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October 2010 - Entrance Examination: Condensed Matter
Multiple choice quizzes

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1. The properties of a simple liquid are simulated on a computer by solving Newton's equation of motion for two isolated systems made of $N_1 = 10,000$ and $N_2 = 20,000$ point particles of given mass interacting through a pair potential and starting from two initial conditions such that the energy per particle is the same for the two systems: $E_1/N_1 = E_2/N_2$. Which of the following is the expected behavior of the simulated system:

   A. The total energy (kinetic plus potential) per particle of the system is constant, because it is a conserved quantity. The kinetic energy per particle, which is proportional to the temperature, fluctuates with an amplitude that is inversely proportional to the number of particles.

   B. Both the total energy and the kinetic energies remain constant, the first because it is a conserved quantity, the second because it is proportional to the temperature.

   C. Neither the total nor the kinetic energies are constant, because conservation laws only hold for infinite systems.

   D. The kinetic energy per particle, which is proportional to the temperature, is constant after thermal equilibrium is reached, whereas the total energy per particle fluctuates with an amplitude that is proportional to the specific heat of the system and inversely proportional to its size.

2. In insulators the displacements of the atoms according to a phonon mode can induce a macroscopic electric polarization. This statement is:

   A. Always false. No phonon can induce ever an electric polarization.

   B. Always true. All phonons induce an electric polarization.

   C. Possibly true only for some acoustic phonons with wave vector $\mathbf{q} = (0, 0, 0)$.

   D. Possibly true only for some optical phonons with wave vector $\mathbf{q} = (0, 0, 0)$.

3. The specific heat of a chemically pure crystal is observed to depend on its isotopic composition. Which statement, among the following, is possibly true.

   A. The strength of anharmonic interactions is very large, because the equilibrium properties of a harmonic crystal cannot depend on atomic masses.

   B. This can only be true at low temperature where quantum effects may be important, because the equilibrium properties of a classical system cannot depend on atomic masses.

   C. There is nothing strange in this behavior because the equilibrium properties depend on the ratio between the average kinetic and potential energies, and the former is inversely proportional to the the atomic mass.

   D. There is nothing strange in this behavior because the specific heat measures the propensity of a material to exchange energy with a thermal bath, and it is therefore a non-equilibrium property.

4. The Grüneisen parameters are essentially derivatives of the phonon frequencies with respect to the crystal volume. You need the Grüneisen parameters of a solid to calculate:

   A. Its constant-volume specific heat.

   B. Its compressibility.

   C. Its electric resistivity.

   D. Its thermal-expansion coefficient.

5. The potential acting on one particle of mass m in one dimension has the form: $V(x) = A(x^2-1)^2$ with $A > 0$. Which statement, among the following, is true?

   A. The ground state of the system is always two-fold degenerate.

   B. The ground-state of the system is never degenerate, and the first excitation energy tends to zero when $m \to \infty$.

   C. The ground-state of the system is never degenerate, and the first excitation energy tends to zero when $m \to 0$.

   D. The degeneracy of the ground state cannot be told without explicitly solving the Schrödinger equation.

6. Among the following molecules identify the one with the vibrational mode of lowest frequency:

   A. An ethylene molecule (C_2H_4).
   B. A ethane molecule (C_2H_6).
   C. A CO molecule.
   D. An acetylene molecule (C_2H_2).

7. In a ferromagnetic solid

   A. there is no inversion symmetry.
   B. there is no gap and the system must be metallic.
   C. there is always a gap and the system must be insulating.
   D. there is no time reversal symmetry.

8. Which technique would you use to measure the vibrational stretch frequency of the N_2 molecule?

   A. Infrared absorption.
   B. Microwave absorption.
   C. Nuclear magnetic resonance.
   D. Raman scattering.

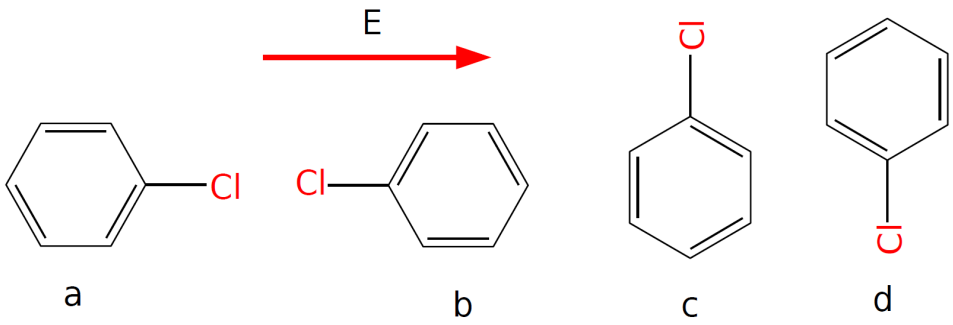
9. At room temperature the entropy of a solid is mainly due to:

   A. The electronic degrees of freedom.
   B. The spin degrees of freedom.
   C. The ionic degrees of freedom.
   D. The electronic degrees of freedom in metals and the ionic degrees of freedom in insulators.

