RADIATIVE PROCESSES IN ASTROPHYSICS

Francesca Perrotta

SISSA, Trieste, Italy

room: A-505
phone: 040 3878 475
e-mail: perrotta@sissa.it

Welcome!

SISSA

- Hallo, I hope everyone is feeling fine! I am glad to meet you!
- This is my first attempt to use slides in my lectures: please, interrupt me if something is unclear!
- **Prerequisites**: basics of Quantum Mechanics, Special Relativity.
- I wish you'll learn a lot. Many exercices will be done together, at the blackboard. Some will be assigned as «homework»: working together is encouraged.
- Attendance is mandatory.
- At the end of the course, a **written exam** asked to assign you 4 credits.
- ENJOY!

Summary

SISSA

Part I. Continuum radiation

- Fundamentals of radiative transfer
- Black Body and thermal radiation
- Statistical mechanics and thermodynamic equilibrium, Part 1
- Thermal radiation from cosmic dust
- Bremsstrahlung
- Synchrotron
- Scattering (diffusion processes)

Part II. Line radiation

- Statistical mechanics and thermodynamic equilibrium, Part 2
- Atomic and molecular structure
- Einstein coefficients (selection rules, transition rates, line profiles and curve of growth)
- Radiative transitions (Bound bound, Bound free transition)
- Collisional excitation and critical density

Textbooks

- Rybicki, G., and Lightman, A., 1979, *Radiative Processes in Astrophysics*, John Wiley and Sons
- Draine, B.T., 2011, *Physics of the Interstellar and Intergalactic Medium*, Princeton University Press
- Nobili, L., 2002, *Processi Radiativi ed Equazione del Trasporto nell'Astrofisica delle alte energie*, ed. Cleup, Padova
- Tielens A.G.G.M., 2006, *The Physics and Chemistry of the Interstellar Medium,* Cambridge University Press
- Longair, M.S., 1992, *High Energy Astrophysics*, Cambridge University Press
- Ghisellini G., 2013, *Radiative Processes in High Energy Astophysics*, Springer ed.
- Jackson J. D., 1962, *Classical Electrodynamics*, John Wiley & Sons
- Spitzer J. L., 1998, *Physical Processes in the Interstellar Medium*, Wiley

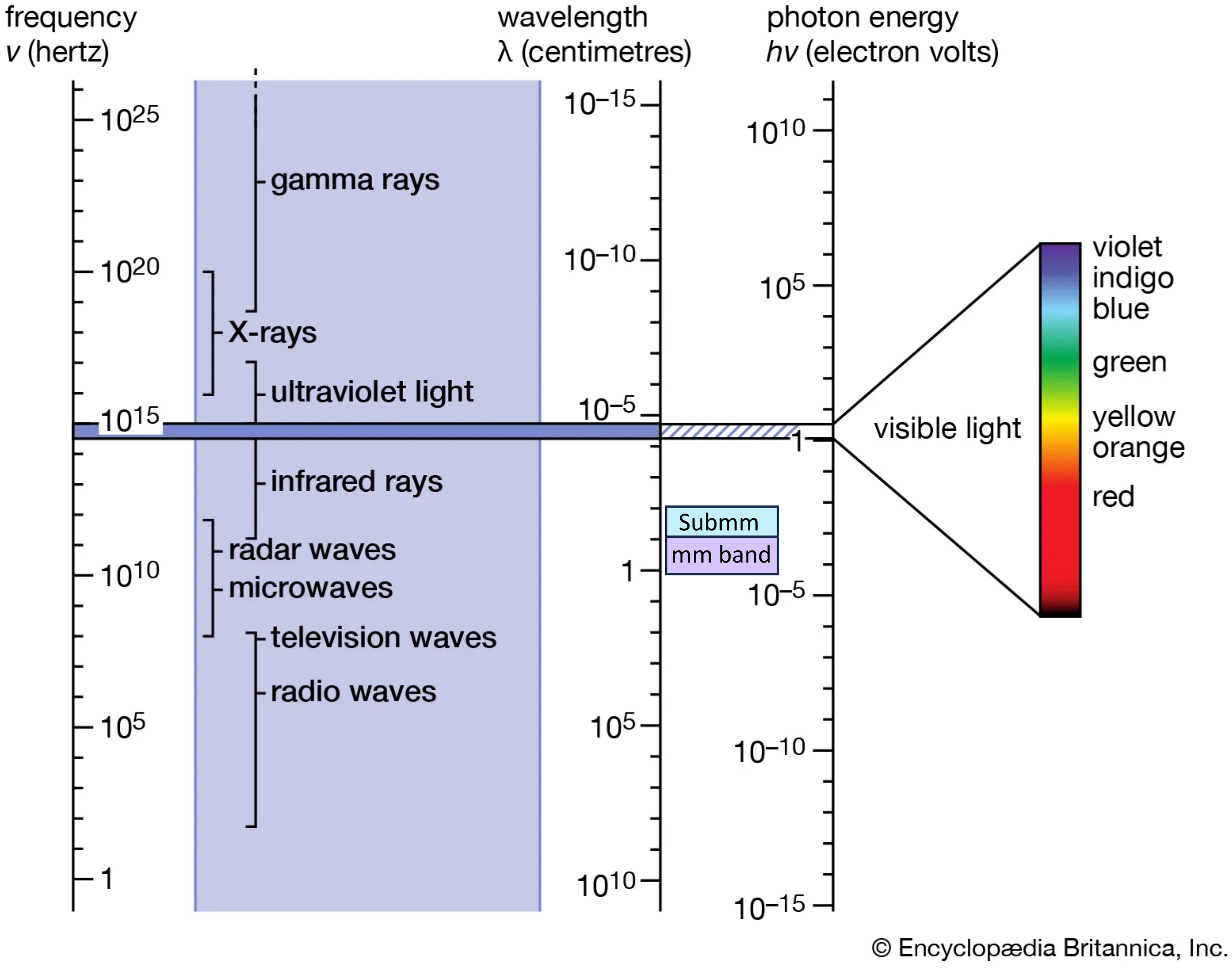
Fundamentals of radiative transfer

SISSA

- The electromagnetic spectrum
- The panchromatic view
- Astrophysical observables
- Emission and absorption coefficients
- Radiative transfer equation

Main ref.: Rybicki, G., and Lightman, A., 1979, Radiation processes in astrophysics, John Wiley and Sons, Ch.1.

The electromagnetic spectrum



Light is characterized by:

Wavelength λ
Frequency ν
Energy E
Temperature T

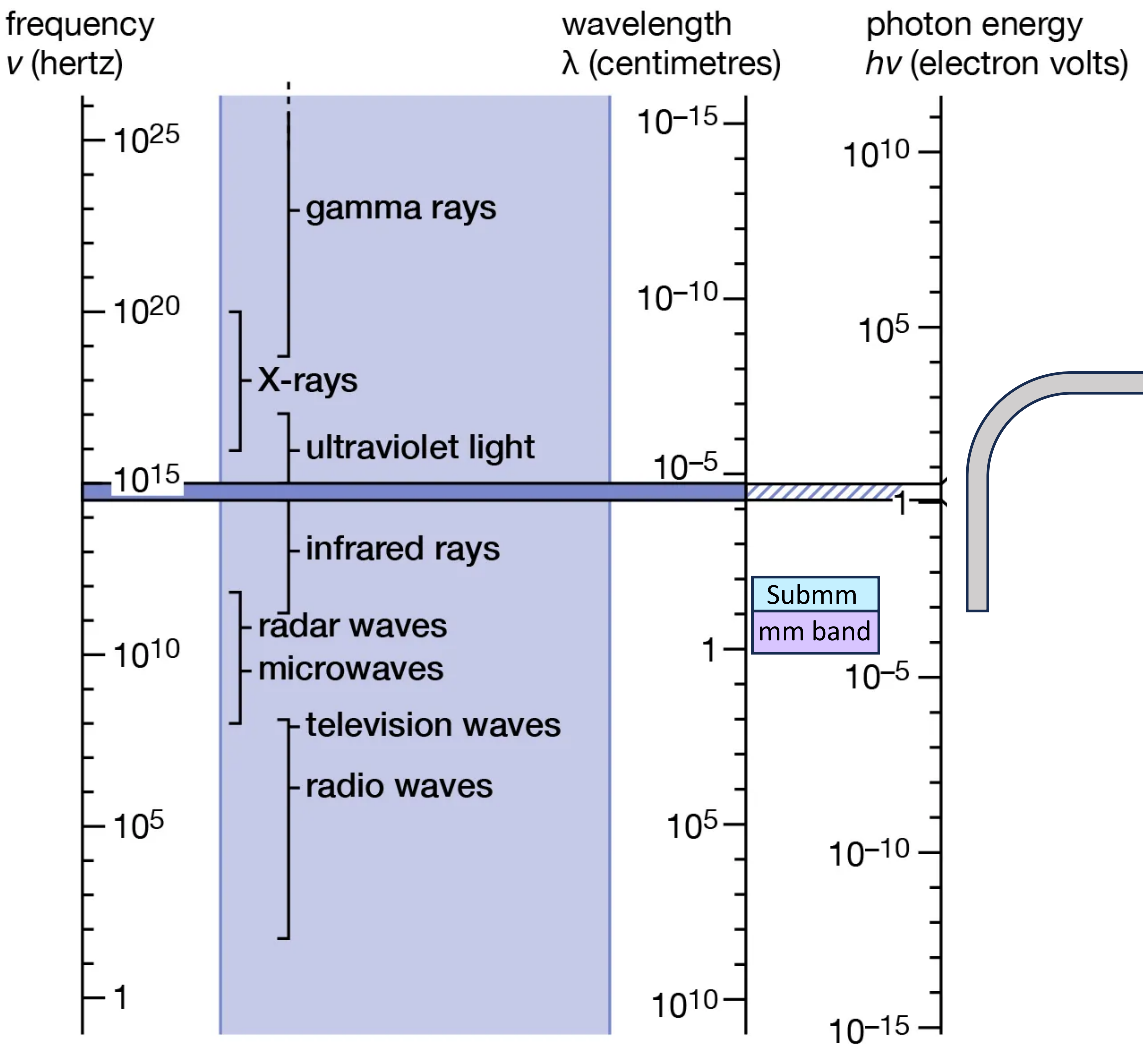
They are related by:
$\lambda\nu=c$
$E=h\nu$
$T=E/k$

Fundamental constants:
$c \approx 2.998 \times 10^{10}$ cm s^{-1}
$h \approx 6.626 \times 10^{-27}$ erg s
$k \approx 1.381 \times 10^{-16}$ erg K^{-1}

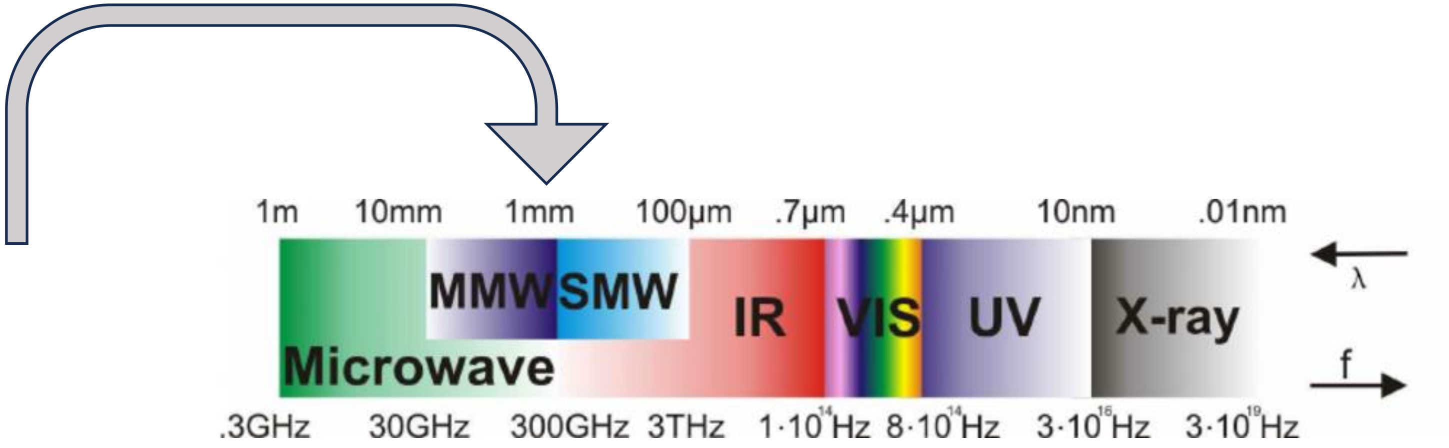
Useful units

in CGS: 1 erg= 1 g cm^{2} s^{-2}
1 eV = 1,6 x10^{-12} erg
1 erg = 10^{-7} Joules
1 eV= 1.6 x10^{-19} J

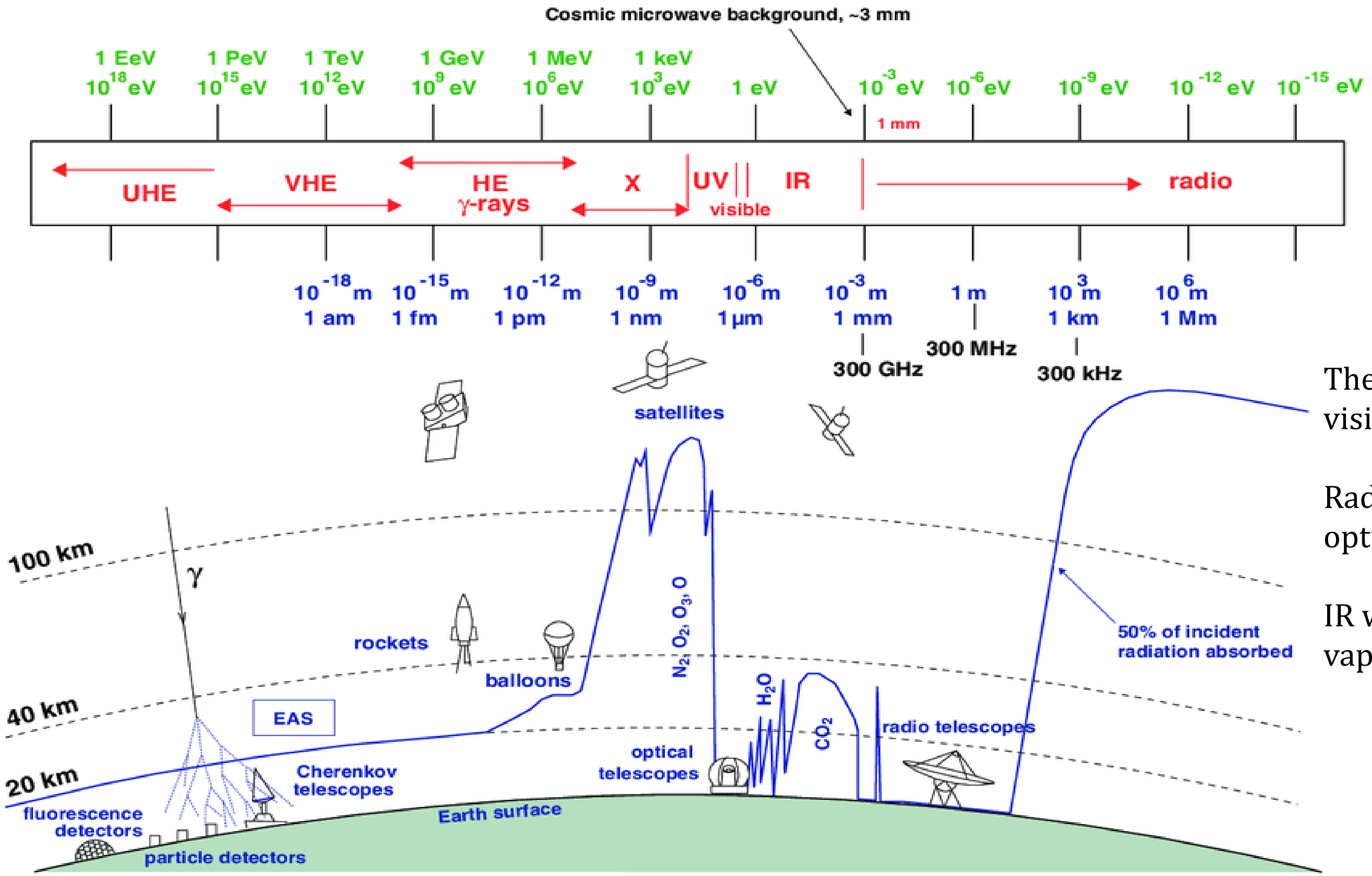
The electromagnetic spectrum



X-rays with high photon energies above 5–10 keV (below 0.2–0.1 nm wavelength) are called *hard X*-rays, while those with lower energy (and longer wavelength) are called **soft X-ray**



Atmospheric opacity



The atmosphere is mostly transparent (0% opacity) in the visible band (380-750 nm) and mostly opaque in the IR.

