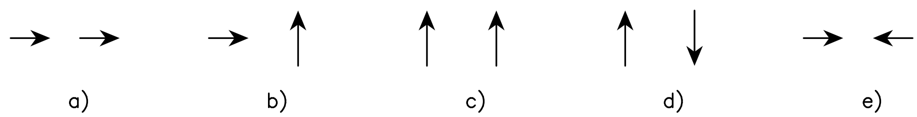
International School for Advanced Studies

Spring 2001 Entrance Examination: Condensed Matter

Multiple choice quizzes

1. Which configuration of two dipoles at a distance a has the lowest electrostatic energy:



2. In atoms with several optically active electrons, in the Hartree approximation, many levels are degenerate. The degeneracy is lifted by interactions not accounted for by the Hartree approximation. Within the LS coupling, which one of the following statements is true:

   (a) The spin-orbit interaction lifts the degeneracy between states which differ only in the total angular momentum J.

   (b) The spin-orbit interaction lifts the degeneracy between states which differ only in the magnetic total angular momentum M_J.

   (c) The spin-orbit interaction lifts the degeneracy between states with different orbital angular momentum L.

   (d) The spin-orbit interaction lifts the degeneracy between states with different spin angular momentum S.

3. The electron of a hydrogen atom in a uniform electric field has a potential energy $-e^2/|\mathbf{r}| + e\mathbf{E} \cdot \mathbf{r}$ which is infinite at large $\mathbf{r}$. Suppose that the electric field is very small. The energy of the system could be lowered if the electron was infinitely far from the nucleus. Why does the atom not ionize immediately?

   (a) The lifetime of the bound state is very long and on the timescale of the measurement no ionization is observed.

   (b) If the electric field is sufficiently small the energy of the electron bound to the nucleus is lower than in any other configuration.

(c) The atoms actually ionize, but the electrons jump from one atom to the other and on average they appear always bound.

(d) The electric field just polarizes the electron and shifts its energy as shown by perturbation theory (Stark effect).

4. In atomic Ti the $3d$ levels are partially filled with two electrons. Select the ground state according to Hund's rules. (Hint: The notation used below is ${}^{2S+1}L_J$.)

   (a) ${}^3G_3$

   (b) ${}^3F_2$

   (c) ${}^1F_3$

   (d) ${}^1S_0$

5. The theory of Bragg diffraction from crystals relies on the symmetry properties of infinite, periodic systems. Actual crystals are finite, and their periodicity broken by defects. Which one among the following statements is true?

   (a) Bragg diffraction spectra are not affected by the size of the sample, nor by the concentration of defects, because the typical X-ray wave-length is much smaller that both the average distance among defects and, *a fortiori*, of the size of the sample.

   (b) The width of Bragg lines decreases with the concentration of defects, while their intensity increases with the size of the sample.

   (c) Bragg diffraction is a thought experiment which cannot be realized for any real finite and/or impure sample.

   (d) The width of Bragg lines decreases when the size of the sample increases, while their intensities increases when the concentration of defects decreases.

6. Two neutral systems whose linear dimensions, l_1 and l_2, are much smaller than their mutual separation, R, have dipole moments, $\mathbf{d}_1$ and $\mathbf{d}_2$, which fluctuate randomly with a stationary probability distribution:

$$P(\mathbf{d}) = (2\pi\sigma^2)^{-\frac{3}{2}} \mathrm{e}^{-\frac{d^2}{2\sigma^2}}.$$

   It is observed that the interaction energy between the two sub-systems decreases as $\sim R^{-6}$ when R grows large. Which one of the following conclusions can be drawn from the this fact?

(a) The correlation between d_1 and d_2, $C_{12} = \langle \mathbf{d}_1 \cdot \mathbf{d}_2 \rangle$, decreases as $\sim R^{-3}$ when R grows large.

(b) The form of the interaction energy indicates that its nature is non-electrostatic. Therefore the two dipoles must be uncorrelated: $C_{12} \equiv 0$.

(c) $C_{12} \sim R^{-6}$.

(d) No conclusions can be drawn from the given elements.

7. If in a thermodynamic system the entropy variation between two equilibrium states is zero it means that:

   (a) The transformation is adiabatic.

   (b) The transformation is reversible.

   (c) The transformation is irreversible.

