An efficient computational framework for atmospheric and ocean flows

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Introduction

Numerical simulations of geophysical flows are not only an essential tool for ocean and weather forecast, but they could also provide insights on the mechanisms governing climate change. A Direct Numerical Simulation (DNS) computes the evolution of all the significant flow structures (eddies and vortices) by resolving them with a properly refined mesh. When a DNS for geophysical flows is possible, it is usually extremely expensive in terms of computational time and memory demand due to the large amount of degrees of freedom to be considered for a proper description of the flow. In addition, often long time intervals have to be simulated. We present two computational pipelines to reduce the computational cost, which could also be used simultaneously: (i) Reduced Order Models (ROMs) that enable fast computations without a significant loss in terms of accuracy and (ii) Large Eddy Simulation (LES) models that allow to use coarser meshes than those required by a DNS thanks to a model for the effect of the small scales that do not get resolved.

ROM for stream function-vorticity formulation [1]

Find vorticity ω and stream function ψ such that

$$\partial_t \omega + \nabla \cdot (\boldsymbol{u}\omega) - \frac{1}{Re}\Delta\omega = F \quad \text{in } \Omega \times (t_0, T),$$
$$-\Delta\psi = \omega \quad \text{in } \Omega \times (t_0, T),$$

where $\omega = \nabla \times \boldsymbol{u}$ and $\boldsymbol{u} = \nabla \times \psi$, $\boldsymbol{u}$ being the flow velocity.

Main features:

- The forcing term is expressed as the product of two functions, one function that depends only on space and the other that depends only on time, i.e.

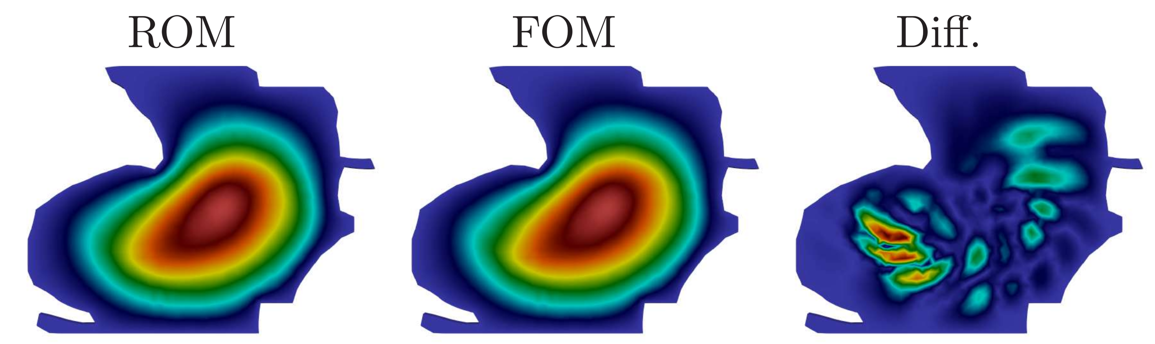
$$F(x, y, t) = F_1(x, y)F_2(t)$$

- For the discretization in time, a Backward Euler scheme is used.
- The discretization in space is carried out by using a Finite Volume method.
- The extraction of the most energetic modes representing the system dynamics is done via Proper Orthogonal Decomposition.
- A Galerkin projection on the space spanned by these most energetic modes is performed for the computation of stream function and vorticity reduced coefficients γ and β, respectively:

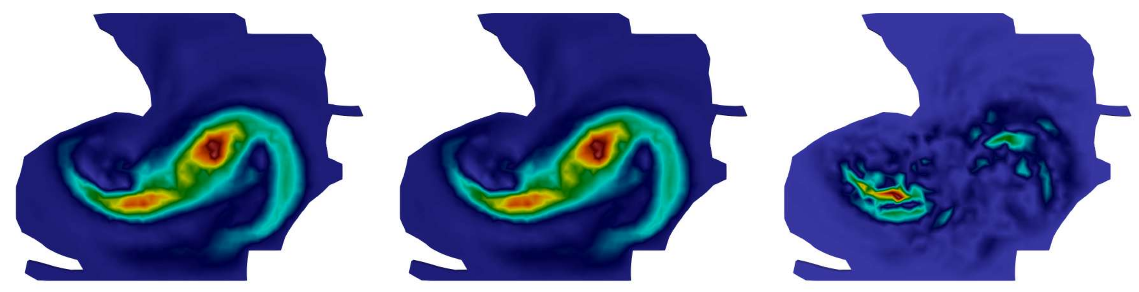
$$\mathbf{M}\dot{\beta} + \gamma^T\mathbf{G}\beta - \frac{1}{Re}\mathbf{A}\beta = \mathbf{H}F_2,$$
$$\mathbf{B}\gamma + \mathbf{M}\beta = 0$$