Radio window: 10 MHz ($\lambda \approx 30$m) to 1 THz ($\lambda \approx 0.3$ mm) in optimal terrestrial observation sites.

IR window: 8-14 μm, depending on water vapor asorption.

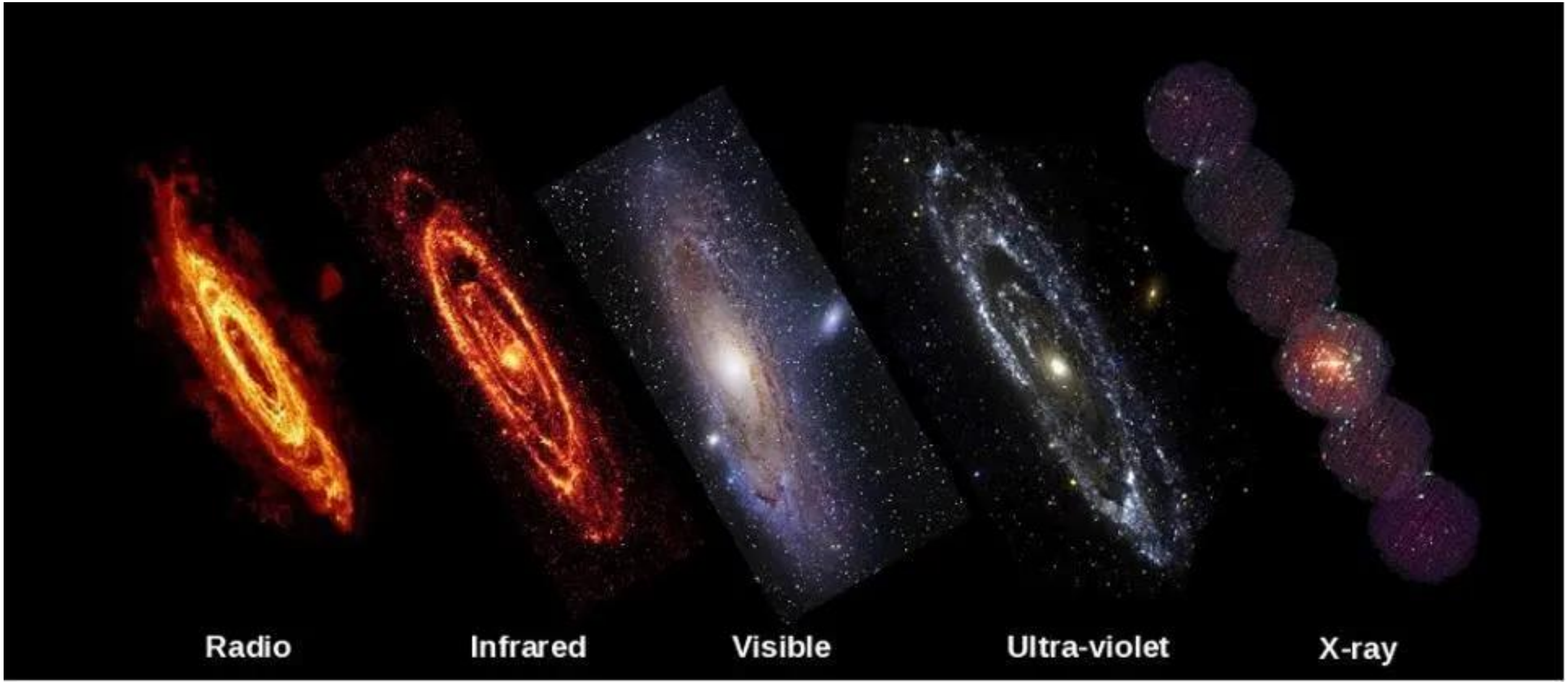
The electromagnetic spectrum

- Radiation is produced in astrophysical sources by many processes: blackbody emission, bremsstrahlung, synchrotron, Compton scattering, as well as line emission from atoms and molecules.

Region	Wavelength (Å)	Wavelength (cm)	Frequency (Hz)	Energy (eV)
Radio	$> 10^9$	> 10	$< 3 \times 10^9$	$< 10^{-5}$
Microwave	$10^9 - 10^6$	$10 - 0.01$	$3 \times 10^9 - 3 \times 10^{12}$	$10^{-5} - 0.01$
Infrared	$10^6 - 7000$	$0.01 - 7 \times 10^{-5}$	$3 \times 10^{12} - 4.3 \times 10^{14}$	$0.01 - 2$
Visible	$7000 - 4000$	$7 \times 10^{-5} - 4 \times 10^{-5}$	$4.3 \times 10^{14} - 7.5 \times 10^{14}$	$2 - 3$
Ultraviolet	$4000 - 10$	$4 \times 10^{-5} - 10^{-7}$	$7.5 \times 10^{14} - 3 \times 10^{17}$	$3 - 10^3$
X-Ray	$10 - 0.1$	$10^{-7} - 10^{-9}$	$3 \times 10^{17} - 3 \times 10^{19}$	$10^3 - 10^5$
Gamma Ray	< 0.1	$< 10^{-9}$	$> 3 \times 10^{19}$	$> 10^5$

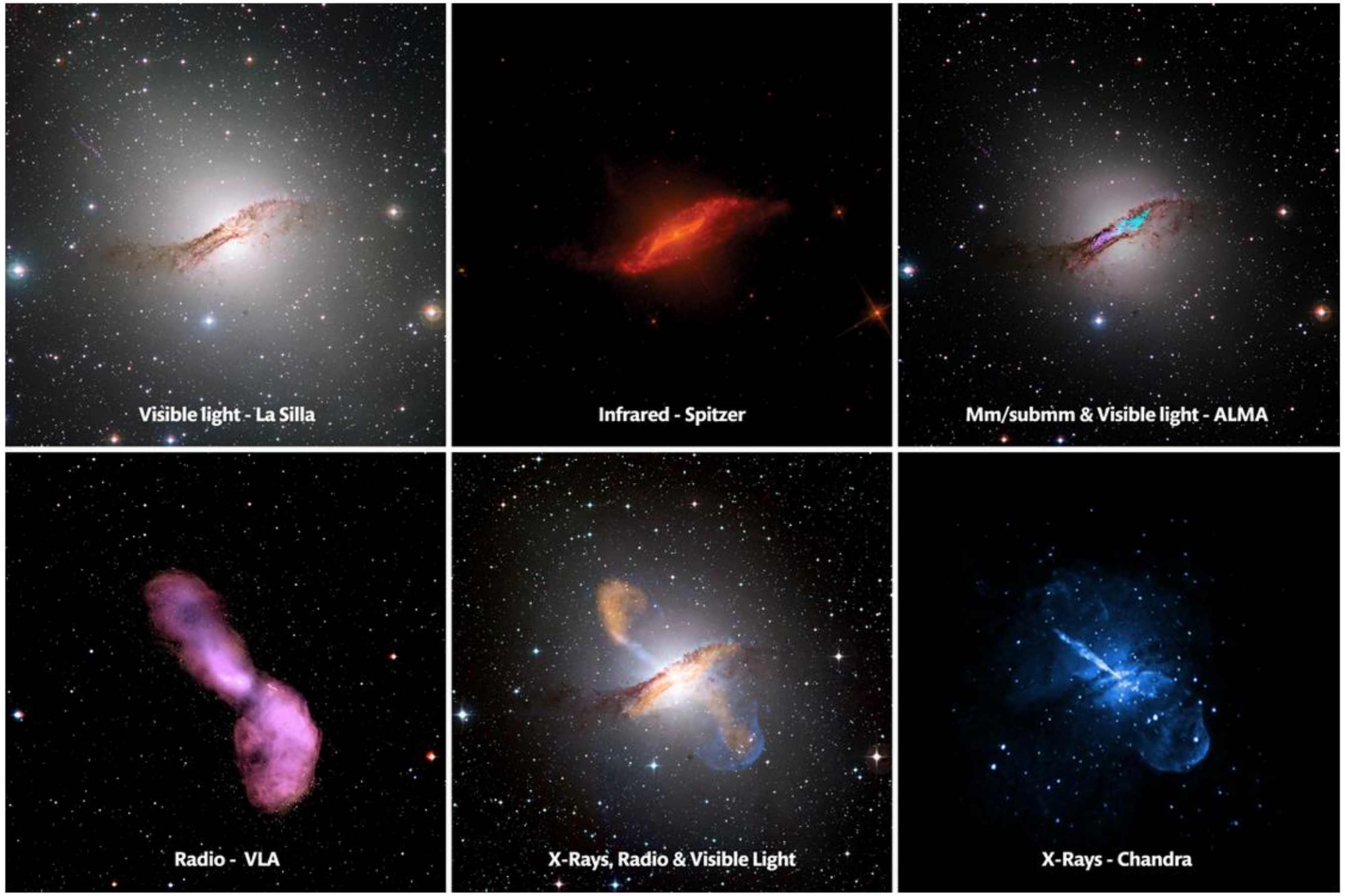
Source: http://csep10.phys.utk.edu/astr162/lect/light/spectrum.html

The panchromatic view



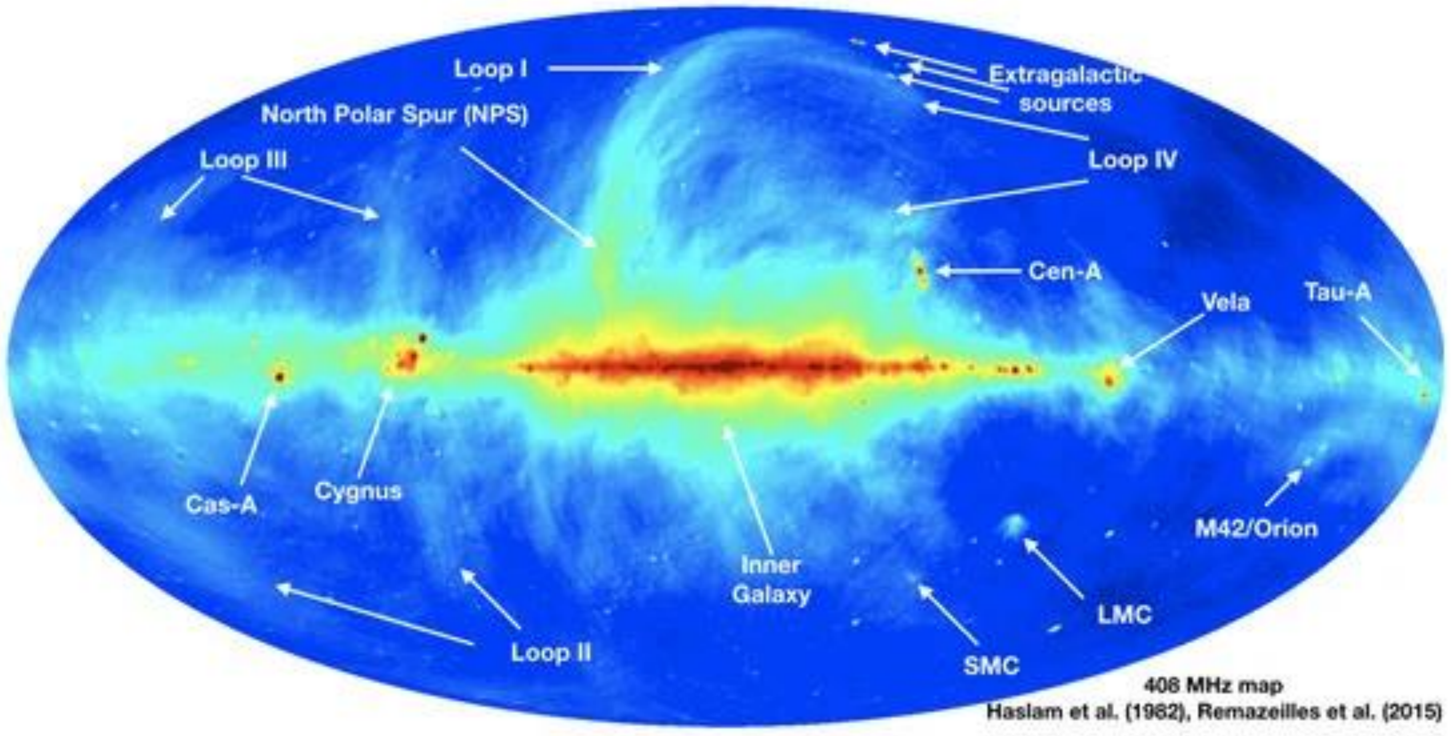
Multiwavelength images of M31, the Andromeda Galaxy. Quite clearly, different wavelengths reveal various details that are unseen in visible light alone. Planck Mission Team / NASA / ESA

The panchromatic view



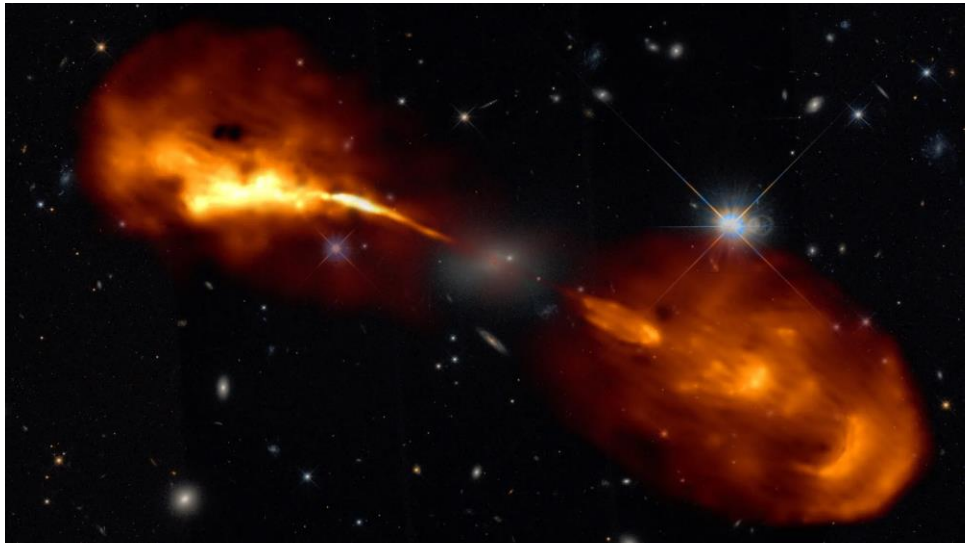
Centaurus A galaxy. By including information from different instruments and wavebands, we can obtain a much more complete image of what we are observing.

