Synchrotron radiation

- Introduction to synchrotron emission
- Power emitted
- Beaming effect
- Polarization
- Spectrum by single electron and by a charge distribution
- Synchrotron self-absorption
- Cooling by radiative losses
- Worked example: the Crab Nebula and other synchrotron sources in astrophysics.

- Longair, M., 1992. High Energy Astrophysics. Second edition; Volume 1, Chapter 3. Cambridge Univ. Press.
- Rybicki, G., and Lightman, A., 1979, Radiative processes in astrophysics, Chapter 6. John Wiley and Sons.
- https://www.cv.nrao.edu/~sransom/web/Ch5.html ; https://www.astro.utu.fi/~cflynn/astroII/l4.html

Synchrotron radiation

Emitted by charges accelerated in a magnetic field.

First observed in early terrestrial particle accelerator experiments, where electrons were moving in a circular path.

It is the dominant emission mechanism in astrophysical radio sources, and important at optical and X-ray wavelengths in AGN:

- radio continuum emission of the Milky Way.
- non-thermal continuum emission of SNRs such as the Crab nebula
- optical and X-ray continuum emission of quasars and AGN.
- transient solar events
- Jupiter magnetosphere

Synchrotron emission reveals the presence of a magnetic field, and the energy of the particles interacting with it.

Synchrotron radiation

- The relativistic electrons in nearly all synchrotron sources have power-law energy distributions, so they are not in local thermodynamic equilibrium (LTE). Consequently, synchrotron sources are often called “nonthermal” sources.

- Space is full of magnetic fields. Typically very weak, but there is plentiful supply of relativistic electrons even in low density environments.

location	Field strength (gauss)
interstellar medium	10^{-6}
stellar atmosphere	1
Supermassive Black Hole	10^{4}
White Dwarf	10^{8}
Neutron star	10^{12}
this room	0.3
Supernova remnants/Crab Nebula	10^{-3}

1 gauss (G) = 10^{-4} tesla (T)

1 tesla (T) = 1 Wb m^{-2}

Synchrotron radiation

A charged particle moving in a magnetic field radiates energy. At non-relativistic velocities, this results in **cyclotron radiation** while at relativistic velocities it results in **synchrotron radiation**.

The relativistic form of the equation of motion of a particle of mass m and charge q in a magnetic field B is dictated by the Lorentz force:

$$\frac{d(\gamma m\,\mathbf{v})}{dt} = \frac{q}{c}\,\mathbf{v} \times \boldsymbol{B}$$

where **v** is the velocity vector of a particle of charge q, and is the Lorentz factor $\gamma = 1/\sqrt{1 - \mathrm{v}^2/c^2}$ >>1 in rel. case.

Being the Lorentz force perpendicular to the motion, the magnetic field cannot change the particle energy, so that |**v**|= const.

Only the direction of **v** can change: $m\gamma \frac{d\mathbf{v}}{dt} = \frac{q}{c}\,\mathbf{v} \times \boldsymbol{B}$

The velocity components parallel and perpendicular to the magnetic field lines, will follow:

$$\frac{d\mathrm{v}_{\parallel}}{dt} = 0 \;\text{ and }\; \frac{d\mathrm{v}_{\perp}}{dt} = \frac{q}{\gamma m c}\,\mathbf{v}_{\perp} \times \boldsymbol{B}$$ $\Longrightarrow |\mathrm{v}_{\parallel}|=$ constant and, being |**v**|= const, also $|\mathrm{v}_{\perp}|=$ constant.

$\Rightarrow$ Circular motion around **B**

Synchrotron radiation

If the velocity along B is non-zero, the particle (e.g. an electron) moves in a helical path. The acceleration of the particle can be related to the centrifugal acceleration:

$\frac{\mathrm{v}^2}{r} = \frac{q}{\gamma m c}\, \mathrm{v}\, B\, \sin\alpha$ where r is the orbit radius (radius of gyration), α is the pitch angle (inclination of **v** w.r.t. B)

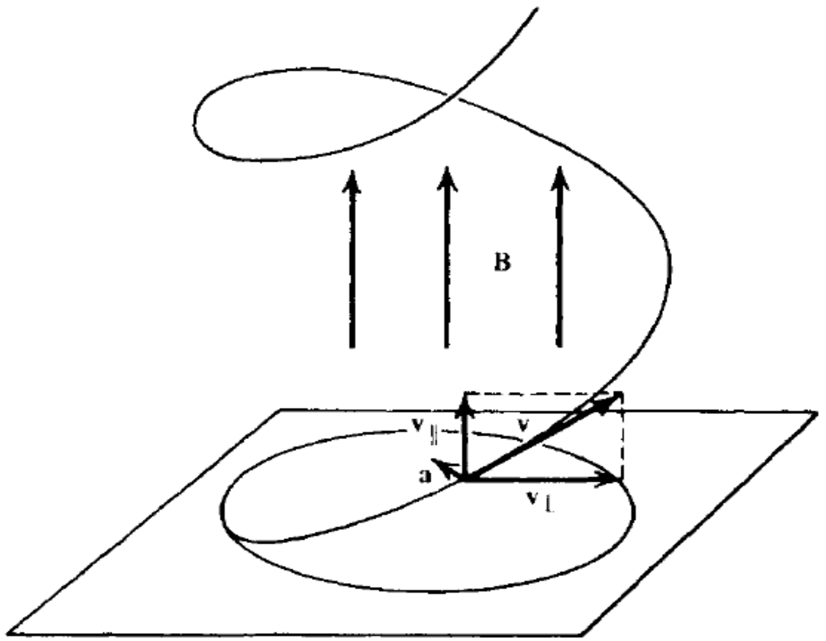
Problem 18: Indicating with $\beta = \mathrm{v}/c$, show that:

the acceleration is given by $a = \frac{q\beta B}{\gamma m}$;

the radius of gyration is $r = \frac{\gamma m c^2 \beta}{qB}$;

the period of gyration is $\mathrm{P} = \frac{2\pi\gamma m c}{eB}$;

the frequency of gyration is $\varpi_B = \frac{2\pi}{P} = \frac{qB}{\gamma m c}$



NOTICE. In the limit v << c, we can obtain all the results of cyclotron emission.

Power emitted

From Larmor's formula, the Power emitted by an accelerated charge is:

$P=\frac{2q^2}{3c^3}\,|\mathbf{a}|^2$. For a relativistic particle, $P=\frac{2q^2}{3c^3}\,\gamma^4\,|\mathbf{a}|^2$

The relativistic expression follows from considering the power emitted in the particle's frame and then transforming it to the laboratory frame using Lorentz transformation.

Problem 19. Show that the radiation emitted by a relativistic electron moving in a magnetic field is $P=\frac{2}{3}r_0^2\,c\,\beta_\perp^2\gamma^2B^2$, with $\beta_\perp=\frac{v_\perp}{c}=\beta\sin\alpha$,

and $r_0=e^2/m_e c^2$ ($\approx 2.8 \times 10^{-15}$ m) is the classical electron radius .

For isotropic distribution of electron relativistic velocities, we average over the pitch angles:

$$\langle \beta_\perp^2 \rangle=\frac{\beta^2}{4\pi}\int \sin^2\alpha\, d\Omega = \frac{2\beta^2}{3}$$

$P_{\rm iso}=\left(\frac{2}{3}\right)^2\, r_0^2\, c\, \beta^2\,\gamma^2\, B^2$, which can be written: $\boxed{P_{\rm iso}=\frac{4}{3}\sigma_T\, c\, \beta^2\,\gamma^2 U_B}$

where $\sigma_T=8\pi\, r_0^2/3$ is the Thomson cross-section, and U_B is the magnetic energy density $U_B=B^2/8\pi$ (in c.g.s.).

For a distribution of particles of mass m and charge q, with isotropic velocity distribution with same |**v**|, we can rewrite

$$P_{iso} = \left(\frac{2}{3}\right)^2 \frac{q^4}{m^2 c^3} \quad \beta^2 \gamma^{\,2} B^2$$

The energy loss is proportional to the magnetic field energy density

The energy loss is largest for low mass particles, electrons radiate more than protons.

Brush up on relativistic angle aberration (Ref. Rybicki Lightman 4.8)

The relativistic aberration formula relates the angle α' at which radiation is emitted, in the particle own frame, with respect to its direction of motion , to the angle α, which an external observer measures between the emission and the direction of motion:

$$\alpha = sin^{-1} \frac{sin\, \alpha'}{\gamma(1+\beta cos\, \alpha')}$$

Problem 20 A relativistic charge in its own reference frame is emitting cyclotron power with a dipolar distribution $P(\theta) = P(0)\cos^2\theta$.

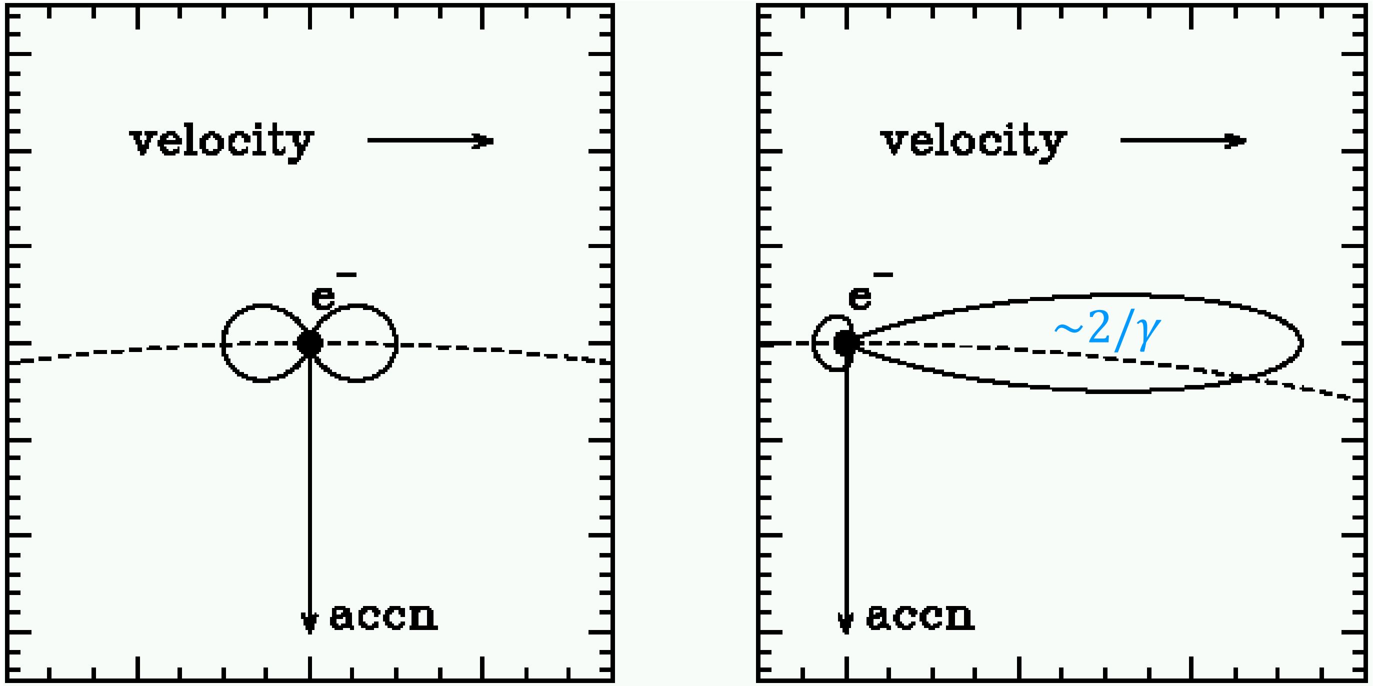
Show that at 99.9% of the speed of light :

- Radiation originally emitted at 45° to the direction of the velocity is seen by an external observer only 1° away from it.
- The points of null radiation, at 90° to the velocity in the electron's frame, appear directed 2.5° away from it to the external observer.
- Show that the null power points of the emission in the electron's frame in general appear at an angle γ^{-1} radians in the external frame.

