Reduced order methods for boundary conditions estimation

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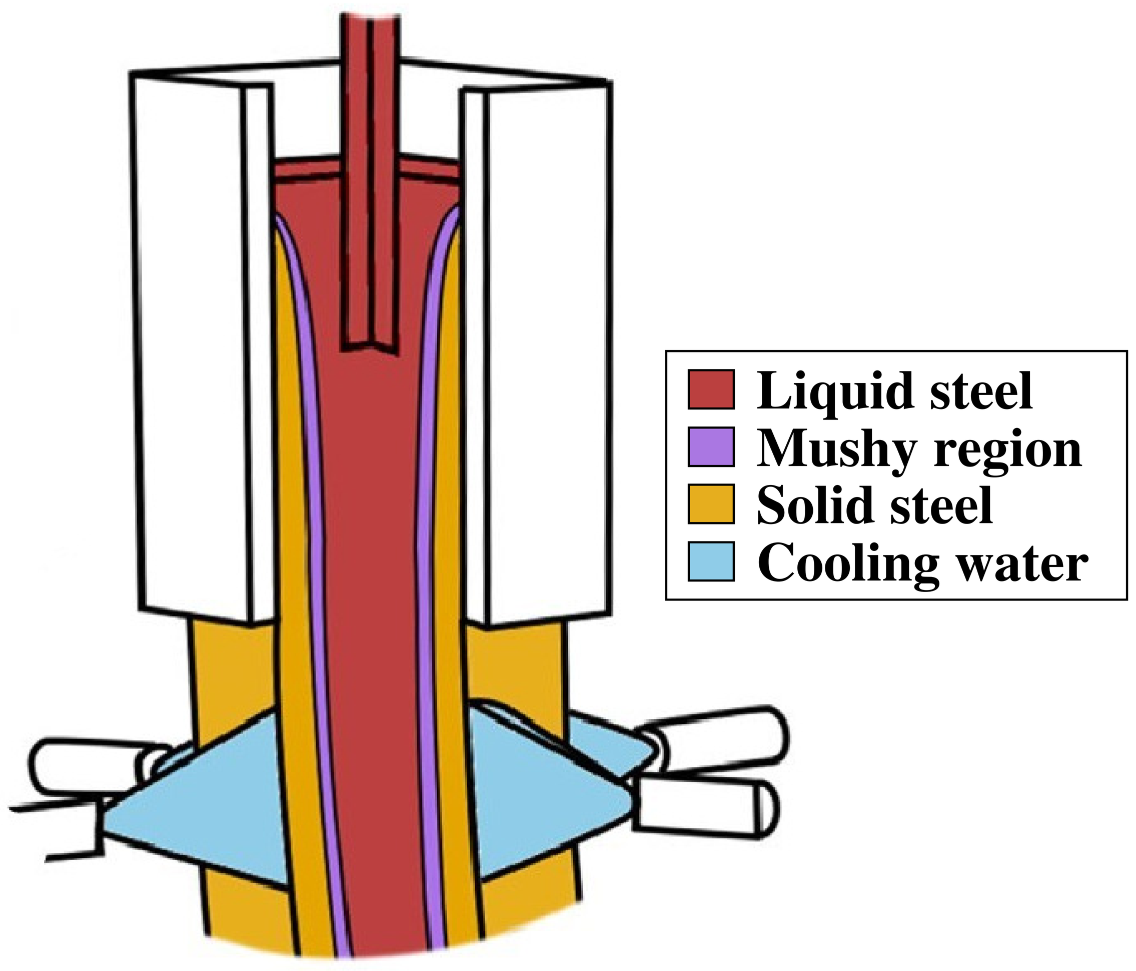
0-Motivation

In continuous casting of steel, the mold is the most critical piece of the process. There, the steel begins to solidify. Its final quality is highly dependent on how this solidification happens. Then, to properly control the proces is necessary to know the heat flux at the mold-steel interface in real time.



Mold schematic [4]

Casting molds are equipped with thermocouples for the measurement of temperatures within the domain. The goal of this research is to develop a methodology for the estimation of the heat flux at the boundary of the mold based on these measures.

