



SISSA mathLab main references and open source software on mathematical modelling for hemodynamics

Prof. G. Rozza's Group December 2019

In the following we provide a comprehensive list (together with a brief introduction) of Prof. Rozza's group activities in the framework of mathematical modelling for hemodynamics. Then we introduce open source software libraries developed by the group in the same context.

Publications

Our research stems from constant developments in the **reduced order modelling** community in the field of **numerical analysis** and **scientific computing**, to which we contribute with our research projects focused on **computational fluid dynamics**. Biomedical engineering applications, especially related to **patient-specific modelling of the cardiovascular system**, provide key motivations and applications to the methodology we develop.

A “**virtual surgery platform**” for patient-specific **coronary artery bypass grafts** has been developed in

- F. Ballarin, E. Faggiano, A. Manzoni, A. Quarteroni, G. Rozza, S. Ippolito, C. Antona, and R. Scrofani, “*Numerical modeling of hemodynamics scenarios of patient-specific coronary artery bypass grafts*”, Biomechanics and Modeling in Mechanobiology, 16(4), pp. 1373-1399, 2017. [\[doi\]](#)
- F. Ballarin, E. Faggiano, S. Ippolito, A. Manzoni, A. Quarteroni, G. Rozza, and R. Scrofani, “Fast simulations of patient-specific haemodynamics of coronary artery bypass grafts based on a POD-Galerkin method and a vascular shape parametrization”, Journal of Computational Physics, 315, pp. 609-628, 2016. [\[doi\]](#)

and allows to compare the effect of parametric changes on the fluid dynamics. Relevant parametric changes are those associated to transient patient's **conditions** (e.g. rest or stress, modeled by Reynolds numbers), **disease** (modeled by severity of stenosis) or **surgical choices** (modeled by geometrical properties of the grafts, e.g. diameters or angles). Model reduction by **POD-Galerkin** and a tailor-made **shape parametrization** allows to perform sensitivity analyses at a fraction of the computational cost that standard discretization techniques would have required.

This is a collaboration with Politecnico di Milano and Sacco Hospital in the framework of PRIN 2012 project.

On the same application of patient-specific coronary artery bypass grafts, integration with **measured experimental (clinical) data** in a reduced optimal flow control and data assimilation framework has been sought in

- Z. Zainib, F. Ballarin, S. Femes, P. Triverio, L. Jiménez-Juan, and G. Rozza, “*Reduced order methods for parametric optimal flow control in coronary bypass grafts, towards patient-specific data assimilation*”, submitted, 2019. [\[preprint\]](#)
- Z. Zainib, “*Reduced order parametrized viscous optimal flow control problems and applications in coronary artery bypass grafts with patient-specific geometrical reconstruction and data assimilation*”. Ph.D. thesis in Mathematical Analysis, Modelling, and Applications, SISSA, Italy, 2019. [\[download\]](#)

The mathematical formulation relies on an **optimal control framework** in which coefficients associated to outflow boundary are tuned so that the numerical simulation matches as best as possible the experimental results.

This is a collaboration with University of Toronto, Sunnybrook Hospital and Politecnico di Torino.

A combination of **model reduction** and **parameter space reduction** techniques has been presented in

- M. Tezzele, F. Ballarin, and G. Rozza, “*Combined parameter and model reduction of cardiovascular problems by means of active subspaces and POD-Galerkin methods*”, in Mathematical and Numerical Modeling of the Cardiovascular System and Applications, D. Boffi, L. F. Pavarino, G. Rozza, S. Scacchi, and C. Vergara (eds.), Springer International Publishing, pp. 185-207, 2018. [[preprint](#)] [[doi](#)]

and applied to an idealized **carotid artery bifurcation**. Given a template healthy carotid artery, we introduce control points associated to a **geometrical parametrization** to represent the presence of a stenosis in the daughter branches. **Active subspaces** are employed in order to reduce the dimension of the parametric space (associated to the geometrical parametrization) based on a pressure drop criterion. Model reduction by **POD-Galerkin** methods benefits from this smaller parameter space both during the offline and online stage. For this studies we use PyGeM (see below)

Preliminary investigations towards **multiphysics problems** are in progress, especially for what concerns **fluid-structure interaction problems** in channels:

- M. Nonino, F. Ballarin, G. Rozza, and Y. Maday, “*Overcoming slowly decaying Kolmogorov n -width by transport maps: application to model order reduction of fluid dynamics and fluid-structure interaction problems*”, submitted, 2019. [[preprint](#)]
- F. Ballarin, G. Rozza, and Y. Maday, “*Reduced-order semi-implicit schemes for fluid-structure interaction problems*”, in Model Reduction of Parametrized Systems, P. Benner, M. Ohlberger, A. Patera, G. Rozza, and K. Urban (eds.), Springer International Publishing, vol. 17, pp. 149-167, 2017. [[preprint](#)] [[doi](#)]
- F. Ballarin and G. Rozza, “*POD–Galerkin monolithic reduced order models for parametrized fluid-structure interaction problems*”, International Journal for Numerical Methods in Fluids, 82(12), pp. 1010-1034, 2016. [[doi](#)]

These methodological developments are motivated by the inherent multiphysics nature of the cardiovascular system, and are specifically tailored for a blood-vessel coupling. Further developments for blood-valve interaction will be scheduled in future. This is a collaboration with Laboratoire Jacques Louis Lions, Sorbonne, Paris.