ROM for stream function-vorticity formulation [1]

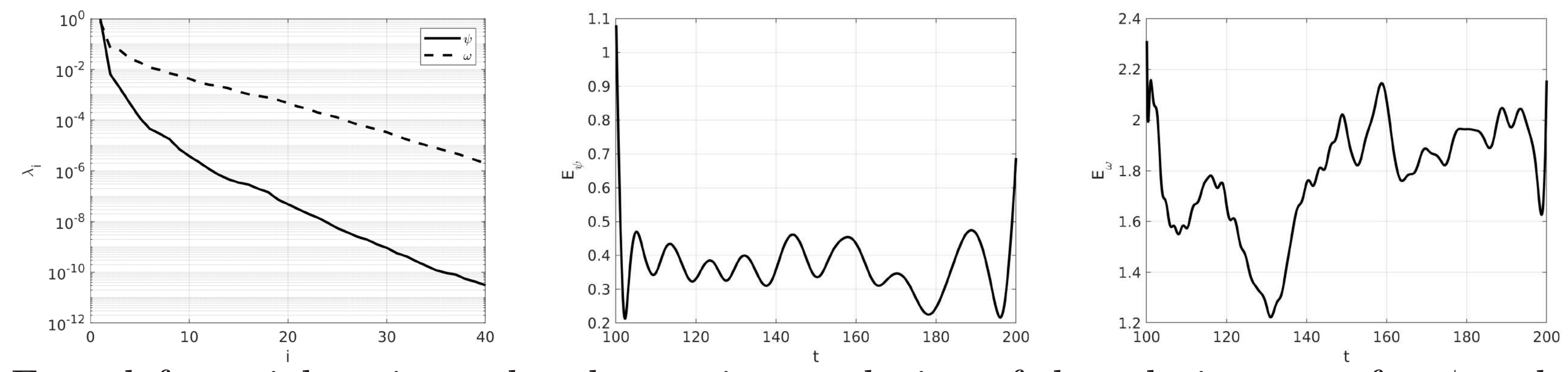
We consider a complex geometry representing the North Atlantic Ocean. This test could be considered as an extension of the classic vortex merger benchmark in a more complex domain and with more complex dynamics. The associated Reynolds number is about 5, which is a typical value for oceanographic simulations.



From left to right: ψ computed by the FOM and the ROM, and their difference.



From left to right: ω computed by the FOM and the ROM, and their difference.



From left to right: eigenvalue decay, time evolution of the relative error for ψ and ω.

LES for quasi-geostrophic equations [2]

The BV-α model reads: find potential vorticity q, *filtered vorticity* $\overline{q}$, and stream function ψ such that

$$\partial_t q + \nabla \cdot (\boldsymbol{u}q) - \frac{1}{\mathrm{Re}}\Delta q = F \quad \text{in } \Omega \times (t_0, T),$$
$$-\alpha^2 \nabla \cdot (a(q)\nabla\overline{q}) + \overline{q} = q \quad \text{in } \Omega \times (t_0, T),$$
$$-\mathrm{Ro}\Delta\psi + y = \overline{q} \quad \text{in } \Omega \times (t_0, T),$$

where

$$q = Ro\ \omega + y, \quad Ro = U/(\beta L^2),$$

α can be interpreted as the *filtering radius* and $a(\cdot)$ is a scalar function such that:

$a(q) \simeq 0$ where the flow field does not need regularization;

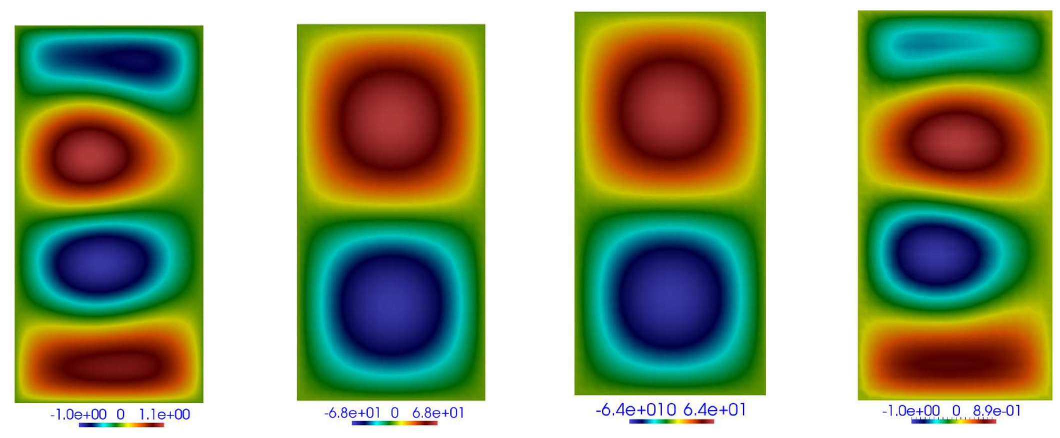
$a(q) \simeq 1$ where the flow field does need regularization.

