, Vol. 14*, 2004.

Dal Maso G.: Γ-convergence and homogenization. *Encyclopedia of Mathematical Physics, edited by Jean-Pierre Françoise, Greg Naber, and Tsou Sheung Tsun*, 449-457, *Elsevier, Oxford*, 2006.

Invited presentations to international conferences and to international advanced schools. Gianni Dal Maso is regularly invited to speak to (an average of) 4 international conferences per year. Main invitation:

Plenary speaker in the Conference "Mathematics and its Applications", Torino, 2006, a Joint Meeting of SIMAI, SMAI, SMF, UMI.

He was also plenary lecturer in the 2001 CNA Summer School Multiscale Problems in Nonlinear Analysis (Carnegie Mellon University, Pittsburgh), and was EMS Lecturer in 2002 (Lectures at MPI MIS in Leipzig and Paris VI).

Organization of international conferences. Gianni Dal Maso has been co-organizer of:

"Variational Methods for Discontinuous Structures", Cernobbio (Como), Italy, 2001;

"Calculus of Variations", Oberwolfach, 2002;

"Calculus of Variations", Oberwolfach, 2004;

"Variational Problems in Materials Science", SISSA, Trieste, 2004;

"Analysis and Numerics for Rate-Independent Processes", Oberwolfach, 2007.

International Prizes. Gianni Dal Maso has been awarded the following prizes after 2000:

Prize of the Minister for the Cultural Heritage for Mathematics and Mechanics 2003, awarded by the Accademia Nazionale dei Lincei;

Prize Luigi and Wanda Amerio 2005, awarded by the Istituto Lombardo Accademia di Scienze e Lettere.

Membership to Editorial Boards of International Journals

1991-present: Mathematical Models and Methods in Applied Sciences.

1992-present: Bollettino della Unione Matematica Italiana.

1993-present: Rendiconti dell'Istituto di Matematica dell'Università di Trieste.

1993-present: Journal of Convex Analysis.

1997-present: Asymptotic Analysis.

2000-present: Annali di Matematica Pura ed Applicata.

2003-present: ESAIM. Control Optimisation and Calculus of Variations.

2005-present: Archive for Rational Mechanics and Analysis.

2008-present: Advances in Calculus of Variations.

2008-present: Set-Valued and Variational Analysis: Theory and Applications.

2009-present: SIAM Journal on Mathematical Analysis.

Section 2: The Research Project

This research project deals with *nonlinear evolution problems* that arise in the study of the *inelastic behaviour* of *solids*, in particular in *plasticity* and *fracture*. The project will focus on selected problems, grouped into three main topics, namely:

1. **Plasticity with hardening and softening,**
2. **Quasistatic crack growth,**
3. **Dynamic fracture mechanics.**

The analysis of the models of these mechanical problems leads to deep mathematical questions originated by two common features: the energies are *not convex* and the solutions exhibit *discontinuities* both with respect to space and time. In addition, plasticity problems often lead to *concentration* of the strains, whose mathematical description requires *singular measures.* Most of these problems have a *variational structure* and are governed by *partial differential equations*. Therefore, the construction of consistent models and their analysis need advanced mathematical tools from the *calculus of variations,* from *measure theory* and *geometric measure theory,* and also from the theory of *nonlinear elliptic* and *parabolic partial differential equations.* The models of dynamic crack growth considered in the project also need results from the theory of *linear hyperbolic equations.* Our goal is to develop new mathematical tools in these areas for the study of the selected problems.

a. State of the art and objectives

In the study of *quasistatic evolution* problems for *rate independent systems* a classical approach is to approximate the continuous-time solution by discrete-time solutions obtained by solving incremental minimum problems (see, e.g., [76], and the review paper [61]). The continuous-time solution obtained in this way satisfies the following conditions:

(GS) *global stability*: at each time the state of the system *minimizes* its internal energy plus the dissipation distance from any other admissible state satisfying the same boundary conditions;

(E) *energy-dissipation balance*: the increment of the internal energy plus the dissipated energy *equals* the work of the external forces.

This energetic approach provides a weak formulation of the problem that requires a very mild regularity of the solutions and, in particular, does not assume the existence of time derivatives. This framework is suitable for problems where the internal energy and all dissipative terms are convex. When nonconvex terms are present, the global stability condition (GS) is questionable, as well as the corresponding condition in the discrete-time approximation. Indeed, general mechanical principles imply only a stability condition, without imposing global minimality. Therefore, the following conditions are more suitable for problems with *nonconvex energy terms*:

(S) *stability*: at each time the state of the system is a *critical point* for the sum of its internal energy and of the dissipation distance from any other admissible state satisfying the same boundary conditions;

(E) *energy-dissipation balance*: the increment of the internal energy plus the dissipated energy *equals* the work of the external forces.

Of course, every solution to (GS)-(E) is also a solution to (S)-(E). In general the converse is not true in the nonconvex case. Condition (GS) can be seen as a *selection criterion* among the solutions to (S)-(E). For many systems with nonconvex energies, different selection criteria are dictated by the physics of the problem.

Indeed, many rate independent processes arise naturally as limits of *rate dependent evolutions*, when a parameter, related to the degree of rate dependence, tends to zero. In most cases the limit of the solutions to these evolution problems satisfies (S) and (E), but not (GS). The most convenient selection criterion is then to consider only the solutions to (S)-(E) that are obtained by this limit process.

a1. Plasticity with hardening and softening

In *small strain linearized elastoplasticity* the strain is additively decomposed as sum of *elastic* and *plastic strain*. The stress is determined by the elastic strain through the standard laws of linear elasticity and is constrained to belong to a prescribed convex set of matrices, whose boundary is interpreted as the *yield surface*. If the stress belongs to the interior of this convex set, the behaviour of the material is purely elastic and no plastic strain is produced. When the stress hits the yield surface, the behaviour of the material becomes inelastic and the plastic strain evolves according to a *flow rule*, depending on the model. In the *associative* case the flow rule implies that the time derivative of the plastic strain is *normal* to the yield surface (at the point corresponding to the stress).

