Can we use Maxwell for the placement of WiFi APs?

Roberto Innocente
SISSA/ITCS

My Answer

No, I don't think.
Of course the equations are there and the methods to solve them are readily available, but it can easily be that the level of detail of the model that we need to use to provide meaningful results that fit to measured data it is outside of our possibilities.
I can imagine difficulties will come from :

- precise characterization of EM behaviour of all real materials involved
- scattering effects
- dispersion
- precise simulation of an IEEE 802.11 modulated carrier channel
- number of grid points to use to make the model accurate.

Anyway the effort is interesting by itself and I think is worth a try.

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Statement of the Problem

Probably you experienced sometimes a not good coverage by the WiFi signal. It would be important to know given the *immutable things*, what can be done to get a better coverage. For instance moving the access points or moving the laptops are possible actions with almost zero cost.
Indoor propagation of decimetric or centimetric em waves (current WiFi uses carriers with λ equal to 12 or 6 centimeters) is a difficult problem. Most of the troubles come with what is called **multipath**. The receiver gets the sum of some signals that travelled along different paths (through reflections and refractions) and therefore present different phases.
Diversity is a technology that tries to alleviate the destructive interference problem using multiple antennas (spaced at least $\lambda/4$) and choosing the one with the best received signal also for transmitting.
MIMO (Multiple Input-Multiple Output) is instead a much more complicate technology that tries even to take an advantage out of multipath, coding separate streams for each uncorrelated path (*spatial multiplexing*). This has been possible only recently with the appearance on the market of cheap, powerful and small DSP chips that do most of the job. IEEE 802.11n. MIMO can't exist in empty space where there are no uncorrelated paths.
The problem to better understand indoor propagation anyway remains especially in the light of the forthcoming pervasive **IOT** (Internet of Things) and their energy and space constraints.

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Methods

Maxwell equations that govern electromagnetic (em) propagation are **PDEs** (Partial Differential Equations). Here, the time derivative parts, are expressed in a form that makes clear their evolution in time (space free of charges, but with possible absorbing, reflecting, refracting materials):

- $\partial_t \vec{\mathbf{D}} \;=\; \nabla \times \vec{\mathbf{H}} - \vec{\mathbf{J}}$ Maxwell-Ampere's Law
- $\partial_t \vec{\mathbf{B}} \;=\; -\nabla \times \vec{\mathbf{E}} - \vec{\mathbf{M}}$ Maxwell-Faraday's Law

Where $\vec{\mathbf{M}}$ is a fictitious magnetic current that can be useful to simplify sometimes computations. The set of Maxwell equation has 2 other equations that say fields are divergence free in space with no charge, but we will see that these contraints are implicit in our algorithm of choice. Other 2 relations are necessary to relate fields in free space to fields in matter. They are called *constitutive relations* :

- $\vec{\mathbf{D}} \;=\; \epsilon \vec{\mathbf{E}}$ where ϵ is *electrical permittivity*
- $\vec{\mathbf{B}} \;=\; \mu \vec{\mathbf{H}}$ where μ is *magnetic permeability*

Considering also materials that can attenuate fields via conversion to heat :

- $\vec{\mathbf{J}} \;=\; \vec{\mathbf{J}}_{src} + \sigma \vec{\mathbf{E}}$ where σ is *electrical conductivity*
- $\vec{\mathbf{M}} \;=\; \vec{\mathbf{M}}_{src} + \sigma_m \vec{\mathbf{H}}$ where σ_m is *equivalent magnetic loss*

We come to Maxwell equations in linear, isotropic, nondispersive, lossy materials :

- $\partial_t \vec{\mathbf{E}} \;=\; \frac{1}{\epsilon} \nabla \times \vec{\mathbf{H}} - \frac{1}{\epsilon}(\vec{\mathbf{J}}_{src} + \sigma \vec{\mathbf{E}})$
- $\partial_t \vec{\mathbf{H}} \;=\; -\frac{1}{\mu} \nabla \times \vec{\mathbf{E}} - \frac{1}{\mu}(\vec{\mathbf{M}}_{src} + \sigma_m \vec{\mathbf{H}})$

Computational Electromagnetics essentially is based on 3 methods : **FDTD** (Finite-Difference Time-Domain) Method, **FEM** (Finite Element Method), **MOM** (Method of Moments). FDTD is the most simple and easy to implement. FEM is needed for complicate geometries but otherwise introduces some complexity. FDTD seems the most appropriate and there is an increasing use of it due to the increase of computational power available.