Synchrotron beaming

For accelerated charges, the emitted power has a characteristic two-lobe distribution around the direction of acceleration. The power is emitted at an angle θ w.r.t. the acceleration direction, according to the following distribution: $P(\theta) = P(0)\cos^2\theta$.

In synchrotron emission, the relativistic aberration of angles makes the emitted power strongly beamed along the direction of motion, which is orthogonal to the (centripetal) acceleration: the power is concentrated into an angle $\sim 1/\gamma$ along the direction of motion, where it is boosted by a factor $\sim \gamma^2$. This has important consequencies on the observed spectrum and on its polarization.



Left: non relativistic electron, emitting in the classical «dipole/doughnut» pattern.

Right: synchrotron emission, with emitted power beamed onto an angle of order $2/\gamma$.

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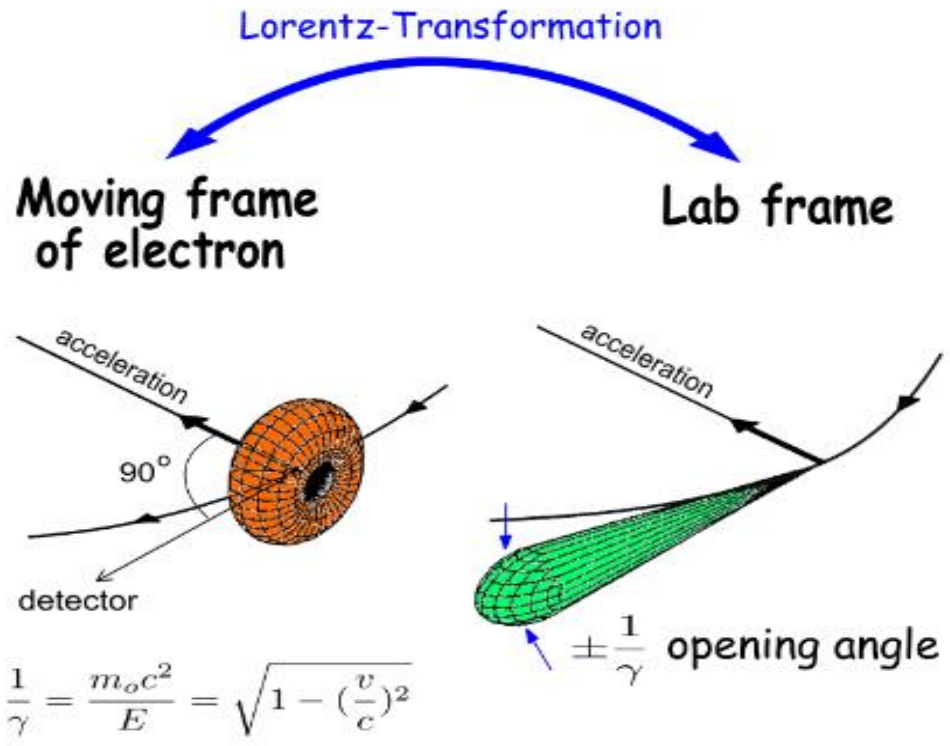

Left: non relativistic electron, emitting in the classical «dipole/doughnut» pattern.

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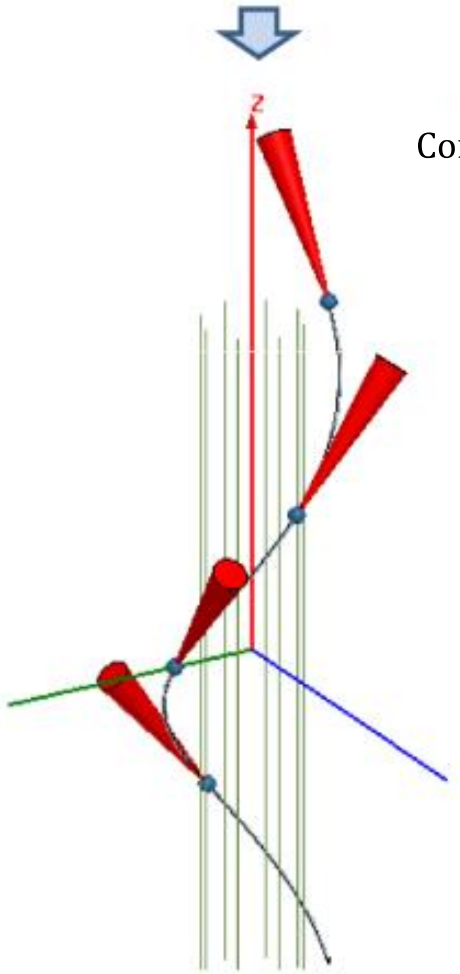
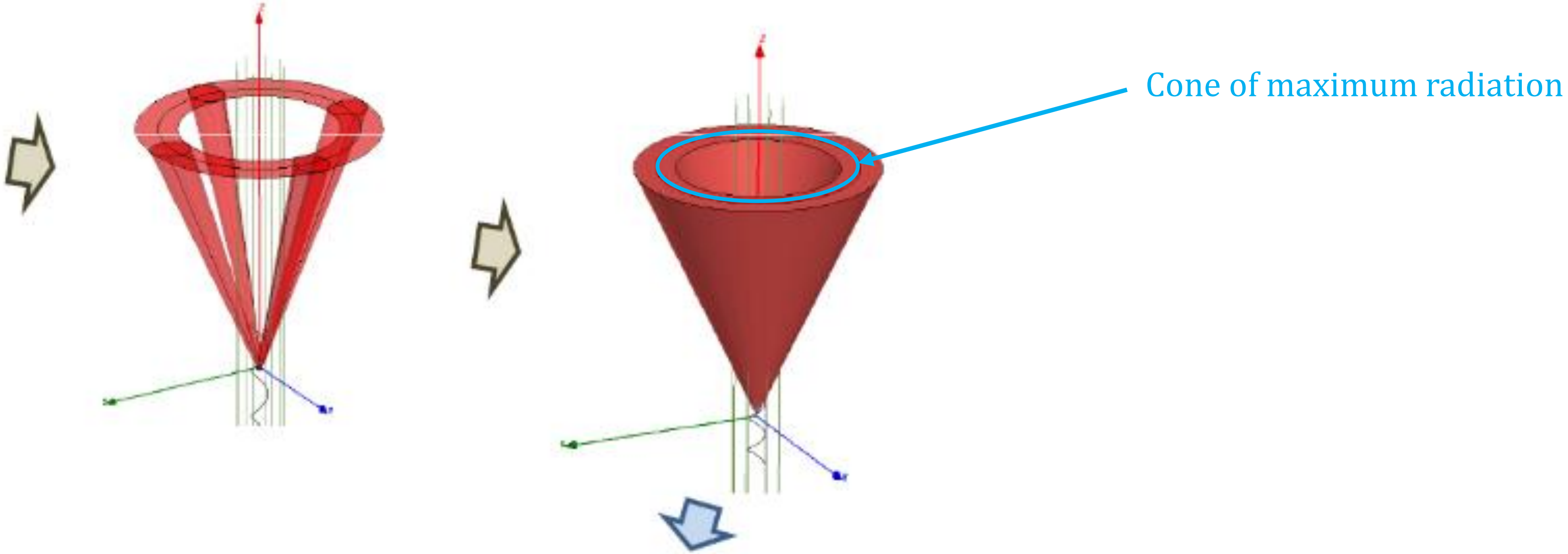
Synchrotron polarization

As a charge spirals about B (counterclockwise if negative charge, and viceversa), its velocity vector describes a velocity cone;



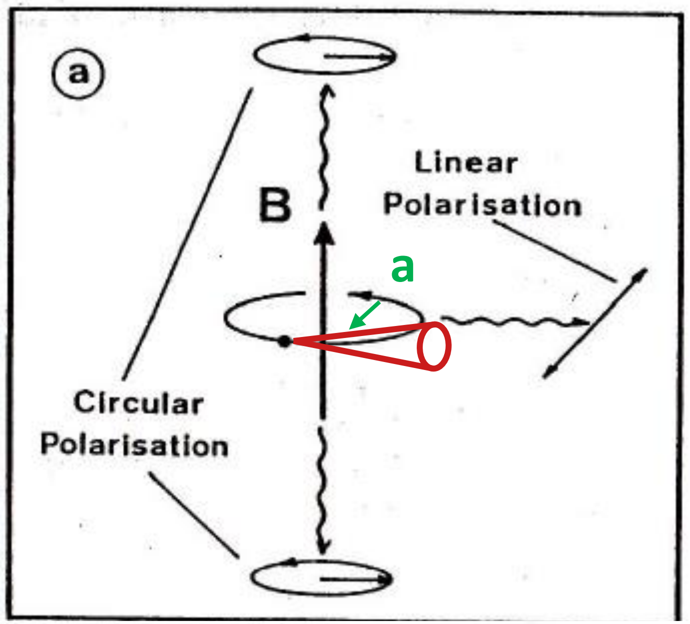
Combining the velocity cone with the radiation beaming:

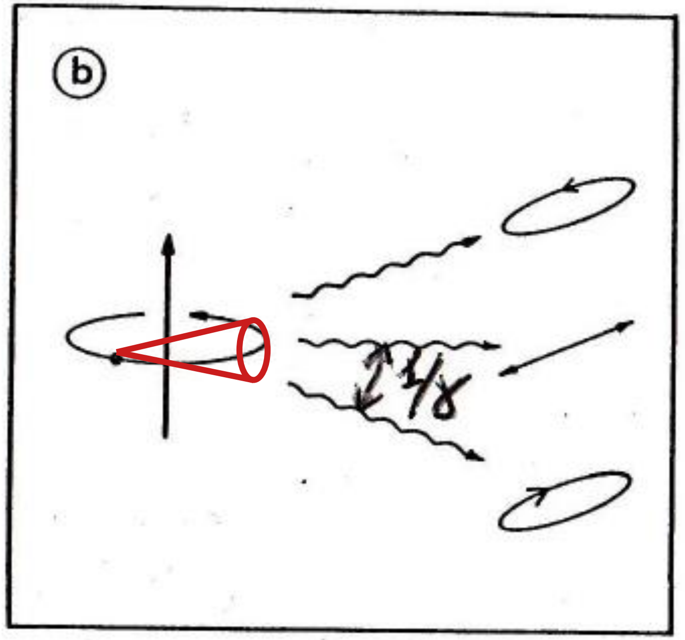


Radiation is confined in an angular region $\pm 1/\gamma$ from the cone of maximum radiation.