In many problems the yield surface is not constant during the evolution process, but depends on some internal variables. We are in the *hardening* regime when the region enclosed by the yield surface expands. On the contrary, we are in the *softening* regime when it contracts. *Perfect plasticity* refers to the case where the yield surface is constant.

Quasistatic evolution problems in associative perfect plasticity [16, 14, 81] and in associative plasticity with hardening can be treated within the framework of rate independent evolution processes governed by (GS) and (E). This has been done also for finite plasticity with hardening [60, 62, 58], where the additive decomposition of the strain is replaced by a multiplicative decomposition of the deformation gradient and the laws of linearized elasticity are replaced by the nonlinear laws of finite elasticity.

On the contrary, plasticity problems with *softening* cannot be treated easily in this framework and are highly unstable. A mathematical explanation of this instability is the presence of a *nonconvex* term in the energetic formulation of the quasistatic evolution. If one wants to study these problems using the standard discrete-time approximation that leads to (GS) and (E), the first difficulty is that the minimum in the incremental problems is not attained, in general, and one has to consider a relaxed formulation. A further difficulty occurs when one tries to pass to the limit as the time step tends to zero. So far these difficulties have been solved either by adding a *regularizing term* that forces the strong convergence of the relevant variables (as in [39]), or by using *Young measures* [17, 19].

Moreover, in these problems with softening the global stability condition (GS) is not justified from the mechanical point of view. Indeed, as observed in [19], in some models global minimality leads to missing the softening phenomenon altogether.

It is natural to consider quasistatic evolution problems in plasticity with softening as the limit case of a suitable *viscous dynamics*, when the viscosity parameter tends to zero. Therefore, it is important to study the behaviour of the limits of the viscous solutions.

For a model problem of small strain associative plasticity with softening, without regularizing terms, this has been done in [18] using Young measures. The limits of the viscous solutions may exhibit *time discontinuities*. This never happens in perfect plasticity or in plasticity with hardening. Moreover, these limits are shown to satisfy the stability condition (S). As for (E), in [18] it is proved only that the increment of the internal energy plus the dissipated energy is *less than or equal to* the work of the external forces. This inequality is due to the fact that only *plastic dissipation* is taken into account, while at jump times the system exhibits also an *instantaneous viscous dissipation*, that has not been considered in that paper.

In the spatially homogeneous case this asymptotic analysis reduces to a singular perturbation problem for a finite dimensional system of ordinary differential equations [82], and the behaviour of the limit process at jump times is obtained by solving a different system of ordinary differential equations, which describes the *fast dynamics.*

A different approach to the problem can be used when the solutions given by the viscous dynamics have *uniformly bounded variation* with respect to time. In this case one can introduce a *time rescaling*, so that the rescaled solutions of the viscous dynamics are uniformly Lipschitz continuous with respect to the rescaled time, and then one can study the limit of the rescaled solutions as the viscosity parameter tends to zero. Jumps in the original time correspond to intervals where the rescaled time is constant. This approach has been followed in [36, 63, 64] for finite dimensional systems governed by ordinary differential equations. The authors of the last two papers have used the same idea to study a similar problem in infinite dimension [78]. One obtains that the limit satisfies the stability condition (S) together with the energy-dissipation balance (E), where now one takes into account also the viscous dissipation occurring at jump times.

In [20] this idea was applied to the study of the *Cam-Clay model* in soil mechanics. This phenomenological model, with one internal variable, exhibits both *hardening* and *softening* behaviour, depending on the loading conditions. Moreover, it is non associative: indeed, the evolution law for the internal variable is not related to the yield surface.

The instabilities that may occur in the softening regime lead to instantaneous jumps of the solutions, and one has to describe the trajectory followed by the system at jump times, as well as the evolution after the jump times. The evolution is defined in terms of the limit of the rescaled solutions of the viscoplastic approximations It turns out that this limit satisfies (S) and (E), taking into account the viscous dissipation occurring at jump times, that can be conveniently expressed in the rescaled formulation.

The main difficulty overcome in this problem is that only the plastic strains of the viscoplastic approximation have uniformly bounded variation in time. This leads to a good control of the plastic strains for the rescaled solutions, while the time dependence of the elastic strains is not controlled in a direct way.

Another feature of this model is that the evolution law for the internal variable is *not variational.* This introduces an additional difficulty in the study of the limit as the viscosity parameter tends to zero, since one cannot prove a compactess result for the internal variable. In [20] this difficulty has been avoided by means of a *regularization* by convolution. It would be interesting to study the problem with different regularizations of the evolution law, based, for instance, on the solution of some diffusion equation. Another option, that should be explored, is to look for a weaker formulation of the limit problem, so that the internal variables are compact in this new environment, and no regularization is needed.

In [21] it is shown that the energetic conditions (S) and (E) satisfied by the rescaled solutions introduced in [20] imply a weak formulation of the flow rule for the plastic strain. The spatially homogeneous case for this model is studied in [15, 28], where we can find two different sets of ordinary differential equations followed by the system during the slow and fast dynamics. These results will be used to develop a *numerical analysis* of the discontinuous solutions of the Cam-Clay model.

We plan to use this general method of viscous approximation and time rescaling to study the quasistatic evolution of other *nonassociative problems* in plasticity in the *small strain* regime.

We hope that the same approach can be used also in the analysis of the quasistatic evolution for *gradient models* [45] in plasticity with hardening and softening.

Another research line that will be explored within this project is a viscosity approach to finite plasticity with hardening and softening.

