SISSA ISAS

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October 2011 - Entrance Examination: Condensed Matter Multiple choice quizzes

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1. Application of a magnetic field to the motion of a free quantum particle of charge q and zero spin modifies its Hamiltonian, as is well known, through the so-called "minimal coupling" – it adds a vector potential term to the momentum of the particle, whose energy remains exclusively kinetic. As a result, the particle's ground state energy

   A. increases – this signals diamagnetism.

   B. decreases – this signals paramagnetism.

   C. is actually not exclusively kinetic, there is also a potential term, in the form of a harmonic oscillator.

   D. remains exactly the same as for zero field – a magnetic field cannot execute any work.

2. The Hamiltonian operator H for an electron of mass m and charge e interacting with a uniform electric field parallel to the z axis,

   A. commutes with the position operator z.

   B. commutes with p_z where $\mathbf{p}$ is the electron momentum operator.

   C. commutes with L_z where $\mathbf{L}$ is the electron orbital angular momentum operator.

   D. commutes with $\mathbf{L}^2$ where $\mathbf{L}$ is the electron orbital angular momentum operator.

3. The ground state of the Beryllium atom has atomic configuration $(1s)^2(2s)^2$. What are the total spin (S) and orbital angular momentum of the excited state of lowest energy?

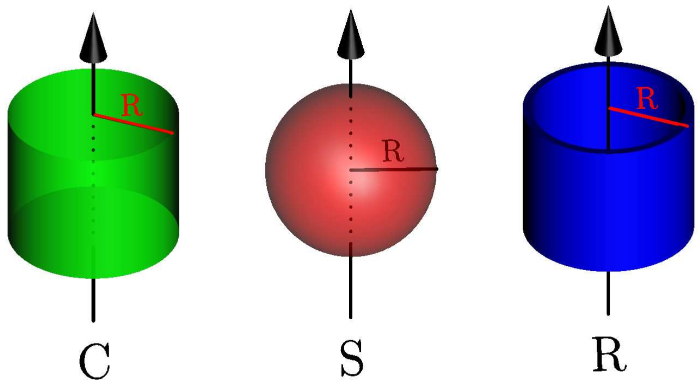
   A. $L = 0$, $S = 0$.

   B. $L = 1$, $S = 0$.

   C. $L = 0$, $S = 1$.

   D. $L = 1$, $S = 1$.

4. The moments of inertia of the solid cylinder (C), sphere (S) and ring (R) of mass M and radius R for the axes shown in the figure are:



   A. $I_C = MR^2$, $I_S = \frac{1}{2}MR^2$, $I_R = \frac{2}{5}MR^2$.

   B. $I_C = \frac{1}{2}MR^2$, $I_S = \frac{2}{5}MR^2$, $I_R = MR^2$.

   C. $I_C = MR^2$, $I_S = \frac{2}{5}MR^2$, $I_R = \frac{1}{2}MR^2$.

   D. $I_C = \frac{2}{5}MR^2$, $I_S = \frac{1}{2}MR^2$, $I_R = MR^2$.

5. Consider a particle of mass m in 1D, subject to a potential $V(x) = x^2 - x^4 + x^6$. The particle undergoes a periodic motion of amplitude A and period T. Let us indicate with $T(A)$ the dependence of the period on the amplitude. Which one of the following statements is true:

   A. $T(A) = \text{constant}$.

   B. $\lim_{A\to 0} T(A) > 0$; $T'(A) > 0$ for small A; $T'(A) < 0$ for large enough A.

   C. $\lim_{A\to 0} T(A) = 0$, $T'(A) > 0$ for $A > 0$.

   D. $\lim_{A\to 0} T(A) = \infty$.

6. Nucleation is a kinetic mechanism through which one phase transforms into another more stable phase. The transformation begins in a small volume or nucleus which then expands to invade the whole system.

   A. It takes place exclusively in second order transitions, but not in first order ones.

   B. It always takes place, both in second order and in first order transitions.

   C. It takes place in first order transitions, but not in second order ones.

   D. It only occurs occasionally, and has nothing general or fundamental to it.

7. Let us consider a linear chain formed by N atoms (with $N > 4$). How many longitudinal vibrational modes (i.e. with displacements parallel to the chain) of finite frequency are there?

   A. $N - 1$.

   B. $N - 2$.

   C. $N - 3$.

   D. $N - 4$.

8. Two samples of germanium (Ge, $Z = 32$) are heavily doped with gallium (Ga, $Z = 31$) and arsenic (As, $Z = 33$). The electric current generated by the action of a homogeneous electric field, $\mathbf{E}$, is recorded in both cases and indicated by $\mathbf{J}_{\text{Ga}}$ and $\mathbf{J}_{\text{As}}$, respectively. Which one of the following relations will be found to hold?