Credit: Giampaolo Pisano

Synchrotron polarization



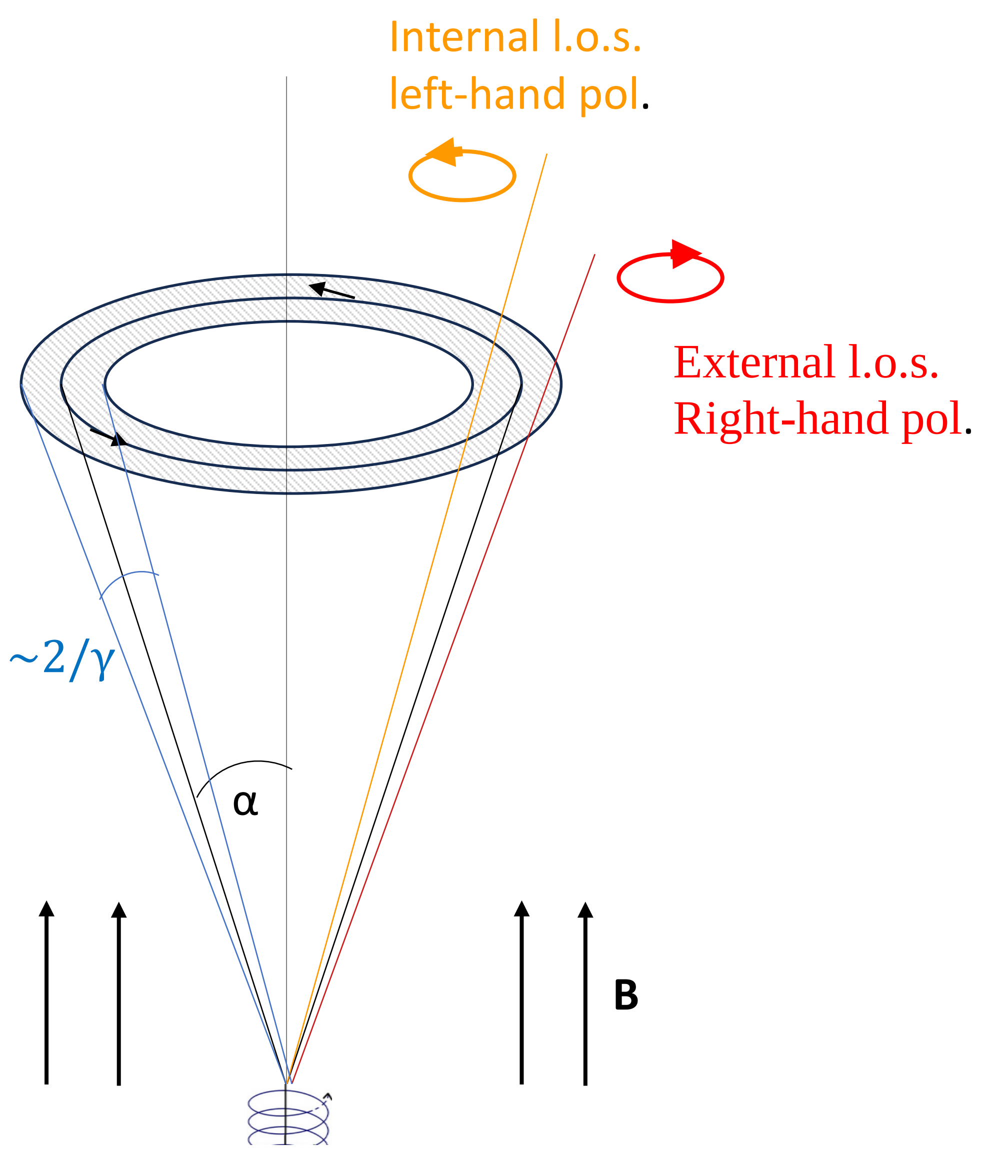


Remember that the electric field $\mathbf{E} \propto \boldsymbol{a} \propto \mathbf{v} \times \mathbf{B}$. If the beaming were along the plane of the orbit, we would see linearly polarized light when looking edge-on, and circularly polarized light when looking face-on (figure a).

However, highly relativistic electrons also have large velocities *along* the magnetic field lines, which means their beamed radiation is viewed neither directly face-on nor edge-on.
This means that synchrotron polarization, for a single electron, is generally elliptically polarized (figure b).

The sense of this elliptical polarization (right or left handed) is determined by whether the observed line of sight lies jus inside or just outside the cone of maximum radiation!

Synchrotron polarization



Electrons spiral counterclockwise in the B lines.

Lines of sight receiving radiation from within the thin ($1/\gamma$) innermost part of the velocity cone, will see the electrons moving around counterclockwise, and the emission is left elliptically polarized.

Seen from the outer part of the emission cone, the electrons move clockwise and the emission is right elliptically polarized.

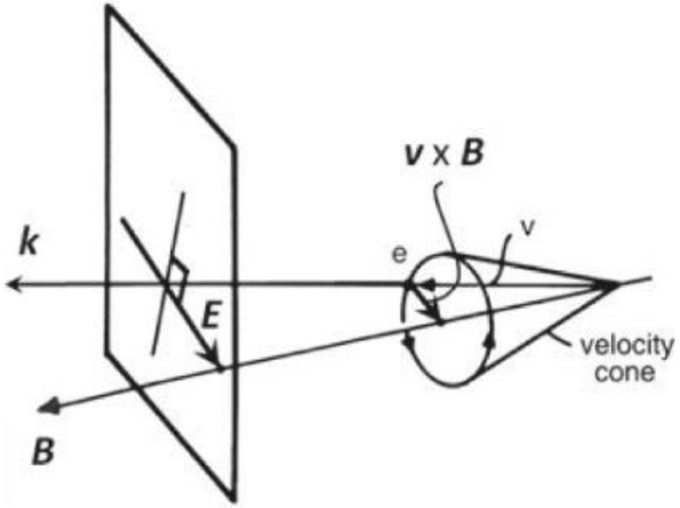
For a reasonable distribution of particles and a smooth distribution of pitch angles, the elliptical component cancels out, as emission cones will contribute equally from both sides of the line of sight.

Thus the radiation will be partially LINEARLY POLARIZED.

Cfr. the cyclotron radiation, elliptically polarized.

Synchrotron polarization

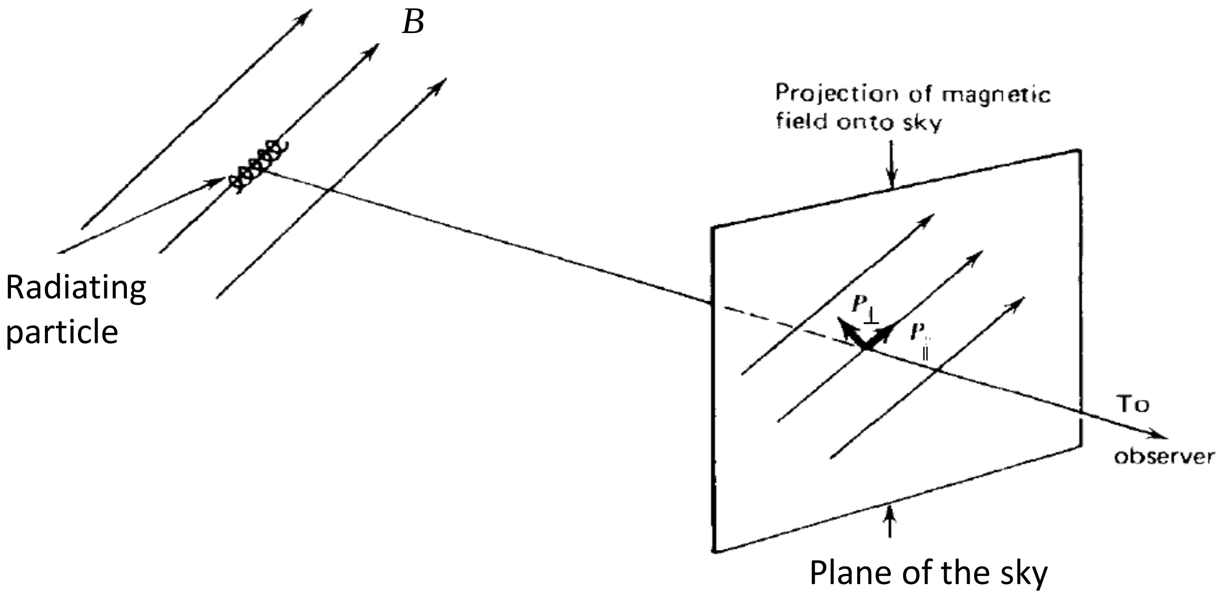
We can characterize the radiation from its powers parallel and perpendicular to the projection of the magnetic field on the plane of the sky:



$$\Pi = \frac{I(\nu)_\perp - I(\nu)_\parallel}{I(\nu)_\perp + I(\nu)_\parallel}$$

This is quite high : $\sim 75\%$ if integrated in frequency.

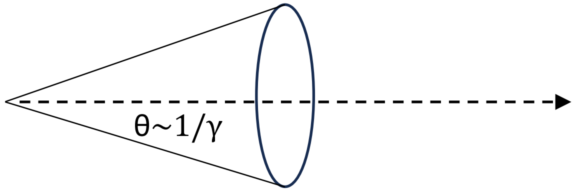
Averaged synchrotron emission is linearly polarized perpendicular to the projection of B on the plane of the sky.

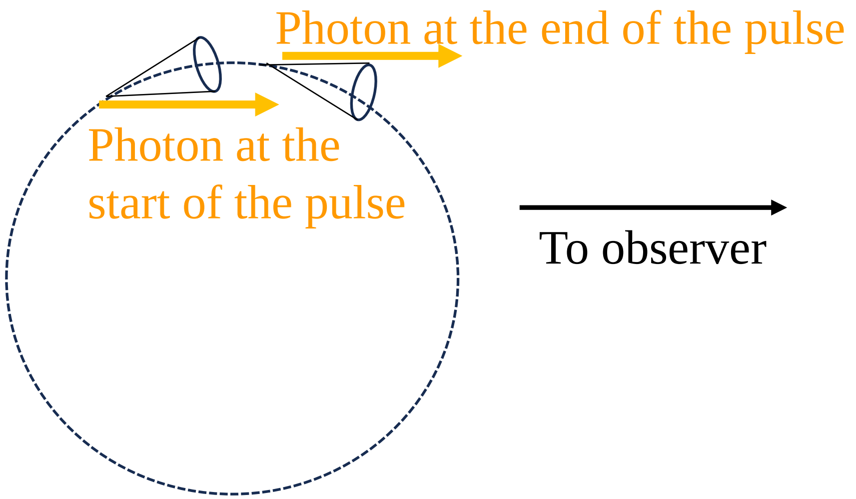


Using a power-law electron energy distribution, with index p, the degree of polarization is:

$$\boxed{\Pi = \frac{p+1}{p+7/3}}$$

Synchrotron spectrum

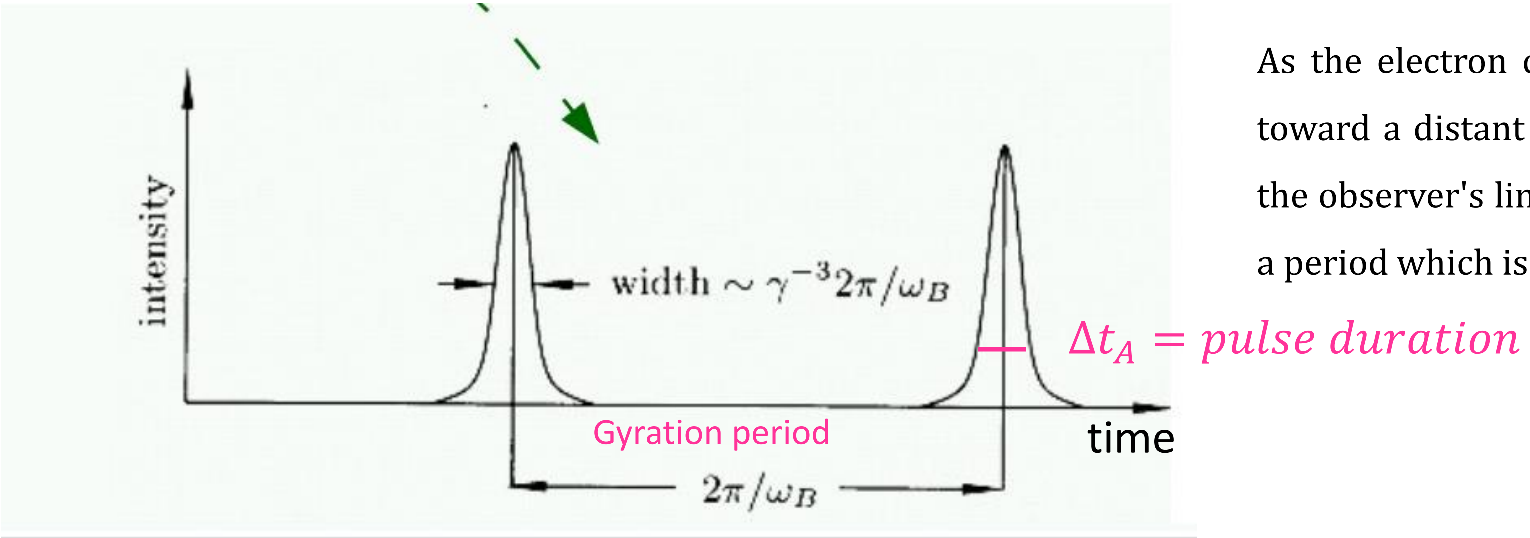




We see radiation from a small portion of the orbit when the cone is pointed toward us - pulse of radiation which becomes shorter for more energetic electrons.

For source moving at v ~ c, photon emitted at end of pulse almost 'catches up' with photon from start of pulse.

A given Line of Sight receives pulsed flashes



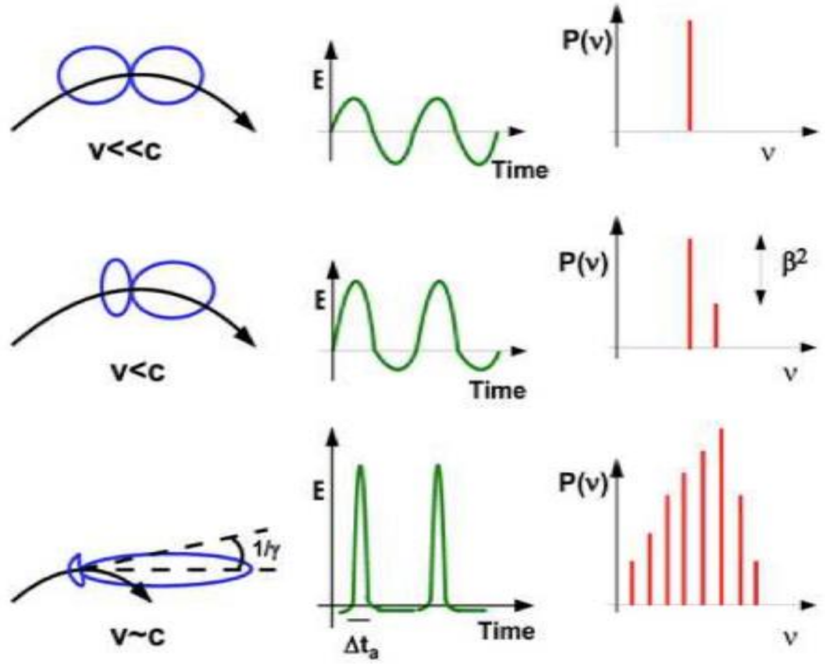
As the electron cycles around the helical path, any emission directed toward a distant observer is seen only when the beam is aligned with the observer's line-of-sight. The observer sees a ``flash'' of radiation for a period which is much shorter than the gyration period.

Synchrotron spectrum

- When the particle is very sub–relativistic, the observed electric field is sinusoidal in time. Correspondingly, the Fourier transform of E(t) gives only one frequency, the first harmonic.

- Increasing the velocity (say, $\beta \sim 0.01$) the emission pattern starts to be asymmetric (for light aberration). E(t) is described by more than just one sinusoid, and higher order harmonics appear. In these cases, the ratio of the power contained in successive harmonics goes as β^2.

- For relativistic ($\gamma \gg 1$) particles, the pattern is so asymmetric that the observers sees only spikes of electric field. They repeat themselves with the gyration period, but all the power is concentrated into Δt_A. To reproduce E(t) we need a large number of sinusoids, with frequencies going at least up to $1/\Delta t_A$. In this case the harmonics are many: the spectrum becomes continuous with any reasonable line broadening effect, and the power is concentrated at high frequencies.



From ref. G. Ghisellini's , Fig.4.5

Synchrotron spectrum

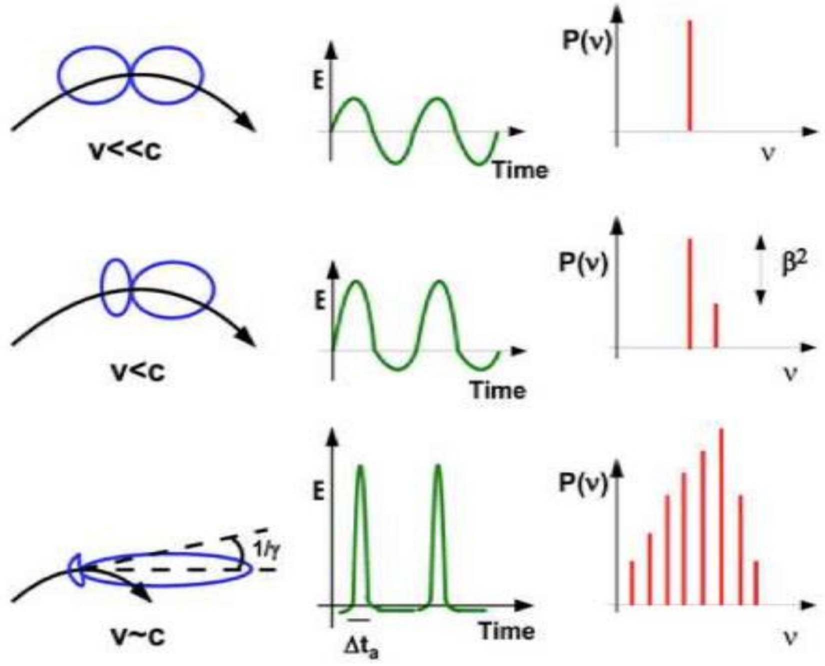
For non-relativistic motion, the gyration frequency ω_B gives the frequency of the emitted radiation directly.

In the synchrotron case, the characteristic frequency of the emission is at **a critical frequency** ν_c:

$$\nu_c = \frac{3\gamma^3\omega_B}{4\pi} = \frac{3\gamma^2 qB}{4\pi mc} \approx 4.2\times 10^{-9}\,\gamma^2\, B[\mu G] \quad \text{GHz}$$

for electrons

Sinchrotron is generally emitted in the radio window: B fields are generally 0.1 - 10 μ G in space. Higher felds in "condensed" objects (e.g. stars).



From ref. G. Ghisellini's , Fig.4.5

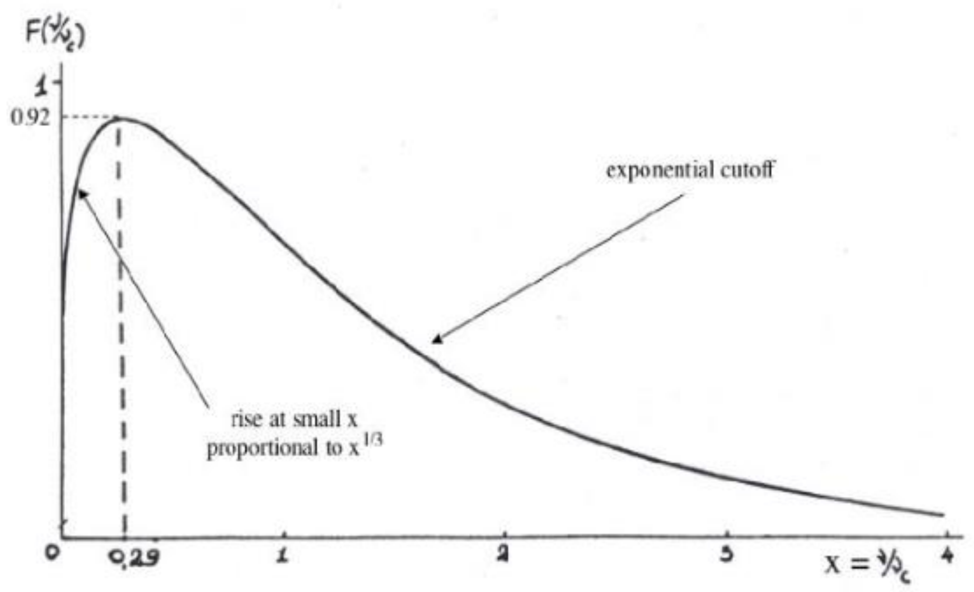
Synchrotron spectrum

The overall spectrum of the emission by a **single charge** consists of the sum of a large number of harmonics of the basic cyclotron emission.

The summed spectrum is relatively peaked, with maximum emission at 0.29 ν_c, and with intensity I(ν) dropping like

$$I(\nu) \propto \left(\frac{\nu}{\nu_c}\right)^{\frac{1}{3}} \qquad \text{for } \nu \ll \nu_c$$

$$I(\nu) \propto \left(\frac{\nu}{\nu_c}\right)^{\frac{1}{2}} \exp\left(-\frac{\nu}{\nu_c}\right) \qquad \text{for } \nu \gg \nu_c$$



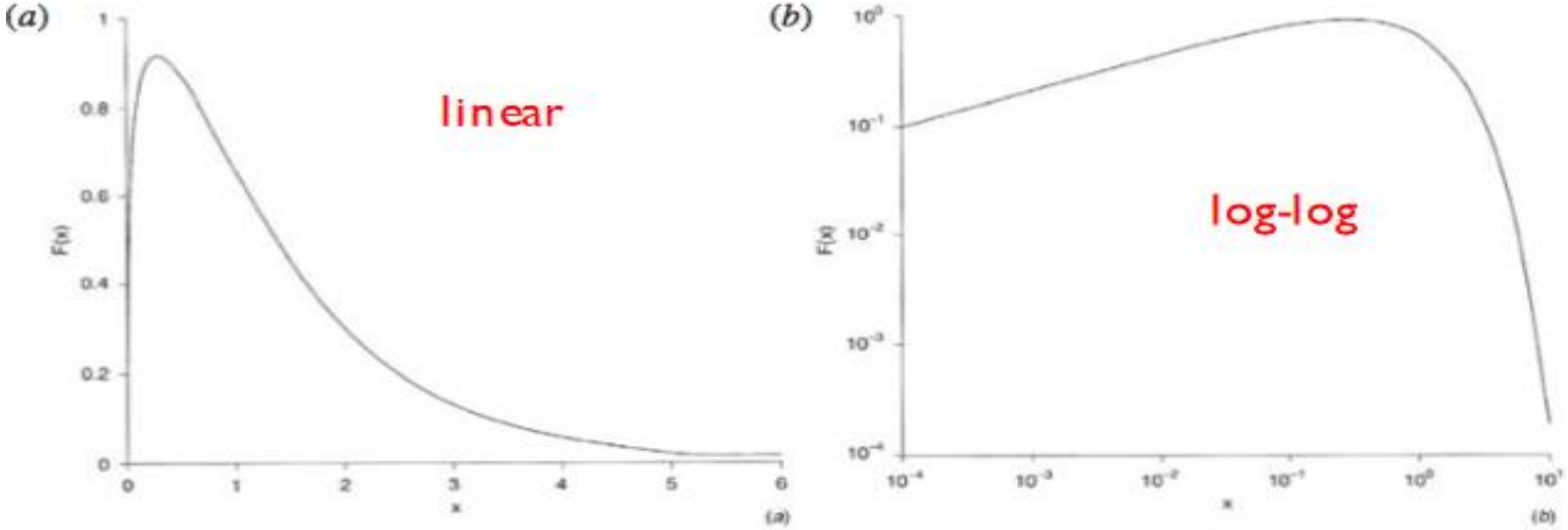
This spectrum is strongly peaked at $\nu \approx \nu_c$.