1-Direct Problem

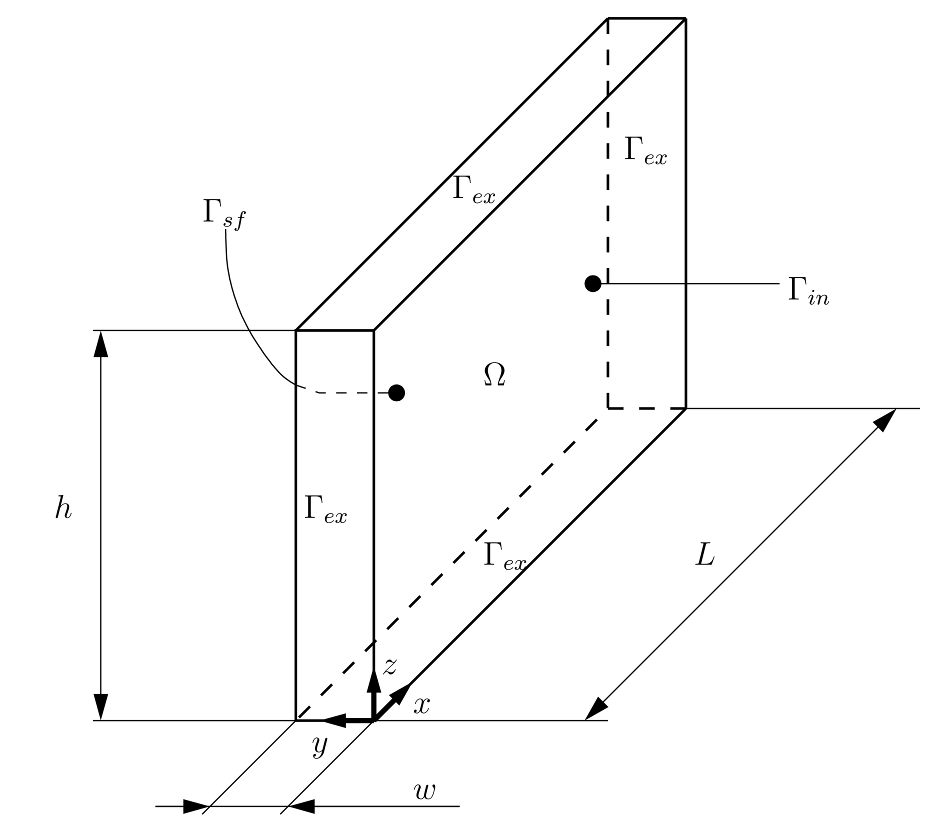
A steady-state heat transfer problem for the mold domain $\Omega \in \mathbb{R}^3$ is considered

Direct problem

$$-k\Delta T(\mathbf{x}) = 0, \quad \forall \mathbf{x} \in \Omega,$$

$$\begin{cases} -k\nabla T(\mathbf{x}) \cdot \mathbf{n} = g(\mathbf{x}) & \forall \mathbf{x} \in \Gamma_{in}, \\ -k\nabla T(\mathbf{x}) \cdot \mathbf{n} = 0 & \forall \mathbf{x} \in \Gamma_{ex}, \\ -k\nabla T(\mathbf{x}) \cdot \mathbf{n} = h(T(\mathbf{x}) - T_f(\mathbf{x})) & \forall \mathbf{x} \in \Gamma_{sf}, \end{cases}$$

$k \in \mathbb{R}^+$, $h \in \mathbb{R}^+$ and $T_f \in H^{1/2}(\Gamma_{in})$.



Given a function $g \in H^{1/2}(\Gamma_{in})$, the previous problem can be numerically solved. The discretization was made using the finite volume method.