The radio sky

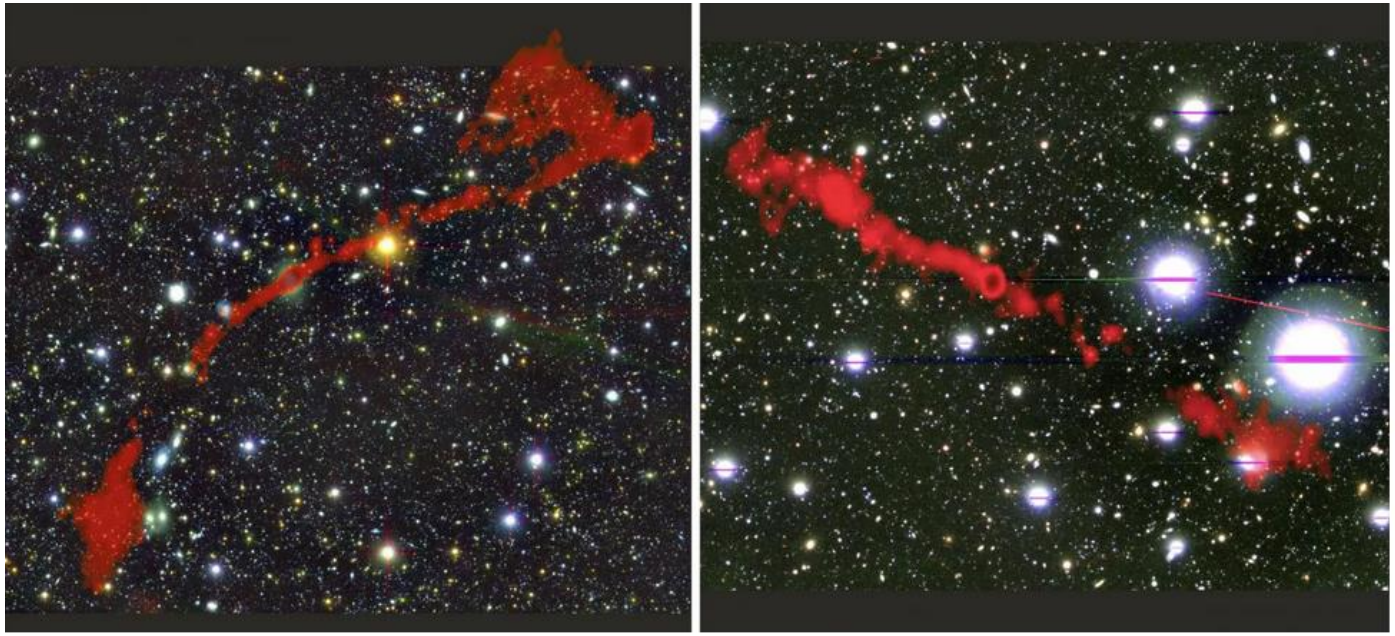


The sky at 408 Mhz , mapped at Jodrell Bank radio telescope. Nearly all of the radio emission from normal galaxies is synchrotron radiation from relativistic electrons and free-free emission from HII regions.

The radio sky



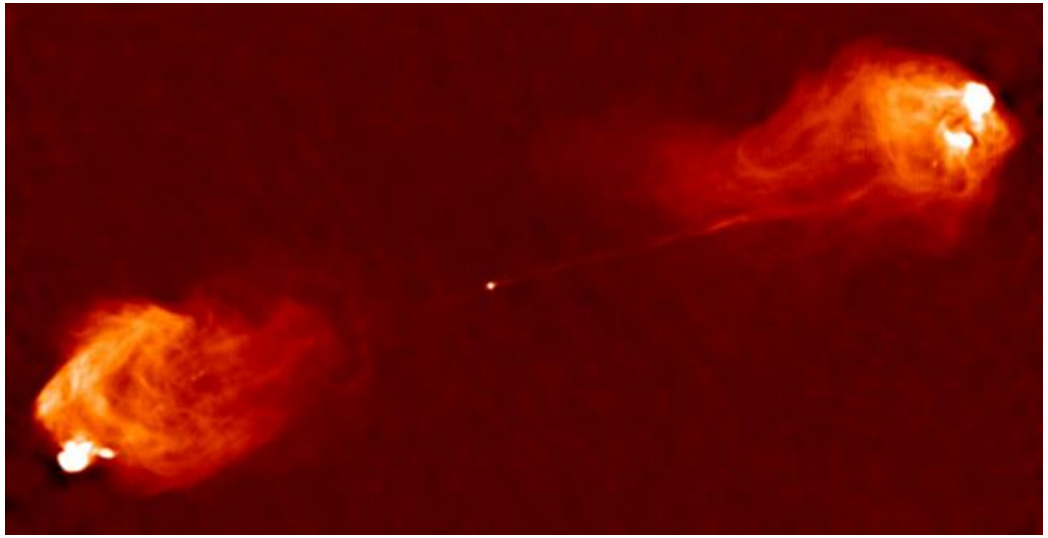
Hercules-A black hole jets captured in a high-resolution image by LOFAR radiotelescope, 10–240 MHz



The two giant radio galaxies found with the MeerKAT telescope. In the background is the sky as seen in optical light. Overlaid in red is the radio light from the enormous radio galaxies, as seen by MeerKAT. Left: MGTC J095959.63+024608.6. Right: MGTC J100016.84+015133.0. Credit: I. Heywood (Oxford/Rhodes/SARAO)

Giant radiogalaxies found with MeerKAT radiotelescope, 544 MHz-3.5 GhZ

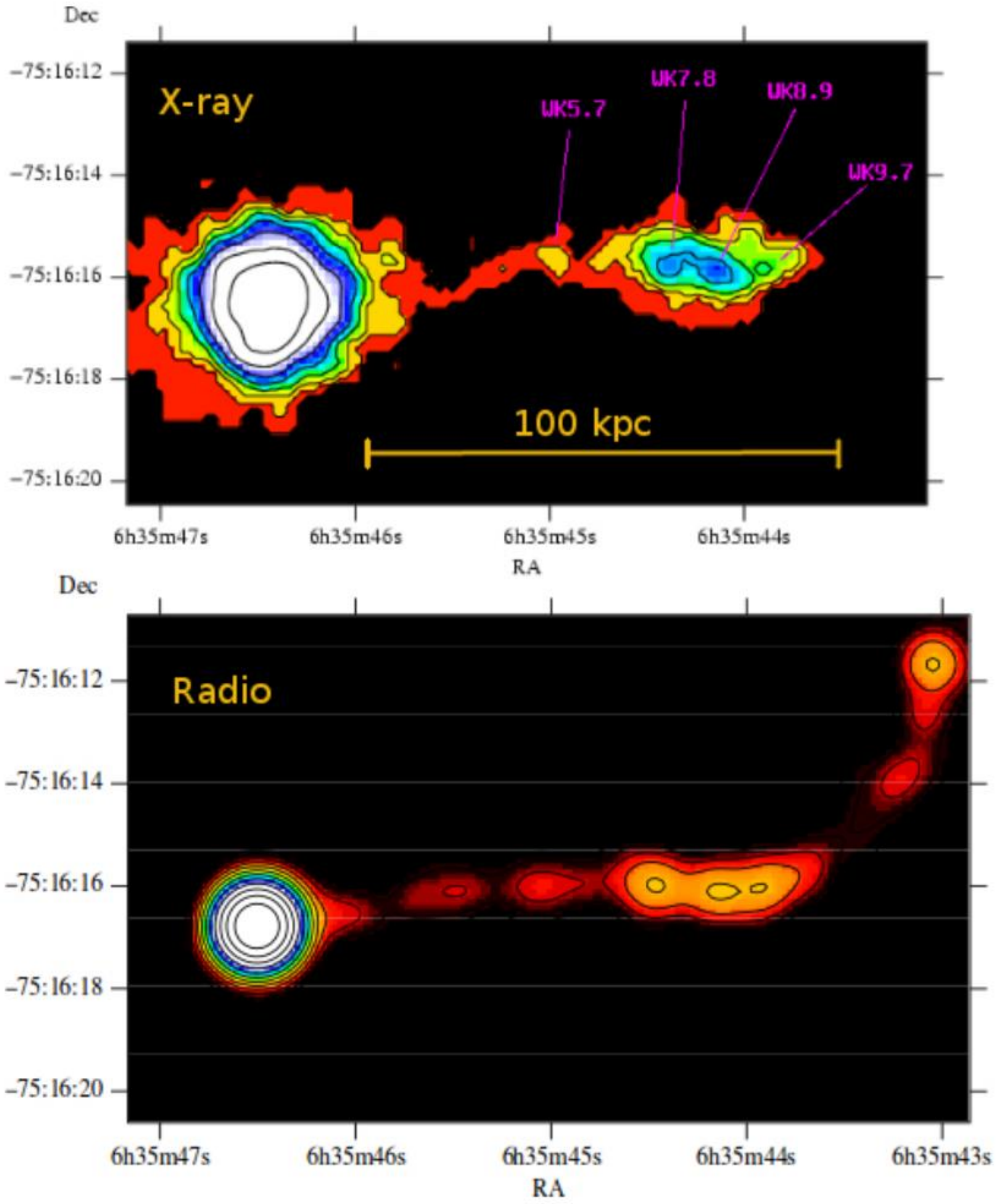
The radio sky



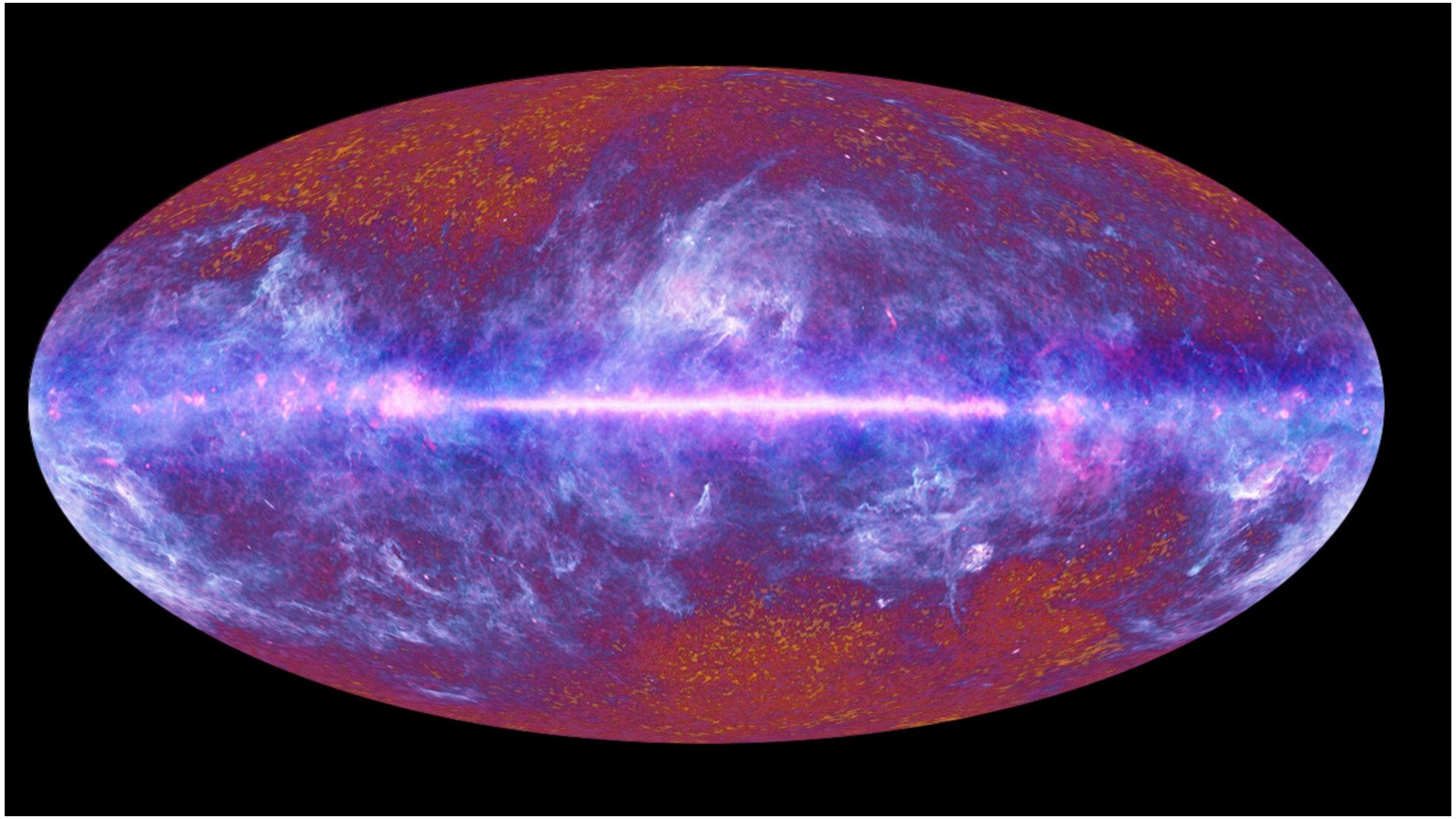
Spoiler alert! The radio emission is due to **the synchrotron process**. The observed structure in radio emission is determined by the interaction between twin jets and the external medium, modified by the effects of relativistic beaming.

Beyond the central point source, there is X-ray emission extending for hundreds of kiloparsecs, and matching the radio emission almost perfectly.

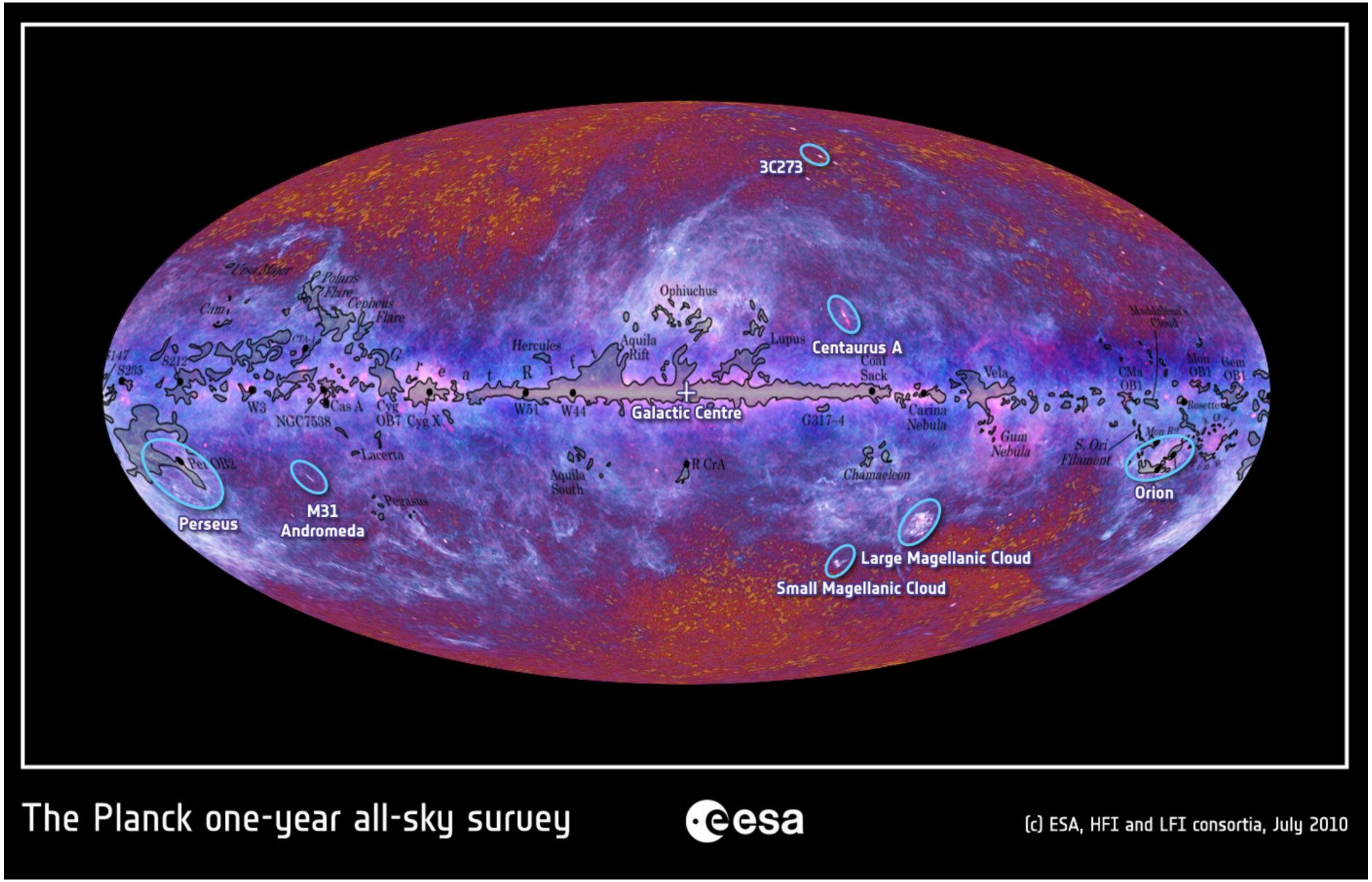
The radio-emitting electrons in the jet are also giving some of their energy to photons of the Cosmic Microwave Background, turning them into the X-rays observed by Chandra. (Inverse Compton effect)