It is then a good approximation to assume that the spectrum of the single charge is of the form : $I(\nu) \propto \delta(\nu - \nu_c)$.

- Notice that there is one value of $\nu_c \propto \gamma^2 \propto \mathcal{E}^2$ for each value of the Lorenz factor. High energy electrons radiate at higher energies!

For a single electron, $\mathcal{E} = \gamma\, m_e c^2$

- To obtain a realistic synchrotron spectrum need to convolve the mono-energetic electron spectrum with an energy distribution function.



The spectrum of the synchrotron radiation of a single electron shown (*a*) with linear axes; (*b*) with logarithmic axes. The function is plotted in terms of $x = \omega/\omega_c = \nu/\nu_c$ where ω_c is the critical angular frequency $\omega_c = 2\pi\nu_c = (3/2)\,(c/v)\,\gamma^2\omega_g \sin\alpha$ where α is the pitch angle of the electron and ω_g is the non-relativistic gyrofrequency, $\omega_g = eB/m_e$.

Spectrum by many particles

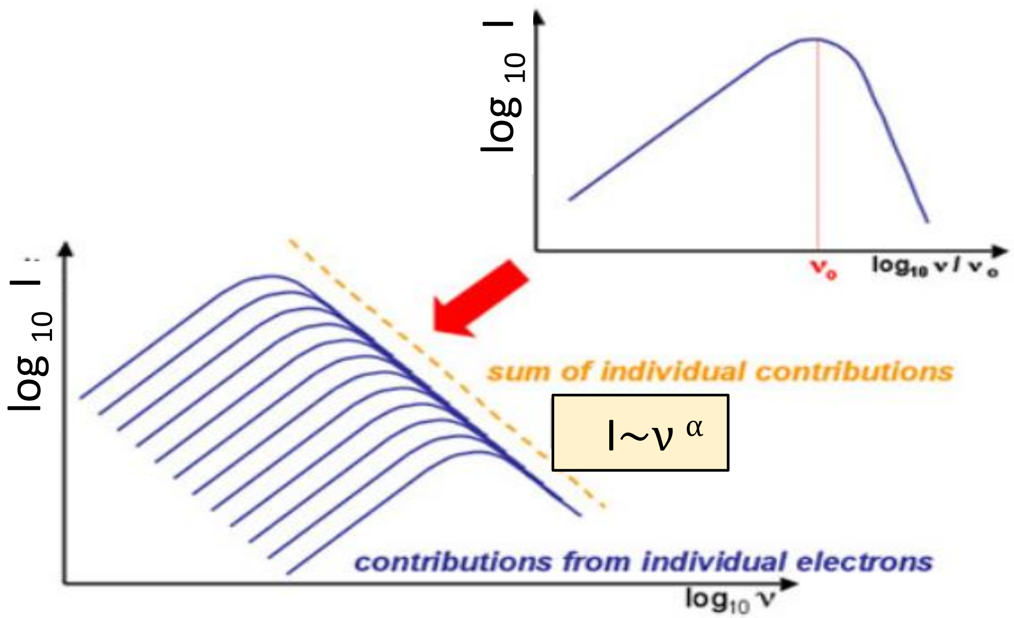
Normally, the electrons which produce synchrotron radiation have a (wide) range of energies. If number of particles with energy between E and E+dE can be written as a power-law in energy (non-thermal synchrotron radiation),

$n(E)\, dE = C\, E^{-p}\, dE$, which can be written as a distribution of particles Lorentz factors:

$n(\gamma)\, d\gamma = C\gamma^{-p}\, d\gamma$ Note: it is a (non thermal) velocity **distribution**, so there are all different values of ν_c.

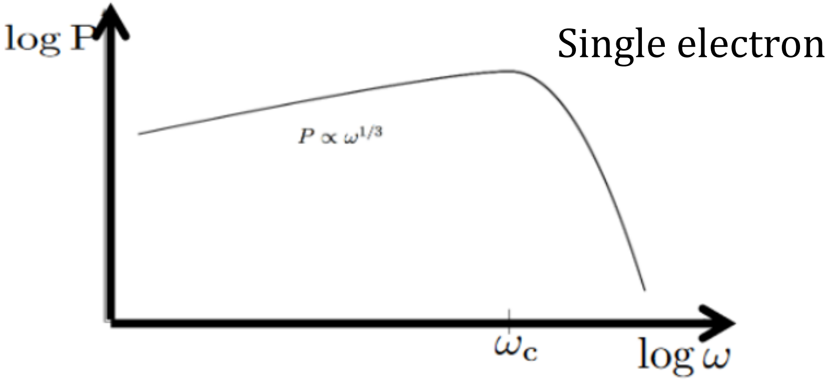
Using the approximation $I(\nu) \approx \delta(\nu - \nu_c)$ for the *single* particle spectrum, and averaging over the energy distribution of the charges, the spectrum of the resulting synchrotron radiation turns out to be also a power-law:

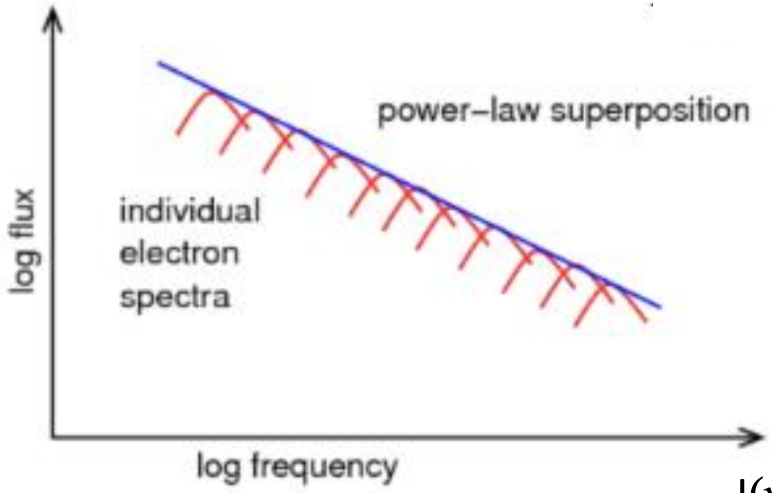
$P(\nu) \propto \nu^{-(p-1)/2} \propto \nu^{-s}$ with $s \equiv (p-1)/2$



Typical values of s:
Milky Way ~0.7
Radio Galaxy 0.7
Pulsar -3 to -2
AGN -1 to +1

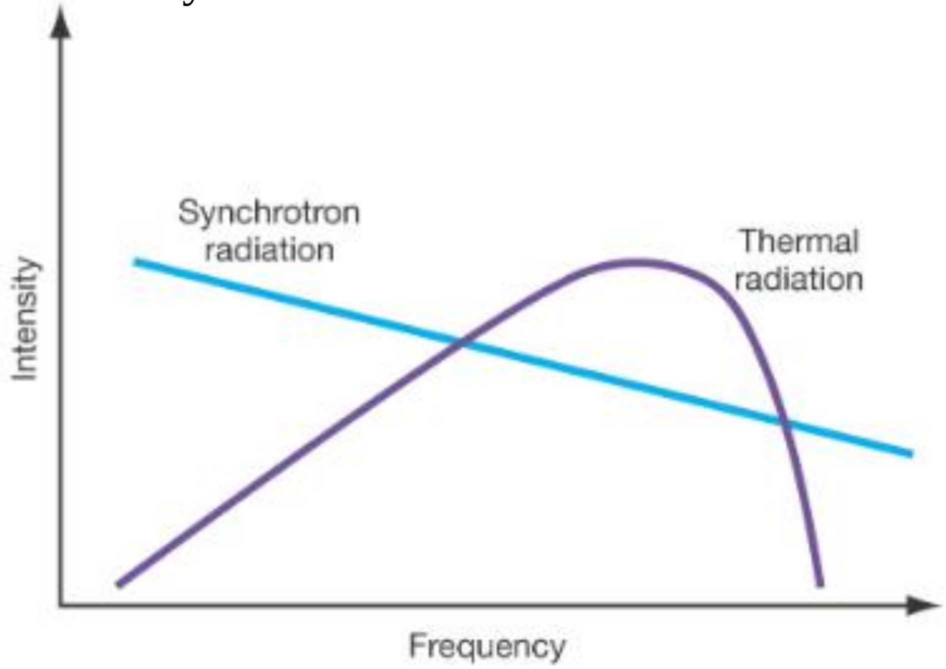
Measuring the spectral index of the radiation (s) gives an indication of the distribution of particle energies (p)!





$I(\nu) \propto \nu^{-(p-1)/2}$

Synchrotron vs. thermal radiation



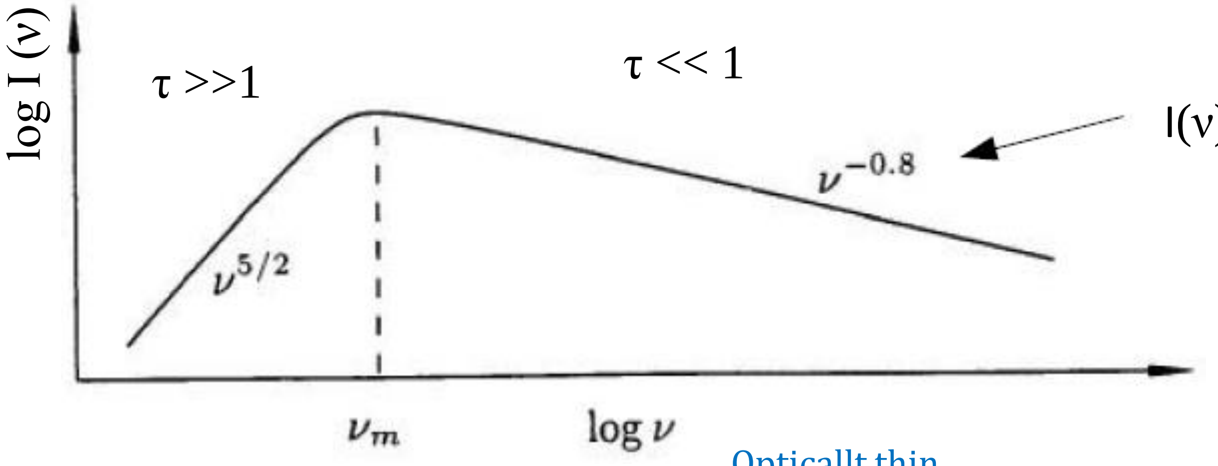
Synchrotron self absorption

- For every emission process there is an associated absorption process.
- Synchrotron emission is accompanied by absorption: a photon interacts with a charge in a magnetic field is absorbed. Below a cut-off frequency, the electrons become opaque to the emitted synchrotron radiation, which is then re- absorbed.
- For a power law distribution of electrons, the optically thick part of the spectrum is

$I(\nu) \propto B^{-1/2} \nu^{5/2}$, remarkably independent on the spectral index p! This index is not equal to 2, the Rayleigh-Jeans value, because the emission is non thermal.

