Integrable systems and Riemann Surfaces

Tamara Grava

November 24, 2016

Period: January -June 2017

Integrable Systems: 2 cycles
The first of part of the course is an introduction to the modern theory of integrable systems and its fundamental concepts. The second part is focused on the study of the Korteweg de Vries equation that is a dispersive equation and the main properties of its solutions. The tool is direct and inverse scattering for the Schrödinger equation. Then we focus on the asymptotic properties of the solution of the Kortweg de Vries equation: namely long time asymptotics and small dispersion asymptotics.

Prerequisites: classical mechanics

- 1.1 Short review of the classical theory of finite-dimensional integrable systems
- 1.2 Bi-Hamiltonian Geometry and Lax pair
- 1.3 The Toda system
- 1.4 The Korteweg de Vries equation: direct and inverse scattering on the line
- 1.5 Long time asymptotic for the solution of the KdV equation and Deift-Zhou steepest descent method
- 1.6 The Cauchy problem for the KdV equation with periodic initial data and action-angle variables.
- 1.7 Whitham modulation equations and small dispersion limits

The course is mostly based on research articles and the following books

- Novikov, S.; Manakov, S. V.; Pitaevski?, L. P.; Zakharov, V. E. Theory of solitons. The inverse scattering method. Translated from the Russian. Contemporary Soviet Mathematics. Consultants Bureau [Plenum], New York, 1984. xi+276 pp. ISBN: 0-306-10977-8

- Magnus, Wilhelm; Winkler, Stanley Hill's equation. Corrected reprint of the 1966 edition. Dover Publications, Inc., New York, 1979. viii+129 pp. ISBN: 0-486-63738-7 34-02

- Miller, Peter D. Applied asymptotic analysis. Graduate Studies in Mathematics, 75. American Mathematical Society, Providence, RI, 2006. xvi+467 pp. ISBN: 0-8218-4078-9

Riemann Surfaces: 2 cycles

This course in an introductory course on the theory of Riemann surfaces. The second part of the courses connects with the course in integrable systems and the their periodic solutions.

Prerequisites: complex analysis and basic notions in topology

- 2.1 Definition, examples, and topological properties

- 2.2 Holomorphic and meromorphic functions and differentials on a Riemann surface.

- 2.3 Jacobi variety and Abel theorem

- 2.4 Riemann Roch theorem and applications

- 2.5 Theta functions and Riemann vanishing theorem

- 2.6 Baker-Akhiezer function and periodic solutions of integrable nonlinear PDEs

The first part of the course on Riemann Surfaces is taken from the following books.

- Kirwan, Frances Complex algebraic curves. London Mathematical Society Student Texts, 23. Cambridge University Press, Cambridge, 1992. viii+264 pp. ISBN: 0-521-41251-X; 0-521-42353-8

- Farkas, H. M.; Kra, I. Riemann surfaces. Second edition. Graduate Texts in Mathematics, 71. Springer-Verlag, New York, 1992. xvi+363 pp. ISBN: 0-387-97703-1

- Springer, George Introduction to Riemann surfaces. Addison-Wesley Publishing Company, Inc., Reading, Mass. 1957. viii+307 pp.

- Griffiths, Phillip; Harris, Joseph Principles of algebraic geometry. Pure and Applied Mathematics. Wiley-Interscience [John Wiley & Sons], New York, 1978. xii+813 pp. ISBN: 0-471-32792-1

The second part of the course is mainly based on research articles.