2-Inverse Problem

Using a least square, deterministic, approach, the boundary condition estimation problem can be stated as:

Inverse problem

Given the temperature measurements $\tilde{T}(\mathbf{x}_i) \in \mathbb{R}^+$, $i = 1, 2, \dots, M$, find $g(\mathbf{x}) \in H^{1/2}(\Gamma_{in})$ which minimizes the functional

$$J[g] = \frac{1}{2}\sum_{i=1}^{M}[T[g](\mathbf{x}_i) - \tilde{T}(\mathbf{x}_i)]^2,$$

where $T[g](\mathbf{x})$ is solution of the direct problem.

Inverse problems are well known for being ill-posed. To solve this problem, we use Alifanov's regularization method which is conjugate gradient method applied on the adjoint equation [1].

The search direction is given by the

Adjoint problem

$$\frac{1}{k}\Delta\lambda(\mathbf{x}) + \sum_{i=1}^{M}(T[g](\mathbf{x}_i) - \tilde{T}(\mathbf{x}_i))\delta(\mathbf{x} - \mathbf{x}_i) = 0, \quad \forall \mathbf{x} \in \Omega,$$

$$\begin{cases} \frac{1}{k}\nabla\lambda(\mathbf{x}) \cdot \mathbf{n} = 0 & \forall \mathbf{x} \in \Gamma_{in} \cup \Gamma_{ex}, \\ \frac{1}{k}\nabla\lambda(\mathbf{x}) \cdot \mathbf{n} + \frac{1}{k^2}h\lambda(\mathbf{x}) = 0 & \forall \mathbf{x} \in \Gamma_{sf}. \end{cases}$$

The step along the search direction is given by the

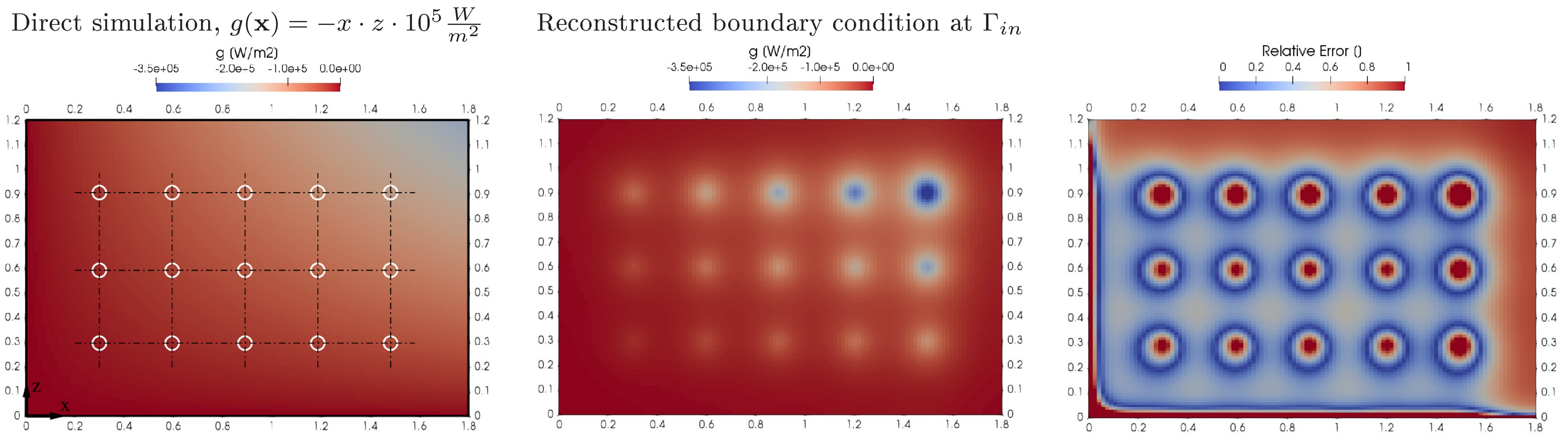
Sensitivity problem

$$-k\Delta\delta T(\mathbf{x}) = 0, \quad \forall \mathbf{x} \in \Omega,$$

$$\begin{cases} -k\nabla\delta T(\mathbf{x}) \cdot \mathbf{n} = \delta g(\mathbf{x}) & \forall \mathbf{x} \in \Gamma_{in}, \\ -k\nabla\delta T(\mathbf{x}) \cdot \mathbf{n} = 0 & \forall \mathbf{x} \in \Gamma_{ex}, \\ -k\nabla\delta T(\mathbf{x}) \cdot \mathbf{n} = h(\delta T(\mathbf{x})) & \forall \mathbf{x} \in \Gamma_{sf}. \end{cases}$$

