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April 2010 - Entrance Examination: Condensed Matter
Multiple choice quizzes

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1. In a certain nanofriction experiment, where a tip is slid on a crystal surface, it is found that the measured friction depends on the isotope mass composition of the surface layer. That seems surprising, because nothing in classical equilibrium statistical mechanics depends on the mass of atoms.

   A. The outcome can only be due to the fact that friction dynamics is not at equilibrium — quantum mechanics has surely nothing to do with it.

   B. The outcome can only be due to the fact that friction phonon dissipation is governed by quantum mechanics — classical dynamics has surely nothing to do with it.

   C. The outcome could have to do both with non-equilibrium dynamics, and/or with quantum mechanics — it is impossible to tell without additional evidence.

   D. The mass dependence described must be false, and is just impossible, because frictional forces are just gradients of (mass independent) free energies.

2. Suppose you were to observe the classical motion of atoms in a warm solid — an N-atom system — at temperature T. According to classical statistical mechanics, each atom contributes a thermal energy $3k_BT$, so that as T tends to zero the specific heat will be constant, and the entropy will not vanish. On the other hand, Nernst's theorem (third law of thermodynamics) states that entropy should vanish at $T = 0$. Which of the following statements is correct?

   A. Quantum mechanics applies to electrons and spins but not to the motion of atoms, for which classical mechanics is perfectly adequate. Nernst's theorem does not apply to condensed systems such as liquids or solids at any temperature.

   B. Nernst's theorem is restricted to systems that exhibit a gap. Solids and liquids have acoustical phonons and no gap, and for them entropy will not vanish.

   C. It is not true that, as stated, the specific heat of a classical system will remain constant as T tends to zero. In the Debye model, which has nothing to do with quantum mechanics and whose result does not contain Planck's constant, the specific heat vanishes like $(T/T_{\rm Debye})^3$, in perfect agreement with Nernst's theorem.

D. Classical mechanics is simply wrong, and must be replaced by quantum mechanics, even if all atoms are classical, at low enough temperatures. In quantum mechanics, the ground state reached at $T = 0$ is essentially non degenerate, and thus has zero entropy.

3. A paramagnetic solid is kept at constant temperature T inside a magnetic field B.

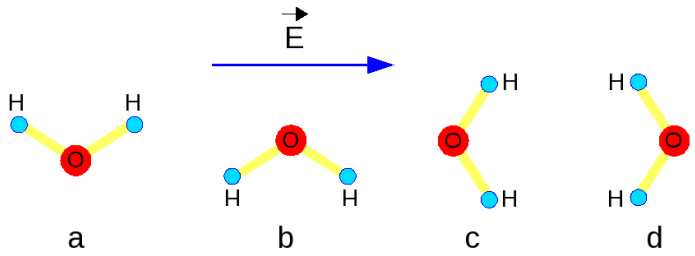
   A. By increasing the magnetic field intensity B, its entropy increases.

   B. By increasing the magnetic field intensity B, its entropy decreases.

   C. Its entropy is independent from the magnetic field intensity.

   D. It depends on the material. For some paramagnetic solids, the entropy increases by increasing the magnetic field, for others it decreases.

4. The figure shows four water molecules in an electric field. One of them is in a stable equilibrium position. Which one?



   A. a.

   B. b.

   C. c.

   D. d.

5. The purely rotational spectrum of a molecule can be detected by absorption of microwave radiation. This is possible only if:

   A. The molecule has inversion symmetry.

   B. The incident light can induce a dipole moment.

   C. The molecule has a permanent dipole moment.

   D. The molecule has no permanent dipole moment.

6. The change of the enthalpy of a thermodynamical system is equal to the heat given to it when:

   A. The pressure is constant.

   B. The volume is constant.

   C. The temperature is constant.

   D. Only at $T = 0$.

7. The electronic configuration of the Pr atom, which has 59 electrons, is $[Xe]4f^3 6s^2$, where $[Xe]$ indicates the ground state configuration of the Xenon atom. According to the Hund's rules, which is its ground state? (we use the notation ${}^{2S+1}L_J$ where S, L and J are the spin, orbital and total angular momenta. For L the spectroscopic notation S, P, D, F, G, H, I stands for $L = 0, 1, 2, 3, 4, 5, 6$, respectively).