Taking inspiration from the large body of work on the Leray-α model, we propose the following indicator function:

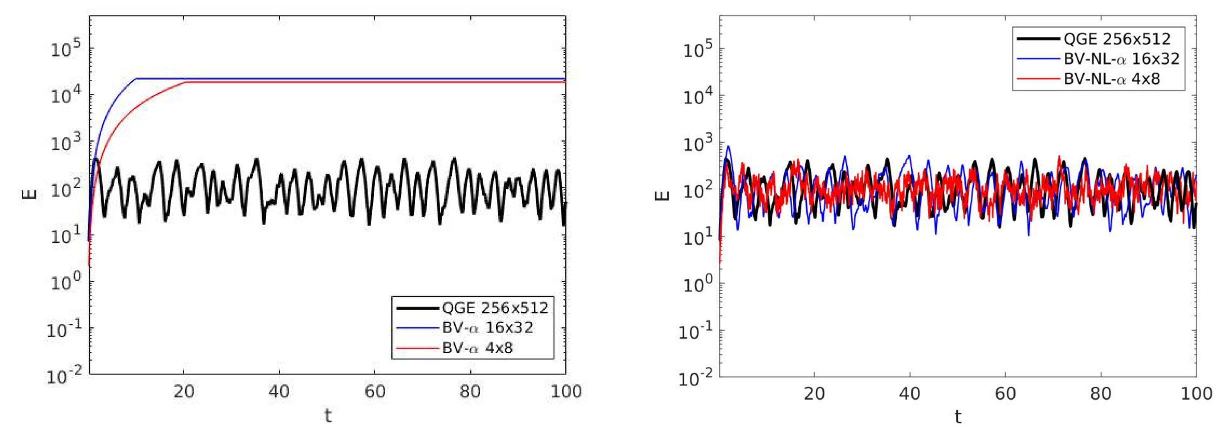
$$a(q) = \frac{|\nabla q|}{\max\left(1, ||\nabla q||_\infty\right)}.$$

LES for quasi-geostrophic equations [2]

For the validation of the BV-α model, we choose the double-gyre wind forcing experiment. We set $F = sin(\pi y)$ and consider $Ro = 0.008$ and $Re = 1000$.



From left to right: DNS on mesh 256x512, DNS on mesh 16x32, BV-α on mesh 16x32 and BV-NL-α on mesh 16x32.



From left to right: time evolution of the kinetic energy and zoomed-in view.

LES for compressible Euler equations [3]