The papers [16, 20, 21] adopt a weak formulation of plasticity problems where the plastic strain is represented by a *measure*. In this framework it will be possible to study an interesting property of the solutions to these evolution problems, that appears mainly in the softening regime: *strain concentration*.

a2. Quasistatic crack growth

A *variational* discrete-time model of *quasistatic growth* of *brittle cracks* was introduced in [38]. It is based on Griffith's idea [44] that the crack growth is determined by the competition between the elastic energy released by the body when the crack grows and the work needed to produce a new crack, or to extend an existing one. The main feature of this model is that the *crack path* is *not prescribed*, but is a result of energy balance.

The first existence result for a continuous-time formulation of the same model was obtained in [29] in the case of antiplane linear elasticity in dimension two. In that paper the admissible cracks are assumed to be connected, or to satisfy a uniform bound on the number of connected components. This restriction simplifies the mathematical formulation of the problem.

These results were extended to the case of *plane linear elasticity* by Chambolle in [12]. In the antiplane case the paper [37] removed the restriction on the connected components of the crack and on the dimension of the space, and introduced a weak formulation in the space *SBV* of *special functions* with *bounded variation* [2]. Using the technical tools developed in [37], in particular the *Jump Transfer Lemma*, some existence results have been obtained in the case of nonlinear elasticity [22, 23, 25] and also for some models of *finite elasticity* with *noninterpenetration* [26, 54, 27]. The numerical approximation of these models has been developed in [67, 68, 69, 7, 40]. It would be interesting to obtain new results in this direction, under different hypotheses on the energy density or on the applied loads.

Although it is frequently used in the engineering community, a rigorous model for *linear elasticity* with *brittle cracks* in dimension three is *still missing*. Even the two-dimensional case is not satisfactory, since the results of [12] require an a priori bound on the number of connected components of the crack. It would be important to develop a rigorous model for this problem, to prove an existence result, and to study the properties of the solutions.

All these results can be expressed in the language developed in [61] for rate independent evolution problems. The solutions are characterized by the global stability condition (GS) and the energy-dissipation balance (E), where now the internal energy is the energy stored in the elastic part, while the dissipation distance and the dissipated energy are proportional to the area of the new portion of crack.

The simultaneous presence of *plastic flow* and *crack growth* has been studied in [31] in the case of linearly elastic - perfectly plastic materials. For technical reasons these results were proved only in dimension two, assuming an a priori bound on the number of connected components of the cracks. It would be interesting to remove the restrictions on the dimension of the space and on the topology of the cracks, and to study the problem of crack growth in more general elastoplastic materials.

Cohesive forces between crack lips appear when the work to produce a new crack depends also on the *crack opening*. Quasistatic models with cohesive forces have been studied within the framework of rate independent systems, but so far the results have been obtained only in the case of a *prescribed crack path* [32, 11]. It would be interesting to study this problem under more general assumptions on the force acting between the crack lips. The main difficulty is that in the most interesting models the materials exhibit a *different response to loading and unloading*, because the dependence of the cohesive force on the crack opening is different in the two cases.

If the crack path is not prescribed, one of the main mathematical difficulties in the study of cohesive zone models is that a *relaxation process* occurs when the force between the crack lips tends to a finite limit as the crack opening tends to zero, for example in the Dugdale-Barenblatt model [34, 4]. This relaxation process can be explained as the macroscopic effect of a large number of microcracks with small openings. It would be interesting to develop a quasistatic evolution model based on the relaxed formulation of the static problem. We expect that this relaxed formulation will imply a *rupture stress limit* for any solution, a property that was already proved for smooth solutions, under suitable hypotheses on the force between the cracks lips [24]. The connection of this problem with *crack initiation* has been studied in [13].

There is an obvious connection between *damage* and fracture. On the one hand many materials undergo a damage process before a crack is formed or grows. On the other hand the macroscopic effect of a large number of microcracks is to weaken the elastic properties of the material. Our purpose is to study this kind of phenomena in some model cases, both in the static and quasistatic case.

In some damage models the *damaged regions* tend to concentrate in *narrow strips* along manifolds of codimension one. In the static case it is expected that the limit problem, as the width of the damaged strip tends to zero, can be interpreted as a crack model, of brittle or cohesive type. It would be interesting to study also the behaviour of the corresponding quasistatic evolutions.

For the static problem the macroscopic effect of a large number of *brittle microcracks* separated by unbreakable fibres has been recently studied in the framework of *homogenization theory*. Depending on the asymptotic behaviour of the relevant parameters (toughness and size of the brittle regions, size of the fibres), the homogenized problem is a fracture problem with *brittle cracks* [5] or with *cohesive forces* [33], or an elasticity problem in a *damaged material* without cracks [79, 80]. It would be interesting to study these problems under more general hypotheses, in the static and quasistatic regime, and to characterize the cohesive zone models that can be obtained in this way.

The choice of a global minimizer in the papers on brittle cracks mentioned at the beginning of this section is not justified by mechanical principles, which lead only to a local condition. However, it is very convenient from the mathematical point of view. Indeed, the incremental minimum problems used in the discrete-time approximation can be solved by using several tools that have already been developed in the study of *free-discontinuity problems*. Nevertheless, the global stability condition (GS) has a drawback: it induces *unnatural time discontinuities* in the crack growth process [70, 73].

Two different ways have been proposed to overcome this difficulty. One [30] is based on a sort of local stability condition, that replaces (GS). The other one [50] is based on global minimization on a suitable set of attainable points. We discuss here a third approach, where the quasistatic evolution is considered as a limit of dynamic evolutions.

From the physical point of view quasistatic evolution models of crack growth arise naturally as limits of *dynamic* models with *inertial* and *friction* terms, when these terms tend to zero. Therefore, the study of the quasistatic limit evolution would require a complete understanding of the dynamic problem. At this level of generality (free crack path, no a priori assumptions on the crack edge, etc.) the study of dynamic crack growth is at the very beginning (see Section a3).