   A. ${}^4I_{9/2}$.

   B. ${}^2S_{1/2}$.

   C. ${}^3P_2$.

   D. ${}^4D_{1/2}$.

8. Let us consider the following solids: bcc-Na, bcc-W, NaCl, solid benzene. Order them in order of increasing cohesive energy:

   A. bcc-Na < solid benzene < NaCl < bcc-W.

   B. solid benzene < bcc-Na < NaCl < bcc-W.

   C. solid benzene < bcc-Na < bcc-W < NaCl.

   D. bcc-Na < solid benzene < bcc-W < NaCl.

9. Let us consider the band structure of a ferromagnetic periodic solid with inversion symmetry. Let us call $\epsilon_{\mathbf{k},\uparrow}$ and $\epsilon_{\mathbf{k},\downarrow}$ the eigenvalues of the Schrödinger equation for electrons with wavevector $\mathbf{k}$ and spin up or down respectively. Which one of the following relationships holds?

   A. $\epsilon_{\mathbf{k},\uparrow} = \epsilon_{-\mathbf{k},\uparrow}$.

   B. $\epsilon_{\mathbf{k},\uparrow} = \epsilon_{-\mathbf{k},\downarrow}$.

   C. $\epsilon_{\mathbf{k},\uparrow} = \epsilon_{\mathbf{k},\downarrow}$.

   D. None of the above.

10. The direction of the magnetization vector in a crystalline ferromagnet is generally oriented along some crystal axis (easy axis), and can only be deviated along another direction by applying a fairly strong external field. About this phenomenon, known as magneto crystalline anisotropy, we know that

    A. It is due to impurities, and is stronger the larger their concentration.

    B. It is only present in metals, where it arises due to dipolar spin-spin interactions, active due to the absence of an electron gap.

    C. It is exclusively present in insulators, where it is due to a lack of screening of Coulomb and exchange interactions.

    D. It is most commonly due to spin-orbit interaction, which is a relativistic effect.

11. The sky is blue because:

    A. The cross section for incoherent scattering of light from the molecules in a gas is proportional to the fourth power of the radiation frequency.

    B. The cross section for incoherent scattering of light from the molecules in a gas is proportional to the fourth power of the radiation wavelength

    C. The oxygen molecule has a strong absorption peak in the blue.

    D. The nitrogen molecule has a strong absorption peak in the blue.

12. A homogeneous electric field is applied to a gas of atomic hydrogen (H) and, independently, to a gas of atomic lithium (Li). The first absorption line in the two gases is:

    A. Slightly shifted in both gases.

    B. Split in three lines in both gases.

    C. Split in three lines in H and slightly shifted in Li.

    D. Split in three lines in Li and slightly shifted in H.

13. The water molecule (H_2O) has the shape of an isosceles triangle. The number of vibrational modes with non zero frequency of H_2O is:

    A. 9.

    B. 4.

    C. 6.

    D. 3.

14. Among the following symmetry operations, identify the one that cannot belong to the point group of a solid.

    A. A mirror reflection.
    B. A rotation of $\frac{2\pi}{5}$ radiants.
    C. A rotation of $\frac{2\pi}{3}$ radiants.
    D. A rotation of $\frac{2\pi}{3}$ radiants followed by a mirror reflection on a plane perpendicular to the rotation axis.

15. Deuterium is an hydrogen isotope with a nucleus composed by a proton and a neutron. Compare the ionization potentials of deuterium I_D and hydrogen I_H.

    A. $I_D = I_H$.
    B. I_D is slightly smaller than I_H.
    C. I_D is slightly larger than I_H.
    D. $I_D = I_H/2$.

16. Let us consider a cubic box of side L. The ground state of an electron inside the box is non degenerate. Which is the degeneracy of the first excited state?

    A. 1.
    B. 2.
    C. 3.
    D. 4.

17. The Fermi energy of a free-electron metal increases with decreasing volume, as

    A. V^{-2}.
    B. $V^{-2/3}$.
    C. $V^{-1/3}$.
    D. It depends on the metal.

18. The volume of a solid changes with temperature due to anharmonic effects:

    A. Increasing the temperature the volume always increases.
    B. Increasing the temperature the volume increases in some solid and decreases in others. If it increases, it does so at all temperatures; if it decreases, it does so at all temperatures.
    C. Increasing the temperature the volume always decreases.
    D. Increasing the temperatures the volume might increase or decrease and this might be temperature dependent.

19. The frequency dependent dielectric constant $\epsilon(\omega)$ of a dispersive medium is, in general, a complex quantity. At positive frequencies, the imaginary part is:

    A. Always positive.

    B. Always negative.

    C. Positive for some frequencies and negative for others.

    D. Always zero. The dielectric constant is a measurable quantity. How can it have an imaginary part?

20. In the Born approximation valid for a weak potential of strength V, the scattering cross section

    A. is proportional to $|\log V|$.

    B. is proportional to $|V|^{-1}$.

    C. is proportional to $|V|^2$.

    D. is proportional to V.