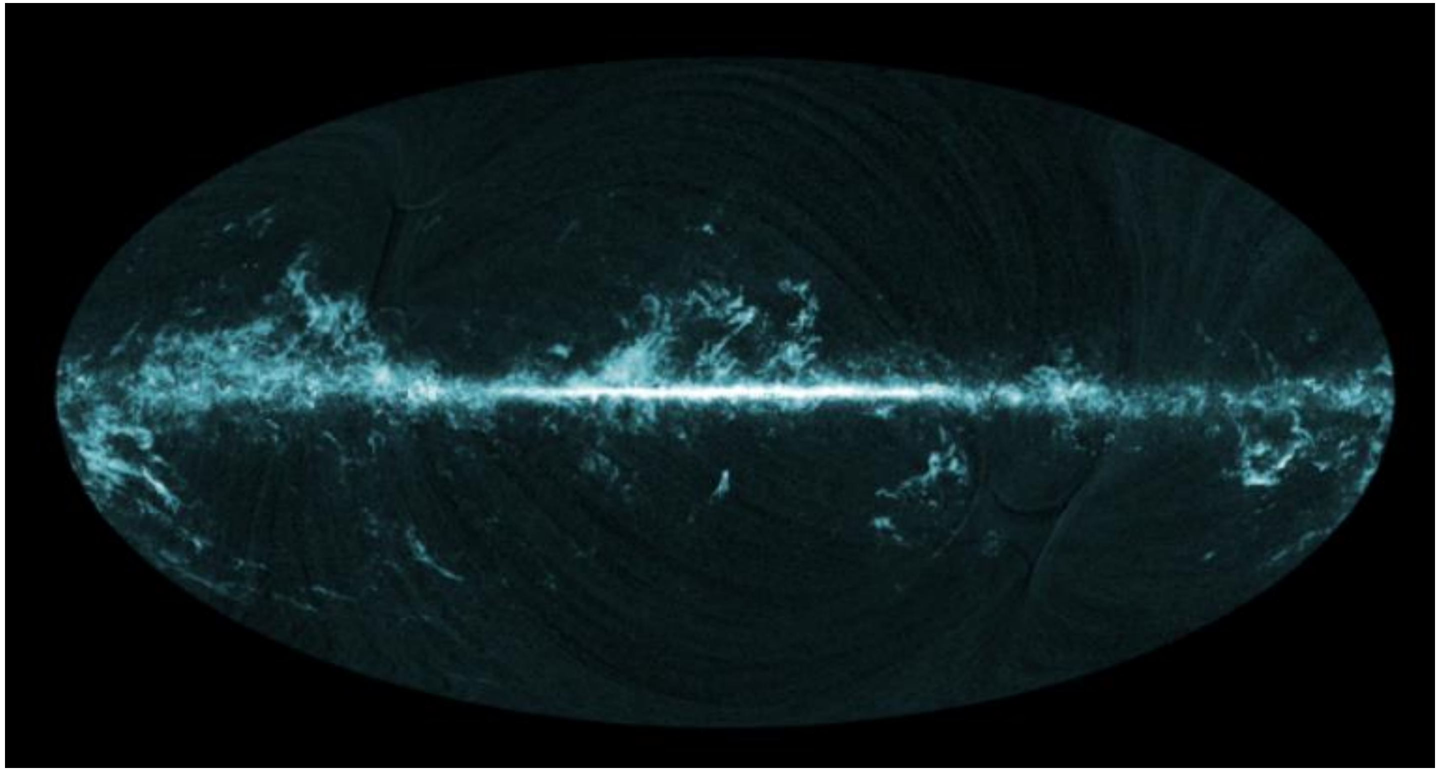
The microwave sky - All-sky image of the microwave sky from Planck satellite , covering the electromagnetic spectrum from 30 GHz to 857 GHz.



The microwave sky - All-sky image of the microwave sky from Planck satellite , covering the electromagnetic spectrum from 30 GHz to 857 GHz.

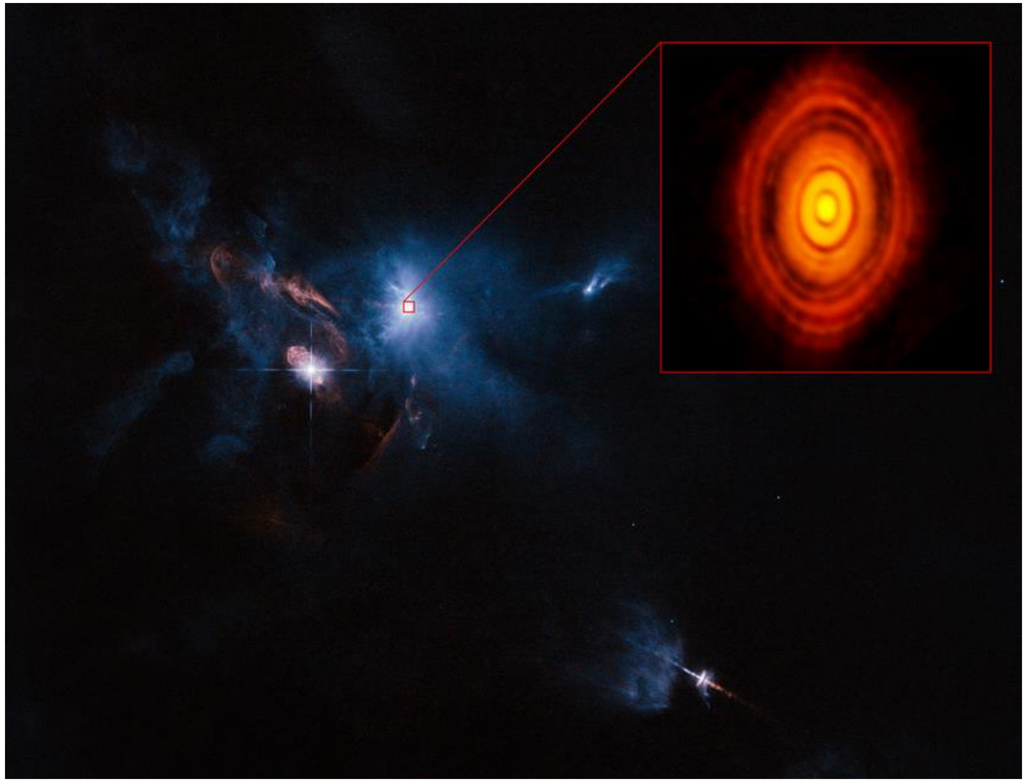


The mm sky : Planck full sky map of J=1-0, at 2.6 mm (115 HGz)

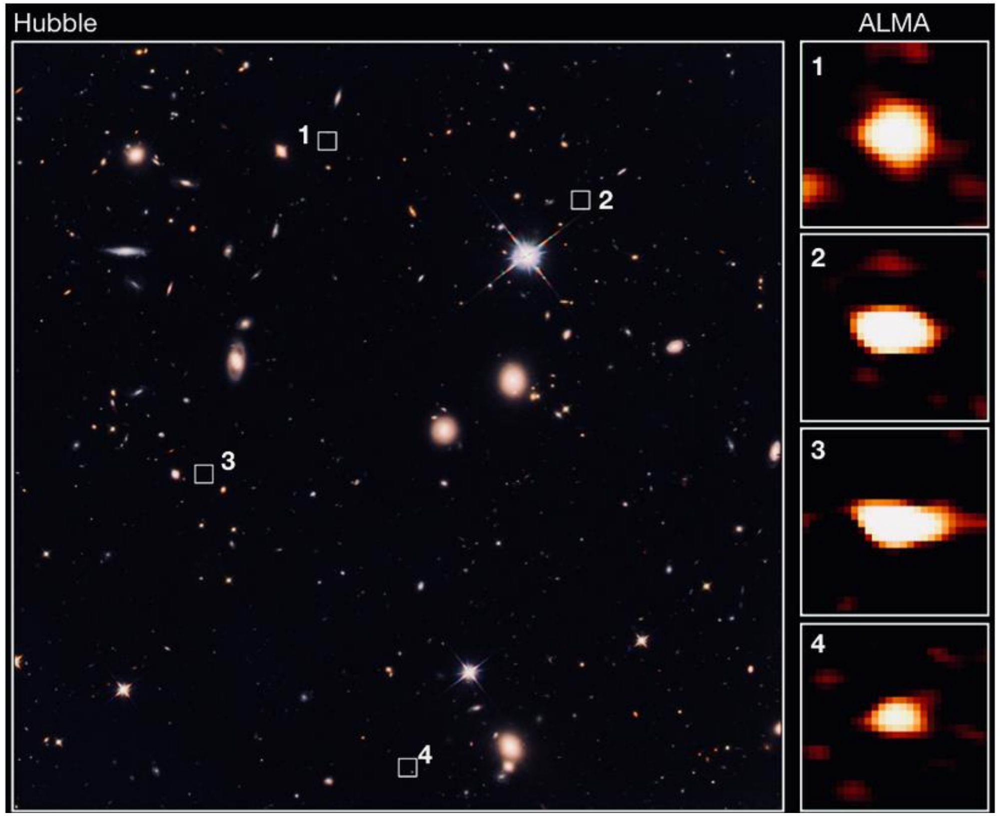


Rotational transition lines of CO are one of the major tracers used to study star forming regions and Galactic structures.
A large number of observations of CO rotational lines are covering the galactic plane.

The mm and submm sky - ALMA array between 35 GHz (68 mm) and 125-163 GHz (1.8-2.4 mm)

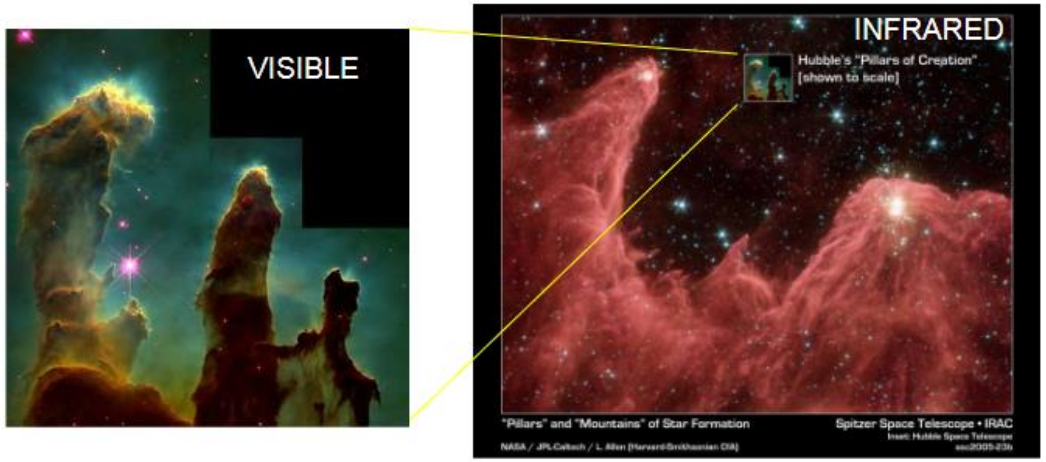


Composite image of the young star HL Tauri (Hubble space Telescope) and its surrounding region, a protoplanetary disk (Atacama Large Millimeter/submillimeter Array)

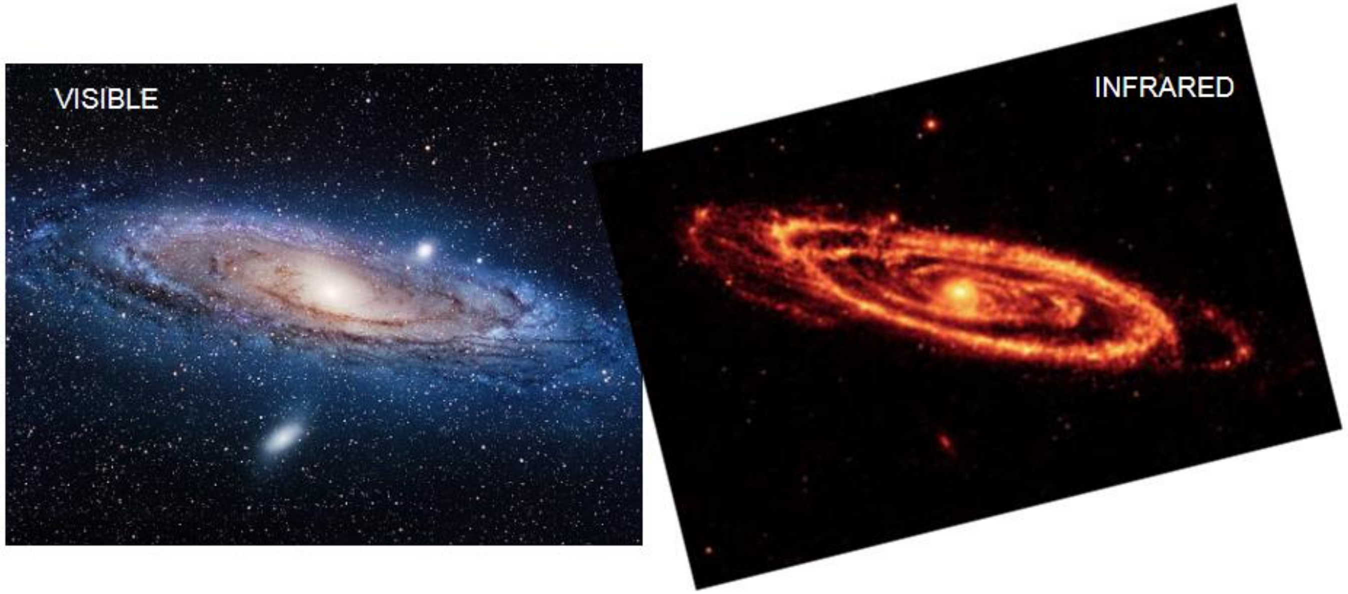


ALMA identified faint galaxies that are not seen with the Hubble Space Telescope's deepest view of the Universe 10 billion light-years away. Here, a comparison of Hubble and ALMA observations.