Therefore, it would be useful to obtain some preliminary results on the limit evolution when inertial terms are negligible with respect to friction terms. This amounts to approximate the quasistatic crack growth by an evolutive model based on a purely viscous dynamics. The same approach has been used for nonconvex plasticity problems. As in that case, the limit models are based on the stability condition (S) and on the energy-dissipation balance (E), where now the notion of critical point is defined in terms of *energy release rate* [46, 47, 55, 72]. So far this method has been applied to the case of a prescribed crack path [10, 48, 73, 83, 49, 71], or assuming a priori bounds on the curvature of the cracks [56]. Our aim is to develop this approach for problems with a free crack path. The first step in this program will be to study the limit of *linearly viscoelastic - perfectly brittle* materials, as the viscosity parameter tends to zero. It would be interesting to consider also different viscous models, with an additional viscous dissipation due to the motion of the *crack tips*.

a3. Dynamic fracture mechanics

For the problem of crack growth in the *dynamic regime* our aim is to develop a model that predicts the crack path as well as the time evolution of the crack along its path, taking into account all inertial effects. This model should combine the system of elastodynamics in the unbroken part of the material with an evolution law for the crack. While the quasistatic regime can be analyzed using the energetic formulation for rate independent processes, so far there is no general framework for a weak formulation of this kind of dynamic evolution problems. Most of the results in the vast literature on dynamic cracks are obtained under simplifying assumptions on the shape or on the path of the crack [74, 66, 41, 57, 59, 75, 77].

So far the only results with a free crack path [9, 53] deal with a regularized crack model, where the crack is replaced by a phase field, as in [3].

The first step towards a general theory of dynamic crack growth with an arbitrary crack path is the study of the *(damped) wave equation* in a domain with a *prescribed growing crack.* An existence and uniqueness result for this problem would be the starting point for a model where the only unknown is the growing crack. The main problem is to find conditions on the growing crack that ensure an *energy-dissipation balance,* taking into account the kinetic energy and the elastic energy of the solution to the wave equation, as well as the energy dissipated by the process of crack growth. However, we expect that this condition alone does not determine the evolution of the crack and that a *further growth criterion* has to be introduced (see the discussion in [51]).

Another approach consists in the study of a *discrete-time* formulation of the dynamic problem of crack growth. Passing to the limit as the time step tends to zero we should obtain a solution of the wave equation in the domain corresponding to the limit growing crack. We hope that it will be possible to prove that it satisfies the energy-dissipation balance and also a further growth criterion.

It is also interesting to study the *quasistatic limit* of the dynamic crack growth when the inertial terms tend to zero. We expect that this limit satisfies the energy-dissipation balance (E) and the stability condition (S) instead of the global stability condition (GS). In many cases the limit of dynamic evolutions with inertial terms will be different from the limit of the viscous approximations, although both satisfy the same conditions (E) and (S). This discrepancy has already been observed in a simplified model [35].

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b. Methodology

For all quasistatic evolution problems considered in the project we shall use an energetic formulation in the spirit of [61]. The *energy-dissipation balance* (E) mentioned at the beginning of Section a is required in all cases. In some problems the *global stability* condition (GS) is replaced by the weaker *stability* condition (S).

b1. Plasticity with hardening and softening

To solve the quasistatic evolution problems considered in this subsection we shall approximate them by *viscoplastic evolution problems*, and we shall study the limit of the solutions to these approximating problems as the viscosity parameter tends to zero. We expect that this approach will overcome the difficulties due to the presence of nonconvex energy terms. The viscoplastic problems are, in general, *nonlinear parabolic problems* that can be solved in the framework of the existing theories. In many cases it is easy to obtain uniform estimates of the total variation with respect to time of some components of the solutions to the viscoplastic problems used in the approximation. Taking advantage of these estimates, we plan to consider a *time rescaling* as in [36, 63, 64, 78, 20, 21], so that these components become uniformly Lipschitz continuous with respect to the rescaled time. This will be one of the technical tools to pass to the limit in the viscous dissipation as the viscosity parameter tends to zero. The limit solutions obtained by this method are expected to satisfy the stability condition (S) and the energy-dissipation balance (E) mentioned at the beginning of Section a. Of course, the dissipated energy considered in (E) includes the limit of the viscous dissipation, which, in many problems, is expected to be concentrated at the jump times of the solution.

This approach has been successful in the study of the Cam-Clay model in soil mechanics. We plan to use this general method for the quasistatic evolution in the *small strain* regime of other *nonassociative problems* in plasticity. Unless a hardening term is present, in this regime we can control only the L^1 norm of the symmetric part of the gradient of the displacement. This leads to a weak formulation of the problem in the space *BD* of functions with *bounded deformation,* studied by Kohn, Strang, and Temam. In this setting the plastic strain is represented by a *measure*. This provides the natural framework for the study of *strain concentration*, that occurs when the plastic strain is supported by a set of codimension one.

The same approach will be used in the study of quasistatic evolution for *gradient models* in plasticity with hardening and softening. In the latter case only the L^1 norm of the gradient of the plastic strain is under control, and this leads to a different weak formulation, where the plastic strain belongs to the space *BV* of functions with *bounded variation.*

In the study of nonassociative models in plasticity, one of the main difficulties is that the flow rule of the plastic strain is *not variational,* so that the available estimates do not guarantee a compactess result for the plastic strains of the solutions of the viscoplastic approximating problems. The simplest way to overcome this difficulty is to introduce a suitable regularization by convolution, as in [20]. We plan to study these problems also with *physically motivated regularizations* of the flow rule based, for instance, on the solutions of suitable *diffusion equations.* Since, in general, in these problems we can only estimate L^1 norms, we expect that the required compactness result can be obtained only through an estimate of the solutions of diffusion equations with L^1 data. The same technique can be used for the study of the Cam-Clay model with a different regularization of the evolution law for the internal variable.

