A continuous trainable filter for convolution with unstructured data

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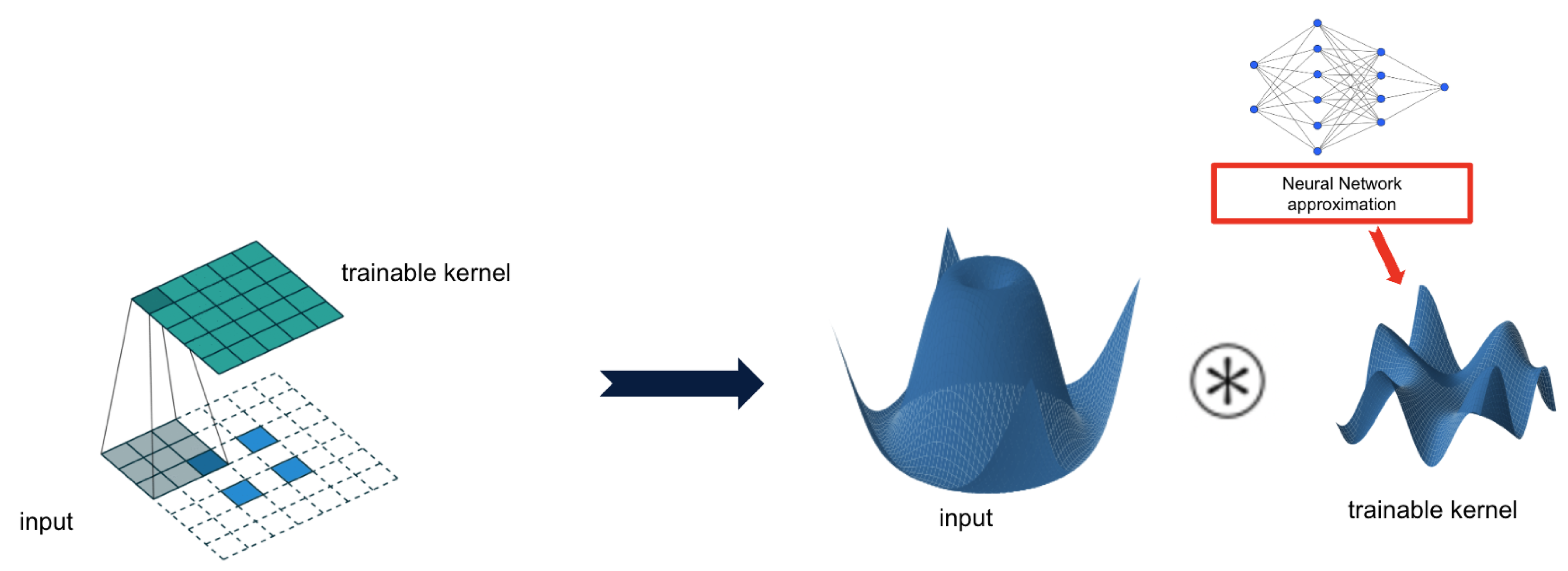
Introduction

Convolutional Neural Networks (CNNs) are one of the most important techniques in deep learning. The fundamental building block of a CNN is a trainable filter, represented as a discrete grid, used to perform convolution on discrete input data. In this work, we propose a continuous version of a trainable convolutional filter able to work also with unstructured data. This new framework allows exploring CNNs beyond discrete domains, enlarging the usage of this important learning technique for many more complex problems. Our experiments show that the continuous filter can achieve a level of accuracy similar to the state-of-the-art discrete filter and that it can be employed in current deep learning architectures as a building block to solve problems with unstructured domains.

Continuous convolutional filter

The basic building block of a Convolutional Neural Network (CNN) is a trainable filter, represented by a discrete grid, which performs cross-correlation, also known as convolution, on a discrete domain [3]. Nevertheless, the idea behind convolution can be easily mathematically extended to unstructured domains, see for reference [2]. The main idea behind a *continuous filter* is to approximate it with a trainable function using a feed-forward neural network and perform standard continuous convolution between the input data and the continuous filter.