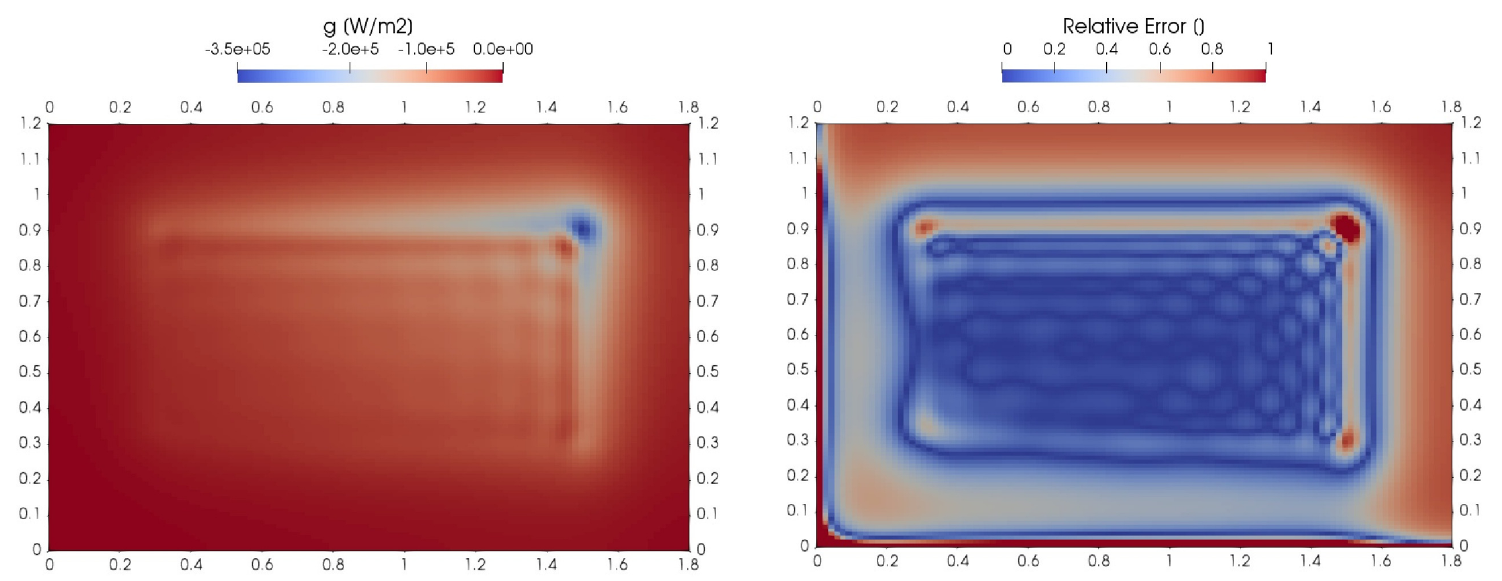
3-Full Order Results

Direct simulation, $g(\mathbf{x}) = -x \cdot z \cdot 10^5 \frac{W}{m^2}$ — Reconstructed boundary condition at Γ_{in}



To improve the heat flux prediction, we interpolate the thermocouples temperatures on the surface they define using Radial Basis Functions. The improvement is confirmed by the reduction of the norms of the relative error.