FDTD Finite-Difference Time-Domain method

The general method consists of 3 steps :

1. Divide the region into a grid of nodes
2. Approximate the PDE by an equivalent Difference-Equation that relates the value of the function there to that at some neighbour nodes
3. Solve the Difference-Equations using Initial/Boundary Conditions

An upper bound on timesteps is given by the *CFL(Courant-Frederichs-Lewy) condition* :

$$\Delta t < C\frac{\Delta s}{\sqrt{n}u} \quad \text{that in 3D becomes} \quad \Delta t < \frac{\Delta s}{\sqrt{3}u}$$

where :

- Δt timestep
- Δs spatial step
- n dimension of the model
- C Courant number, usually $= 1$
- u speed of light in the specific material $u = c/n$

Yee's algorithm

Kane Yee, in 1966, proposed some important variations of the general method that gave to the **FDTD** em algorithm a particular robustness. After 50 years from its publication and many proposed modifications it still remains the basic pillar of numerical em. He used *centered differences* to approximate all first order spatial derivatives, so achieving a second order bound on errors. Then these are his other modifications :

- because Maxwell $\nabla\times$ equations relate one field in a point to the curl of the other field, he proposed to construct two separate grids for the magnetic field and the electric field components such that they are separated by $\Delta s/2$ (**Yee lattice**, **staggered grids**)
- he proposed the **leapfrog** way of marching in time : solve at time t for the electric field components based on $\mathbf{H}$ at half integer times, then for time $t + 1/2\Delta t$ solve for the magnetic field components based on $\mathbf{E}$ at integer times and so on.

Due to these characteristics, the Yee's algorithm *implicitly* can be shown to obey the 2 divergence relations of the Maxwell's equations.

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Yee lattice/cell

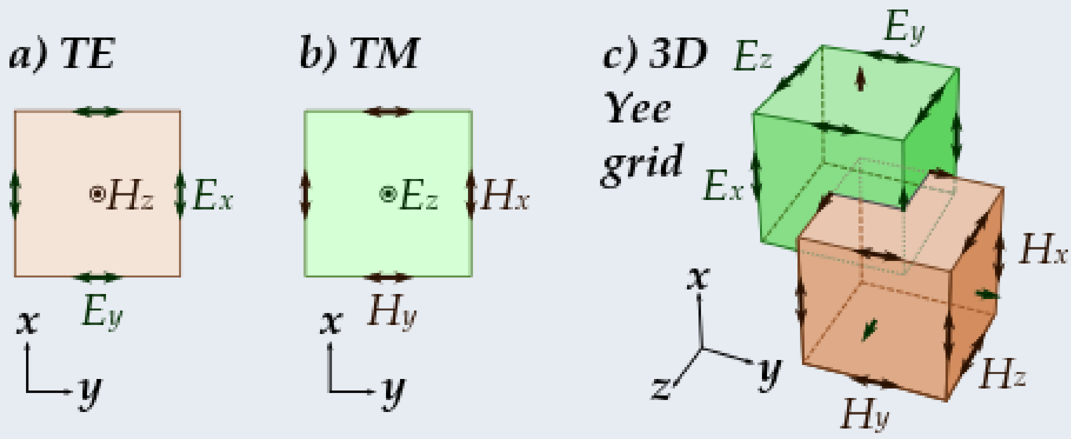

Figure 1: Yee cell

The Yee cell

Leapfrog algorithm

In the game called *leapfrog* each player jumps over all the others squatted ahead of him and then he squats too. Then the last rises up and does the same and so on. (If anyone is interested the italian name of the game is *cavallina* and refers to a horse instead of a frog.) The name of the algorithm comes from the fact that **E** and **H** *leap over* each other.

