International School for Advanced Studies

Spring 2001 Entrance Examination: Condensed Matter

Solve the following problems. Write out solutions clearly and concisely. State each approximation used. Diagrams welcome. Number page, problem, and question clearly. Do not write your name on the problem sheet, but use extra envelope.

Problem: Two level system. Write down the evolution of the spin of a single electron in presence of a costant external field (set $\mu_B = 1$ for simplicity). The spin of the electron at the time $t = 0$ is directed perpendicular to the field. Say if the spin will ever come back to its initial state, and if so after how long.

Problem: A one-dimensional potential. Consider the one-dimensional real wave-function $\psi(x) = (A + Bx)e^{-\alpha x^4}$. Under what conditions on A and B is $\psi(x)$ the ground state wave-function of a local potential with energy $E = 0$? What is such a potential?

Problem: Photoelectric effect. When light impinges on a metal surface, it may induce a current of electrons escaping from the sample. The figure below depicts the maximum (continuous line) and minimum (broken line) kinetic energies, E_{kin}, of the outgoing electrons as functions of the energy of the incoming photons, $\hbar\omega$ for a simple metal with a single band. Numerical values are roughly appropriate to the case of sodium (Na). Assuming that the free-electron model applies to the present case,

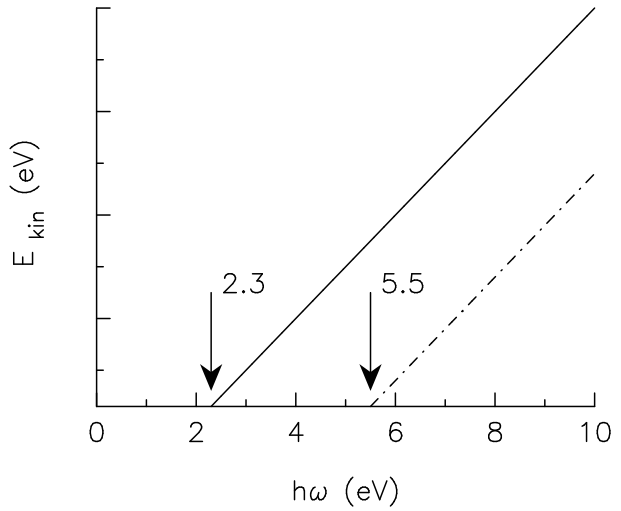

Figure 1: Figure for the problem on the photoelectric effect.

calculate the density of valence electrons of the metal under investigation. (Hint: Do

calculations in atomic units, else use the following constants: 1 eV $= 1.602 \times 10^{-19} J$, $\hbar = 1.055 \times 10^{-34} Js$, electron mass $m = 9.1 \times 10^{-31} kg$.)

Problem: Rotational lines. Let us consider the KCl molecule. The rotational lines of the molecule are equally spaced in frequency with $h\Delta\nu = 2.91 \times 10^{-5} eV$. Determine the distance between the two nuclei of the molecule. Assume that the mass of K is $39u$, and the mass of Cl is $35u$, where u is the atomic mass unit, $u = 1.661 \times 10^{-27} kg$. (Hint: $1eV = 1.602 \times 10^{-19} J$ and the Planck constant is $\hbar = 1.055 \times 10^{-34} Js$.)

Candidates interested in the bio-simulation curriculum can substitute the problem number 4 with the following.

Problem: Cyclobutadiene. Calculate the energy levels and the resonance energy of cyclobutadiene (C_4H_4) using Hueckel MO theory (tight binding). a) Is cyclobutadiene an aromatic system? b) Assign the point group symmetry according to your results of the resonance energy calculation. c) How many isomers would you expect for 1,2-dimethyl-cyclobutadiene?