   A. $\mathbf{E} \cdot \mathbf{J}_{\text{Ga}} > 0$, $\mathbf{E} \cdot \mathbf{J}_{\text{As}} > 0$.

   B. $\mathbf{E} \cdot \mathbf{J}_{\text{Ga}} > 0$, $\mathbf{E} \cdot \mathbf{J}_{\text{As}} < 0$.

   C. $\mathbf{E} \cdot \mathbf{J}_{\text{Ga}} < 0$, $\mathbf{E} \cdot \mathbf{J}_{\text{As}} > 0$.

   D. $\mathbf{E} \cdot \mathbf{J}_{\text{Ga}} < 0$, $\mathbf{E} \cdot \mathbf{J}_{\text{As}} < 0$.

9. Spaghetti cook faster, the higher the temperature of the boiling water. Which of the following tricks does not alter the cooking time of spaghetti:

   A. Adding more salt to the water.

   B. Raising the power of the fire that heats the water.

   C. Putting a heavy cover on top of the pot.

   D. Cooking on the top of a mountain.

10. Suppose two people, A and B, agree to play a series of fair games until one person has won three games. They each have wagered the same amount of money, the intention being that the winner will be awarded the entire pot. But suppose, for whatever reason, the series is prematurely terminated, at which point A has won two games and B just one. How should the stake be divided?

   A. The entire stake to A.

   B. 2/3 to A and 1/3 to B.

   C. 3/4 to A and 1/4 to B.

   D. 1/2 to A and 1/2 to B.

11. The water molecule, H_2O:

    A. Has nonzero spin and magnetic dipole moment, and zero electrical dipole moment.

    B. Has zero spin and magnetic dipole moment, and nonzero electrical dipole moment.

    C. Has nonzero spin and magnetic dipole moment, and nonzero electrical dipole moment.

    D. Has zero spin and magnetic dipole moment, and zero electrical dipole moment.

12. Is it possible to distinguish a metal from an insulator from the analysis of its electron charge density?

    A. Yes. In insulators the charge density is localized around the atoms, while in metal it is spread all over the unit cell.

    B. Yes. In insulators the charge density has gaps, while in metals there is no gap.

    C. Yes. In insulators the charge density forms bonds and it is localized mainly on the lines joining the atoms, while in metals it is spread uniformly.

    D. No.

13. The rate of a chemical reaction can be approximated by the formula: $\kappa \approx A e^{-\frac{E^*}{k_B T}}$, where E^* is the reaction barrier, A is a constant with the dimension of the inverse of time, T is temperature, and k_B the Boltzmann constant. A same reacton is observed in two samples wich only differ by the isotopic composition of a given atomic species. Let us label by “1” the heavier sample and by “2” the lighter one. Which one of the following relations will be found to hold:

    A. $A_1 = A_2$, $E_1^* = E_2^*$.

    B. $A_1 < A_2$, $E_1^* = E_2^*$.

    C. $A_1 = A_2$, $E_1^* < E_2^*$.

    D. $A_1 < A_2$, $E_1^* < E_2^*$.

14. According to the Bloch theorem, each solution $\psi_{\mathbf{k}n}(\mathbf{r})$ of the Schrödinger equation in a periodic potential with the periodicity of a Bravais lattice can be labeled by a wave vector $\mathbf{k}$ in the first Brillouin zone and a band index n. Moreover $\psi_{\mathbf{k}n}(\mathbf{r}) = e^{i\mathbf{kr}} u_{\mathbf{k}n}(\mathbf{r})$ where $u_{\mathbf{k}n}(\mathbf{r})$ has the periodicity of the Bravais lattice. Assuming $n \neq n'$ the following condition holds:

    A. $\psi_{\mathbf{k}n}(\mathbf{r})$ is orthogonal to $\psi_{\mathbf{k}'n'}(\mathbf{r})$ for any $\mathbf{k}$ and $\mathbf{k}'$.

    B. $u_{\mathbf{k}n}(\mathbf{r})$ is orthogonal to $u_{\mathbf{k}'n'}(\mathbf{r})$ for any $\mathbf{k}$ and $\mathbf{k}'$.

    C. $\psi_{\mathbf{k}n}(\mathbf{r})$ is orthogonal to $\psi_{\mathbf{k}'n'}(\mathbf{r})$ only for $\mathbf{k} = \mathbf{k}'$.

    D. $u_{\mathbf{k}n}(\mathbf{r})$ is orthogonal to $u_{\mathbf{k}'n'}(\mathbf{r})$ only for $\mathbf{k} \neq \mathbf{k}'$.

