SISSA
SCUOLA INTERNAZIONALE
SUPERIORE di STUDI AVANZATI
International School
for Advanced Studies
via Bonomea 265 • 34136 Trieste • ITALY
T +39 040 3787 111 • F +39 040 3787 249
www.sissa.it

October 2013 - Entrance Examination: Condensed Matter Multiple choice quizzes

1. A cubic meter of H_2 and a cubic meter of O_2 are at the same pressure (p) and at the same temperature (T_1) in their gas phases. Assuming both gases as ideal, suppose you increase, at constant volume, their temperature to T_2. Which one of the following sentences is true?

   A. H_2 requires more heat than O_2.
   B. H_2 requires less heat than O_2.
   C. It cannot be said. It is necessary to perform the experiment.
   D. Both gases require the same amount of heat.

2. Consider a system in thermal equilibrium with a heat bath. The mean square deviation of the energy of the system is:

   A. Zero, because the energy is a constant of motion.
   B. A constant independent of the size of the system and different from zero.
   C. Proportional to the square of the size of the system.
   D. Proportional to the size of the system.

3. Let us consider a system described by a generalized coordinate, q, whose free energy is given by: $F(q) = \frac{1}{2}kq^2$ giving rise to a generalized force: $f = -\frac{\partial F}{\partial q} = -kq$. Let us indicate by ΔF and ΔG the works needed to change the elastic constant k adiabatically from $k = k_1$ to $k = k_2 > k_1$ in two distinct processes keeping the coordinate (ΔF) or the force (ΔG) fixed. Which one of the following statements is true?

   A. $\Delta F > \Delta G > 0$.
   B. $\Delta G > \Delta F > 0$.
   C. $\Delta F < \Delta G < 0$.
   D. $\Delta G < \Delta F < 0$.

4. The Hamiltonian of a system of interacting electrons contains a term $V \propto \mathbf{L} \cdot \mathbf{S}$, where $\mathbf{L} = \sum_i \mathbf{l}_i$ and $\mathbf{S} = \sum_i \mathbf{s}_i$ are the total orbital and spin angular momenta, respectively. Let us indicate with $\mathbf{J} = \mathbf{L} + \mathbf{S}$ the total angular momentum of the system. Which of the following operators commute with V?

   A. J^2, J_z, L^2, and S^2.

   B. L^2, L_z, S^2, and S_z.

   C. J^2, J_z, L_z, and S_z.

   D. None of the above.

5. Consider a particle of mass m inside a cubic box of side L, which is itself inside a cubic box of side $L_1 > L$. The particle is in the ground state. Suppose that suddenly the box of side L is removed. In order to pass to the new ground state in the wider box, the particle emits photons of energy E. Among the following, choose a possible energy of a photon emitted by the particle:

   A. $E = \frac{3\hbar^2}{2m}\left[\left(\frac{\pi}{L_1}\right)^2 - \left(\frac{\pi}{L}\right)^2\right]$.

   B. $E = \frac{3\hbar^2}{2m}\left[2\left(\frac{\pi}{L_1}\right)^2 - \left(\frac{\pi}{L_1}\right)^2\right]$.

   C. $E = \frac{3\hbar^2}{2m}\left[2\left(\frac{\pi}{L}\right)^2 - \left(\frac{\pi}{L}\right)^2\right]$.

   D. $E = \frac{3\hbar^2}{2m}\left[\left(\frac{\pi}{L}\right)^2 - \left(\frac{\pi}{L_1}\right)^2\right]$.

6. The magnetization of a normal metal, i.e. not superconducting, in the presence of a small uniform magnetic field B is

   A. finite and antiparallel to B;

   B. finite and parallel to B;

   C. zero;

   D. finite and perpendicular to B.

7. Silicon is a semiconductor with an energy gap of about 1.1 eV. Its dielectric constant is about $\varepsilon_0 = 12$. Suppose you put a cube of Si in an external electric field of 100 V/m. Which is the minimum length L necessary to close the gap?

   A. About 1.3 mm.

   B. About 1.3 cm.

   C. About 13 cm.

   D. About 1.3 m.

8. A free electron with mass m_e and kinetic energy E_e and a free proton with mass m_p and kinetic energy E_p (both assumed to be non-relativistic) have the same wavelenght. What about their kinetic energy?