	$\Vert\epsilon\Vert_{L^2}$	$\Vert\epsilon\Vert_{L^\infty}$
no inter	0.1039	1.944
inter	$1.287 \cdot 10^{-2}$	1.474



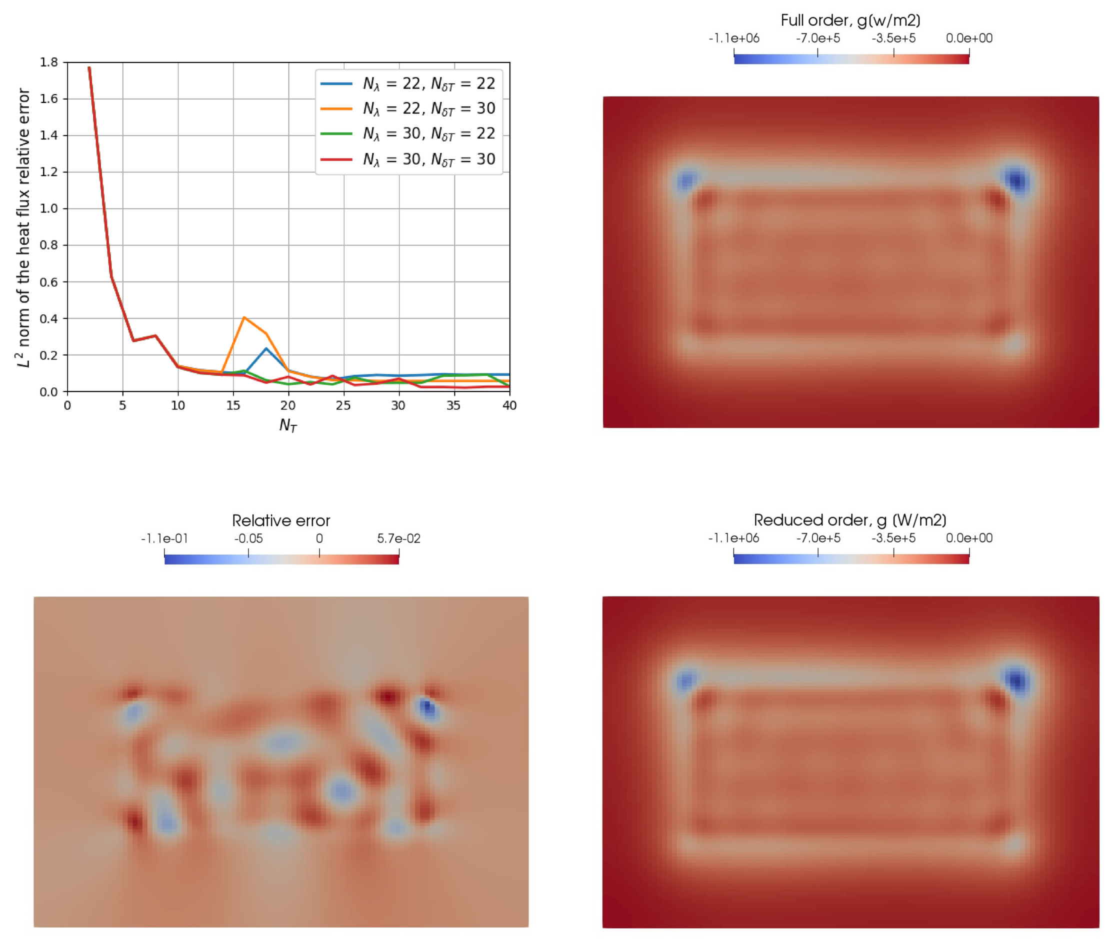
4-Reduced Inverse Problem

We apply a segregated approach for the reduction, i.e. we make a POD-Galerkin projection of each problem (direct, adjoint and sensitivity) on the respective reduced basis space.

The parameters of the inverse problem are the measured temperatures. To reduce the number of parameters, we perform a SVD on a set of experimentally measured temperatures the first few SVD modes. The values of the parameter for the training set are then chosen randomly accordingly to the probability density function of the SVD modes.

- Creation of snapshots using experimentally measured temperatures.
- Reduced basis spaces constructed using a POD approach [3].
- Projection of each problem onto the respective reduced basis spaces.
- Creation of a reduced Alifanov's regularization algorithm.

The reduction process is made using the software ITHACA-FV.



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