At very high frequency, the cutoff is due to radiative losses.

Low energy photons cannot escape regions with high electron densities. NOT in thermal equilibrium, then Kirchhoff's law does not apply.



$I(\nu) \propto \nu^{-(p-1)/2}$

Opticallt thick self absorption $I(\nu) \propto \nu^{5/2}$

Opticallt thin $I(\nu) \propto \nu^{-s}$

Synchrotron Self-absorption spectrum: Brightness temperature of a non-thermal source

At low frequencies, where absorption is most important, an optically thick source cannot be brighter than the Rayleigh-Jeans spectrum, $B_\nu = 2kT_B\nu^2/c^2$. For a non thermal source, the trick is to define a brightness temperature at each frequency ν using the non thermal energy $\gamma m_e c^2$ of the electron emitting at that frequency.

Let $kT_B = \gamma m_e c^2$ and $\nu = \frac{3\gamma^2 eB}{4\pi m_e c}$

Eliminating γ, T_B is now a function of frequency:

$$kT_B = m_e c^2 \left(\frac{4\pi m_e c\nu}{3eB}\right)^{1/2}$$

And $B_\nu = 2kT_B\nu^2/c^2 \propto \nu^{5/2}$.

Synchrotron cooling and spectral distortions from energy losses

A plasma made by electrons emitting synchrotron radiation will cool down in a timescale given by the electron energy $E = \gamma\, m_e\, c^2$, divided by the synchrotron radiation rate, P_{iso} :

Cooling time: $\tau = \dfrac{3 m_e c^2}{4 \sigma_T c U_B \gamma \beta^2}$ Or $\tau \sim 16 \text{ yr} \left(\dfrac{1\ Gauss}{B}\right)^2 \dfrac{1}{\gamma}$

Higher Lorentz factor of the electrons implies higher frequency of the emitted photon ($\nu_c \leftrightarrow \gamma$) and a shorter cooling time.

The **rate** at which an electron loses energy $\mathcal{E}$ in synchrotron emission is proportional to $\mathcal{E}^2$ ($P_{iso} \propto \gamma^2 \propto \nu_c$) , so higher-energy electrons are depleted more rapidly.

The (critical) frequency at which photons are emitted is also proportional to $\mathcal{E}^2$ ($\propto \gamma^2 \propto \nu_c$), so synchrotron losses eventually **steepen source spectra** at higher frequencies.

Location	Typical B (Gauss)	Cooling time
Interstellar medium	10^{-6}	10^{10} yrs
Stellar atmosphere	1	5 days
Super massive Black Hole	10^{4}	10^{-3} sec
White dwarf	10^{8}	10^{-11} sec
Neutron star	10^{12}	10^{-19} sec

Typical cooling times

For typical extended powerful radio sources, $\gamma \sim 10^3$ and $B \sim 10^{-9}$ T and so the lifetimes of the electrons are expected to be $\tau \leq 10^7 - 10^8$ years. In the case of X-ray sources, for example, the diffuse X-ray emission from the Crab Nebula and the jet of M87, the energies of the electrons are very much greater, the inferred magnetic field strengths are greater and so the relativistic electrons have correspondingly shorter lifetimes. Since the lifetimes of the electrons are shorter than the light travel time across the sources, the electrons must be continuously accelerated within these sources.

Useful summary and formulas to solve numerical problems:

- A distribution of electrons having the same velocity $|\mathbf{v}|$, than the same γ, corresponds to a value of the critical frequency, which can be approximated to be the «almost monochromatic» frequency of the emitted photons. Measuring ν_c, and assuming B, one infers γ for those electrons:

$$\nu_c = \frac{3\gamma^3\omega_B}{4\pi} = \frac{3\gamma^2 eB}{4\pi m_e c} \quad \Longrightarrow \quad \gamma = \sqrt{\frac{4\pi m_e\, c\, \nu_c}{3eB}}$$

- The electron energy is then $E = \gamma\, m_e\, c^2$
- If the electron $\mathbf{v}$ distribution is isotropic, their kinetic energ is radiated in photons of frequency $\approx \nu_c$ (and energy $h\nu_c$) at a rate:

$$P_{iso} = \left(\frac{2}{3}\right)^2 \frac{q^4}{m_e^2 c^3}\, \beta^2\, \gamma^2\, B^2 \longleftrightarrow P_{\rm iso} = \frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_B \longleftrightarrow P_{\rm iso} = \frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_B$$

The cooling time calculation is then straightforward.

Useful units and formulas in CGS (Gaussian units)

Electric charge is measured in statC (or «esu»):

1 statC = 1 $g^{1/2}$ $cm^{3/2}$ s^{-1}

B(magnetic field) in Gauss, with 1 G = 1 g ½ $cm^{-1/2}$ s^{-1} ; 1 Tesla= 10^4 G

1 erg = 1 g cm^{2} s^{-2}

1 eV= 1,60218 × 10^{-12} erg = 1,60218 10^{-19} J ; 1 eV= 2.418×10^{14} Hz (from E=hν)

$r_0 = e^2/m_e c^2$ (≈ 2.8 × 10^{-13} cm);

$\sigma_T = 8\pi r_0^2/3$= 6.6524 ×10^{-25} cm^2 ;

e = electron charge = 4.8032 ×10^{-10} statC

m_e= 9.1093937 ×10^{-28} g;

h=4.13 ×10^{-15} eV s = 6.26 ×10^{-27} erg s

c ≈ 2.998 × 10^{10} cm s^{-1}

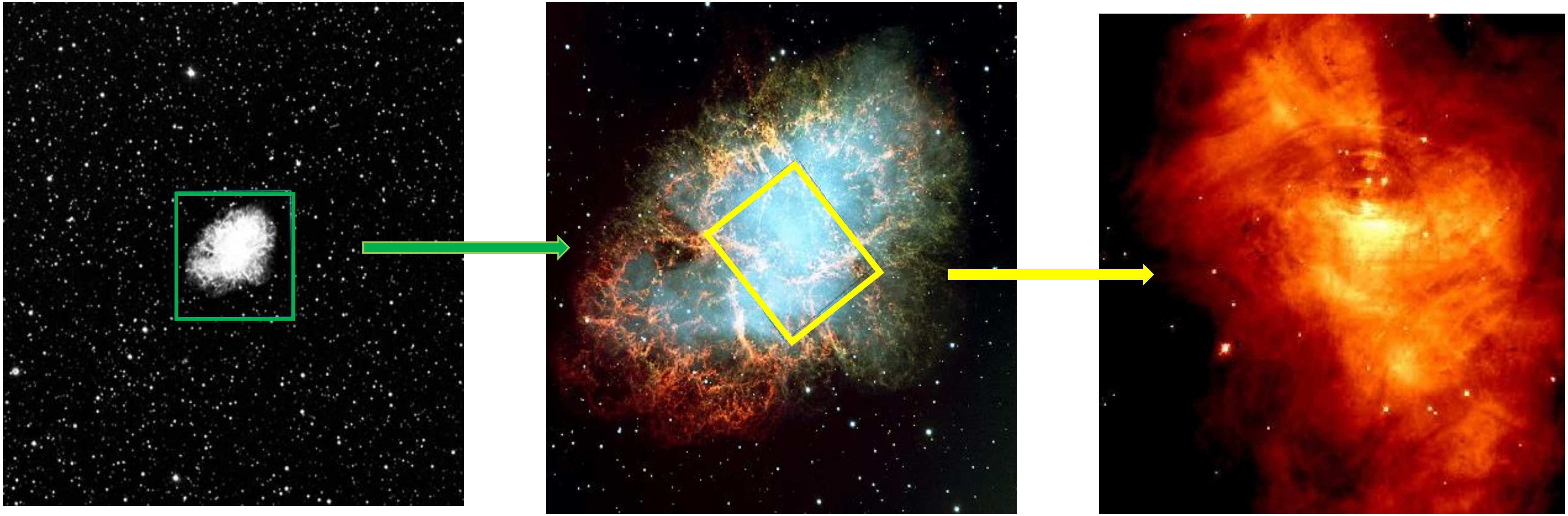
h ≈ 6.626 ×10^{-27} erg s

Boltzmann constant k ≈ 1.381 × 10^{-16} erg K^{-1}

$m_e c^2$ ≈ 34.3 erg

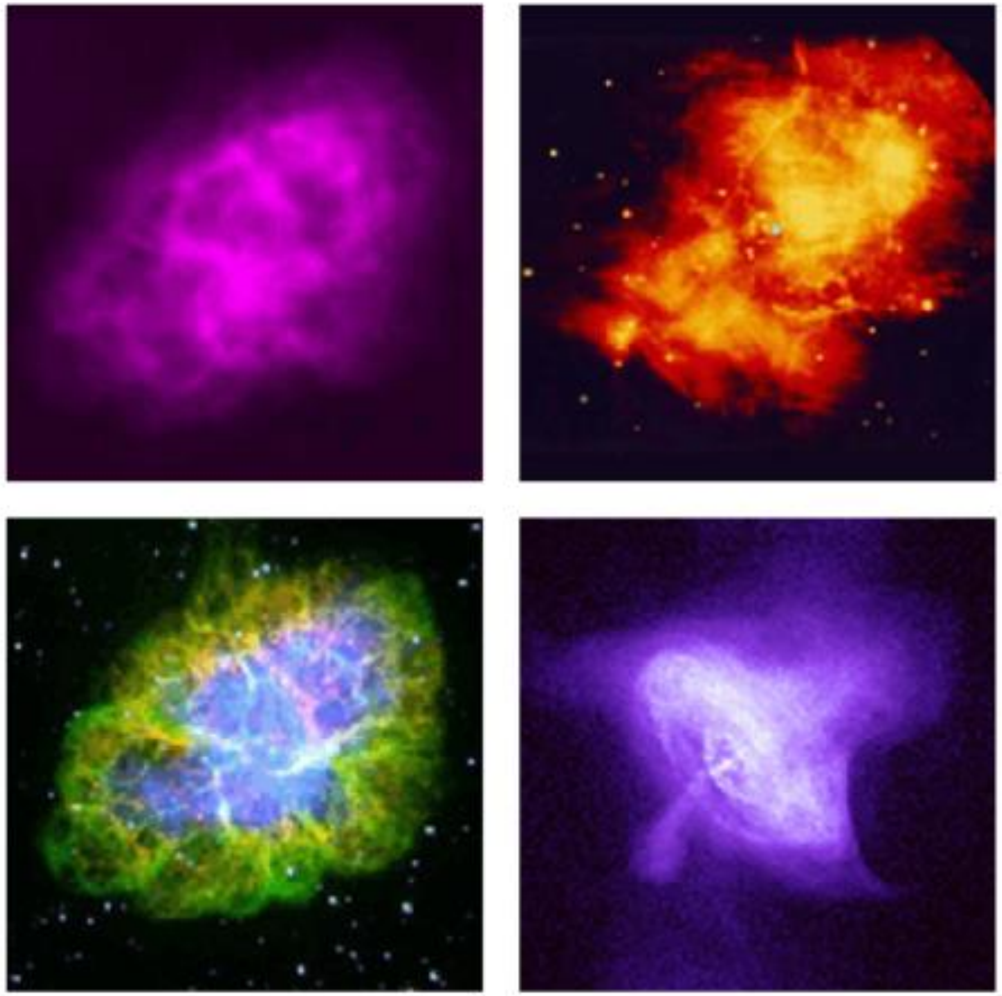
Worked example: The Crab nebula (M51) in Taurus.