   A. They have the same kinetic energy.

   B. $E_p/E_e{=}m_p/m_e$.

   C. $E_p/E_e{=}m_e/m_p$.

   D. $E_p/E_e{=}(m_p/m_e)^2$.

9. The Fe atom has a ground state electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$. According to Hunds rules its ground state is: (we indicate each term with the symbol ${}^{2S+1}L_J$ where S is the total spin angular momentum, L the total orbital angular momentum and J the total angular momentum)

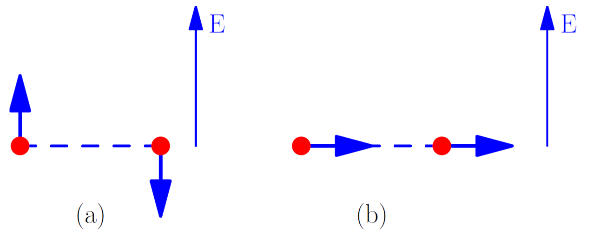
   A. ${}^5D_4$.

   B. ${}^2D_{3/2}$.

   C. ${}^2D_{5/2}$.

   D. ${}^5P_0$.

10. Consider two electric dipoles at fixed distance d in a uniform electric field E in the two configurations shown in the figure:



Which one of the following statements is true?

   A. The electrostatic energy difference between the two configurations is independent from E and configuration (b) has the lowest energy among all possible configurations only when $E = 0$.

   B. The electrostatic energy difference between the two configurations is independent from E and configuration (b) has the lowest energy among all possible configurations.

   C. The electrostatic energy difference between the two configurations depends on E and configuration (a) has the lowest energy among all possible configurations when E is very large.

   D. The electrostatic energy difference between the two configurations depends on E and configuration (b) has the lowest energy among all possible configurations when E is very large.

11. Consider the following quantum mechanical operators for a free particle: position $\mathbf{r}$, momentum $\mathbf{p}$, orbital angular momentum $\mathbf{L}$, spin angular momentum $\mathbf{S}$. Which ones commute with the parity operator?

    A. $\mathbf{L}$ and $\mathbf{S}$.

    B. $\mathbf{r}$ and $\mathbf{p}$.

    C. $\mathbf{p}$ and $\mathbf{L}$.

    D. $\mathbf{r}$ and $\mathbf{S}$.

12. Consider two electronic levels of an idealized system (for example a molecule) whose geometry could be changed parametrically, and suppose the symmetry of the two levels is known all along. As the geometry changes, the energies of the levels will change parametrically, but

    A. The two levels can never cross, no matter what their symmetry.

    B. The two levels can always cross, no matter what their symmetry.

    C. The two levels cannot cross if their symmetry is the same, but can cross if their symmetry is different.

    D. The two levels can cross if their symmetry is the same, but cannot if their symmetry is different.

13. Consider a one dimensional potential $V(x)$ with two identical minima, separated by a barrier of height U and width d. Given an electron initially in the first minimum, compare the probabilities to jump to the second minimum, neglecting prefactors, either (i) classically, due to a finite temperature T, ignoring quantum effects; or (ii) at $T = 0$, by pure quantum mechanical tunneling. The two probabilities:

    A. Both are dependent on U but independent on d

    B. Both are dependent on U and on d

    C. The classical probability depends only on U, the quantum mechanical on both U and d.

    D. The classical probability depends on U and d, the quantum mechanical only on U

14. At long wavelenghts, acoustic phonons have a linear dispersion, their frequency ω is proportional to the wavenumber $|\mathbf{q}|$: $\omega(\mathbf{q}) = c_s|\mathbf{q}|$ where c_s is the speed of sound in the direction of $\mathbf{q}$. In three dimensions the contribution of this mode to the density of phonon states is proportional to $\rho(\omega) \propto \omega^2$. What happens in two dimensions:

    A. $\rho(\omega)$ is constant.

    B. $\rho(\omega) \propto \omega$.

    C. $\rho(\omega) \propto \sqrt{\omega}$.

D. $\rho(\omega) \propto \frac{1}{\sqrt{\omega}}$.

15. The sky at sunset is reddish because:

A. The atmospheric temperature is lower than at midday, and the maximum of black-body radiation shifts from shorter wavelengths (blue) to longer wavelengths (red) when the temperature decreases.

B. Absorption of short wavelength radiation (blue) is enhanced when the atmospheric temperature decreases because fewer molecules in the air are thermally excited. At higher temperature, the transition from the molecular ground state to a higher energy level that is thermally excitated would in fact be hindered by the Pauli exclusion principle.

C. Molecules in the air absorb light more in the blue than in the red, so that the red component of the solar spectrum is enhanced by this selective absorption.

D. The scattering cross section of electromagnetic radiation from molecules in the air is a decreasing function of the radiation wave-length. As the distance traveled by a solar light in the atmoshpere is longer ad sunset than at midday, the long wavelength component of the solar light reaching the planet's ground is reacher in red at sunset than at midday.