Advantages of continuous filters

1. Applicable for not complete images, useful for training image reconstructors or sparse incomplete data
2. Generalization to different problem (not only images)
   - Find latent spaces for complex manifolds by the use of continuous autoencoders
   - Capturing non trivial relationship in unstructured data
   - Solving parametric problems for finite set and generalize the solution

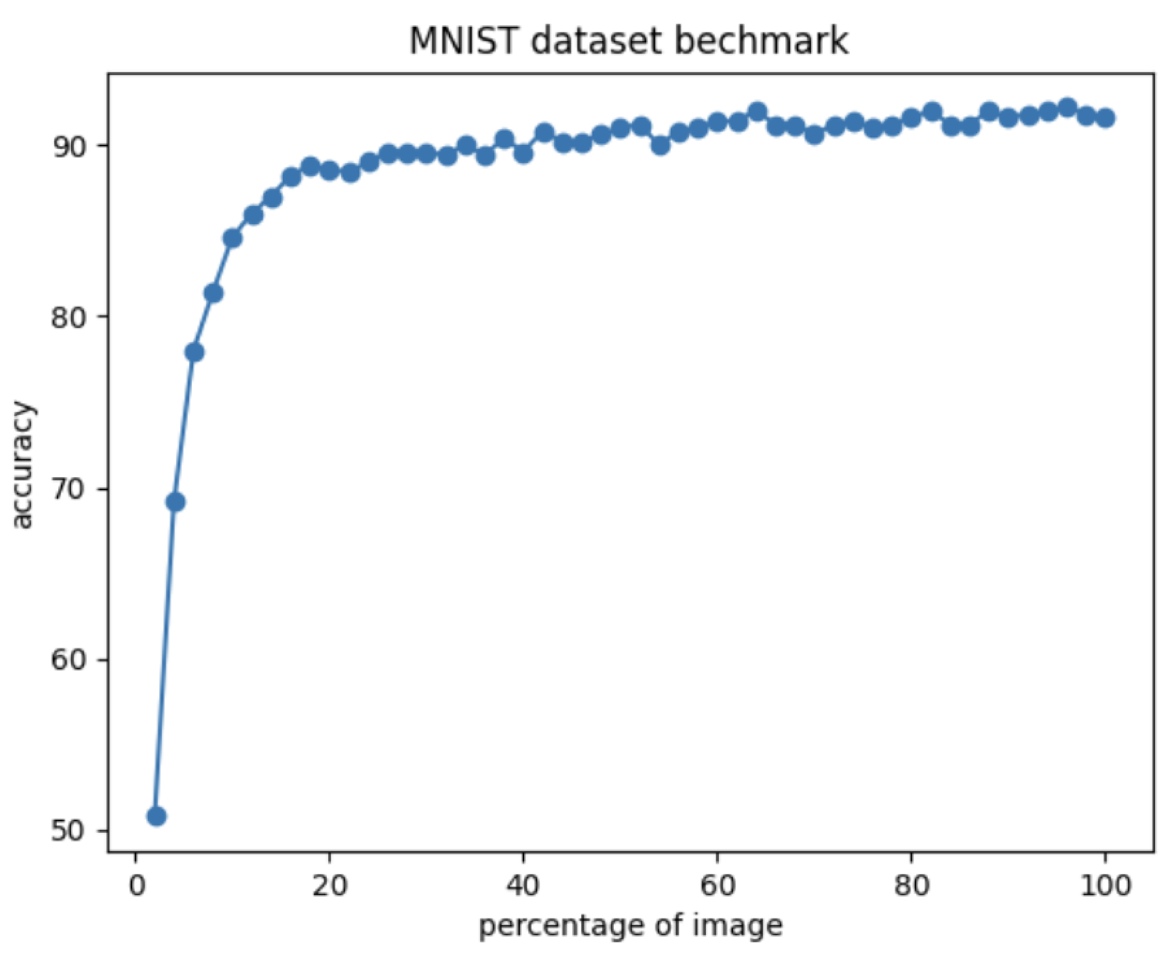
MNIST dataset and partially-completed images

Continuous filters behaves the same as discrete filters for structured domains.

Test on MNIST dataset

- Classic CNN: 93% accuracy on test set
- Continuous CNN: 92% accuracy on test set