The Crab is a **900 yrs old** Supernova remnant appearing today as an expanding, roughly spherical gas cloud with a very filamentary structure. It is about 2 kpc distant and has an angular size of about 4 arc-minutes. The total **optical** emission of the remnant is dominated by the synchrotron process.

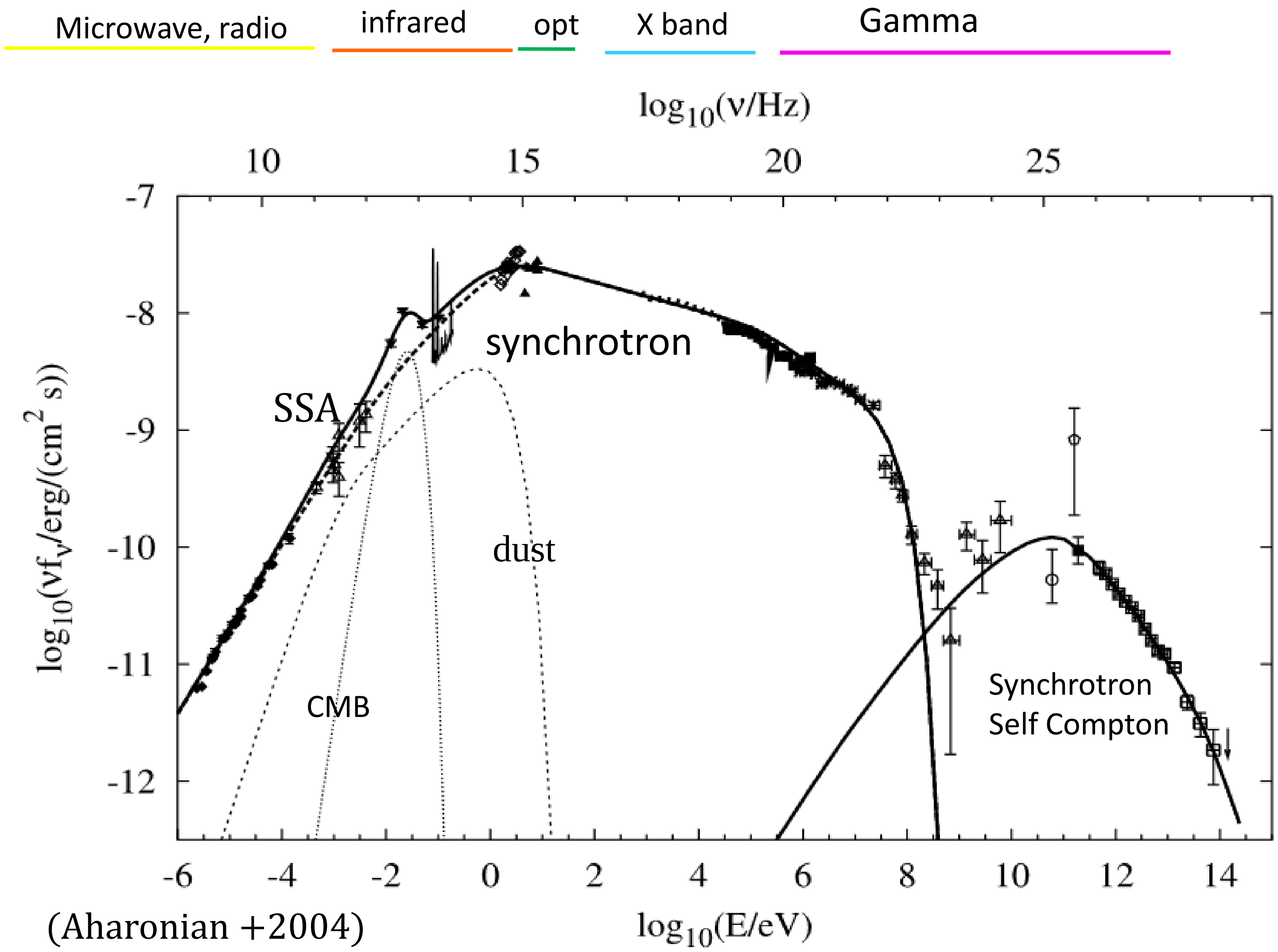


HST Optical images of Crab Nebula. The emission from filaments is mostly in Hydrogen and Oxygen lines, although in the central regions the emission is dominated by electrons and the **synchrotron process**. The source of the electrons is probably the **central pulsar, or neutron star.**

Crab nebula emits **from radio up to TeV Gamma rays**. Its synchrotron spectrum shows a turnover at about 100 keV. There is possibly a inverse-Comptonised spectrum at very high energies (10^{10}-10^{12}) eV. *(Compton scattering later in the course)*. The total luminosity of the Crab is $L \sim 5 \times 10^{38}$ erg sec^{-1} and the magnetic field is thought to be of order **10^{-4} Gauss** .



Images of the Crab in radio, infrared, optical and X-ray.
see http://chandra.harvard.edu/photo/0052/what.html.



(Aharonian +2004)

Let us consider, for example, the Crab photons at 20 keV (X band) (1eV = 1.602×10^{-12} erg) or $\nu \approx \nu_c = 4.8 \times 10^{18}$ Hz, for a magnetic field strength of order 10^{-4} Gauss.

- For these electrons, the Lorentz factor is:

$$\gamma = \sqrt{\frac{2\, me\, c\, \nu_c}{3eB}} \approx \sqrt{\frac{2 * 9.11 * 10^{-28}\mathrm{g} * 2.998\ 10^{10}\ \frac{\mathrm{cm}}{\mathrm{s}} * 4.8 * 10^{18}\ \mathrm{s}^{-1}}{3 * 4.8 * 10^{-10}\ \mathrm{statC} * 10^{-4}\ \mathrm{g}^{1/2}\ \mathrm{cm}^{-1/2}\ \mathrm{s}^{-1}}} \approx \sqrt{18.2 * 10^{14}} \approx 4.2 * 10^{7}$$

  - The single electron energy is : $E = \gamma\, m_e\, c^2 \approx 34$ erg ~ 21 TeV
  - The emitted power at this frequency ν_c is (using $\beta \approx 1$)

$$P_{iso} = \left(\frac{2}{3}\right)^2 \frac{e^4}{m^2 c^3} \beta^2 \gamma^2 B^2 = \frac{4 * (4.8 * 10^{-10})^4}{9 * (9.11 * 10^{-28})^2 * (2.998\ 10^{10})^3} * (4.2\ 10^{7})^2 * (10^{-4})^2\ \ \mathrm{erg/s} \approx 1.8 * 10^{-8}\ \mathrm{erg\ s^{-1}}$$

It follows that the cooling time (in which the 20 keV electron radiates away all its kinetic energy) is:

$\tau = \frac{E}{P} \approx 50$ yr , which is **much shorter than the age of the nebula**. This indicates that there must be a fresh supply of high energy electrons into the nebula: the central pulsar is the usual culprit implicated in this matter.

Problem 21: The nebula emits X-rays at 10^5 eV ($\sim 10^{19}$ Hz) , which can be associated with the peak of the synchrotron spectrum at 0.29 ν_c. The magnetic field in the nebula has a strength of $B \approx 10^{-4}$ Gauss.

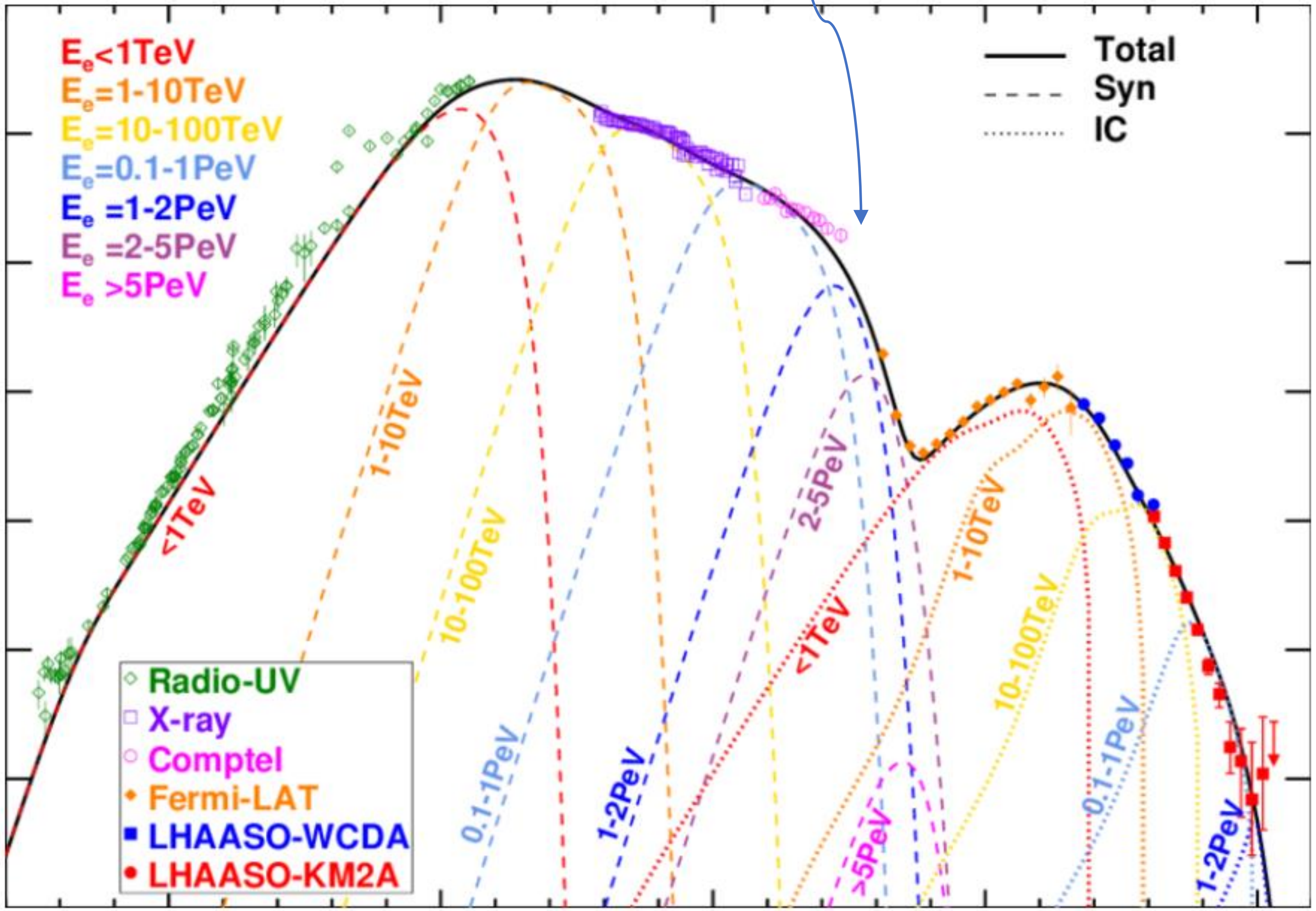
Compute the energy of the electrons producing this radiation, and the synchrotron power radiated per electron if they are isotropic. How long can the electrons radiate X-rays at this rate? Show that this is much shorter than the known age of the Crab (900 years).

