A hybrid reduced-order model for segregated FSI in an ALE approach at $Re = 10^7$

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Introduction

This poster presents a hybrid reduced-order model capable of reproducing the behavior of the characteristics response of the system (lift and drag forces, amplitude/ frequency of the displacement) of a flow passing rotational and translation airfoil in turbulence regime in Finite Volume Method.

Methodology: Hybrid reduced-order model

In this poster, the **separability** of a given quantity of interest s is assumed using the Proper Orthogonal Decomposition (POD):

$$s(\boldsymbol{x},t) \approx \sum_{i=1}^{n} a_i(t)\boldsymbol{\phi}_i(\boldsymbol{x})$$

The spatial mode $\boldsymbol{\phi}_i$ is given by $\boldsymbol{\phi}_i = \lambda_i^{-1/2}\mathbf{S}\boldsymbol{\psi}_i$. $\mathbf{S} = [s(\boldsymbol{x},t_1),\cdots,s(\boldsymbol{x},t_N)]$ is the snapshot matrix with $N \geq n$. $\boldsymbol{\psi}_i$ and λ_i are respectively the eigenvectors and eigenvalues of the matrix $\mathbf{S}^T\mathbf{S}$ i.e $\mathbf{S}^T\mathbf{S}\boldsymbol{\psi}_i = \lambda_i\boldsymbol{\psi}_i$. The temporal modes $a_j(t)$ is given by $a_j(t) = (s,\boldsymbol{\phi}_j)_{L^2}$.

The POD and the Galerkin projection are used to construct the reduced systems of both the velocity and pressure. In order, to build a complete low-dimensional model independent of the mesh motion technique and the turbulence model used. At the *online phase*,

- POD and *radial basis functions* (RBF) are put together to reconstruct the mesh motion and also reduce the computational time due to the grid motion;
- POD and Neural Networks are combined to predict the Eddy viscosity.

1- Fluid and structure motions

Considering the incompressible Navier-Stokes in ALE formulation:

$$\nabla \cdot \bar{\boldsymbol{u}} = 0$$

$$\frac{\delta \bar{\boldsymbol{u}}}{\delta t} + \nabla \cdot (\bar{\boldsymbol{u}} \otimes (\bar{\boldsymbol{u}} - \boldsymbol{u}_g)) = \nabla \cdot \left[\frac{1}{\rho}\nu_{eff}\nabla\bar{\boldsymbol{u}}\right] - \frac{1}{\rho}\left(\nabla\bar{p} + \frac{2}{3}\rho\nabla k\right).$$

In addition, **space conservation law** is enforced, plus *initial and boundary conditions*.

- $\frac{2}{3}\rho\nabla k$ is the normal stresses arising from the Boussinesq hypothesis;
- ν_t is the so-called turbulent viscosity and $\nu_{eff} = \nu + \nu_t$ is the effective viscosity;
- k is the turbulent kinetic energy;
- $\boldsymbol{u}_g$ is the grid velocity.

$$m\ddot{h} + c_h\dot{h} + k_h h - mb\ddot{\theta}\cos\theta + mb\dot{\theta}^2\sin\theta = F_h(t)$$

$$\boldsymbol{I}_\theta\ddot{\theta} + c_\theta\dot{\theta} + k_\theta\theta - mb\ddot{h}\cos\theta = \boldsymbol{M}_\theta(t)$$

F_h is the lift force, $\boldsymbol{I}_\theta$ is the moment of inertia of the foil, θ is the pitch rotation, h(t) is the plunge displacement, b is the distance between the pivot location and the center of mass. The structural stiffness of the plunge and pitch is designated by k_h and k_θ.

2A - Time series of the plunge

The time series of the displacement of the center of rotation.