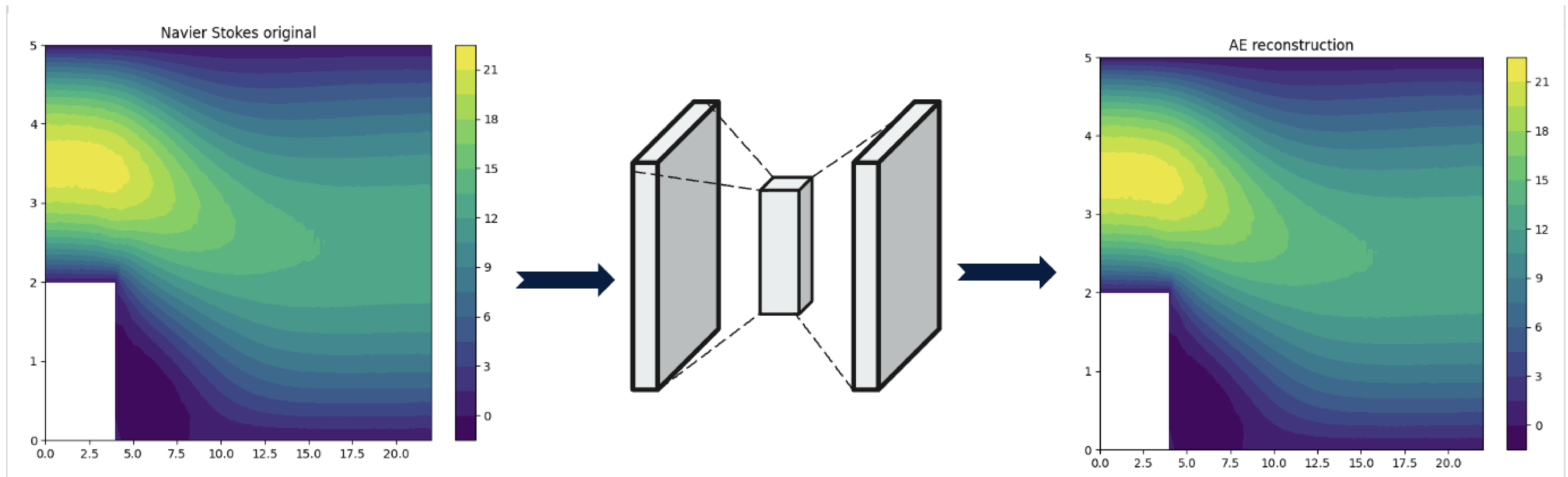
Continuous filters have the additional advantages to work with partially-completed images. Information is usually distributed on a lower dimensional manifold, hence using less informative pixels should not reduce drastically accuracy. We report how accuracy change by using a continuous CNN on the MNIST data-set.



Continuous convolution for Navier Stokes problem

Continuous filter application

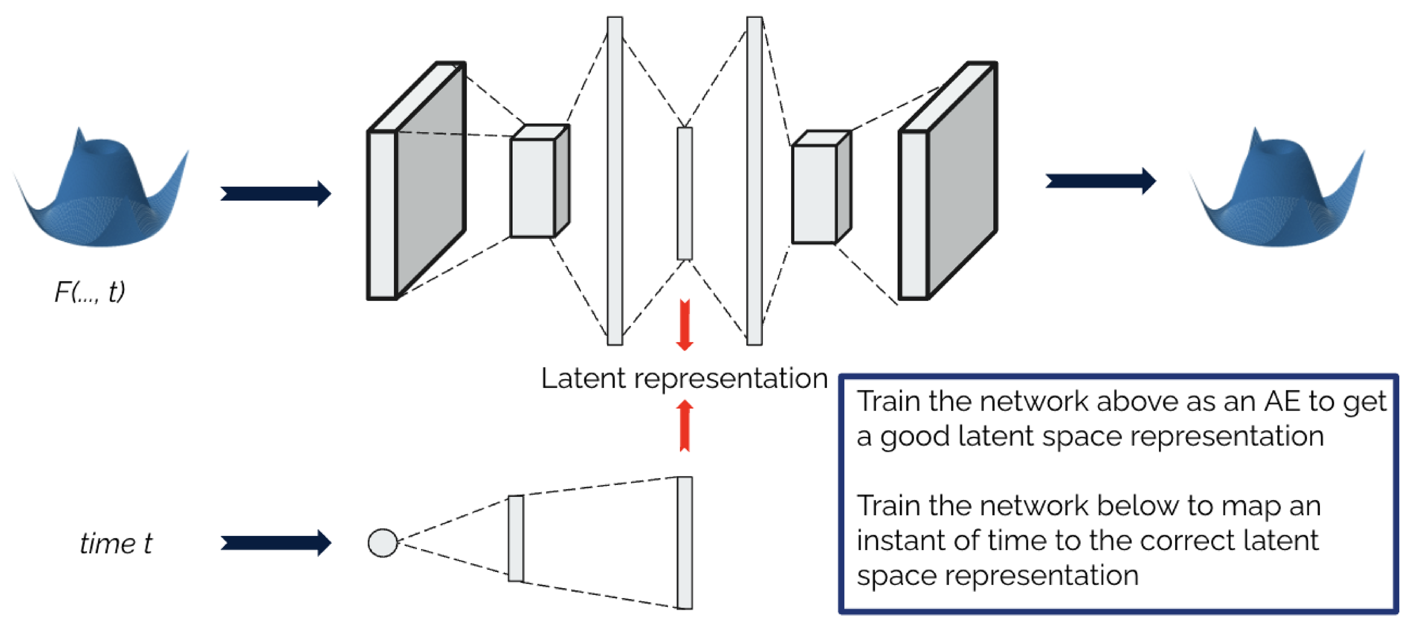
- A continuous convolution network, convolution + transpose convolution, based on the continuous filter has been applied to the step Navier Stokes problem.
- The training phase is done by considering just 20 out of 250 velocities, while the testing is done on 80 randomly choosen velocities.
- A basic architecture composed by only one continuous filter can achieve a 0.03% L^2 norm error on training, while 0.03% L^2 norm error on testing with unseen snapshots.