Reduced order methods for bifurcation of parametric flows (Coanda Effect in mitral valves) is undergoing with University of Houston:

- G. Pitton, G. Rozza. On the Application of Reduced Basis Methods to Bifurcation Problems in Incompressible Fluid Dynamics. J. Scie.Comp., 2017, 73, p.157. [<https://arxiv.org/abs/1801.00923>]
- G. Pitton, A. Quaini, G. Rozza. Computational reduction strategies for the detection of steady bifurcations in incompressible fluid-dynamics: applications to Coanda effect in cardiology, J. Comp. Phys., 2017, 344, p.544. [<https://arxiv.org/abs/1708.09718>]
- M. Hess, A. Quaini, G. Rozza. Reduced Basis Model Order Reduction for Navier-Stokes equations in domains with walls of varying curvature, Int. J. CFD, 2019 [<https://arxiv.org/abs/1901.03708>]

SISSA mathLab Open Source Software and Tools



RBniCS is an implementation in **FEniCS** of several reduced order modelling techniques (and, in particular, **certified reduced basis** method and **Proper Orthogonal Decomposition-Galerkin** methods) for parametrized problems. An object-oriented approach and an intuitive and versatile python interface greatly simplifies the definition of the mathematical problem. Several tutorials show the capabilities of the library. [[gitlab](#)] [[github](#)] [[SISSA mathLab](#)].



multiphenics is a python library that aims at providing tools in **FEniCS** for an easy prototyping of **multiphysics** problems on conforming meshes. In particular, it facilitates the definition of **subdomain/boundary restricted** variables and enables the definition of the problem by means of a **block structure**. Several tutorials show the capabilities of the library. [[gitlab](#)] [[github](#)] [[SISSA mathLab](#)].



PyGeM (Python Geometrical Morphing) is a package that allows you to deform a given geometry or mesh with different deformation techniques such as Free Form Deformation, Radial Basis Functions and Inverse Distance Weighting. [[github](#)] [[SISSA mathLab](#)].

F. Salmoiraghi, A. Scardigli, H. Telib, and G. Rozza, **Free Form Deformation, mesh morphing and reduced order methods: enablers for efficient aerodynamic shape optimization**, Int. J. CFD, 2018 [[arxiv](#)].



ARGOS, the Advanced Reduced Groupware Online Simulation platform, is the online platform for the reduced order scientific computing framework ITHACA "In real Time Highly Advanced Computational Applications".

A gallery of applications and tutorials (from RBniCS) is available online and allows computation to be run from standard web browsers. In particular, the **ATLAS** project will collect all cardiovascular applications. A proof of concept application on a parametrized flow in a carotid artery is currently available in collaboration with University of Pavia (Comp. Mech.Group) and Policlinico San Donato.

[[argos.sissa.it](#)] [[argos.sissa.it/atlas](#)]

The complete list of SISSA mathLab software is available on [GitHub](#) and on [SISSA mathLab website](#).

Collaborations



**SUNNYBROOK HEALTH SCIENCES CENTRE, TORONTO,
CANADA & UNIVERSITY OF TORONTO, CANADA**

Clinical collaboration on the study of patient-specific aortic coarctation cases and coronary artery bypass grafts configurations. Methodological development of reduced order modelling strategies for data assimilation based on experimental measurements.

OSPEDALE LUIGI SACCO, MILANO, ITALY

Clinical collaboration on the study of patient-specific coronary artery bypass grafts and left ventricular assist devices. Methodological development on reduced order modelling strategies for shape parametrization of bypass grafts.



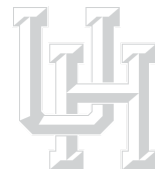
**AZIENDA OSPEDALIERA SAN CAMILLO FORLANINI, ROMA,
ITALY**

Clinical collaboration on the study of left ventricular assist devices. Methodological development of shape parametrization and non intrusive reduced order modelling techniques.



UNIVERSITY OF HOUSTON, TEXAS, U.S.

Methodological development of reduced order modelling techniques for problems with bifurcations.



**UNIVERSITY OF PAVIA and POLICLINICO SAN DONATO,
ITALY.**

Developments of ATLAS and data driven non-intrusive model reduction techniques for parametric flows in haemodynamics.