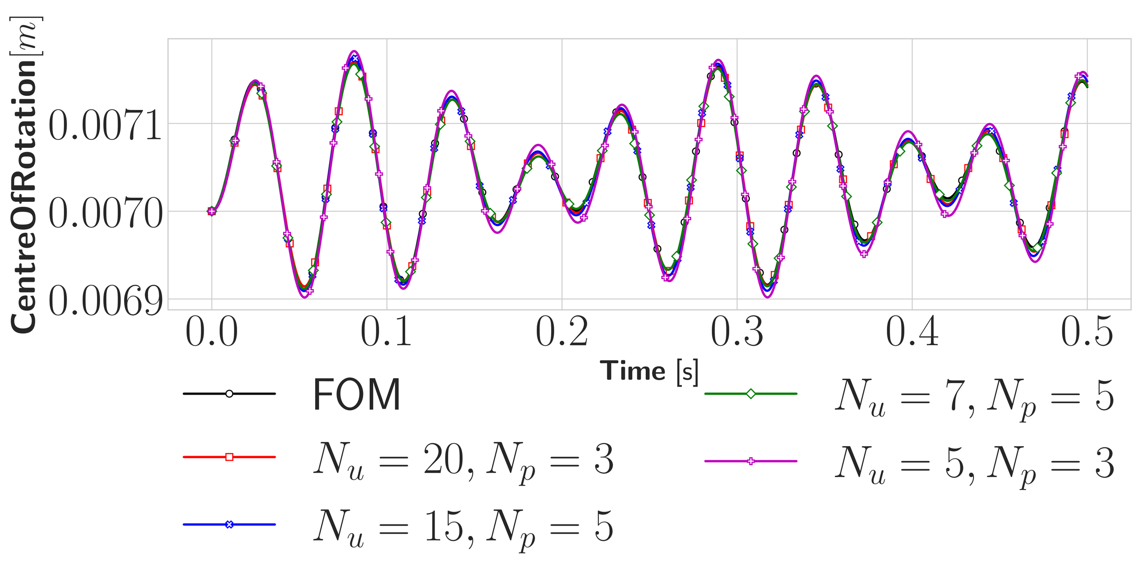

Figure 1: N_u and N_p are the number of spatial modes used to reconstruct for both the velocity and pressure.

2B - Lift forces analysis

The figure shows the effect of the number of modes on the accuracy.

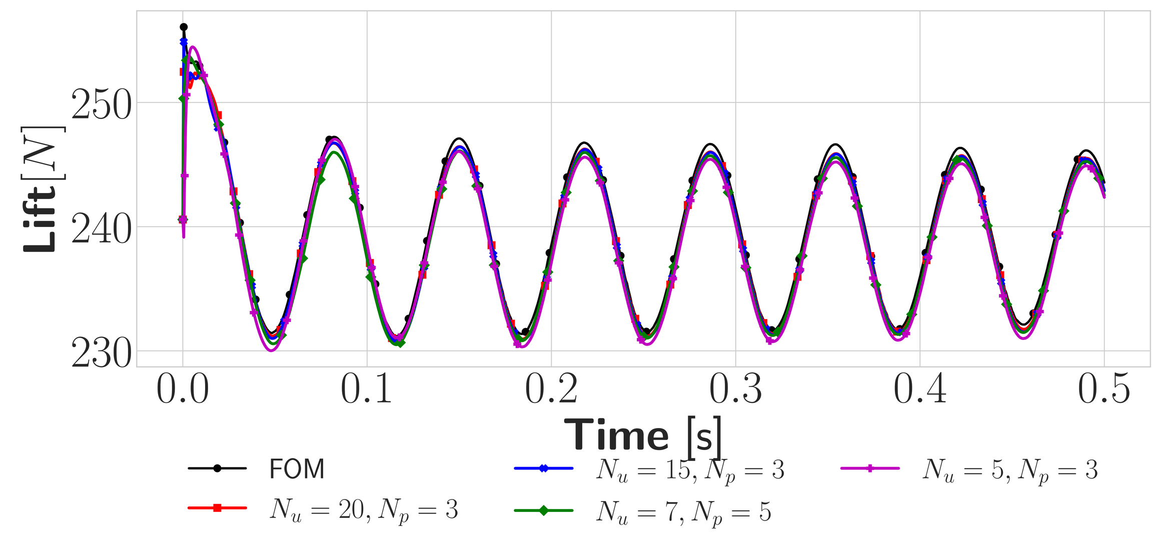

Figure 3: N_u and N_p are the number of spatial modes used to reconstruct for both the velocity and pressure