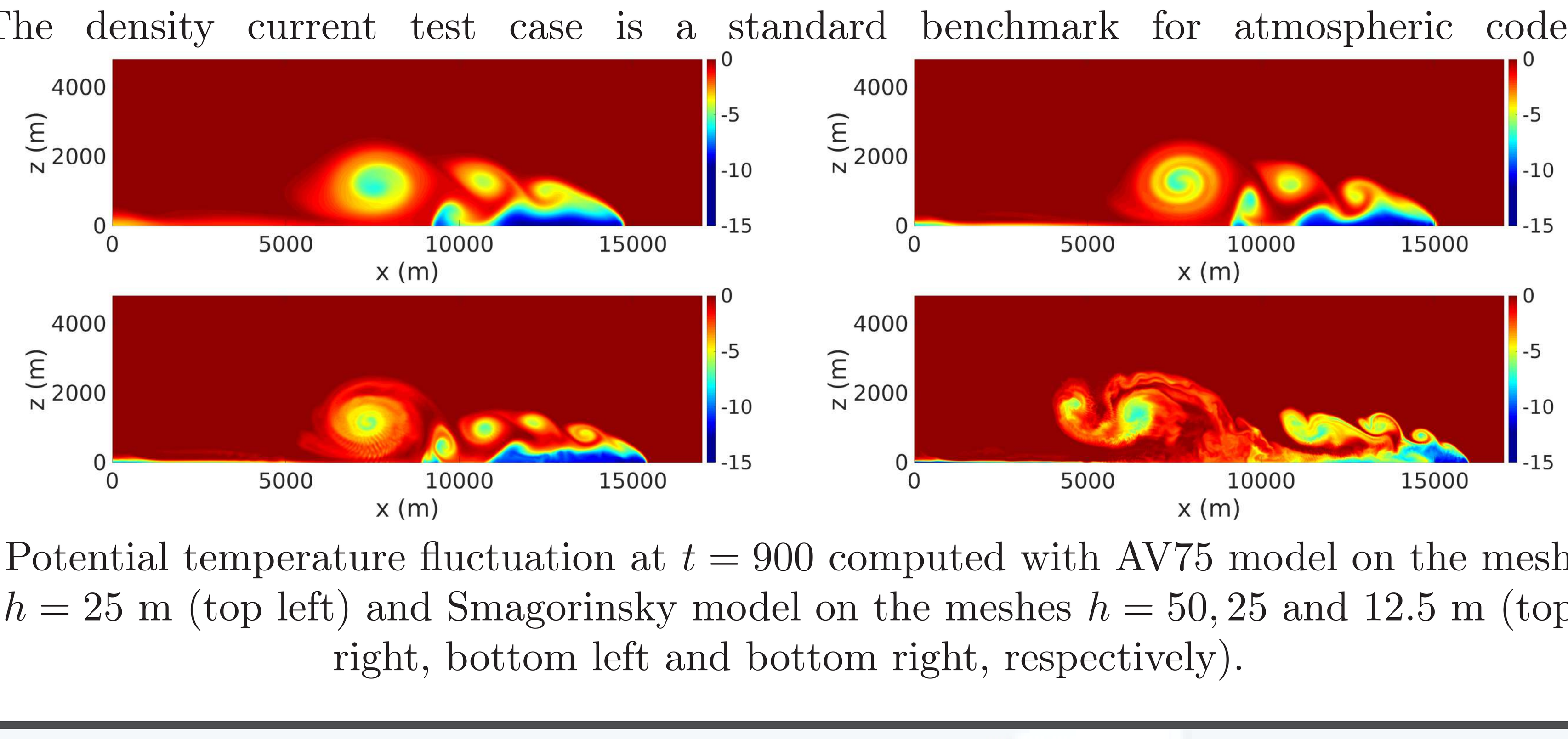
$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\boldsymbol{u}) = 0 \quad \text{in } \Omega \times (0, t_f],$$
$$\frac{\partial(\rho\boldsymbol{u})}{\partial t} + \nabla \cdot (\rho\boldsymbol{u} \otimes \boldsymbol{u}) + \nabla p' + gz\nabla\rho - \nabla \cdot (2\mu_a \boldsymbol{\epsilon}(\boldsymbol{u})) = 0 \quad \text{in } \Omega \times (0, t_f],$$
$$\frac{\partial(\rho e)}{\partial t} + \nabla \cdot (\rho e\boldsymbol{u}) + \nabla \cdot (p\boldsymbol{u}) + \rho g\boldsymbol{u} \cdot \widehat{\boldsymbol{k}} - \nabla \cdot \left(\frac{\mu_a c_p}{Pr}\nabla T\right) = 0 \quad \text{in } \Omega \times (0, t_f],$$

Main features:

- The pressure is expressed as the sum of a fluctuation p' with respect to a background state: $p = p' + \rho g z$.
- Two stabilization techniques are considered: i) artificial viscosity with $\mu_a = 75$ (referred to as AV75 model) and ii) LES Smagorinsky model.

LES for compressible Euler equations [3]

The density current test case is a standard benchmark for atmospheric codes.



Potential temperature fluctuation at $t = 900$ computed with AV75 model on the mesh $h = 25$ m (top left) and Smagorinsky model on the meshes $h = 50, 25$ and 12.5 m (top right, bottom left and bottom right, respectively).

References

[1] Girfoglio, M. and Quaini, A. and Rozza, G. A POD-Galerkin reduced order model for the Navier-Stokes equations in stream function-vorticity formulation. *Comput. Fluids*, 2022.

[2] Girfoglio, M. and Quaini, A. and Rozza, G. A novel Large Eddy Simulation model for the quasi-geostrophic equations in a finite volume setting. *J. Comput. Appl. Math.*, 2022.

[3] Girfoglio, M. and Quaini, A. and Rozza, G. Validation of OpenFOAM solvers for the Euler equations using benchmarks for mesoscale atmospheric modeling. *in preparation*, 2022.