A reduced order model for segregated FSI solvers based on a ALE approach

Valentin Nkana, Giovanni Stabile, Andrea Mola and Gianluigi Rozza
Mathematics Area, mathLab, SISSA, International School of Advanced Studies, Trieste, Italy

Introduction

The flow past a circular cylinder is a benchmark problem in fluid dynamics that serves to illustrate a large variety of applications many engineering situations.

1 - Physical model of the fluid and the structures

1 Physical model of the fluid

In dynamic meshes (specifically when working with moving/deforming bodies), the integral form of the general transport equation is written as follows,

$$\int_{V_P} \frac{\partial(\rho\phi)}{\partial t} dV + \int_{V_P} \nabla \cdot (\rho\phi(\boldsymbol{u} - \boldsymbol{u}_g)) dV - \int_{V_P} \nabla \cdot (\rho\gamma_\phi(\nabla\phi)) dV = \int_{V_P} S_\phi(\phi) dV$$

where the volume V_P is a function of time. ρ is the **density**, and $\boldsymbol{n}$ is the outward unit normal vector on the boundary surface, $\boldsymbol{u}$ is the **fluid velocity**, $\boldsymbol{u}_g$ is the **boundary mesh velocity**, γ_ϕ is the diffusive coefficient, and S_ϕ is the volume source/ sink . From the aforementioned general transport equation, we can write down the NSEs. For instance, by setting the variables to $\phi = 0$ $\gamma_\phi = 0$ and $S_\phi = 0$; $\phi = \boldsymbol{u}$ $\gamma_\phi = \mu$ and $S_\phi = -\nabla p$
we obtain the continuity equation and momentum equation respectively. It's essential that the calculation of $\boldsymbol{u}_g$ obeys the **geometric conservation law**:

$$\frac{d}{dt}\int_V dV + \oint_S \boldsymbol{u}_g \cdot \boldsymbol{n} dS = 0 \qquad \Leftrightarrow \qquad \frac{\partial V}{\partial t} + \nabla \cdot \boldsymbol{u}_g = 0.$$

2 Physical model of structures

The modeling equation of the solid is given in Lagrangian frame.

$$m\ddot{y} + c\dot{y} + ky = F_y$$

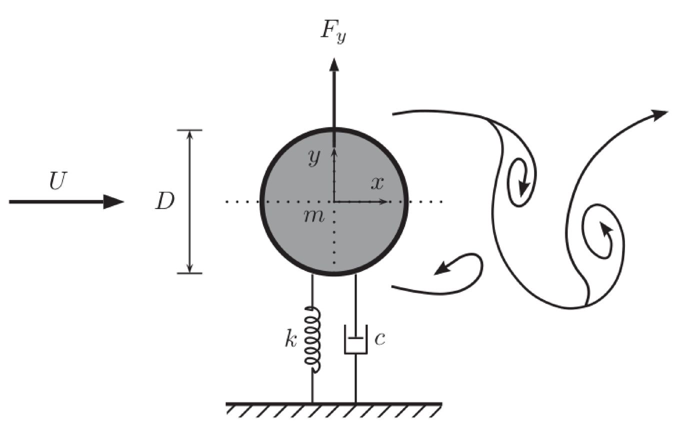
or in terms of natural frequency and damping ratio of the system by

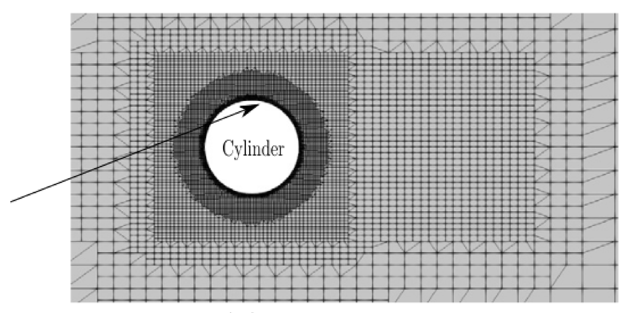
$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = \frac{F_y}{m}$$

m is the system's mass, c is the structural damping, and k is the spring's stiffness. $\omega_n = \sqrt{k/m} = 2\pi f_n$ is the natural pulsation of the system $\zeta = c/(2m\omega_n)$ is the fraction of structural damping with respect to critical or simply damping ratio, and F_y is the lift force in the direction transverse to the free-stream.