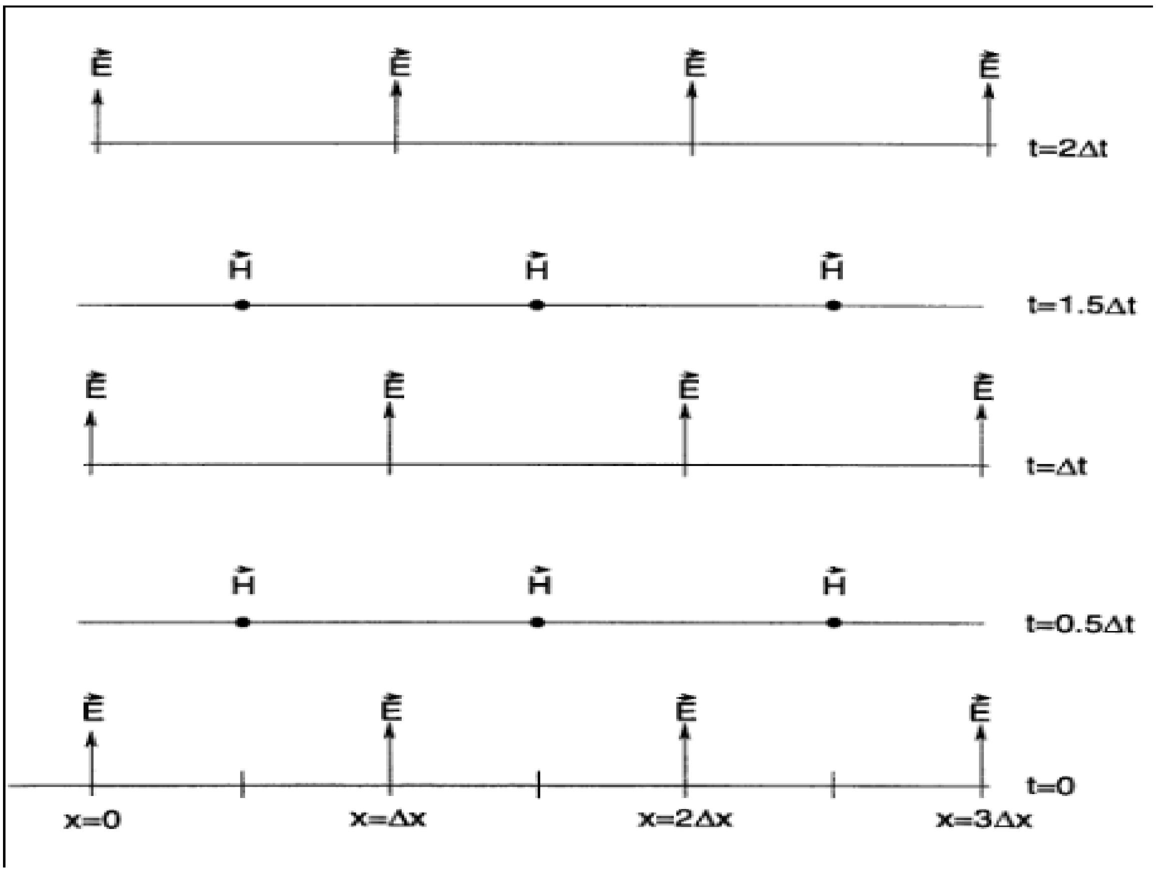

Figure 2: From Taflove : space-time chart for 1d wave prop

Chart for 1D wave propagation using centered differences for the space derivatives and *leapfrog* for the time derivatives.