The infrared and visible sky

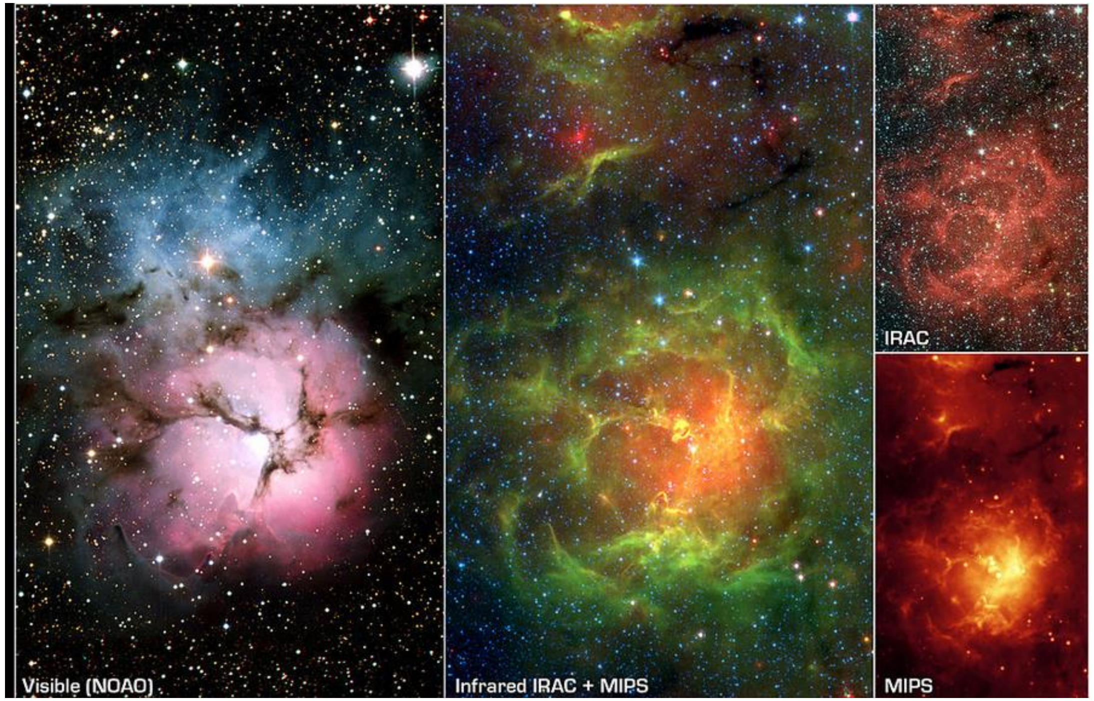


Reflected starlight and dust associated to the birth of stars in our Galaxy



Stars and dust in the spiral arms of external galaxies

The infrared and visible sky



The **Trifid Nebula** is an HII region. It is a combination of an open cluster of stars, an emission nebula (the relatively dense, reddish-pink portion), a reflection nebula (the mainly blue portion), and a dark nebula(the apparent 'gaps' in the former that cause the trifurcated appearance, also designated **Barnard 85**).

Helix Nebula. Planetary nebula, region of dust and gas from the cast-off layers of a dying star.

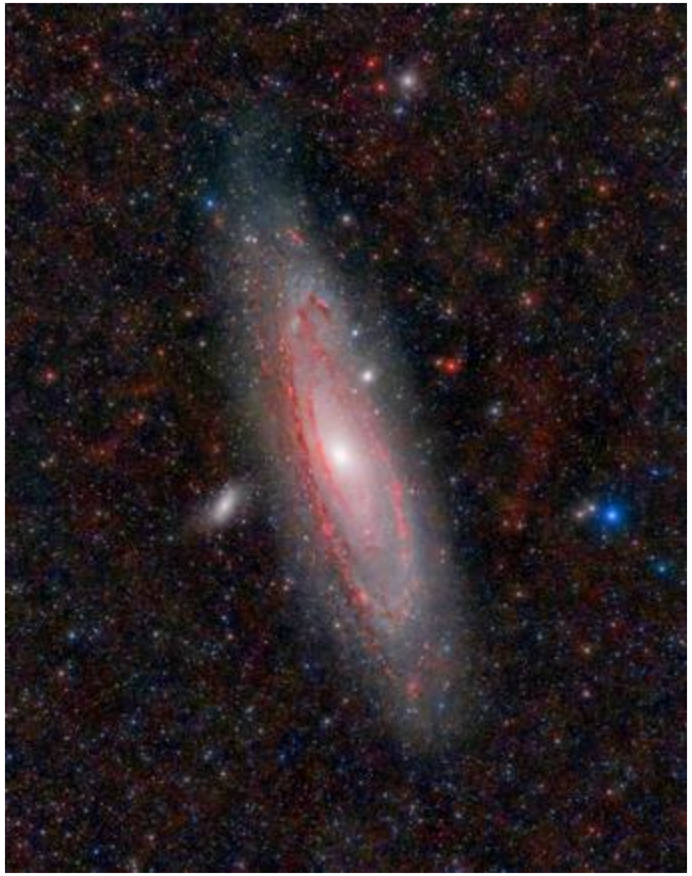
Oxygen atoms excited by the UV of the central star: Forbidden lines @500 nm of OIII is the blue-green

Red color is due to H-alpha ad Nitrogen.

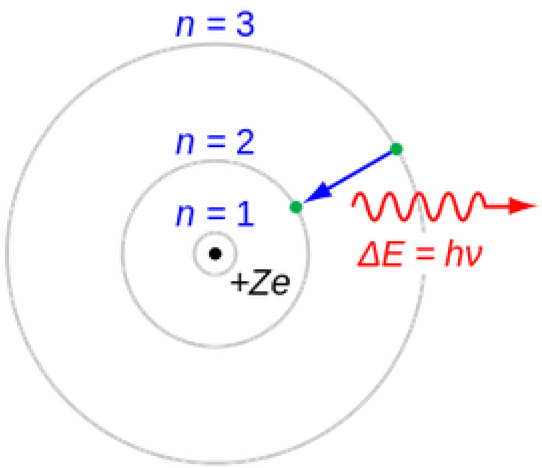
H-alpha line at 656 nm (optical, red)

Horsehead nebula in Hα

Lagoon nebula in Hα



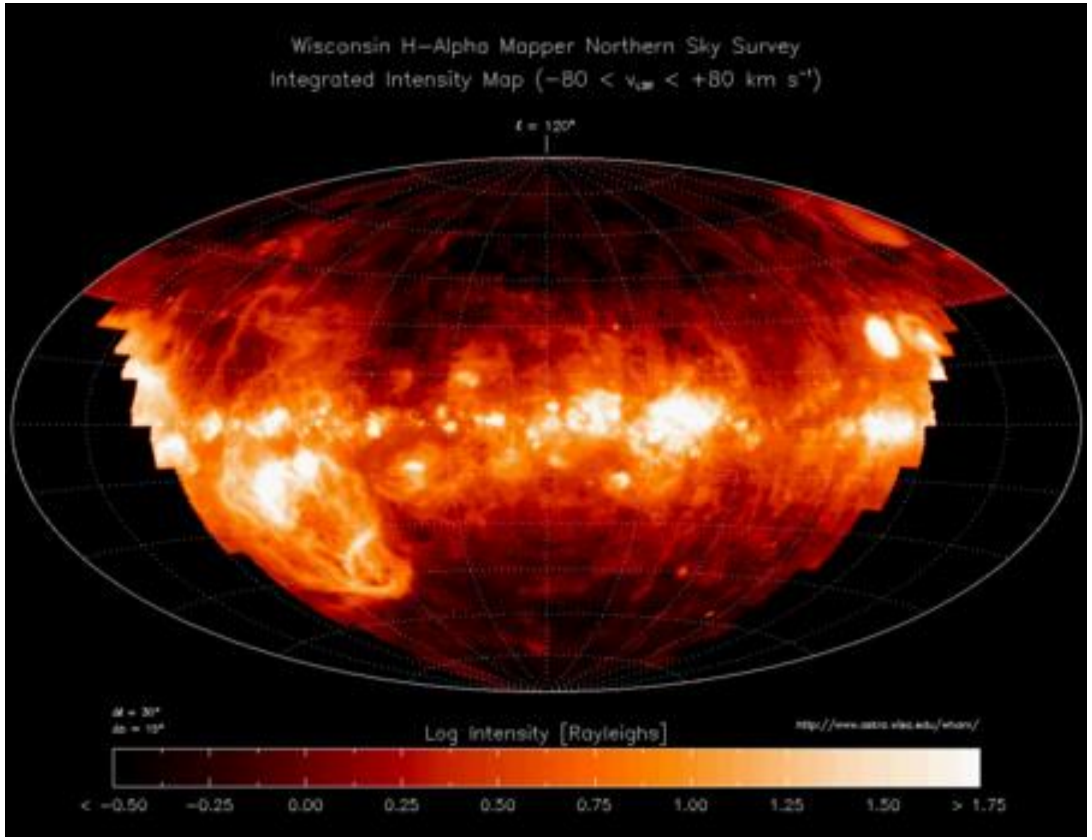
Andromeda in Hα



H_α is a Balmer series line. When the electrons and protons recombine in a HII region, they generally recombine to upper energy levels (large n), and then cascade down, emitting Balmer lines

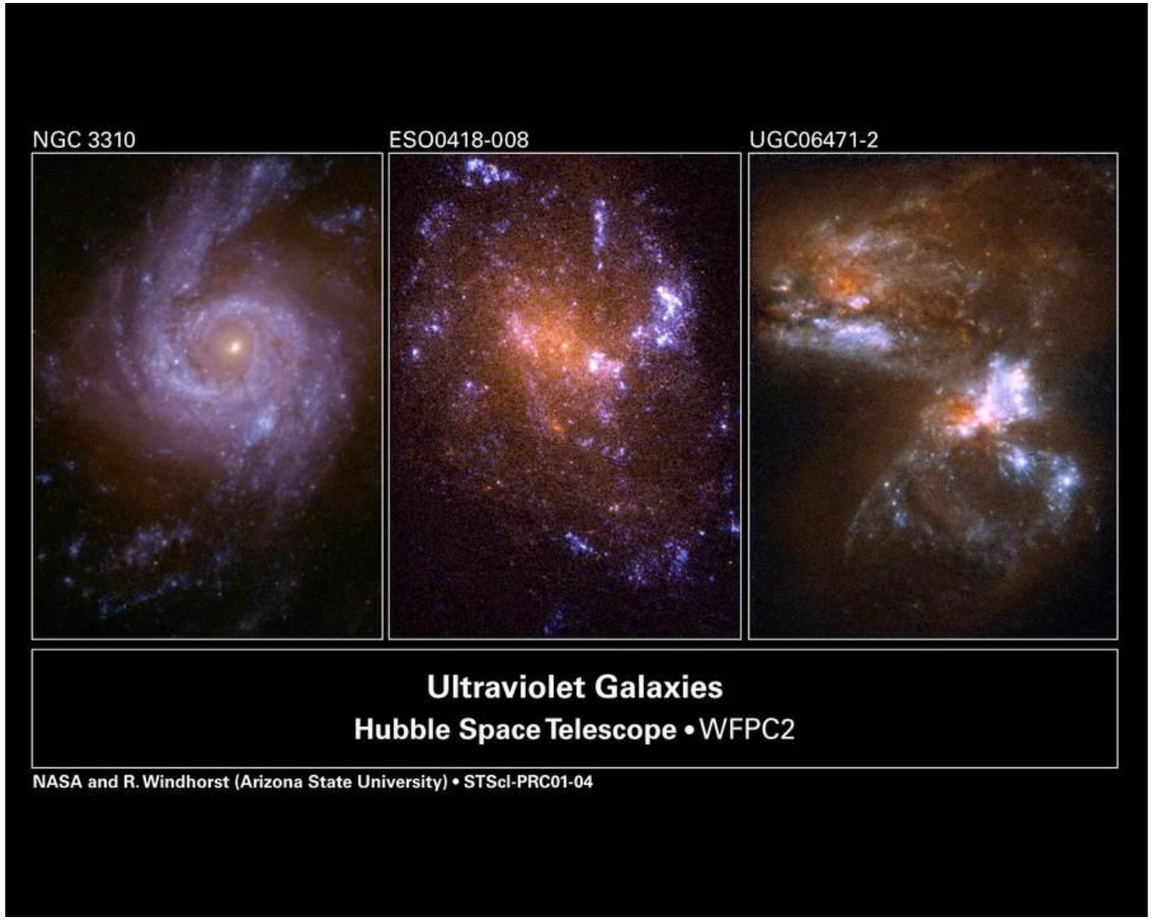
Emission nebulae called HII regions

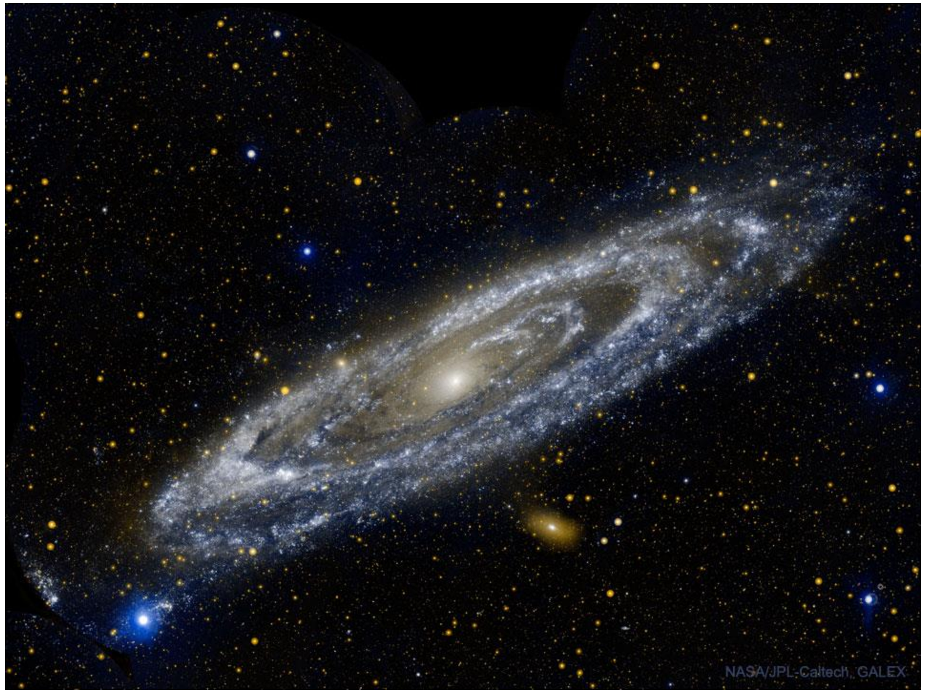
H-alpha line at 656 nm (optical, red)



The distribution of ionized hydrogen, HII in the parts of the Galactic interstellar medium visible from the Earth's northern hemisphere, as observed with the Wisconsin Hα Mapper (Haffner et al. 2003).

The Visible/UV Sky





NASA's Galaxy Evolution Explorer (GALEX) satellite UV image of Andromeda. Star formation in Spiral arms.

The X-ray Sky eROSITA X-ray telescope onboard the Russian-German SRG spacecraft: 0.5-11 keV (soft and hard)

Blue : highest energies, followed by green, yellow and red.

The red foreground glow comes from hot gas near the solar system.

Milky Way plane: gas and dust in the disk absorb all but the most energetic X-rays.