Problem 22: The synchrotron spectrum of the Crab shows a **change of slope** at around 10^{15} Hz ($\sim$ 4 eV), thought to be due to the high energy electrons, which radiate their energy away in a time shorter than the age of the nebula. Show that at this frequency the electrons have a lifetime which is *about* the same as the age of the Nebula, if the magnetic field strength is $B = 10^{-4}$ Gauss.

Examples of electron lifetimes in the Crab nebula (as in 2018 papers)

	Photon frequency ν_{syn} (Hz)	Electron energy U (eV)	Electron lifetime τ (yr)
Radio (0.5 GHz)	5×10^8	3.05×10^8	109,000
Optical (600 nm)	5×10^{14}	3.05×10^{11}	109
X ray (4.1 keV)	1.0×10^{18}	1.37×10^{13}	2.4
Gamma ray (41 MeV)	1.0×10^{22}	1.37×10^{15}	0.024 (9 d)

The Crab synchrotron spectrum turns over at about 10^{15} Hz (or 4 eV). The cooling time at this frequency is about 1000 years, which is of the same order as the age of the nebula. **"Ageing" of the spectrum. (see Problem 21)** The search for a break in the high frequency spectrum is a tool to estimate the radiative age of thedominant population of relativistic electrons.



The LAHASO Collaboration, Science, 2021, Vol 373, Issue 6553, pp. 425-430 .

Different regions of the same radio source may have different radiative age based on:

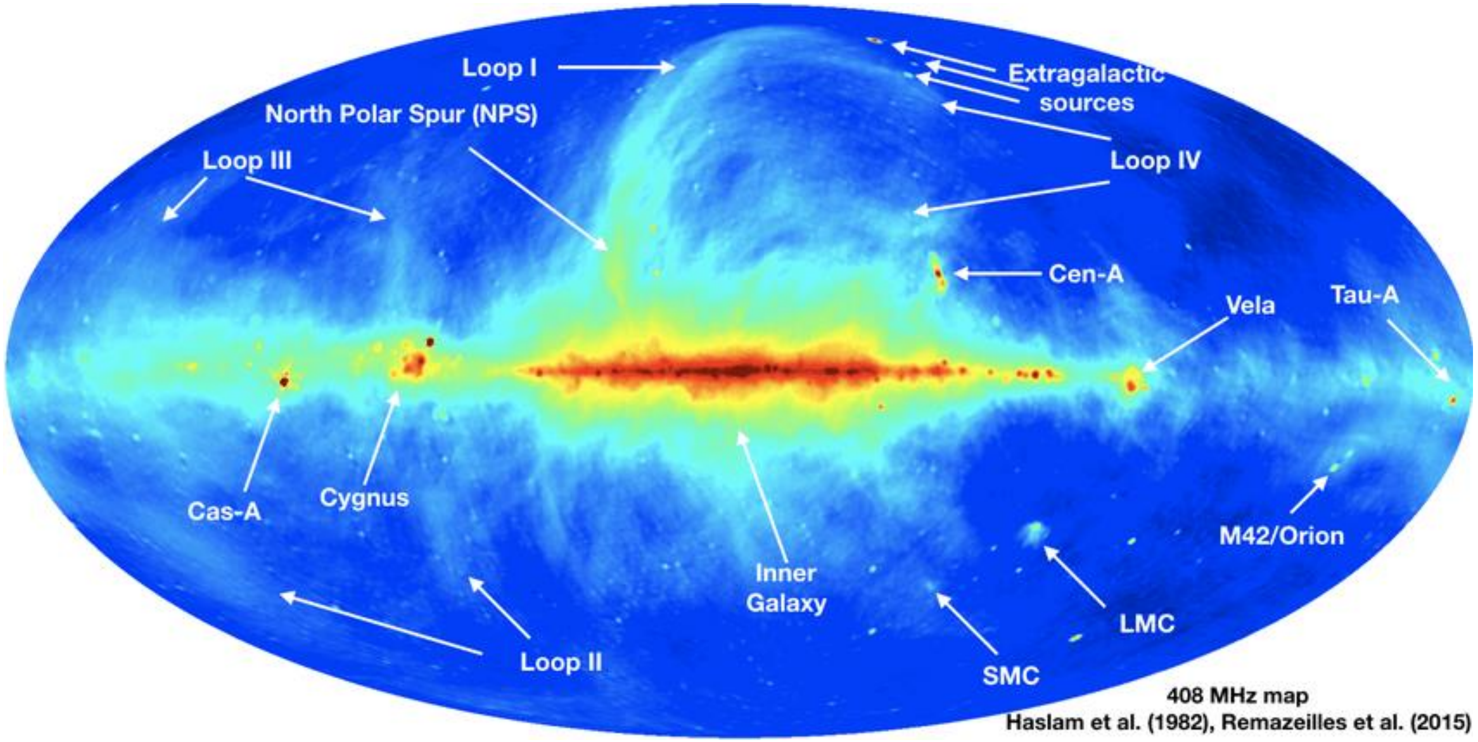
1. injection time

2. location within the source

3. Individual radio source evolution

Other synchrotron sources: the Milky Way

Synchrotron emission at 408 MHz, from relativistic electrons (positrons) within the Milky Way. In a field of 1 μG, their energies correspond to $\gamma = 10^4$.

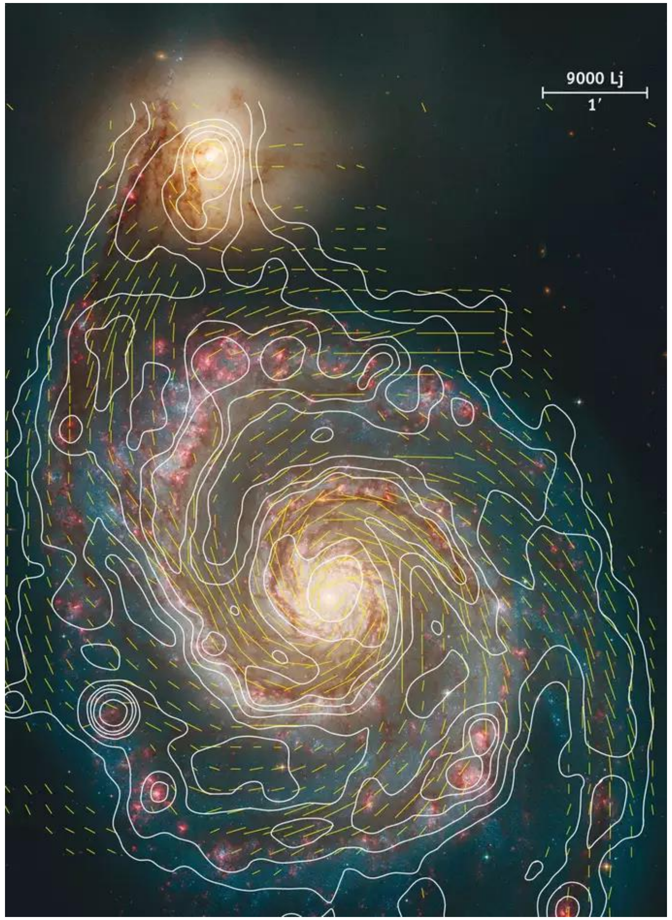


(Taurus A is the Crab Nebula)

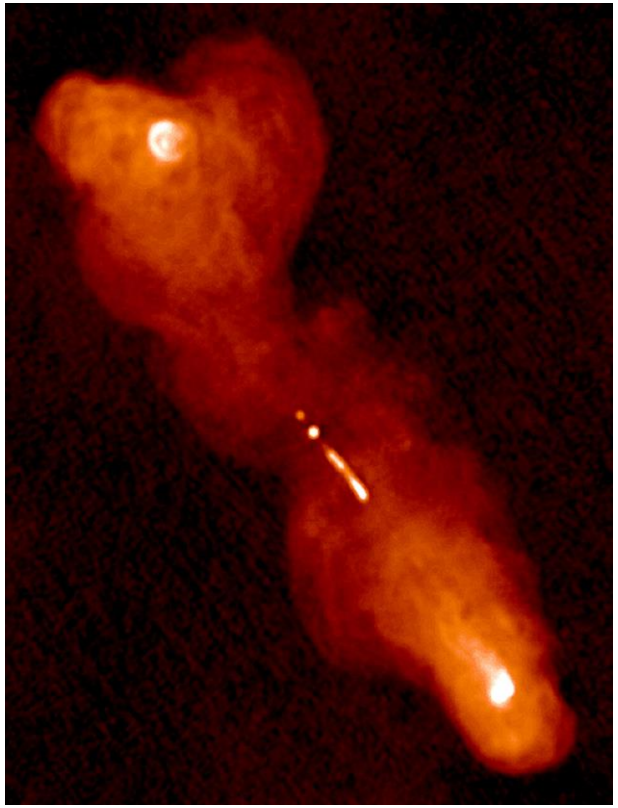
Problem 22: Compute the Larmor radius of a relativistic proton in our galaxy, assuming $B \sim 10\mu$ G and $\gamma = 10^4$.

Will it escape the Galaxy?

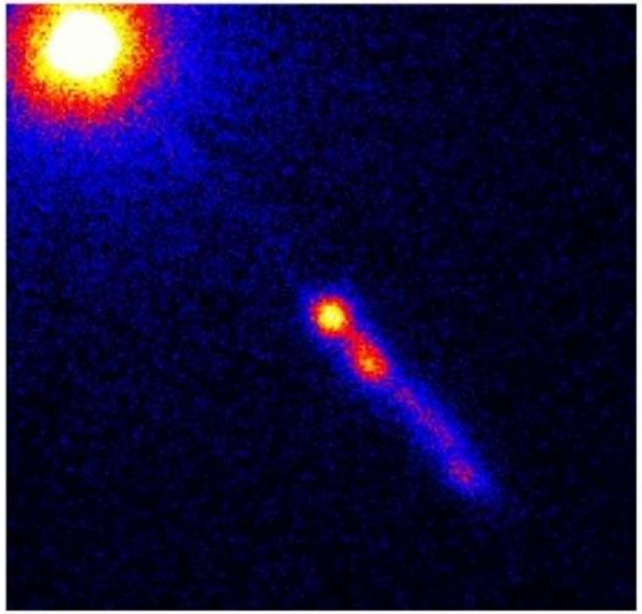
Other synchrotron sources



M51 (Vortex Galaxy)



Radio image 3C219 Seyfert Galaxy (VLA)



Chandra image of the radio jet of 3C 273