Data-assimilation, parameter space reduction and reduced order methods in applied sciences and engineering

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Introduction
We introduce a new framework for parameters space reduction in naval and biomedical engineering obtained by coupling: Active Subspace property to identify lower dimensional structure in the parameters space [1]; Free Form Deformation (FFD) and Radial Basis Functions (RBF), to morph the geometry; Boundary Element Method as high fidelity solver for the wave resistance [2]; Response surface method (RSM) and POD.

The Active Subspace Property
Consider a function, its gradient vector and a sampling density

\[ f(x), \quad x \in \mathbb{R}^m, \quad \nabla f(x) \in \mathbb{R}^m, \quad \rho : \mathbb{R}^m \rightarrow \mathbb{R}_+ \]

Take the average outer product of the gradient and partition its eigendecomposition,

\[ C = E[(\nabla f(x)(\nabla f(x))^T)\rho dx] = W AW^T \]

\[ A = [A_1, A_2], \quad W = [W_1, W_2], \quad W_i \in \mathbb{R}^{m \times n} \]

Rotate and separate the coordinates,

\[ x = WW^T x = W_1W_1^T x + W_2W_2^T x = W_1y + W_2z. \]

A parametric version of the DTMB 5415 hull
The DTMB 5415 is a hull conceived for preliminary design of a US Navy Combatant in 1980 and it is a very common benchmark for the validation of CFD models. The hull geometry includes both a sonar dome and transom stern.

As geometrical parameters we select 6 components of 4 control points of a FFD lattice over one side wall of the hull and we apply the deformation to the other side. The structural parameter is the displacement of the hull and the physical one is the velocity.

Eigenvalues and error analysis
We approximate the gradients of the wave resistance with respect to the parameters and look for a spectral gap of the \( C \) matrix.

We underline the presence of a major gap between the first and the second eigenvalue and a minor one between the second and the third.

The sufficient summary plot \((f(x) against W_1^T x)\) confirms the presence of an active subspace of dimension 1 and 2.

Spectral and POD analysis
The presence of an active subspace of dimension one is clear both from the spectral analysis and the sufficient summary plot.

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The three dimensional active subspace spanned by the first three eigenvectors of the covariance matrix seems to better capture the behaviour of the output function. We use this information to perform a further reduction by a POD-Galerkin ROM.

Carotid parametrization
Vessels geometry strongly influences hemodynamics behaviour. We study the influence of the vessel shape on blood flow. (In collaboration with Francesco Ballarin).

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= f \\
\nabla \cdot \mathbf{v} &= 0 \\
\mathbf{v} &= v_g \quad \text{on } \Gamma_{in} \\
\rho \mathbf{n} \cdot \mathbf{v} &= 0 \quad \text{on } \Gamma_{out}
\end{align*}
\]

In particular we want to simulate an occlusion.

We deform the carotid after the bifurcation moving 10 RBF control points solving an interpolation system.

The output function is the relative pressure drop of the two branches, computing the integral of the pressure on the highlighted sections.

References

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