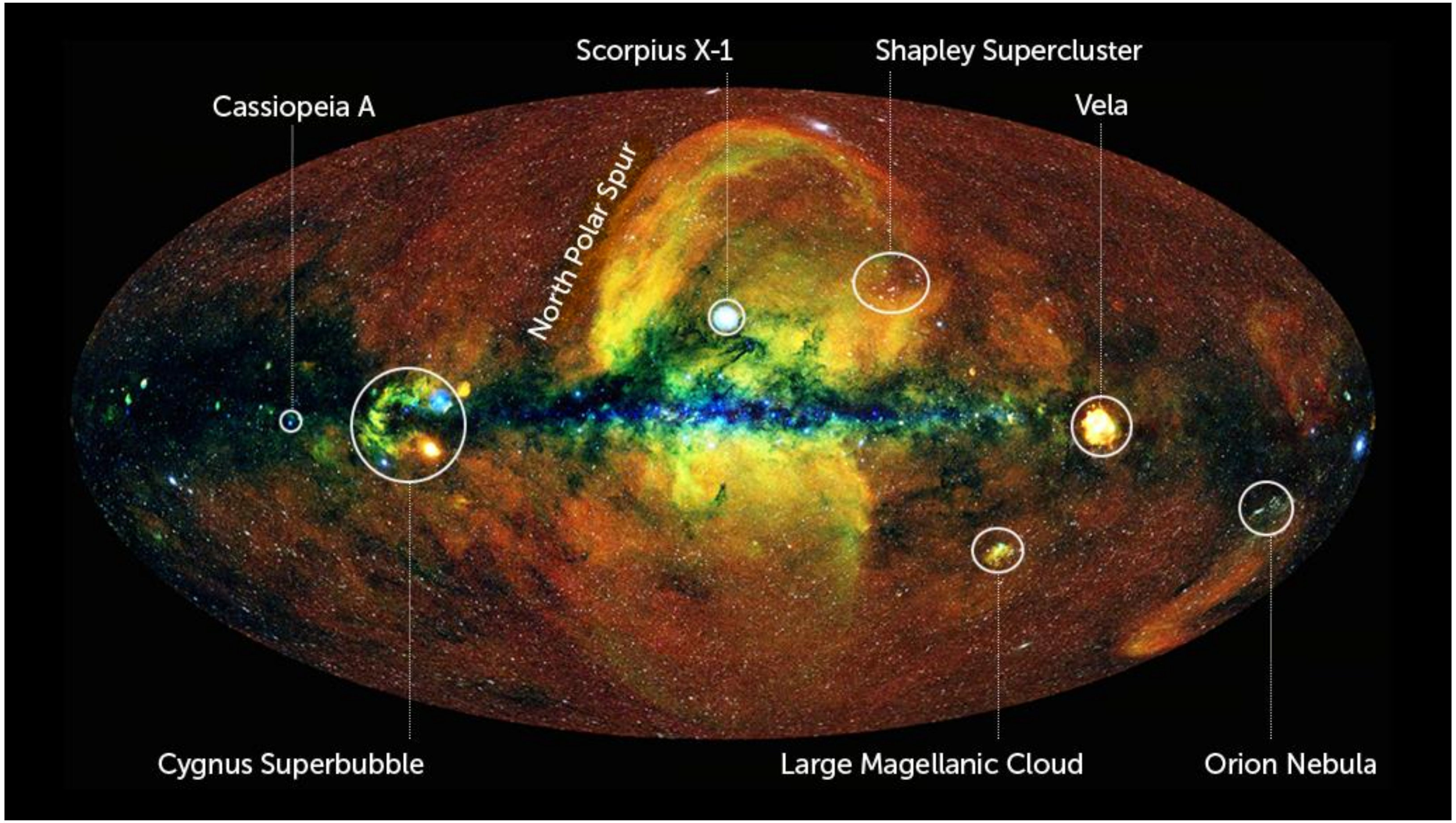
Supernova remnant: Cassiopeia A and Vela, and a star system called Scorpius X-1.

Orion Nebula (Star forming region), Cygnus Superbubble.

A mysterious arc of X-rays called the North Polar Spur.

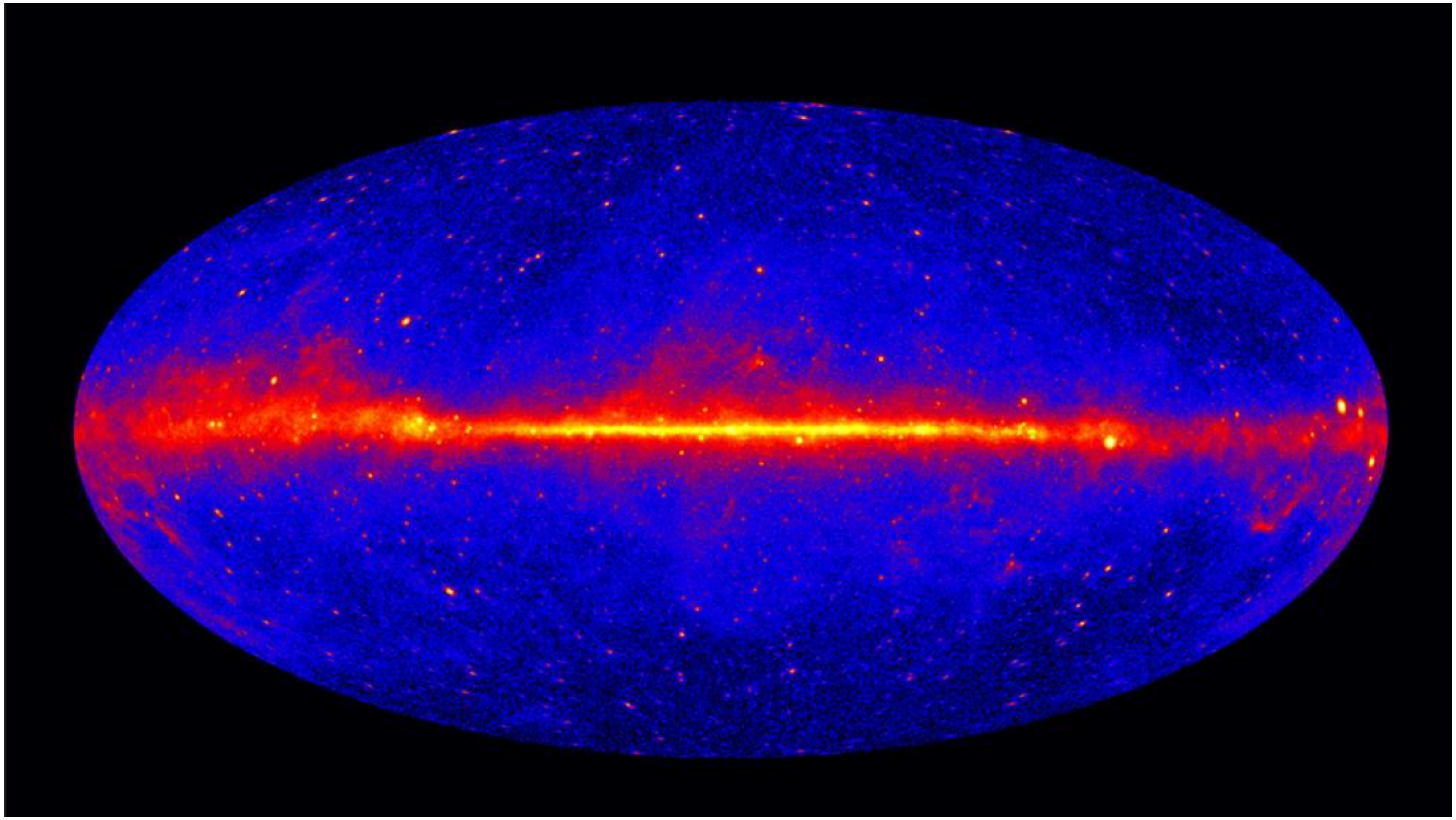
Extra galactic, LMC and Shapley Supercluster.

Supermassive black holes accreting in the centers of other galaxies

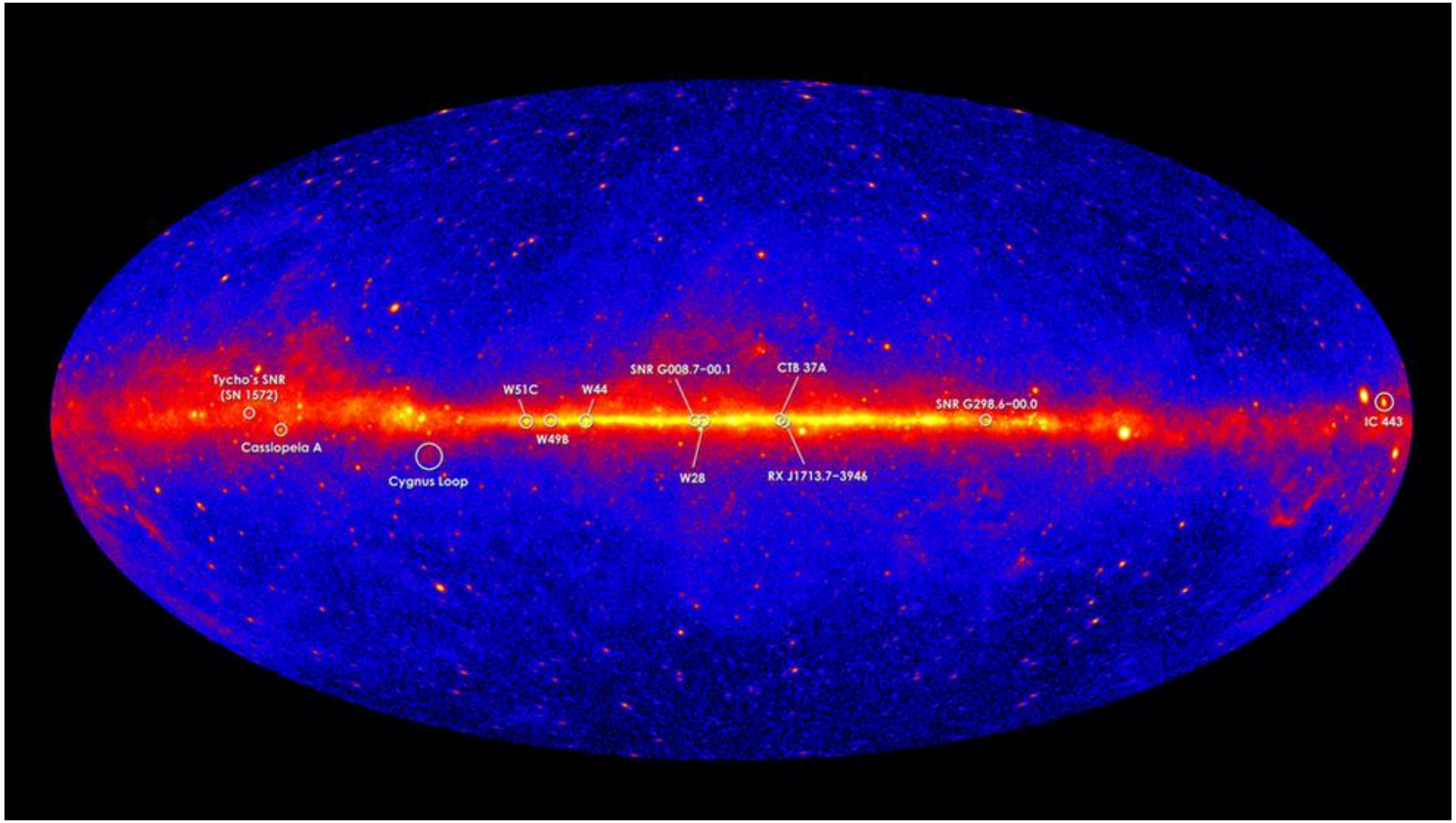


credits: https://www.sciencenews.org/article/xray-map-sky-erosita-telescope-milky-way

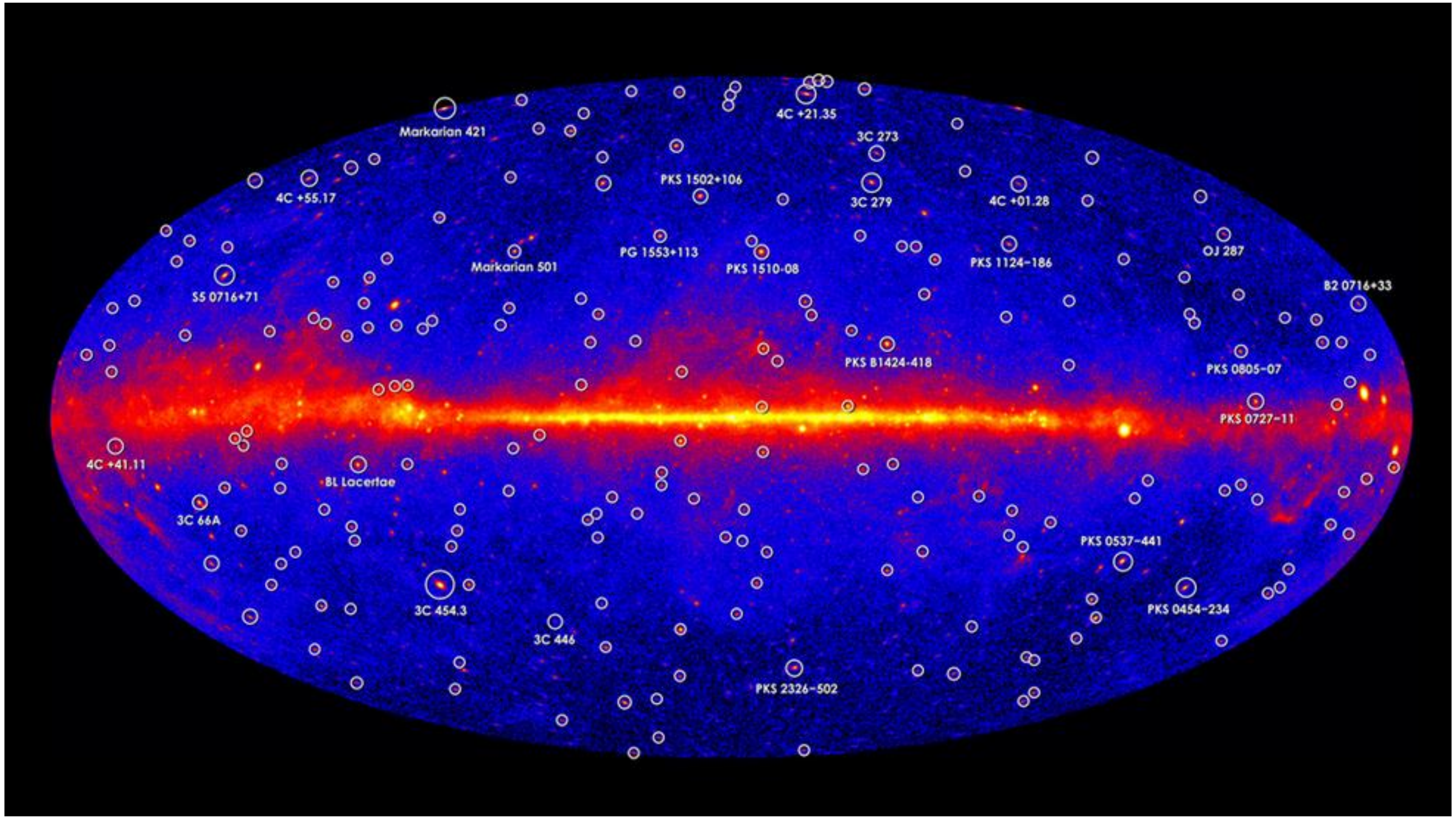
The Gamma-ray Sky - NASA's Fermi spacecraft. Fermi-LAT : 20 MeV-300 GeV
credits : https://svs.gsfc.nasa.gov/11545