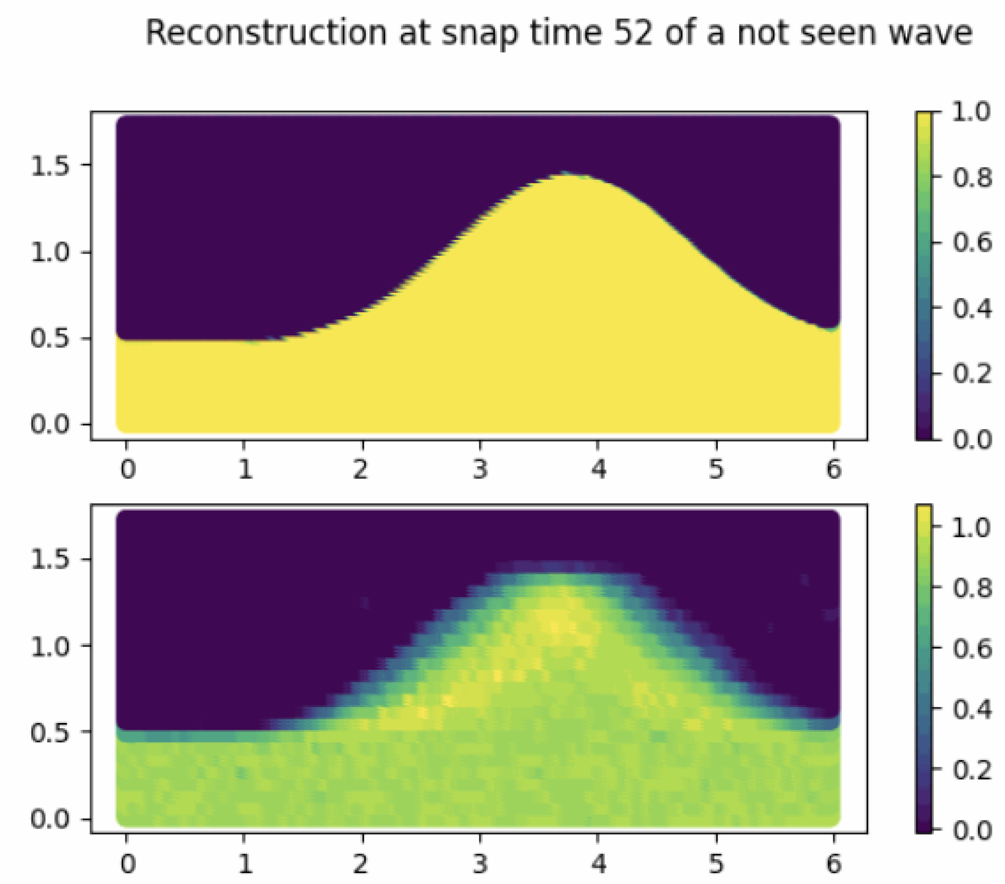
Liquid-Gas phase problem predictions

The Liquid-Gas problem is hardly decomposed using classical ROM such as PCA, due to its discontinuity. By using continuous auto-encoder and a feed forward network, and exploiting continuous filters, it is possible to reproduce by only knowing the time of the snapshot the complete liquid-gas state.

Deep network architecture



Problem results



References and Acknowledgements

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