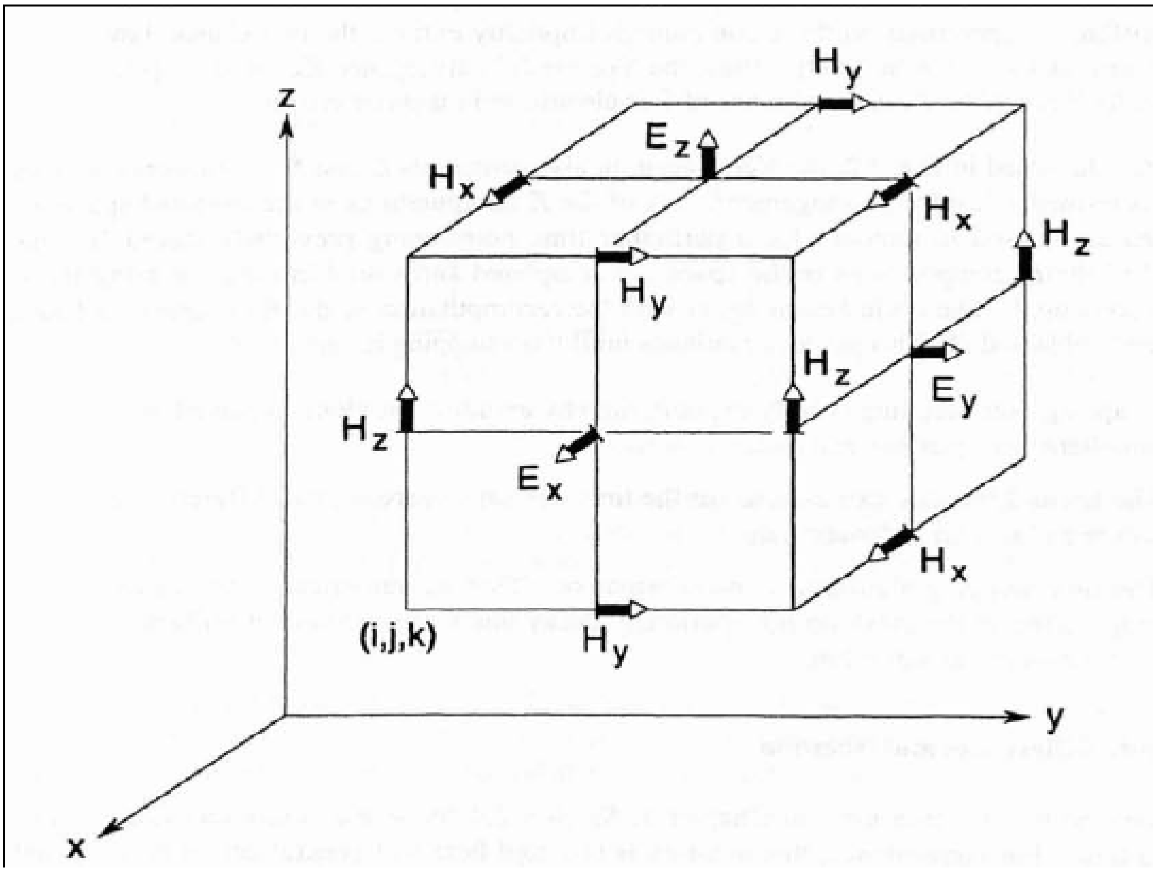

Figure 3: From Taflove : 3D Yee cell

802.11n MIMO

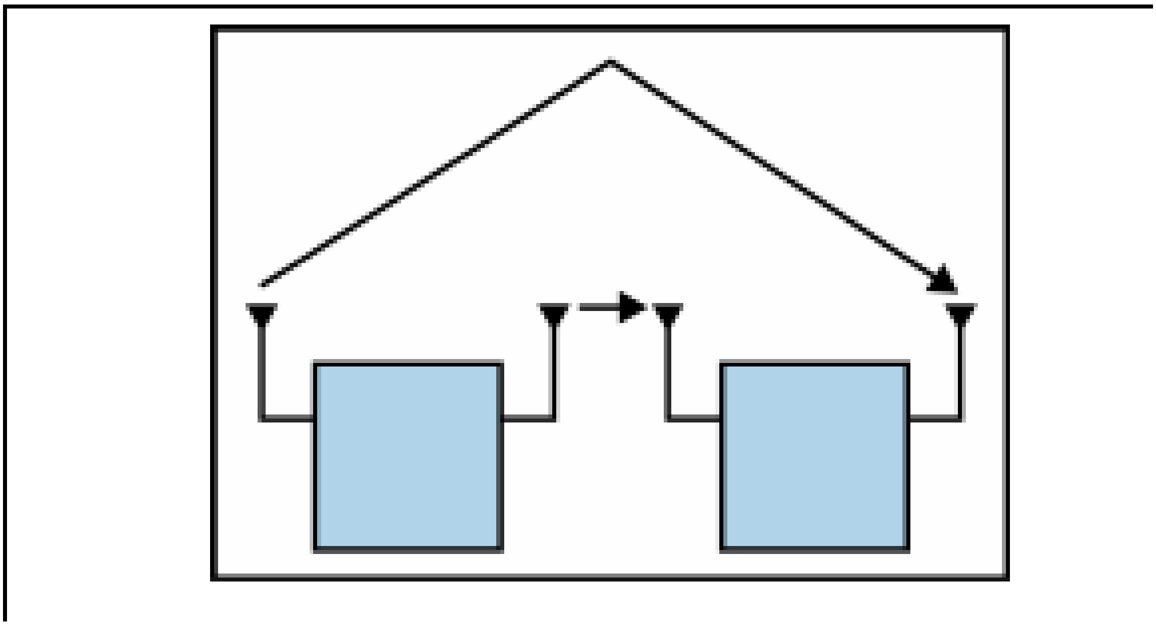

Figure 4: MIMO multipath (From O'Reilly 802.11n)

MIMO notation : **TxR:S** Transmitters, Receivers, Spatial streams
In 802.11n up to 4 spatial streams are supported. **2x2:2** refers to a system in which there are 2 transmitters, 2 receivers, but only 2 spatial streams.

Commercial / Opensource software

Yee's FDTD is a relatively easy algorithm and many packages are already commercially available . The most known commercial implementations are **XFDTD** and **MWS** (Microwave Studio). The first is an implementation of standard FDTD, the second is in fact a suite of codes . Anyway commercial packages often are not flexible enough. Therefore many groups, mainly in academia, developed in house their