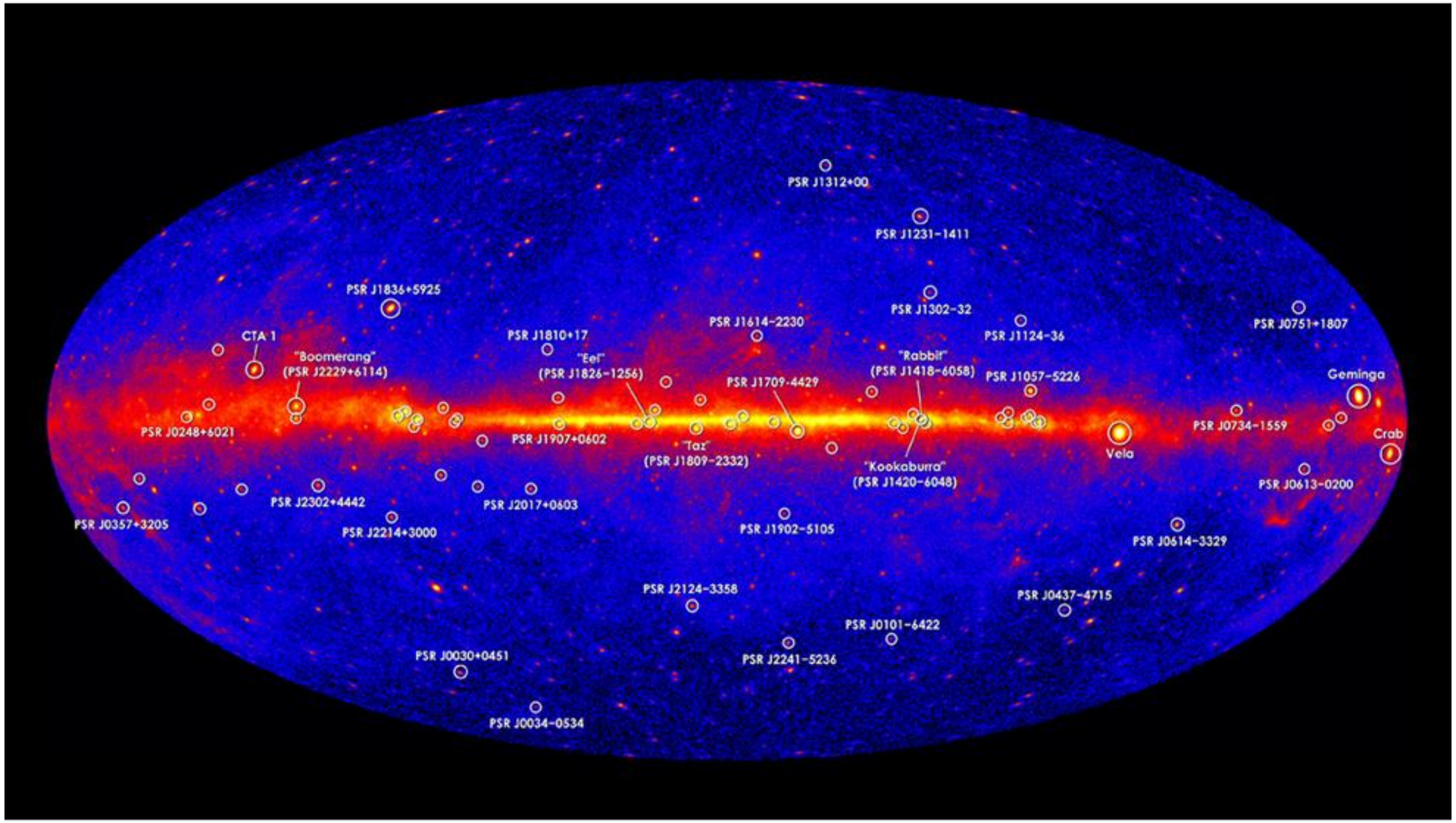
Gamma rays released from supernovae



Material falling into supermassive black holes accounts for the majority of gamma ray sources detected by Fermi



Gamma rays emitted from pulsars



Astrophysical radiation emission processes

Radiation is produced in astrophysical sources by many processes: blackbody emission, bremsstrahlung, synchrotron, Compton scattering, as well as line emission from atoms and molecules.

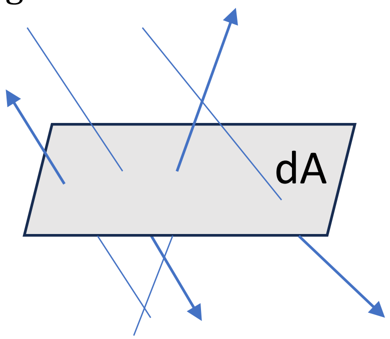
Region	Wavelength (Å)	Main Sources
Radio	$> 10^9$	Supernova remnants, galaxies, galactic center, HII regions, quasars
Microwave	$10^9 - 10^6$	Sun, Crab nebula, microwave background, Milky Way, radio galaxies, quasars
Infrared	$10^6 - 7000$	Sun, planets, Galactic center, nebulae, dust, quasars, active galaxies
Visible	$7000 - 4000$	Sun, planets, stars, galaxies, quasars
Ultraviolet	$4000 - 10$	Sun, hot stars, active galaxies, quasars
X-Ray	$10 - 0.1$	Sun, compact binaries, black holes, hot gas in galaxy clusters, AGN, quasars
Gamma Ray	< 0.1	crab pulsar, vela pulsar, galactic disk, matter anihilation, extragalactic sources(?)

Astrophysical observables

Energy flux.

When the scale of a system is much larger than the radiation wavelength, we can consider radiation to travel in straight lines (rays). The energy flux F of the radiation represents the energy passing through a given area in the unit time:

dE= F dA dt

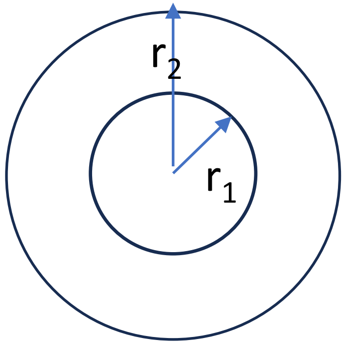


The flux is a measurable quantity(the energy carried out by the intercepted photons).

An isotropic source is a source emitting equal amounts of energy in all directions. Energy conservation implies that the flux through two shells around an isotropic source is the same (inverse square law):

$$4\,\pi r_1^2\ F(r_1) = 4\,\pi r_2^2\ F(r_2)$$

$$F(r) = \frac{constant}{r^2}$$



CGS Flux units: erg s^{-1} cm^{-2}.

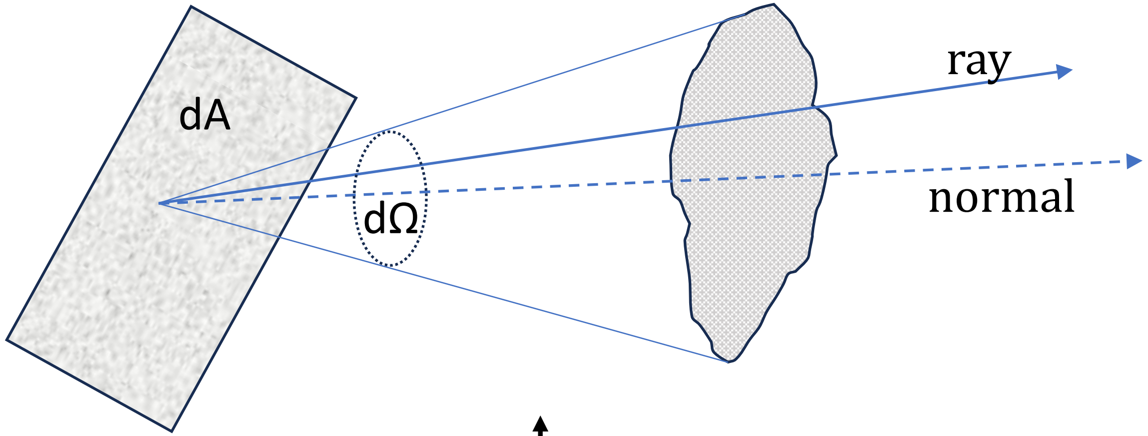
Astrophysical observables

Specific intensity.

The flux is a measure of the energy carried by all rays passing through a given area A. More information lies in the energy carried along a specific **direction** of an individual ray or, better, the energy carried by a beam.

The energy crossing dA in the time dt and in frequency range dν , carried by the beam (all the rays whose direction lies within the solid angle dΩ with respect to the normal), is given by:

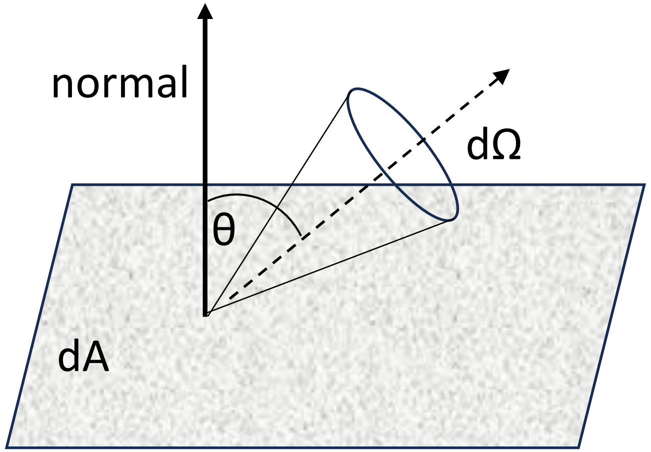
$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$ (I_ν is a $I_\nu(\mathbf{n})$,) it has units in cgs: erg cm^{-2} s^{-1} sr^{-1} Hz^{-1})



I_ν is the specific intensity or brightness. It is an intrinsec property of the radiation, and depends on location in space, on direction and on frequency, but is constant along the line of sight (if in free space).

- In an isotropic radiation field, I_ν =const for all directions.
- Relation between intensity and flux collected by an observer in a given direction (normal to the surface):

$$F_\nu = \int I_\nu \cos\theta \; d\Omega_{obs} \qquad \text{CGS } F_\nu \text{ units: erg } s^{-1} \; Hz^{-1} \, cm^{-2}.$$



Assuming rays arrive ⊥ , cosΘ=1, and , in solid angles in which the specific intensity is not varying, $F_\nu = I_\nu \, \Delta\Omega_{obs}$

Astrophysical observables

Flux at the surface of of a uniform brightness sphere

Calculate the flux at an arbitrary distance from a sphere of uniform brightness (or *specific intensity*) I_ν.

All rays leaving the sphere have the same brightness. This is an isotropic source.

The (specific) intensity is conserved along rays in free space, for energy conservation.

At the observer's position P (distance r from source), the intensity is B if the ray intersects the sphere, and zero otherwise:

$$F_\nu(r) = \int I_\nu \cos\theta \, d\Omega_{obs} = I_\nu \int_0^{2\pi} d\varphi \int_0^{\theta_c} \sin\theta \cos\theta \, d\theta \qquad \textit{units:} \quad \text{erg s}^{-1} \text{ Hz}^{-1} \text{ cm}^{-2}$$

where $R = r \sin\theta_c$, so θ_c is the angle at which a ray from P is tangent to the sphere.

The energy collected in 1 cm^2 of the *observer* area, per unit frequency, is

$$F_\nu(r) = \pi I_\nu (1 - \cos^2\theta_c) = \pi I_\nu \sin^2\theta_c \qquad \text{or} \qquad \boxed{F_\nu(r) = \pi I_\nu \left(\frac{R}{r}\right)^2}$$

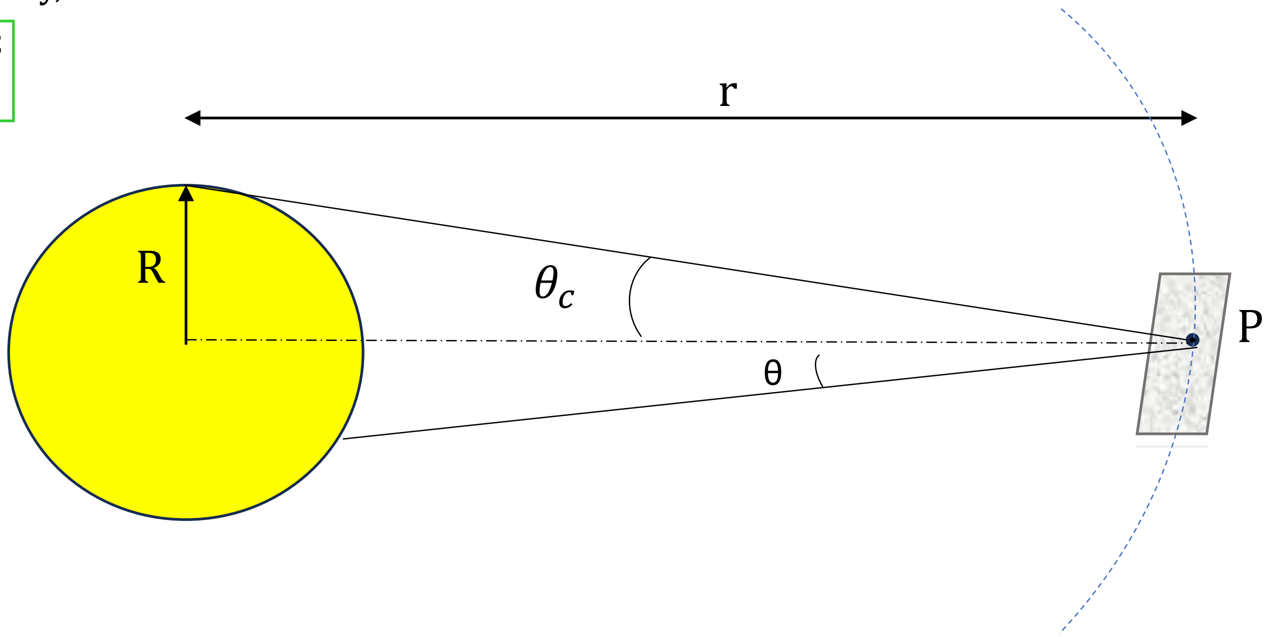
Setting r= R, we get the flux *emerging from the unit surface area*:

$$\boxed{F_{\nu,\text{surface}} = \pi I_\nu}$$

intrinsic source property!

The flux collected by the unit area at the observer position is then:

$$F_\nu(r) = F_{\nu,\text{surface}} \left(\frac{R}{r}\right)^2$$



Luminosity of a sorce

The flux arising from the surface of a source is not anymore dependent on the observer or on the observing distance. It is an intrinsic property of the source, which allows to define another intrinsic source property: its luminosity.

Remember: at the *surface* of the emitting source,

$F_{\nu,\,surface} = \pi I_\nu$ erg s^{-1} Hz^{-1} cm^{-2}

Similarly, integrating over the whole spectrum,

$F_{surface} = \int F_\nu d\nu = \pi \int I_\nu \, d\nu = \pi I$ erg s^{-1} cm^{-2} (Watt m^{-2} in the ISU)

Assuming the source is a *sphere* of radius R, homogeneously emitting, we define its luminosity as the total (=solid angle integrated) energy radiating from its surface per unit frequency interval:

$L_\nu = 4\pi R^2 F_{\nu,\,surface}$ in CGS, erg s^{-1} (Watt , in the ISU)

An observer at distance r from the source, can infer the intrinsic source luminosity from the observed flux:

$$F_\nu(r) = F_{\nu,\,surface} \left(\frac{R}{r}\right)^2 = \frac{L_\nu}{4\pi R^2}\left(\frac{R^2}{r^2}\right) = \frac{L_\nu}{4\pi r^2}$$

Integrating over the frequency spectrum, $F(r) = \frac{L}{4\pi r^2}$ = power collected from the unit area at the observer distance.

Problem 1. Knowing that the bolometric solar luminosity is $L_\odot = 3.832\times 10^{26}$ W, the Sun radius is 6.96×10^{5} km, and the distance Sun-Earth is about 93 million kilometers, calculate (in ISU):

1) the flux *received* by a square meter of area on Earth.
2) The flux *emitted* by a square meter of the Sun's surface. How many LED light bulbs (typical wattage, 10 Watts) would you need to get the same flux?

Astrophysical observables

Radiative energy density

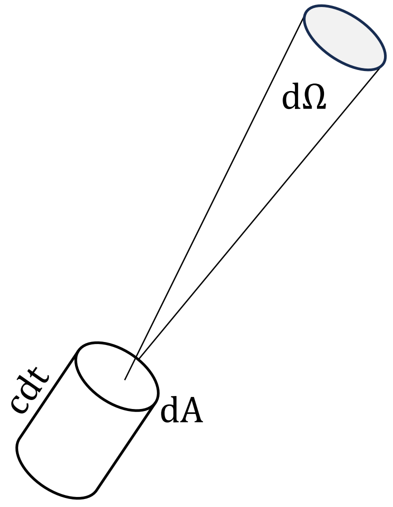
For a certain direction, and given volume element, the energy density is defined via

$$dE = u_\nu(\Omega)\, dV\, d\Omega\, d\nu$$

where the volume element for light is $dV = c\, dA\, dt$. It follows that

$$u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$$

$u_\nu(\Omega)$ units in cgs: erg cm^{-3} sr^{-1} Hz^{-1}



The total energy density at a given frequency is

$$u_\nu = \int u_\nu(\Omega)\, d\Omega = \frac{1}{c} \int I_\nu(\Omega)\, d\Omega \equiv \frac{4\pi}{c} J_\nu$$

u_ν units in cgs: erg cm^{-3} Hz^{-1}

where J_ν represents the mean intensity of the radiation field:

$$J_\nu = \frac{1}{4\pi} \int I_\nu(\Omega)\, d\Omega$$

→ For an isotropic field, $J_\nu = I_\nu$

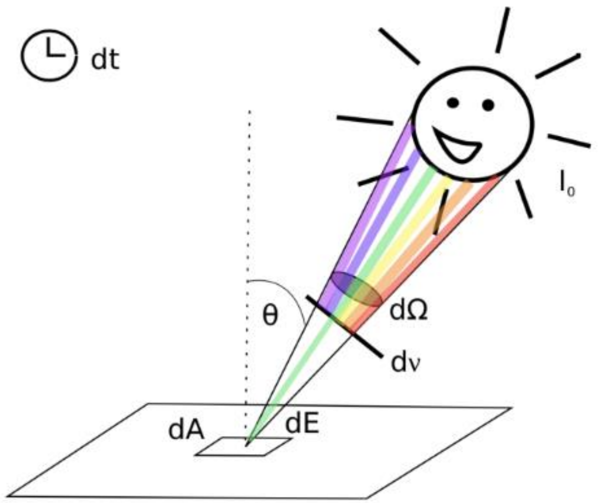
Also J_ν has units (in cgs) of erg cm^{-2} s^{-1} sr^{-1} Hz^{-1}.

