SISSA ISAS

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Fall 2002 – Entrance Examination: Condensed Matter
Multiple choice quizzes

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1. To a first approximation the two electrons in a helium atom occupy the same 1s orbital with opposite spins. (Call E_0 the energy expectation value in that approximate state). In the exact ground state however:

   (a) due to correlations there is a small probability for the two 1s spins to be parallel, and this lowers the energy below E_0.

   (b) correlations favor the two electrons to be preferably on opposite side of the nucleus, and this lowers the energy below E_0.

   (c) electrostatic repulsion of the two electrons introduces a small positive correction, which raises the energy slightly above E_0.

   (d) the first Hund's rule applies, and the configuration switches to parallel 1s electrons with total spin 1, with energy slightly below E_0.

2. Suppose you could compress a simple material, say hydrogen, to an unlimited high pressure. What would you expect to find in the end:

   (a) a hard solid of protons (that are heavy and always remain classical), insulating because the electrons can no longer avoid the static nuclei.

   (b) a hard solid of protons (that are heavy and always remain classical), metallic because the electron kinetic energies grow faster than their potential energies.

   (c) a total collapse with condensation of squeezed bosonic hydrogen atoms to zero volume.

   (d) a quantum Fermi liquid of protons and electrons, the solid structure disrupted by proton zero point motion, the kinetic energies raising according to their respective Fermi energies.

3. Given any real system, such as an atom, a molecule, a piece of solid or liquid matter, one can define the ionization energy I as the energy to be provided to extract an electron out to vacuum, and the electron affinity A as the energy released when adding an electron from vacuum to the system. What general statement do you think is correct:

   (a) $I \geq A$, A positive or negative

   (b) $A \geq I \geq 0$

   (c) $I = A$

(d) I positive, A negative

4. The ground state of a system of bosons has no nodes (node = zero of the wavefunction); that of a system of fermions generally has nodes, which imply an extra kinetic energy.

   (a) neither of these two statements is generally true

   (b) only the first statement is true, as fermions can also become nodeless as in superconductors

   (c) only the second statement is true, as bosons will in general will also have nodes as in helium.

   (d) both statements are true, the fermion nodes due to the Pauli principle, and the extra kinetic energy the Fermi energy.

5. Consider the He atom with two electrons in the configuration $1s_2$. What happens to the two electrons wave function if the two electron coordinates are interchanged (without changing the spins)?

   (a) the total wave function changes its sign.

   (b) the total wave function changes sign and amplitude.

   (c) the total wave function changes only the amplitude but not the sign

   (d) there is no change.

6. Approximately, how many H_2O molecules are contained in a liter of water ?

   (a) 3×10^{15}

   (b) 3×10^{20}

   (c) 3×10^{25}

   (d) 3×10^{30}

7. A cyclotron shoots protons of 1 MeV kinetic energy against an Aluminum (Z=13) target. What is the closest distance from the Aluminum nuclei the electrons can reach ? (Hint: Recall that $e^2/a_{Bohr} \approx 27.2eV$.)

   (a) 2×10^{-10} m

   (b) 2×10^{-12} m

   (c) 2×10^{-14} m

   (d) 2×10^{-16} m

8. Consider the simple ionization process:

$$h\nu + \mathrm{He} \longrightarrow \mathrm{He}^+ + \mathrm{e}^-,$$

   where the Helium atom is initially in its ground state (whose total energy is $E = -78.88$ eV) and the He^+ ion in the final state is also in its ground state. What is the minimum energy the incoming photon must have in order to ionize the system ?

   (a) 24.46 eV

(b) 51.66 eV

(c) 65.27 eV

(d) 78.88 eV

9. A classical particle of charge Q sits at distance d from a flat metallic surface. How much energy is needed to remove the particle to infinite distance?

(a) Q^2/d

(b) $Q^2/2d$

(c) $Q^2/4d$

(d) It depends on the work function of the metal.

10. Magnetic insulators often have spin magnetization moments directed along preferred orientations. What is the essential physical ingredient behind such anisotropy?

(a) The small magnetic field due to the earth.

(b) The surfaces of any real material will break spin-rotational invariance, inducing preferred orientations.

(c) The spin-orbit interaction.

(d) The presence of the crystalline potential which breaks rotational invariance.

11. The spins of a magnetic insulator interact among each other with interactions that under many circumstances can be taken to be of the Heisenberg form $H = \sum_{i,j} J_{ij}\vec{S}_i \cdot \vec{S}_j$. The interaction is mainly due to:

(a) The dipolar field seen by each spin due to all the other spins.

(b) The combined effect of Pauli principle and Coulomb interactions between the electrons.

(c) The spin-orbit interaction.

(d) Each spin feels the others through the conduction electrons.

12. The resistivity of a piece of normal (non superconducting) metal does not go to zero as the temperature goes to zero. This is due to:

(a) Zero-point quantum oscillations of the lattice which scatter the electrons

(b) Presence of impurities

(c) Scattering due to phonons

(d) Both a) and c).

13. Given the reduced mass of a diatomic molecule, which one of the following properties cannot be deduced from the potential energy curve $V(r)$.

(a) The momentum of inertia.

(b) The peak positions of the infrared absorption spectrum.

(c) The peak positions of the ultraviolet absorption spectrum.

(d) The peak positions of the roto-vibrational spectrum.

14. The external appearance of a solid (trasparent, colored, shiny, etc) is mainly determined by:

    (a) its internal optical properties, in turn detemined by the interband transition of its electrons in the bulk solid

    (b) its internal optical properties, in turn determined by the optical phonon vibrations of its atoms in the bulk solid.

    (c) its first layer surface electronic properties, in turn detemined by the interband transition of its electrons localized in the first layer only, the penetration length of light being similar to the lattice spacing.

    (d) its first layer surface vibrational properties, in turn detemined by the optical phonon vibrations of its atoms localized in the first surface layer only, the penetration length of light being similar to the lattice spacing.

15. Suppose you were given a piece of metallic material on which you could perform a single measurement of very modest accuracy at a single temperature not chosen by you, and you had to tell whether the material is or is not a superconductor at that temperature. What would you choose as the safest among these four possible measurements:

    (a) resistivity

    (b) conductivity

    (c) magnetic susceptibility

    (d) specific heat

16. All modern electronics is based on semiconductors, principally silicon. This is because:

    (a) semiconductors permit currents that are neither too small (as in insulators) nor too large (as in metals);

    (b) silicon, one of the most abundant elements in nature, is one of the cheapest materials available;

    (c) only for semiconductors, particularly silicon, the industrial chip technology processes became so massively sophisticated to supplant all other technologies, in particular the old fashioned vacuum tubes;

    (d) electron conductance in semiconductors can be controlled by doping, and by external voltages.

17. An electron in a rigid box with hard walls may be seen as exerting a pressure on the walls. In classical mechanics, that pressure exists and is due to temperature, that drives frequent collisions of the electron with the walls. In quantum mechanics, and at zero temperature:

    (a) the pressure becomes negative, as the electron falls in a bound state with the walls

(b) the pressure vanishes as there are no more collisions

(c) the pressure is positive, the electron kinetic energy decreasing with increasing volume;

(d) the pressure is equal to $\hbar$.

18. Take a free molecule made of $N > 2$ atoms, and consider its low energy spectrum, due to the classical motion of the N nuclei. The number of nonzero frequency modes will be

    (a) N

    (b) 3N

    (c) 3N-3

    (d) 3N-6