own variations, but usually they did'nt open them to the public. MIT developed an Yee FDTD program for electromagnetics simulations called **MEEP** and distributed it as opensource. It was first released in 2006 and it is still under continuous development. We will experiment with it to see if it can fit our needs.

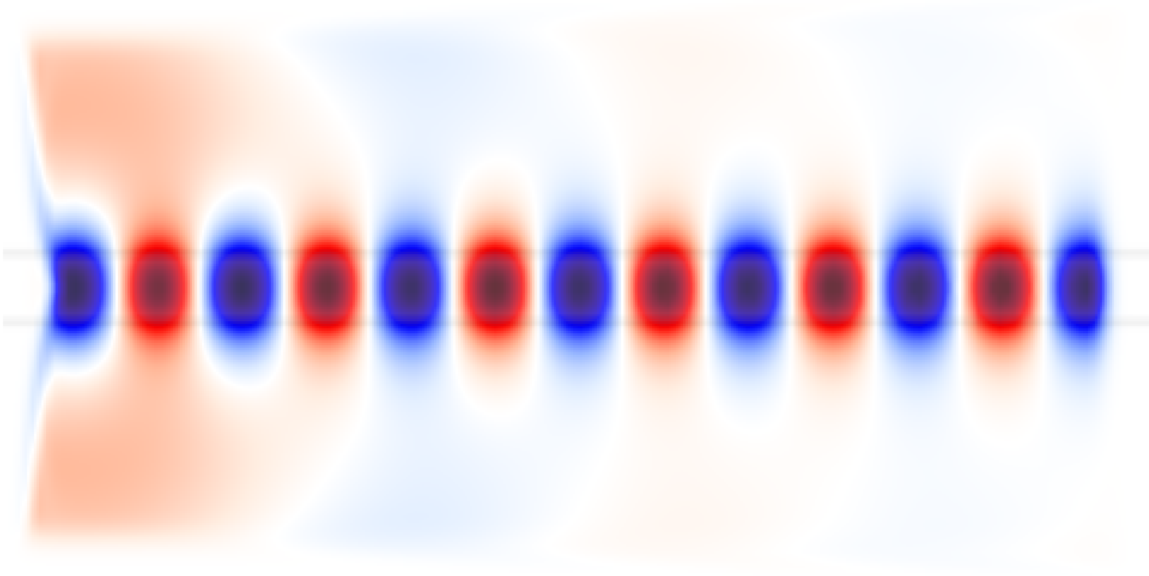

Figure 5: MEEP Waveguide propagation, src on the left

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Experimentally

It is possible to check with cheap and matchbox sized tools the signal in air and I experienced very large spatial differences in just a few centimeters (size comparable to wavelength). In this way I plan to verify what would be computed with simulations.

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Contact Information

- Email: *inno@sissa.it*