15. In a periodic solid described by Bloch wavefunctions $\psi_{\mathbf{k}n}(\mathbf{r})$, the Fock potential is a nonlocal potential that can be written as:

$$V_x(\mathbf{r}, \mathbf{r}') = -e^2 \sum_{\mathbf{k}n} \frac{\psi_{\mathbf{k}n}(\mathbf{r})\psi^*_{\mathbf{k}n}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \tag{1}$$

    where the sum is over all $\mathbf{k}$ vectors inside the first Brillouin zone, and all the occupied bands n. Calling $\mathbf{R}$ the direct lattice vectors, the potential $V_x(\mathbf{r}, \mathbf{r}')$ has the following property:

    A. $V_x(\mathbf{r} + \mathbf{R}, \mathbf{r}' + \mathbf{R}') = V_x(\mathbf{r}, \mathbf{r}')$ for any $\mathbf{R}$ and $\mathbf{R}'$.

    B. $V_x(\mathbf{r} + \mathbf{R}, \mathbf{r}') = V_x(\mathbf{r}, \mathbf{r}')$ for any $\mathbf{R}$.

    C. $V_x(\mathbf{r} + \mathbf{R}, \mathbf{r}' + \mathbf{R}) = V_x(\mathbf{r}, \mathbf{r}')$ for any $\mathbf{R}$.

    D. None of the above. $V_x(\mathbf{r}, \mathbf{r}')$ has no periodicity.

16. Two identical plane metal plates carrying an opposite charge $\pm Q$ are held parallel to each other at a distance h, thus generating a potential drop $V = 4\pi Q \frac{h}{S}$ (in c.g.s.- Gaussian units), where S is the area of the plates. Let ΔW_Q and ΔW_V the work done to increase the distance between the plates by Δh, while keeping the charge of the plates constant, or the potential drop between them, respectively. Which one of the following relation between ΔW_Q and ΔW_V holds?

    A. $\Delta W_V = \Delta W_Q$.

    B. $\Delta W_V = -\Delta W_Q$.

    C. $\Delta W_V = \Delta W_Q \left(1 + \frac{\Delta h}{h}\right)$.

    D. $\Delta W_V = \Delta W_Q \left(1 + \frac{\Delta h}{h}\right)^{-1}$.

17. The Hamiltonian of a particle of mass m and charge q constrained onto the surface of a sphere of radius R and interacting with a homogeneous magnetic field, $\mathbf{B}$, can be approximated as: $H = \frac{1}{2mR^2}\mathbf{l}\cdot\mathbf{l} - \frac{q}{2m}\mathbf{l}\cdot\mathbf{B}$, where $\mathbf{l}$ is the angular momentum of the particle. Which one of the following (set of) operators does not commute with the Hamiltonian:

   A. $\mathbf{l}\cdot\mathbf{B}$.

   B. the parity.

   C. $\mathbf{l}\cdot\mathbf{l}$.

   D. $\mathbf{l}\times\mathbf{B}$.

18. In a solid, only one among the following effects can be calculated without accounting for the anharmonicity of the potential energy surface in which the ions move.

   A. The thermal expansion.

   B. The melting temperature.

   C. The isobaric specific heat.

   D. The isochoric specific heat.

19. Let's consider the five d orbitals of an atom. When the atom is isolated they are fivefold degenerate. When the atom is in an external potential with octahedral symmetry, this degeneracy splits.

   A. There are two groups one threefold (d_{xy}, d_{xz}, d_{yz}) and one twofold degenerate $(d_{x^2-y^2}, d_{3z^2-r^2})$.

   B. There are three groups, two twofold degenerate $(d_{xz}, d_{yz}$ and $d_{xy}, d_{x^2-y^2})$ and one nondegenerate $(d_{3z^2-r^2})$.

   C. There are two groups one threefold $(d_{3z^2-r^2}, d_{xz}, d_{yz})$ and one twofold $(d_{x^2-y^2}, d_{xy})$ degenerate.

   D. There is no splitting, the orbitals remain fivefold degenerate.

20. The interaction potential between two neutral particles (atoms or molecules) is usually parametrized through the Lennard-Jones potential, with a short-range repulsion $(\sigma/r)^{12}$ and a long-range attraction $-(\sigma/r)^6$. Which of the following sentences is correct:

   A. Both power laws have a physical justification.
   B. Only the short-range part has a physical justification. The long-range part is arbitrary.
   C. Only the long-range part has a physical justification. The short-range part is arbitrary.
   D. None of them have a real physical justification.