The total emission is simply $u = \int u_\nu\, d\nu = \frac{4\pi}{c} \int J_\nu d\nu$ cgs units: erg cm^{-3}

Glossary of observational quantities

- Spectral/Monocromatic Luminosity (for isotropically emitting source) :

(*) $L_\nu = 4\,\pi\, D^2\, F_\nu\,(D)$ [W Hz^{-1}] / [erg s^{-1} Hz^{-1}]



- Power (Bolometric / Absolute Luminosity) : $L = \int_0^\infty L\nu\, d\,\nu$ [W] / [erg s−1]

- Flux density: $F_\nu\,(D)= L_\nu/\,4\,\pi\,D^2$ [W Hz^{-1} m^{-2}] / [erg s^{-1} Hz^{-1} cm^{-2}]

[1 Jy=10^{-26} W Hz^{-1} m^{-2}]

- ($I_\nu \equiv B_\nu$ at the source surface, surface Brightness) $B_\nu = F_\nu$ (Source radius)ΔΩ [W Hz^{-1} m^{-2} $ster^{-1}$] / [erg s^{-1} Hz^{-1} cm^{-2} $ster^{-1}$]

  (That is I_ν, assuming cosΘ= 1, rays arriving perpendicular to the collecting g area).

  (Flux density per unit solid angle).

- Specific emissivity $\varepsilon_\nu = \frac{L_\nu}{dV}$ [W Hz^{-1} m^{-3}] [erg s^{-1} cm $^{-3}$ Hz $^{-1}$]

(*) If we consider a star as the source of radiation, the flux emitted by the star into a solid angle ω is L=ωD^2F, where F is the flux density observed at a distance D from the star (it is also usual to refer to the total flux from a star as the **luminosity**, L). If the star radiates isotropically then radiation at a distance D will be distributed evenly on a spherical surface of area 4πD^2 and hence L= 4πD^2F.
For an extended luminous object such as a nebula or galaxy, we use the **surface brightness** , i.e. the flux density per unit solid angle.

Radiative transfer

If a ray passes through matter, energy may be added or subtracted from it by emission or absorption, and the specific intensity will not in general remain constant. Also «scattering» can change the beam intensity, by deflecting photons off the beam or adding photons to it.

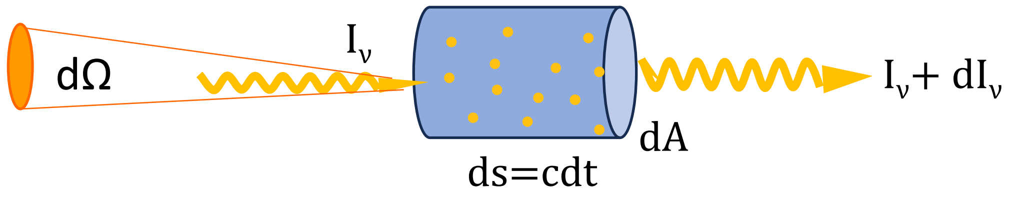
Spontaneous emission.

The spontaneous **emission coefficient** j_ν is defined as the energy emitted per unit time per unit solid angle, unit frequency and per **unit volume**:

$$dE = j_\nu \, dV \, d\Omega \, dt \, d\nu$$

where j_ν has units of erg s^{-1} cm^{-3} Hz^{-1} $ster^{-1}$

Remember the definition of spec.intensity, $dE = I_\nu dA \, d\Omega \, dt \, d\nu$; in going a distance ds, a beam of cross section dA travels through a volume $dV = dA \, ds$, thus adding intensity to the beam: $dI_\nu = j_\nu \, ds$



Also often used, the **emissivity**, defined as energy spontaneously emitted per unit time, unit frequency and unit volume:

$dE = \varepsilon_\nu \, dV \, dt \, d\nu$ → $\varepsilon_\nu = \int j_\nu \, d\Omega$ and if emission is isotropic, $j_\nu = \frac{\varepsilon_\nu}{4\pi}$; ε_ν has units of erg s^{-1} cm^{-3} Hz^{-1}

Absorption

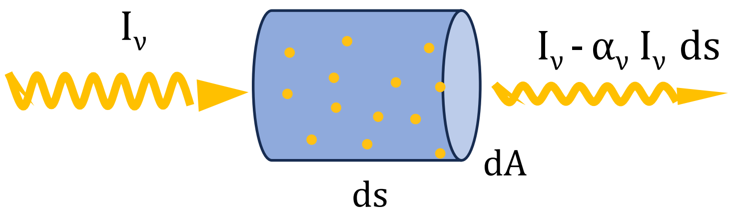
Radiation can also be absorbed. Consider a medium with given particle (microscopical absorbers) density n (cm^{-3}), with each particle having a given microscopic absorbing area σ_ν (cm^2).

The total absorbing area is: $n\, \sigma_\nu\, dAds$.

The energy absorbed out of per unit the beam solid angle per unit crossed area, per unit time and frequency is

$dE= - dI_\nu\, dA\, d\Omega\, dt\, d\nu = - I_\nu\, n\, \sigma_\nu\, dA\, ds\, d\Omega\, dt\, d\nu \rightarrow dI_\nu= I_\nu\, n\, \sigma_\nu\, ds$

$\rightarrow$ $dI_\nu \equiv \alpha_\nu I_\nu\, ds$



where $\alpha_\nu \equiv n\, \sigma_\nu$ is the **absorption coefficient.** In cgs units, it is in cm^{-1}.

The energy subtracted from the beam is $dE = - \alpha_\nu I_\nu\, ds\, dA\, d\Omega\, dt\, d\nu$ out of the beam

Often used, the mass absorption coefficient, or **opacity κ_ν**, given by

$\alpha_\nu = \rho\, \kappa_\nu$, with ρ the mass density. The opacity has units cm^{-2} g.

Thus, obviously $\frac{\rho}{n} = \frac{\sigma_\nu}{k_\nu}$ μ= mass per absorber $= \frac{\sigma_\nu}{\kappa_\nu}$

We define the **mean free path** as the mean physical distance traveled in a homogeneous medium, $\ell_\nu = \frac{1}{\alpha_\nu}$

Note: this is a *phenomenological* law. We'll see later on the course that the absorption coefficient has a deeply quantistic nature.

Equation of radiative transfer

In presence of emission of absorption,the energy is no more conserved, and , neglecting scattering,

$$\frac{d\,I_\nu}{ds} = -\,\alpha_\nu\, I\nu\; + \;j_\nu$$

- For pure emission, $I_\nu\,(s) = I_\nu\,(0) + \int j_\nu(s')ds'$ brightness increases along the l.o.s.

- For pure absorption, $I_\nu\,(s) = I_\nu\,(0)\,\exp\,[-\int_0^s \alpha_\nu(s')ds'] \equiv I_\nu\,(0)\,e^{-\tau_\nu}$ brightness decreases exponentially with the **optical depth** τ_ν**,** defined as

$$d\tau_\nu = \alpha_\nu\, ds$$

Note, τ is a more natural «line of sight» unit than s.

When scattering is included, this gets complicated, because photons can be added/ removed from the beam, and often numerical solutions are required.

$\frac{d\,I_\nu}{ds}$ represents the variation of specific intensity of a beam of solid angle dΩ when crossing a volume dAds:

Power **absorbed** per unit volume, solid angle, frequency interval: $dE/\,dA\,ds\,d\Omega\,dt\,d\nu = -\,\alpha_\nu\, I\nu$

Power **added** per unit volume, solid angle, frequency interval: $dE/dA\,ds\,d\Omega\,dt\,d\nu = j_\nu$

Optical depth

The optical depth is a dimensionless quantity, depending on the density of the medium, on the properties of the medium particles, as well as on the traveled length and on the light frequency. It is defined as:

$$\tau_\nu(s) \equiv \int_0^s \alpha_\nu(s')ds' \sim n\,\sigma_\nu\, s$$

(the last step only holds for an homogeneous medium).

A medium whith $\tau_\nu > 1$ is called thick or opaque; A medium whith $\tau_\nu < 1$ is called thin or transparent.

- The probability for a photon to travel a given distance s in the medium without being absorbed is proportional to $\exp(-\tau_\nu(s))$. Thus one usually defines the **mean free path** of a photon as $l_\nu = \frac{1}{n\,\sigma_\nu} = \frac{1}{\alpha_\nu}$.

Formal solution to radiative transfer equation

We rewrite $\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$ having defined the **Source function** $S_\nu \equiv \frac{j_\nu}{\alpha_\nu}$;

a formal solution is then:

$$I_\nu(\tau_\nu) = \underbrace{I_\nu(0)\, e^{-\tau_\nu}}_{\text{Attenuated background light}} + \overbrace{\int_0^{\tau_\nu} S_\nu(\tau'_\nu)\, \underbrace{e^{-(\tau_\nu - \tau'_\nu)}}_{\text{Self absorption}}\, d\tau'}^{\text{Glowing medium}}$$

The intensity emerging from the medium is the irradiated intensity attenuated by absorption, plus the integrated source function, also attenuated by absorption. But scattering and stimulated emission have to be also accounted for.

Case of constant source function

If S_ν is independent on τ'_ν (e.g. a blob of uniform composition, T, n) , then

$$I_\nu(\tau_\nu)= I_\nu(0)\, e^{-\tau_\nu} + S_\nu(1-e^{-\tau_\nu}) = S_\nu + e^{-\tau_\nu}\,(I_\nu(0) - S_\nu)$$

Interesting limits:

Optically thick regime: $\tau_\nu >> 1 \quad I_\nu \longrightarrow S_\nu$

(e.g., a rock): the background source if fully absorbed, photons only from the upper layes of the cloud

Optically thin regime: $\tau_\nu \rightarrow 0 \quad I_\nu \rightarrow I_\nu(0) + S_\nu \tau_\nu$ linear behaviour: unattenuated background source + photons from the cloud.

Radiation Transfer as a relaxation process

From the transfer equation, rewritten as

$$\frac{d I_\nu}{ds} = -\alpha_\nu\,(I_\nu - S_\nu) = \alpha_\nu(S_\nu - I_\nu)$$

we see that:

- if $I_\nu < S_\nu$ then $dI_\nu/ds>0$: → intensity increases along path
- if $I_\nu > S_\nu$, intensity decreases

The equation is "self regulating" : I_ν relaxes to attractor S_ν and the characteristic length scale for relaxation is mean free path.

Radiative transfer in presence of elastic scattering

In addition to emission and absorption, photons can also be scattered in the medium.

The emission of a volume element in the presence of scattering depends on the incident flux.

Remember the mono-directional relations: (ds=c dt)

Power **absorbed** per unit volume, solid angle, frequency interval: $dE/\, dA\, ds\, d\Omega\, dt\, d\nu = -\alpha_\nu I\nu = \frac{dI_\nu}{ds}$ negative

Power **added** per unit volume, solid angle, frequency interval: $dE/dA\, ds\, d\Omega\, dt\, d\nu = j_\nu = \frac{dI_\nu}{ds}$ positive

(erg s^{-1} cm^{-3} Hz^{-1} $ster^{-1}$)

Assume isotropic scattering (efficiency of scattering independent on Ω) and *coherent* scattering (energy received per frequency interval dν= energy scattered in the same frequency interval; also named *elastic* or *monochromatic* scattering). Integrating over dΩ,

$$\Delta E_\nu \text{(scattered over } 4\pi) = \underbrace{4\pi j_\nu^{sc}}_{\text{Power scattered in all directions per unit volume, time dt, d}\nu} \overbrace{dA\, ds}^{dV}\, dt\, d\nu = \underbrace{[\int \alpha_\nu^{sc}\, I_\nu(\Omega)\, d\Omega\,]}_{\text{Power subtracted from all directions per unit volume, time dt, d}\nu} \cdot \overbrace{dA\, ds}^{dV}\, dt\, d\nu = \Delta E_\nu \text{(received from all directions)}$$

to find

$$j_\nu^{sc} = \alpha_\nu^{sc} \frac{1}{4\pi} \int I_\nu d\Omega = \alpha_\nu^{sc} J_\nu \qquad \text{(erg s}^{-1}\text{ cm}^{-3}\text{ Hz}^{-1}\text{ ster}^{-1})$$

Thus the intensity change due to scattered radiation *in a beam* (monodirectional ray of light in solid angle dΩ) is

$$dI_\nu = -\alpha_\nu^{sc} (I_\nu - J_\nu)\, ds$$

Part of the incoming beam is deviated, subtracting intensity. More intensity is added by rays from other directions, deflected into the direction of our beam.

Extinction

Combining scattering and absorption, the monodirectional equation for a monochromatic beam is

$$dI_\nu = -\alpha_\nu^{sc}\,(I_\nu - J_\nu)\,ds + \alpha_\nu^{abs}\,(S_\nu - I_\nu)ds \qquad \text{where} \qquad S_\nu = \frac{j_\nu}{\alpha_\nu^{abs}} \quad \text{(source function for absorption/emission only)}$$

We can define a Source function for the whole **extinction** process, due to absorption+scattering:

$$S_\nu^{ext} \equiv \frac{j_\nu + \alpha_\nu^{sc} J_\nu}{\alpha_\nu^{abs} + \alpha_\nu^{sc}}$$

Extinction: sum of absorption and scattering

And the transfer equation becomes

$$\frac{dI_\nu}{ds} = (\alpha_\nu^{abs} + \alpha_\nu^{sc})(S_\nu^{ext} - I_\nu)$$

The mean free path becomes $\ell = \frac{1}{\alpha_\nu^{abs} + \alpha_\nu^{sc}}$ (path of a photon before being absorbed or scattered).

During the random walk, the probability that a free path end with a **true absorption** is $\varepsilon_\nu = \frac{\alpha_\nu^{abs}}{\alpha_\nu^{abs} + \alpha_\nu^{sc}}$

The corresponding probability for scattering is $1 - \varepsilon_\nu = \frac{\alpha_\nu^{sc}}{\alpha_\nu^{abs} + \alpha_\nu^{sc}}$ (*single-scattering albedo*)

A photon has $N = 1/\varepsilon_\nu$ scatterings (number of free paths) before absorption, so the total path, before being absorbed, is

$$\ell_* = \sqrt{N}\,\ell = \frac{\ell}{\sqrt{\varepsilon_\nu}} = \frac{1}{\sqrt{\alpha_\nu^{abs}(\alpha_\nu^{abs} + \alpha_\nu^{sc})}}$$ = *thermalization length* or *diffusion length* or *effective mean free path* **(*)**

(*) In a random walk, the net displacement of a photon is $\mathbf{R} = \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \ldots \mathbf{r}_N$. The average of these displacements would be 0.

Thus we use the mean square displacement $\ell_*^2 \equiv \langle \boldsymbol{R}^2 \rangle = N\ell^2$, where $\ell = \langle \mathbf{r}_i^2 \rangle$ is the root mean quare of the photon free path