10. Consider the operator $\hat{t} = \hat{\boldsymbol{\sigma}} \cdot \hat{\mathbf{p}}$, where $\hat{\boldsymbol{\sigma}} = \{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$ is a vector whose components are the Pauli matrices and $\hat{\mathbf{p}}$ is the (vector) momentum operator, and let us indicate by t its eigenvalues. Which statement, among the following, is true:

    A. $t = \pm|\mathbf{p}|$.
    B. $t = \pm p_z$.
    C. $t = \pm\sqrt{p_x^2 + p_y^2}$.
    D. $t = \pm\sqrt[3]{|p_x p_y p_z|}$.

11. The figure shows four clorobenzene molecules (C_6H_5Cl) in an electric field. Only one of them is in a stable equilibrium position. Which one?



A. a.

B. b.

C. c.

D. d.

12. According to the Bloch theorem, the eigenfunctions of the one-particle Hamiltonian of a crystalline solid are classified by a vector $\mathbf{k}$ and a band index v, so that each of them can be written as $\psi_{\mathbf{k},v}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k},v}(\mathbf{r})$ where $u_{\mathbf{k},v}(\mathbf{r})$ has the periodicity of the Bravais lattice. Which sentence, among the following, is wrong?

A. $u_{\mathbf{k},v}(\mathbf{r})$ and $u_{\mathbf{k},v'}(\mathbf{r})$ are orthogonal.

B. $\psi_{\mathbf{k}+\mathbf{G},v}(\mathbf{r})$ and $\psi_{\mathbf{k},v}(\mathbf{r})$ can be chosen equal ($\mathbf{G}$ is a reciprocal lattice vector).

C. $\psi_{\mathbf{k},v}(\mathbf{r})$ and $\psi_{\mathbf{k},v'}(\mathbf{r})$ are orthogonal.

D. $u_{\mathbf{k}',v}(\mathbf{r})$ and $u_{\mathbf{k},v}(\mathbf{r})$ are orthogonal.

13. At equilibrium, the acetylene molecule (C_2H_2) is linear. The number of normal modes for small oscillations around equilibrium with non-vanishing frequencies is:

A. 12.

B. 9.

C. 7.

D. 6.

14. Identify, among the following solids, the one with the highest entropy per mole at room temperature.

    A. bcc-Na.

    B. fcc-Ni.

    C. Carbon in the diamond structure.

    D. Silicon in the diamond structure.

15. Consider a particle in a one-dimensional potential $V(x)$ such that $V(x) < 0$ for all x and $V(\pm\infty) = 0$:

    A. There are no bound states.

    B. All solutions are bound states.

    C. There is always at least one bound state.

    D. It depends on the shape of the potential.

16. Flip a coin $2N$ times, where N is large. Let $P(x)$ be the probability of obtaining exactly $N + x$ heads, with $x \ll N$:

    A. $P(x) \propto \exp(-x^2/N)$.

    B. $P(x) \propto \exp(-x/N)$.

    C. $P(x) \propto \exp(-x^2/N^2)$.

    D. $P(x) \propto \exp(-x/N^2)$.

17. The most common isotope of Carbon has nuclear mass A = 12. The sound velocity is measured in two chemically pure diamond samples, whose isotopic composition is however different: let us indicate with $A_1 > A_2$ the average nuclear masses in the two samples, and with v_1 and v_2 the two corresponding sound velocities. Which of the following will be found:

    A. $v_1 > v_2$.

    B. $v_1 < v_2$.

    C. $v_1 = v_2$.

    D. One cannot tell, because the sound velocity is not a monotonic function of the nuclear mass.

18. The acetylene molecule (C_2H_2) is linear, whereas the water molecule (H_2O) is triangular, methane (CH_4) is tetrahedral, and benzene C_6H_6 is hexagonal. Which one of the following molecular liquids has the largest static electric permittivity?

    A. H_2O.

    B. C_2H_2.

    C. CH_4.

    D. C_6H_6.

19. Consider a Fluorine atom, F (electronic configuration $1s^2 2s^2 2p^5$), and its negative ion F^-, and let us indicate by ϵ°_{1s} and ϵ^-_{1s} the 1s energy levels of the neutral and ionic species, respectively. Which one of the following statements is true:

    A. $|\epsilon^\circ_{1s}| > |\epsilon^-_{1s}|$.

    B. $|\epsilon^\circ_{1s}| < |\epsilon^-_{1s}|$.

    C. $|\epsilon^\circ_{1s}| = |\epsilon^-_{1s}|$.

    D. One cannot tell without solving the Schrödinger equation because the orbital energies of inner atomic shells are not monotonic functions of the outer-shell occupancies.

20. Let $\mathbf{s}$, $\mathbf{p}$, and $\mathbf{r}$ be the spin angular momentum, linear momentum, and position operators of an electron, respectively. Which one of the following operators breaks both the parity and time-reversal invariance?

    A. $\frac{1}{2}(\mathbf{r}\cdot\mathbf{p} + \mathbf{p}\cdot\mathbf{r})$.

    B. $\mathbf{s}\cdot\mathbf{p}$.

    C. $\mathbf{s}\cdot\mathbf{r}$.

    D. $\mathbf{s}\cdot\mathbf{s}$.