An alternative approach to this problem could be to find a very weak formulation of the evolution law in a suitable space of *Young measures*, where the solutions to the viscoplastic problems are compact. The hope is that all terms of this new formulation are continuous, or lower semicontinuous, with respect to the weak* convergence of Young measures.

The numerical analysis of the Cam-Clay model will begin from the study of the *spatially homogeneous* case, where the evolution is governed by two different systems of ordinary differential equations, describing the slow and the fast dynamics [28]. The transition times are determined by the zeros of some known functions depending on the state of the system. In the general case we hope that the difficulty due to the presence of jump times can be solved using the *rescaled viscosity formulation* studied in [20, 21], where all singularities in time disappear and the rates of elastic and plastic strains are under control.

The quasistatic evolution problem in *finite plasticity* is more difficult, because of the *multiplicative decomposition* of the strain and of the *logarithmic growth* of the dissipation distance. We plan to *approximate* the problem by a sequence of *nonlinear parabolic partial differential equations*, which should provide a sort of regularized version of the evolution problem. We expect that, if the approximation is conveniently chosen, one can prove the existence of a solution. The hard problem will be to find good estimates for these solutions, in order to pass to the limit as the approximation parameter tends to zero. The big challenge will be the study of this problem in the softening regime, where time discontinuities are expected. We hope that a time rescaled formulation of the problem can be useful also in this case.

b2. Quasistatic crack growth

In the study of most *quasistatic problems* of *crack growth* we shall approximate the continuous-time solution by discrete-time solutions obtained by solving incremental minimum problems. The limit will satisfy the energetic formulation based on the global stability (GS) and energy-dissipation balance (E) mentioned at the beginning of Section a. The internal energy of the system is now the elastic energy stored in the unbroken region. As for the dissipation distance and the dissipated energy, they depend on the problem, and take into account the area of the crack increment, the crack opening, and the plastic dissipation (if present).

The incremental minimum problems will be solved by means of tools that have already been developed in the study of *free-discontinuity problems* [2]. Most of them are based on *geometric measure theory.*

The main difficulty in the study of crack growth in *linearly elastic - perfectly brittle* materials in arbitrary space dimension is that we cannot control the full gradient of the displacement, but only its symmetric part, because the constants in Korn's inequality depend on the unknown geometry of the cracks. However, if we assume that the displacements are bounded in L^∞, we can apply a compactness theorem [6] in the space *SBD* of *special functions* with *bounded deformation*, introduced in [1]. This leads to a weak formulation of the problem in *SBD*. The main drawback of this approach is that the L^∞ bound of the displacements is not natural: it does not follow from any hypothesis on the boundary conditions, since the geometry of the cracks is unknown. To study the problem in the general case we need to reformulate it in a new function space, where a similar compactness result could be obtained without L^∞ bounds.

In the models for brittle crack growth in elastic materials the crack at a given time is essentially the union of the discontinuity sets of the deformation at all past times. This is not the case in elastoplastic materials, where plastic slips can determine further discontinuities of the deformation. This is one of the main difficulties in the modeling of crack growth in these materials. A preliminary result [31] in this direction has already been obtained for the *linearly elastic - perfectly plastic* case in dimension two, working in the space *BD* of functions with bounded deformation and assuming an a priori bound on the number of connected components of the cracks. It would be interesting to remove these restrictions on the dimension of the space and on the topology of the cracks. We expect that the main difficulty will be to find a convenient notion of *convergence for the cracks*, in order to study the limit of the approximate solutions obtained in the discrete-time formulation.

In the case of cracks with a *cohesive zone* the construction of consistent models of quasistatic growth is much more difficult than in the case of brittle cracks. Indeed, one of the main mathematical tools for the study of crack growth, the *Jump Transfer Lemma* in [37], was proved only in the brittle case. The extension of this result to some cohesive zone models would be a very important achievement. Another difficulty is that the energy functionals for cohesive models with free crack path are *not lower semicontinuous* in the natural function space for a weak formulation of the problem, unless the force exerted between the crack lips tends to infinity as the crack opening tends to zero. So far no complete theory of crack growth has been developed for cohesive zone models with free crack path.

For this reason it would be interesting to obtain new results for these models also under the simplifying assumption of a *prescribed crack path.* In this case we can formulate the problem in a suitable Sobolev space and we can focus on the main difficulty: the different response to loading and unloading, due to the fact that the dependence of the cohesive force on the crack opening is different in the two cases. A weak formulation of this problem has already been studied in [11], under special hypotheses, using Young measures to overcome the lack of compactness in suitable function spaces. We hope that stronger formulations of this kind of problems can be obtained.

If the force between the crack lips tends to a finite constant when the crack opening tends to zero, as in the case of the Dugdale-Barenblatt model, a *relaxation process* occurs. Indeed, the minimizing sequences of the incremental minimum problems are compact only in the space BV of functions with bounded variation, while the sum of the elastic energy and of the energy dissipated by the crack is *not lower semicontinuous* in BV. The corresponding relaxed energy is finite on the whole space BV. For smooth displacements with large gradients the relaxed energy is smaller than the original elastic energy, due to the asymptotic effect of a large number of microcracks. The main difficulty is that in the relaxed formulation of the static problem we have to consider also displacements whose distributional gradient has a *Cantor part,* i.e., a singular part that is not concentrated on macroscopic cracks (of codimension one). The techniques introduced in [13, 24] can be used to determine the *rupture stress limit* for piecewise C^1 solutions of the relaxed static problem. They should be extended to arbitrary BV solutions.

