Radiative processes in Astrophysics- F. Perrotta, SISSA. 2023-2024
Problem set #1.

Problem 1. Knowing that the bolometric solar luminosity is $L_\odot$ =3.832× 10^{26} W, the Sun radius is 6.96× 10^5 km, and the distance Sun-Earth is about 93 million kilometers, calculate (in ISU):

1) the flux *received* by a square meter of area on Earth.
2) The flux *emitted* by a square meter of the Sun's surface. How many LED light bulbs (typical wattage, 10 Watts) would you need to get the same flux?

Problem 2. Explain why the peak of frequency in the Planck spectrum does not correspond to the inverse of the peak wavelength multiplied by c.

Hint: $B_\lambda(T)$ is the amount of energy (erg) put out each second (s) in a wavelength range (Angstrom) which is radiated by a surface area (cm^2)

Problem 3. If the background blackbody radiation scales with cosmological redshift as $T(z)=T_0(1+z)$, and if the recombination occurred at a redshift z ~ 1100, determine the temperature of the CMB at the recombination epoch and its peak frequency at that time. In which band of the electromagnetic spectrum was the CMB emission peak?

Problem 4. What is the temperature at the solar surface? Use both the intensity of radiation on Earth (from Problem 1) and that the spectrum peaks about 500 nm to get answers.

Problem 5. Show that, defining the emission efficiency as $Q_{em} \equiv \frac{j_\nu}{j_\nu^{BB}}$ and the absorption efficiency as $Q_{abs} \equiv \frac{\alpha_\nu}{n\pi a^2}$ (with n = number density of absorbers, and πa^2 = geometric section of the absorber) in LTE $Q_{em} = Q_{ab}$. (I omitted the index ν in Q_{em} and Q_{abs}, but they are generally function of ν)

Problem 6. In open, inhomogeneous systems, in rarefied matter partially permeable to radiation (i.e., optically thin medium), the absence of a mutual equilibrium matter/radiation allows the formation of spectra different from the Planck distribution. Assuming the matter in thermal equilibrium, consider the transfer equation in presence of scattering. Write it in the two cases:

1) Absorption dominates over scattering;
2) Scattering dominater over absorption.

In case 2), what happens if the radiation field is isotropic?

Problem 7. Compare the emission spectrum of a grey body at temperature T with the spectrum of a blackbody at the same temperature.

1) Is the brightness peak of the two spectra occurring at the same frequency? Why?

2) How do you expect the photon occupation number to be for the grey body radiation with respect to that of the blackbody radiation with the same temperature? And the chemical potential?

3) Is it correct to define the grey body radiation at temperature T as a gas of photons in thermodynamic equilibrium with itself ?

Problem 8. Consider a body in LTE, emitting thermal radiation. Suppose its absorption efficiency Q_{abs} is a function of the frequency, $f(\nu)$. Is it correct to define the thermal radiation as radiation in thermodynamic equilibrium with itself? If not, what would be necessary to make it become a Planckian distribution?

Problem 9. Consider the setting of the Kirchhoff's ideal experiment. Suppose the walls of the enclosure are opaque (no transmission through the walls), but the material of the walls is partially reflecting light (coherent diffusion), so that the absorption efficiency of the walls is Q_{abs} <1 while $\alpha_{scattering} \neq 0$. Piercing the walls without perturbing the equilibrium, we know that we detect Blackbody radiation, which is radiation emitted by a perfect absorber/emitter (i.e., the effective Q_{abs} of the system is = 1). Why doesn't the scattering inside the box prevent the effective Q_{abs} to be equal to unity?

Problem 10. The dust "temperature" derived from the observed spectrum depends critically on the form of assumed emissivity law. Suppose the power law index of Q_{abs} steepens (e.g., β changes from β =1 to β=2). How will the emergent spectrum plausibly change?