2A- Coupling conditions and computational domain

$\boldsymbol{u} = \boldsymbol{u}_g = \frac{\partial \boldsymbol{d}}{\partial t}$ $\qquad \boldsymbol{\sigma}_s \boldsymbol{n}_s = \boldsymbol{\sigma}_f \boldsymbol{n}_f$ $\qquad \boldsymbol{d}_{new} = \boldsymbol{d}_{old} + \Delta t \boldsymbol{u}_g$





Going from left to right, an observation of definition sketch for cross-flow VIV of a circular cylinder. The cylinder undergoes free vibrations constrained in the transverse y direction to the free-stream U in x direction; and the computational domain.

2B -Proper Orthogonal Decomposition (POD)

We assume that a given field is written as follow: $\boldsymbol{F} = \boldsymbol{F}(\boldsymbol{x}, t; \boldsymbol{\eta})$

$$\boldsymbol{u} = \sum_{i=1}^{N_u} a_i(t;\boldsymbol{\eta})\boldsymbol{\phi}_i(\boldsymbol{x}) \qquad p = \sum_{i=1}^{N_p} b_i(t;\boldsymbol{\eta})\psi_i(\boldsymbol{x}) \qquad \boldsymbol{d} = \sum_{i=1}^{N_d} c_i(t;\boldsymbol{\eta})\xi_i(\boldsymbol{x}).$$

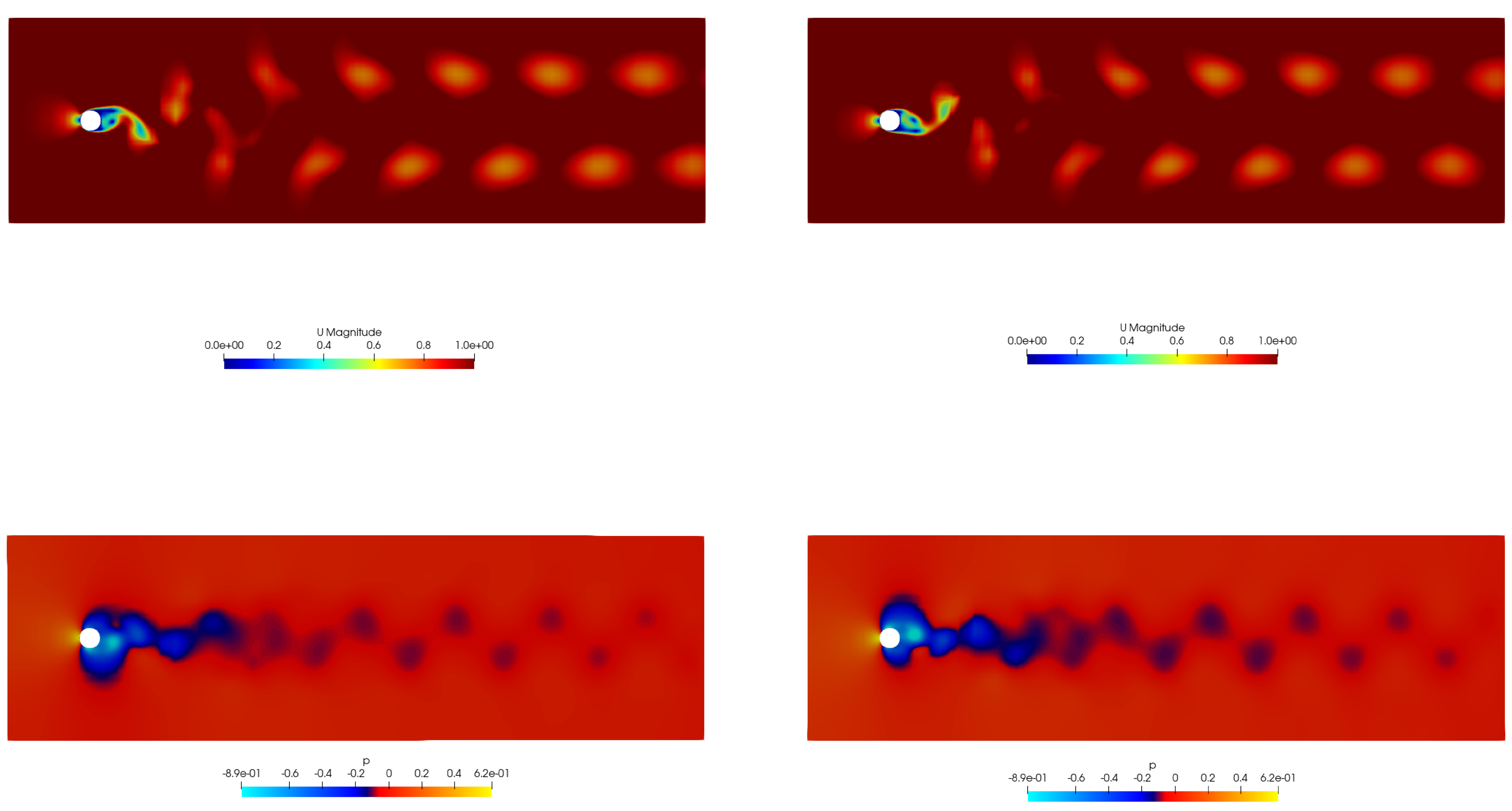
Which can be rewritten as:

$$\boldsymbol{u}(\boldsymbol{x}, t; \boldsymbol{\eta}) = \Phi \boldsymbol{a} \qquad p(\boldsymbol{x}, t; \boldsymbol{\eta}) = \Psi \boldsymbol{b} \qquad \boldsymbol{d}(\boldsymbol{x}, t; \boldsymbol{\eta}) = \Xi \boldsymbol{c}$$

$$(\phi_l, \phi_k)_{L^2} = \delta_{lk} \quad \text{and} \quad a_k = (\boldsymbol{u}, \phi_k)_{L^2}$$

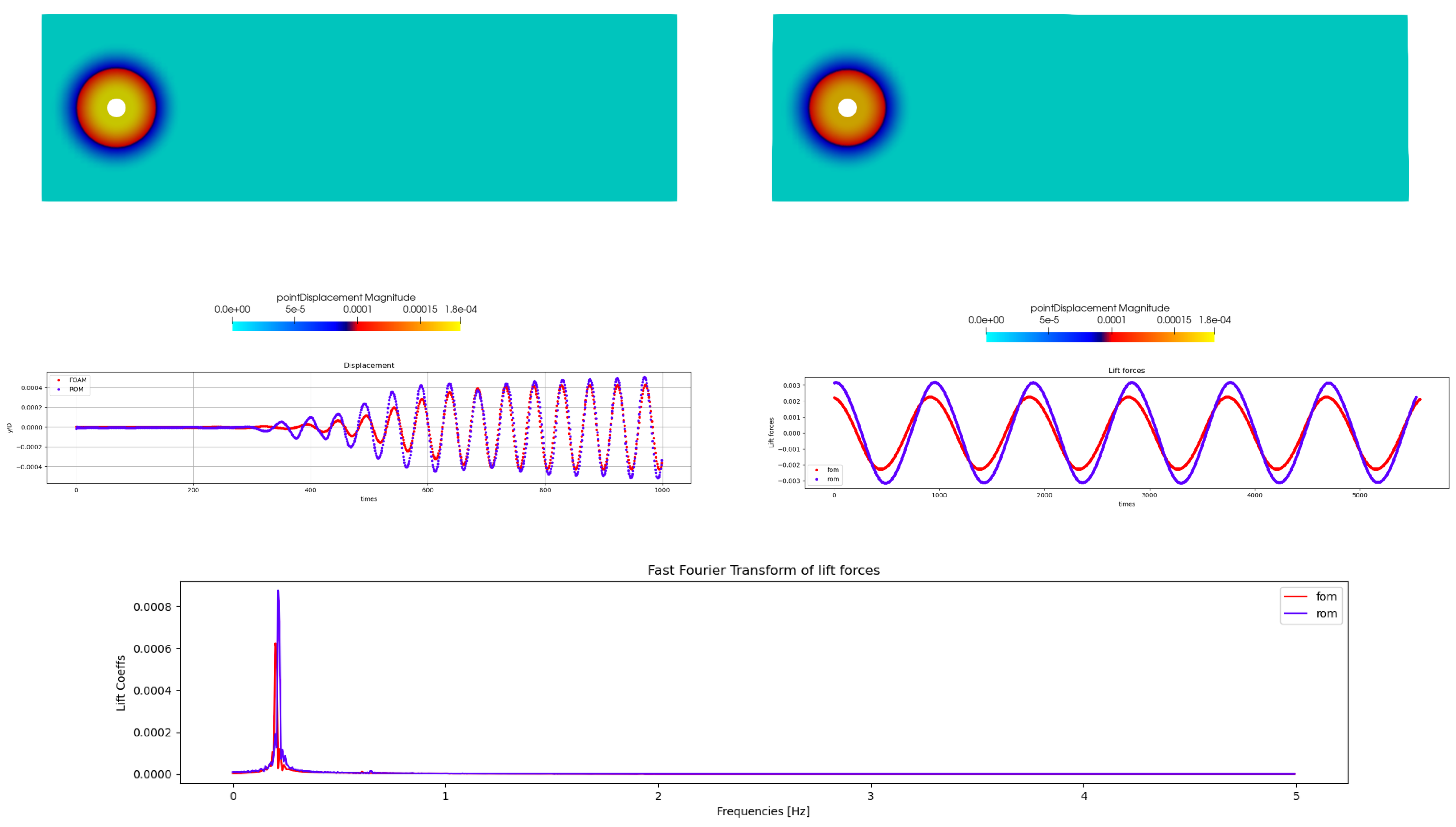
3A- Results: Part I

From left to right, the first row compares **velocities**, and the second row compares **pressure** solutions obtained from OpenFOAM and ITHACA-FV, respectively.



3B- Results: Part II

Going from left to right, first row shows a comparison of the solution of the point displacement field obtained from OpenFOAM and ITHACA-FV, respectively.Second row shows comparison the **displacement of the cylinder's center** and the lift forces acquired from OpenFOAM and ITHACA-FV. Finally, we compare frequencies response.



4 - Computational science and engineering softwares `mathlab.sissa.it/cse-software`

QR code
`mathlab.sissa.it/cse-software`
To see more on mathlab computational science and engineering software.

PyDMD
`github.com/mathLab/PyDMD`
`mathlab.github.io/PyDMD`
PyDMD is a Python package that uses Dynamic Mode Decomposition for a data-driven model simplification based on spatiotemporal coherent structures.

ITHACA-FV
`github.com/mathLab/ITHACA-FV`
`mathlab.github.io/ITHACA-FV`
ITHACA-FV is an implementation in OpenFOAM of several reduced order modeling techniques based on Finite Volume method.

EZyRB
`github.com/mathLab/EZyRB`
`mathlab.github.io/EZyRB`
EZyRB is a python library for data-driven (non-intrusive) model order reduction with POD with interpolation.

References

[1] H. Jasak and Ž. Tuković. Dynamic mesh handling in openfoam applied to fluid-structure interaction simulations. In *Proceedings of the V European Conference on Computational Fluid Dynamics ECCOMAS CFD 2010*, 2010.

[2] G. Stabile, M. Zancanaro, and G. Rozza. Efficient geometrical parametrization for finite-volume-based reduced order methods. *International journal for numerical methods in engineering*, 121(12):2655–2682, 2020.