To construct a quasistatic evolution model based on the BV relaxed formulation we shall consider first a discrete-time model, whose solutions are obtained by solving incremental relaxed minimum problems. The results for the static problems could be used to prove some properties of these discrete-time solutions. We hope that these properties will enable us to pass to the limit as the time step tends to zero. We expect that this limit satisfies (GS) and (E) for the relaxed energy, thus providing a continuous-time solution.

The relationships between *damage* and *fracture* and between *cohesive cracks* and *microcracks* can be considered in the framework of *homogenization theory* and will be studied by Γ-*convergence* techniques. In particular, damage problems with *damage concentration* along manifolds of codimension one will be considered first in the static case. We shall use Γ-convergence techniques to study the limit behaviour of these stationary solutions when the width of the damaged strip tends to zero. It is expected that the limit problem can be interpreted as a crack model, of brittle or cohesive type. The behaviour of the corresponding quasistatic evolutions will be studied using the techniques developed in [42, 43, 65].

We shall continue the study of the macroscopic effect of a large number of microcracks started in [5, 33, 79, 80]. We shall consider different hypotheses on the geometry, on the toughness, and on the elasticity coefficients of the regions containing the microcracks and of the regions separating them. The *homogenization limit* should correspond to a damage problem or a crack problem, depending on the hypotheses. By this procedure we expect to obtain a large class of crack models with cohesive zone. Also in this case we shall use Γ-*convergence* techniques for the static problems, and we shall try to adapt the ideas of [42, 43, 65] for the quasistatic evolutions.

To overcome the drawbacks (see Section a2) of the quasistatic evolution models based on global stability (GS) and energy-dissipation balance (E), we shall develop a *vanishing viscosity* approach to quasistatic crack growth, as for problems in plasticity. This will lead to models based on stability (S) and energy-dissipation balance (E). Our final goal is to develop this approach for problems with a free crack path, but some preliminary results will be obtained in the case with prescribed crack path. We shall first study the limit of *linearly viscoelastic - perfectly brittle* materials, as the viscosity parameter tends to zero. Then we shall consider different viscous models, with an additional dissipation due to the motion of the *crack tips* (for two-dimensional elasticity with one-dimensional cracks) or of the *crack fronts* (for three-dimensional elasticity with two-dimensional cracks), as in [52].

Once the viscosity method for the linear case with brittle cracks has been understood, several extensions of the viscosity approach are possible: linear elasticity and cracks with cohesive forces, nonlinear elasticity and brittle cracks, nonlinear elasticity and cracks with cohesive forces, etc.. If the viscosity approach to finite plasticity mentioned at the end of Section b1 is fully developed, it will be possible to attack the problem of quasistatic crack growth in materials governed by finite plasticity, which is relevant in the understanding of ductile crack growth.

b3. Dynamic fracture mechanics

Since we want to focus on the interaction between wave propagation and crack growth, without restrictions on the shape or on the path of the crack, and only few preliminary results are available at this level of generality, we plan to study dynamic fracture mechanics under the simplest hypotheses on the elastic response and on the mechanism of crack formation: we shall consider only the case of *linear elasticity* with *brittle cracks*. We shall examine mainly the *antiplane case,* so that the displacement is scalar and the system of elastodynamics reduces to the *wave equation.*

The first step is the study of the *(damped) wave equation* in a domain with a *prescribed growing crack.* If the crack is not sufficiently regular, the uncracked region may not be open. This rules out the standard formulation of the wave equation in Sobolev spaces (constructed on the uncracked part of the domain), and requires a weaker formulation of the problem in spaces containing discontinuous functions, like SBV. Moreover, the Galerkin method cannot be used directly, since a complete orthonormal system with respect to the space variables would depend on time. A possible alternative is to use a *time discretization approach,* as in [53].

Once an existence and uniqueness result for this problem has been obtained, the next step is to find conditions on the growing crack that ensure an *energy-dissipation balance,* taking into account the kinetic energy and the elastic energy of the solution to the wave equation, as well as the energy dissipated by the process of crack growth. This step probably requires a preliminary study of the continuous dependence of the solution of the wave equation on the growing crack. As mentioned in [51], the energy-dissipation balance does not determine the evolution of the crack in the dynamic regime. We have to introduce a *further growth criterion*, based on general mechanical principles.

The proof of the existence of a growing crack satisfying the energy-dissipation balance and this additional criterion is a great challenge. We shall consider first the case of a *prescribed crack path,* which simplifies the functional setting, and allows us to focus on the motion of the crack tip in its prescribed trajectory. We shall use the results already obtained for the *regularized crack model* [53]: either we shall adapt the ideas and the techniques developed in that paper to the case of a growing crack, or we shall try to pass to the limit as the regularization parameter tends to zero.

Of course, the dynamic fracture problem with a free crack path is much more difficult. However, the existence theorem for the regularized crack model proved so far does not assume a prescribed crack path, and we hope that it can be used as a tool to obtain the desired result.

The alternative approach based on *discrete-time approximations* requires some compactness estimates on the approximate solutions in suitable function spaces. Since the problem with prescribed crack path can be formulated in Sobolev spaces, in this case it should not be difficult to prove that the limit function is a solution of the wave equation in the domain corresponding to the limit growing crack. The proof of the energy-dissipation balance seems to be more difficult. The case with free crack path requires new ideas.

As for the quasistatic limit of the dynamic crack when the inertial terms tend to zero, it will be studied first in the case of a prescribed crack path and then in the general case. We expect that it will be easy to prove that this limit satisfies the stability condition (S) and the energy-dissipation balance (E). The main issue is a characterization of the time discontinuities of the limit of the dynamic evolutions. Adapting to the dynamic case the time rescaling technique used in the viscous approximation [20, 56], we expect, at least in the case of a prescribed path, that the behaviour of the limit at jump times can be described by a *rescaled dynamic evolution.*

c. Resources

The financial support requested will foster the collaboration among the team members on the subjects of the project and will promote new scientific contacts with other groups working on similar problems with a different expertise. The latter goal will be obtained also through the organization of two multidisciplinary conferences, that will bring together mathematical analysts, numerical analysts, mechanicians and applied mathematicians with expertise in plasticity problems or in fracture mechanics. The Grant would also allow us to add to the existing team three post docs, that will be selected among the most promising young researchers in this field.