   (d) It may be anyone of the previous.

8. The mean free path of a molecule in a dilute gas

   (a) is a decreasing function of the cross section,

   (b) is an increasing function of the cross section,

   (c) does not depend on the cross section.

9. Suppose that electrons were spin-3/2 particles. What would be the ground-state electronic configuration of the Carbon atom (Z=6)?

   (a) $1s^2 2s^2 2p^2$

   (b) $2p^6$

   (c) $1s^4 2s^2$

   (d) $1s^1 2s^1 2p^1 3s^1 3p^1 3d^1$

10. Knowing the energy range of visible light is from 1.7 eV to roughly 3.0 eV and that the energy gaps of Silicon and Diamond are, respectively, 1.12 eV and 5.5 eV, which one of the following statements is correct?

    (a) Both Silicon and Diamond appear transparent

    (b) Diamond appears to reflect light as a metal while Silicon is transparent

(c) Silicon appears to reflect light as a metal while Diamond is transparent

(d) Both appear to reflect light as a metal

11. You are given two identical pieces of the same elemental metal. Knowing that the two metal pieces differ only by one mass unit (i.e., they are consecutive isotopes of the same element), and measuring very precisely the speed of sound ratio (v_1/v_2):

    (a) You can tell which is the heavier isotope but nothing else

    (b) You can tell the atomic weight and what element it is

    (c) You can tell the atomic weight but in principle not the element

    (d) You cannot tell anything

12. In the semiclassical theory of transport, an electric field $\mathbf{E}$ causes a band electron of crystal momentum $\mathbf{k}$ to move according to $\hbar\dot{\mathbf{k}} = -e\mathbf{E}$. Consider two ideally perfect crystals, a metal and an insulator. Suppose (unrealistically, of course) that collisions could be totally ignored so that the above equation of motion is valid at all times. Then:

    (a) The electrons in the metal will accelerate indefinitely, until approaching the speed of light. The electrons in the insulator will not move at all.

    (b) There will be no current in either case. The insulator does not conduct, while in the perfect metal $|\mathbf{E}| = 0$.

    (c) The metal Fermi surface will be displaced proportionally to $|\mathbf{E}|$, that of the insulator will not be displaced at all.

    (d) Each electron will execute an oscillatory motion of frequency $\propto |\mathbf{E}|$. The total oscillating current will cancel out to zero in the insulator but not in the metal.

13. Consider metallic sodium (Na-Na distance 3.66 Å), a good metal with a nearly spherical Fermi surface. Imagine expanding artificially and uniformely the whole lattice by a factor 10^8 so that the new Na-Na distance becomes 3.66 cm. With all that vacuum between an atom and the next, we suppose that Na would cease to conduct. In fact:

    (a) That's not true. The band-width will be extremely narrow but non-zero and there will be no energy gap.

(b) The coupling of electrons to ultra-soft lattice vibrations will destroy conduction.

(c) There will be an energy gap due to electron-electron repulsion, causing the metallic conduction to disappear.

(d) Expanded Na will insulate because electrons will be immobilized by the impossibility to tunnel through extremely large vacuum.

The following questions are intended for candidates interested in the bio-simulation curriculum

14. Biological membranes contain all but one of the following major classes of biomolecules:

    (a) Lipids

    (b) Carbohydrate

    (c) Protein

    (d) Nucleic acid

15. Which of the following is most important in the formation of an alpha helix from a polypeptide chain?

    (a) Hydrogen bonds

    (b) Salt bridges

    (c) Disulfide bridges

    (d) Hydrophobic interactions

16. The amino acid with the bulkiest side chain is

    (a) Alanine

    (b) Valine

    (c) Tyrosine

    (d) Histidine

    (e) Tryptophan

17. Amino acids with bulky side chains are most easily accommodated by which structure?

(a) alpha-helix

(b) beta-pleated sheet

(c) collagen-like triple helix

(d) beta-turn

18. What is the least amount of information required to calculate the pH of a solution containing a mixture of weak acid and conjugate base? (Let [HA] = the concentration of acid, and [A-] = the concentration of base.)

    (a) pKa

    (b) $[HA]$

    (c) $[HA]$ and $[A-]$

    (d) pKa and $[HA]$

    (e) pKa, $[HA]$, and $[A-]$