3A - Definition of the test case

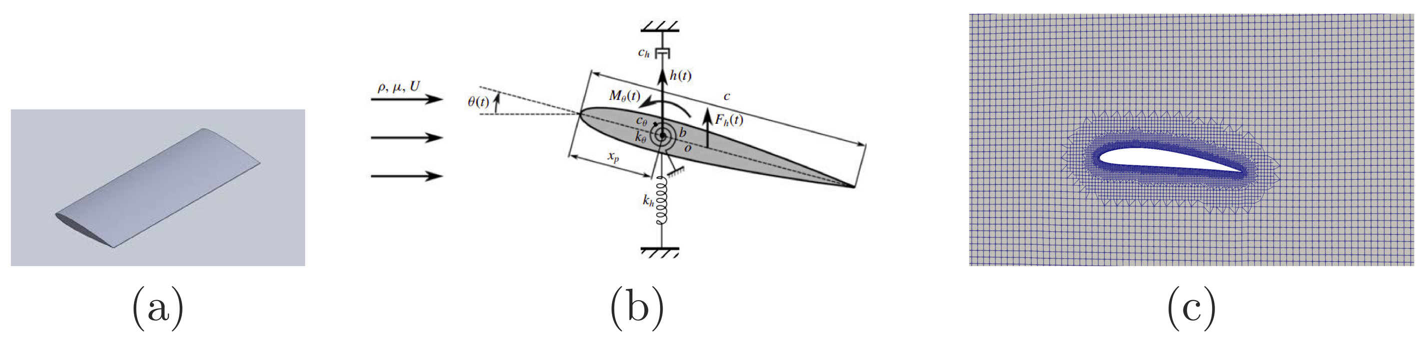

Figure 2: Naca 0012 3D airfoil model (a), a schematic of the fluid-structure system considered (b): a foil allowed to undergo 2 degrees of freedom fully passive plunging and pitching motion, and zoom mesh around the airfoil (c)

3B - Quantities of interests

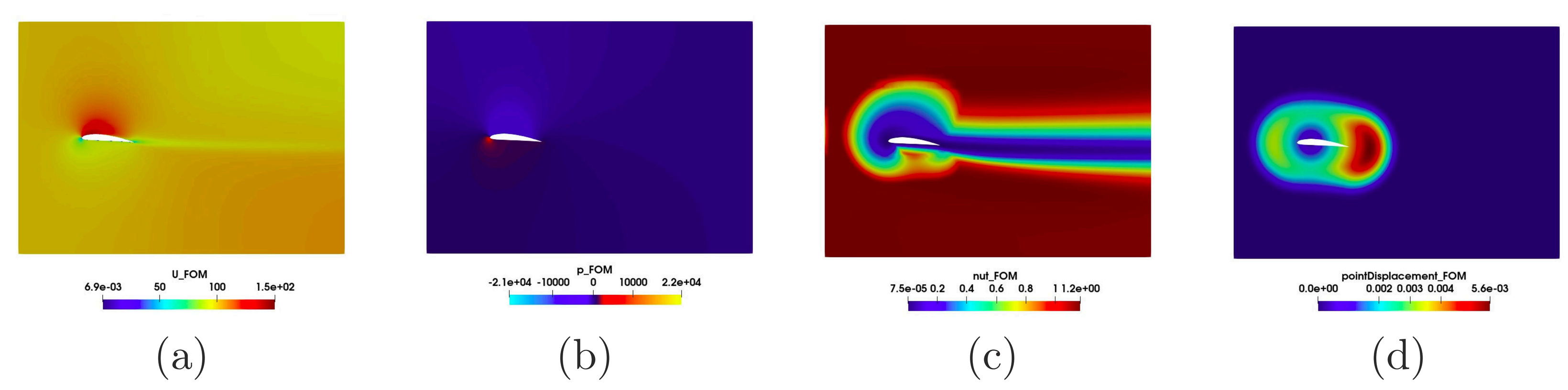

Figure 4: velocity (a), pressure (b), Eddy viscosity (c), and point cloud displacement of the grid (d)

4 - Computational science and engineering softwares: `mathlab.sissa.it/cse-software`

PyGeM
`github.com/mathLab/PyGeM`
`mathlab.github.io/PyGeM`

PyGeM is a python package using Free Form Deformation, Radial Basis Functions, and Inverse Distance Weighting to morph complex geometries.

PyDMD
`github.com/mathLab/PyDMD`
`mathlab.github.io/PyDMD`

PyDMD is a Python package that uses Dynamic Mode Decomposition for a data-driven model simplification based on spatiotemporal coherent structures.

ITHACA-FV
`github.com/mathLab/ITHACA-FV`
`mathlab.github.io/ITHACA-FV`

ITHACA-FV is an implementation in OpenFOAM of several reduced-order modeling techniques based on the Finite Volume Method.

EZyRB
`github.com/mathLab/EZyRB`
`mathlab.github.io/EZyRB`

EZyRB is a Python library for data-driven (non-intrusive) model order reduction with POD with interpolation.

References and Acknowledgements

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[2] V. N. Ngan, G. Stabile, A. Mola, and G. Rozza. A reduced-order model for segregated fluid-structure interaction solvers based on an ALE approach. *arXiv preprint arXiv:2305.13613*, 2023.

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