The **Principal Investigator – Gianni Dal Maso –** will coordinate the entire project and will have full responsibility of the project development.

i. Team members

The scientific project will be developed by the P.I. with the contribution of the following team members:

Flaviana Iurlano: Ph.D. Student of the P.I. at SISSA (first year). She works on the interaction between damage and fracture.

Christopher J. Larsen: Former Ph.D. Student of Irene Fonseca at Carnegie Mellon University (thesis defense in 1996), now Associate Professor at Worcester Polytechnic Institute (USA). A leading expert in mathematical models in fracture mechanics. 13 papers in MathSciNet.

Maria Giovanna Mora: Former Ph.D. student of the P.I. at SISSA (thesis defense in 2001), now Research Assistant at SISSA. She has a deep expertise in free discontinuity problems, in quasistatic evolution problems, and in dimension reduction problems. 18 papers in MathSciNet.

Matteo Negri: Former Ph.D. student of the P.I. at SISSA (thesis defense in 2001), now Research Assistant at the University of Pavia (Italy). His expertise includes the numerical approximation of free discontinuity problems and the analysis of quasistatic crack growth. 13 papers in MathSciNet.

Simone Racca: Ph.D. Student of the P.I. at SISSA (second year). He works on the viscous approximation of crack growth.

Francesco Solombrino: Former Ph.D. student of the P.I. at SISSA (thesis defense in 2010), now Research Scientist at RICAM, Johann Radon Institute for Computational and Applied Mathematics (Austria). His expertise includes the Cam-Clay models in soil mechanics and the method of viscous approximation and rescaling for the solution of rate independent problems. 3 papers in MathSciNet.

Rodica Toader: Former Ph.D. student of the P.I. at SISSA (thesis defense in 1997), now Research Assistant, University of Udine (Italy). Her expertise includes homogenization and relaxation problems, approximation and density results for free discontinuity problems, quasistatic problems in fracture mechanics. 19 papers in MathSciNet.

The team members not working at SISSA have long lasting collaboration with the P.I. on the subjects of the project. The fact that their working place is different from SISSA does not imply additional costs for the project (except for the travel costs for mutual collaboration).

ii. Existing resources that will contribute to the project

My hosting institution is SISSA in Trieste and, more specifically, the Functional Analysis and Applications Sector. SISSA offers excellent research facilities in a good research environment, with a very rich mathematical library and a continuous flux of visitors, even for relatively long terms. Also, The Abdus Salam International Centre for Theoretical Physics (ICTP) in Trieste has an active Mathematics Section with many visiting positions and a large number of scientific activities. In addition, SISSA has a strong interaction with the Mathematics Department of the University of Trieste.

The faculty at SISSA is very small, but the complementary expertise of my colleagues might be very useful in the development of the project. I would like to mention in particular my colleagues: Andrei Agrachev (Control Theory), Antonio Ambrosetti (Nonlinear Analysis), Stefano Bianchini (Transport Problems), Antonio DeSimone (Mechanics of Materials), and Andrea Malchiodi (Geometric Analysis).

The numerical aspects of the project will also benefit of the activity of MathLab, a recently established SISSA laboratory for mathematical modeling and scientific computing, directed by Antonio DeSimone and Alfio Quarteroni, and devoted to the interactions between mathematics and its applications.

iii. Budget

The funds will be used to cover the following costs:

- **Post-docs.** The largest budget item is for post-doctoral researchers. The cost for a post-doc fellowship is 50.000 € per year. The net salary (after taxes) will be about 38.600 € per year. There will be 11 man-years of post-docs. There will be three two-year post-doc positions that will start their activity in years 1, 2, 3. The positions starting in years 1 and 2 can be extended up to four years. The position starting in year 3 can be extended up to three years. In case some position is not extended, a new post-doc position will be offered for the remaining time. The total amount is 550.000 €.
- **Equipment.** Computer equipment will be purchased in the first three years, for an amount of 3.000 € per year. In the table below the cost charged every year corresponds to the depreciation amount computed according to our internal rules. The total amount is 9.000 €.
- **Travel (SISSA Team Members).** The travel and subsistence expenses for SISSA team members and post-docs will be for participation in conferences and for scientific collaboration on subjects related to the project. The average estimated cost is 3.000 € per year per person. The total amount is 93.000 €.
- **Travel (Other Team Members).** The travel and subsistence expenses for the team members not employed by SISSA (Christopher J. Larsen, Matteo Negri, Francesco Solombrino, and Rodica Toader) will be for for participation in conferences and for scientific collaboration on subjects related to the project. These costs will be payed directly by SISSA. The average estimated cost is about 2.500 € per year per person. The total amount is 49.000 €.
- **Workshops.** We plan to organize two international conferences in years 3 and 5 (last semester), and to spend 15.000 € for each event for the invitation of the main speakers. The total amount is 30.000 €. Possible additional costs for the conferences will be funded by other sources.
- **Visitors.** We plan to invite some external experts to SISSA or to the institutes of the team members for scientific collaboration in the fields of the project. In all cases these costs will be payed directly by SISSA. These invitations will strengthen the scientific links with other research groups in these fields or in related areas and will also allow us to obtain updated information on the most recent advances on topics related to the project. The travel expenses for short and long term invitations of external experts are 13.000 € per year. The total amount is 65.000 €.
- **Publications.** The average estimated cost is 1.100 € per year, plus 1.100 € in the year of the first conference and 1.150 € in the year of the second conference. The total amount is 7.750 €.
- **Subcontracting costs.** These are for two Certificate on Financial Statements (CFS) and the cost is limited to the second and fourth financial reporting period, according with the rules of FP7 for the CFS (a CFS is mandatory for every claim in the form of reimbursement of costs whenever the amount of the EC contribution is equal or superior to 375.000 €). The total amount is 4.000 €.

The PI will dedicate 40% of his time to the project over the period of the Grant.

There will be two key intermediate goals, called *intermediate results* and *final results*, which will be completed in years 3 and 5. The results of the intermediate goals will be discussed in the international conferences oraganized within the project.

Budget - Table 1 **Please enter duration in months** | **60**

	Cost Category	Months 1 to 18	Months 19 to 36	Months 37 to 54	Months 55 to 60	Total
Direct Costs:	**Personnel:**					
	PI	0,00	0,00	0,00	0,00	**0,00**
	Senior Staff	0,00	0,00	0,00	0,00	**0,00**
	Post docs	100.000,00	200.000,00	200.000,00	50.000,00	**550.000,00**
	Students	0,00	0,00	0,00	0,00	**0,00**
	Other	0,00	0,00	0,00	0,00	**0,00**
	Total Personnel:	**100.000,00**	**200.000,00**	**200.000,00**	**50,000,00**	**550.000,00**
	Other Direct Costs:					
	Equipment	2.000,00	4.000,00	2.500,00	500,00	**9.000,00**
	Consumables	0,00	0,00	0,00	0,00	**0,00**
	Travel (SISSA Team Members)	24.000,00	30.000,00	30.000,00	9.000,00	**93.000,00**
	Travel (Other Team Members)	15.000,00	14.500,00	14.500,00	5.000,00	**49.000,00**
	Workshops	0,00	15.000,00	0,00	15.000,00	**30.000,00**
	Visitors	19.500,00	19.500,00	19.500,00	6.500,00	**65.000,00**
	Publications	1.650,00	2.750,00	1.650,00	1.700,00	**7.750,00**
	Total Other Direct Costs:	**62.150,00**	**85.750,00**	**68.150,00**	**37.700,00**	**253.750,00**
	Total Direct Costs:	**162.150,00**	**285.750,00**	**268.150,00**	**87.700,00**	**803.750,00**
Indirect Costs (Overheads):	20% of Direct Costs	32.430,00	57.150,00	53.630,00	17.540,00	**160.750,00**
Subcontracting Costs:	(No Overheads)	0,00	2.200,00	0,00	1.800,00	**4.000,00**
Total Requested Grant:	**(by Reporting Period and Total)**	**194.580,00**	**345.100,00**	**321.780,00**	**107.040,00**	**968.500,00**

For the above cost table, please indicate the % of working time the PI dedicates to the project over the period of the grant: | **40 %**

Budget - Table 2

"Key Intermediate Goal" as Defined on Page 23	Estimated % of Total Requested Grant	Expected to be Completed on Month:	Comment
Intermediate Results	56%	Month 36	The intermediate results will be discussed in the international conference organized in the third year of the project
Final Results	44%	Month 60	The final results will be discussed in the international conference organized in the last semester of the project
Total	**100%**		

d. Ethical issues

	Research on Human Embryo/ Foetus	YES	Page
	Does the proposed research involve human Embryos?	NO	
	Does the proposed research involve human Foetal Tissues/ Cells?	NO	
	Does the proposed research involve human Embryonic Stem Cells (hESCs)?	NO	
	Does the proposed research on human Embryonic Stem Cells involve cells in culture?	NO	
	Does the proposed research on Human Embryonic Stem Cells involve the derivation of cells from Embryos?	NO	
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	YES	

	Research on Humans	YES	Page
	Does the proposed research involve children?	NO	
	Does the proposed research involve patients?	NO	
	Does the proposed research involve persons not able to give consent?	NO	
	Does the proposed research involve adult healthy volunteers?	NO	
	Does the proposed research involve Human genetic material?	NO	
	Does the proposed research involve Human biological samples?	NO	
	Does the proposed research involve Human data collection?	NO	
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	YES	

	Privacy	YES	Page
	Does the proposed research involve processing of genetic information or personal data (e.g. health, sexual lifestyle, ethnicity, political opinion, religious or philosophical conviction)?	NO	
	Does the proposed research involve tracking the location or observation of people?	NO	
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	YES	

	Research on Animals	YES	Page
	Does the proposed research involve research on animals?	NO	
	Are those animals transgenic small laboratory animals?	NO	
	Are those animals transgenic farm animals?	NO	
	Are those animals non-human primates?	NO	
	Are those animals cloned farm animals?	NO	
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	YES	

	Research Involving non-EU Countries (ICPC Countries)	YES	Page
	Is the proposed research (or parts of it) going to take place in one or more of the ICPC Countries?	NO	
	Is any material used in the research (e.g. personal data, animal and/or human tissue samples, genetic material, live animals, etc) : a) Collected in any of the ICPC countries?	NO	
	b) Exported to any other country (including ICPC and EU Member States)?	NO	
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	YES	

	Dual Use	YES	Page
	Research having direct military use	NO	
	Research having the potential for terrorist abuse	NO	
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	YES	

Section 3: The Research Environment

Host Institution

My hosting institution is SISSA in Trieste and, more specifically, the Functional Analysis and Applications Sector. As already mentioned in Section 2.c.ii, SISSA offers excellent research facilities in a good research environment, with a very rich mathematical library and a continuous flux of visitors, even for relatively long terms.

The faculty at SISSA is very small, but the complementary expertise of my colleagues might be very useful in the development of the project. I would like to mention in particular my colleagues: Andrei Agrachev (Control Theory), Antonio Ambrosetti (Nonlinear Analysis), Stefano Bianchini (Transport Problems), Antonio DeSimone (Mechanics of Materials), and Andrea Malchiodi (Geometric Analysis).

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