NEUTRINOS AND LARGE SCALE STRUCTURES

MATTEO VIEL
INAF & INFN – Trieste

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OUTLINE

- Quantifying the impact of neutrinos on cosmological observables

- Simulating neutrinos beyond linear theory: neutrinos and LSS

- Review of (tightest) constraints on neutrino masses

- Sterile neutrinos and the coldness of cold dark matter at small scales
EVOLUTION of COSMOLOGICAL LSS – I: methods

**Linear theory** -- use popular codes like CAMB [http://camb.info/]

**Non-linear evolution** -- approximations (e.g. Lognormal modelling /Peacock & Dodds, PT etc.) or N-body/hydrodynamic/adaptive mesh refinement techniques

Early simulations: **direct summation** method for the gravitational N-body problem (still useful for stellar systems) Holmberg 1941, Aarseth 1979, Peebles, White etc.

Improvement made in the 90s to compute large scale force via Fourier/mesh techniques

**Tree algorithms** arrange particles in groups and compute forces by summing over multipole expansions.

These two have been combined into **Tree+PM** codes, that could include hydrodynamic processes using for example the smoothed particle hydrodynamics (**SPH**, Lucy 1977).

Hydrodynamic processes are important at small scales
EVOLUTION of LSS –II : dynamics in the linear regime

Effects in terms of matter clustering, Hubble constant, Energy density

(see Lesgourgues & Pastor 2006)

Different evolution in terms of dynamics and geometry as compared to massless neutrino universes
Note that the equation above is not exact but it is a good approximation (e.g. Komatsu et al 11)
EVOLUTION of LSS - IV: individual neutrino masses do matter

Lesgourgues & Pastor 2006
**SIMULATION of LSS – I: basic equations**

### DM

\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0
\]

Collisionless Boltzmann Equation

\[
\nabla^2 \Phi(\mathbf{r}, t) = 4\pi G \iiint f(\mathbf{r}, \mathbf{v}, t) \, d\mathbf{v}
\]

Poisson equation

N-body problem: follow the Newton’s equation of motions for a large number of particles under their own self-gravity

### GAS

\[
\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0
\]

Continuity

\[
\frac{du}{dt} = -\frac{P}{\rho} \nabla \cdot \mathbf{v} - \frac{\Lambda(u, \rho)}{\rho}
\]

Energy

\[
\frac{d\mathbf{v}}{dt} = -\frac{\nabla P}{\rho} - \nabla \Phi.
\]

Euler

\[
P = (\gamma - 1)\rho u
\]

Gas eq.of state
SIMULATION of LSS – II: basic equations for DM

Tree method – expansion of the gravitational potential in multipoles

\[ \Phi(r) = G \int_{V'} \frac{\rho(r')dV'}{|\mathbf{r}' - \mathbf{r}|} \approx \frac{1}{r} + \sum_i \frac{\partial (1/r)}{\partial r_i} + \frac{1}{2} \sum_{ij} \frac{\partial^2 (1/r)}{\partial r_i \partial r_j} - \frac{1}{6} \sum_{ijk} \frac{\partial^3 (1/r)}{\partial r_i \partial r_j \partial r_k} \]

\[ \Phi(r) = \left\{ M(1/r) - \mathbf{P} \cdot \nabla (1/r) + \frac{1}{2} \mathbf{Q} : \nabla \nabla (1/r) - \frac{1}{6} \mathbf{S} : \nabla \nabla \nabla (1/r) + \cdots \right\} \]

P dipole moment, Q tensor quadropole moment, S usually not considered
1980: Bond et al. 1980 – linear theory (also Russian school with Zeldovich)
1983: Bond et al. – Evolution of Boltzmann-Einstein equations. Clustering properties of galaxies not reproduced if the universe is dominated by neutrinos (White et al. 1983) – numerical experiment

1992: Davis et al. HDM or CHDM models P3M codes with neutrino particles placed as the dark matter ones (same CDM spectrum + velocities): $32^3$ particles

1993: Klypin et al. $2 \times 128^3$ particles at $z_{IC}=14$ with the right power spectrum

1994: Ma & Bertschinger approximate linear scheme evolved at $z=13$ and after that pure N-body
SIMULATION of LSS – IV: the distribution of matter

Pure HDM not allowed. However CHDM is still viable and impacts on the cosmic web

Comparison: COBE Compatible Bias
$b = 1.5$

FILAMENTS

CHDM1

CLUMPS

$\text{a, b, c, d}$

CDM1

Brodbeck et al. 98
**SIMULATION of LSS – V: Initial Conditions**

![Graph showing the linear power spectrum with black, blue, and green lines, indicating matter power, neutrino power, and shot noise level respectively.](image)

- **Black** is matter power
- **Blue** is matter power in neutrino simulations
- **Thick blue** is neutrino power
- **Green dashed** is shot noise level for a typical run

**Zeldovich displacement of the particles**

\[ x(q, z) = q + D_+(z) \nabla_q \phi_i(q) \]

\[ \phi_i(x) = -\frac{3}{2} H^2 a^3 \Phi_i(x) \]
N-body simulations – I: particles

Simulation of neutrinos as an independent set of particles that interact gravitationally

COLD DM       NEUTRINOS 0.6 eV       NEUTRINOS 0.3 eV

Brandbyge et al 08
N-body simulations – II: velocities

Draw velocity from Fermi-Dirac distribution

Brandbyge et al 08a
N-body simulations – III: effects in terms of non-linear power

Brandbyge et al 08

\[ \frac{\Delta P}{P} \bigg|_{\text{max}} \sim -9.8 \frac{\Omega_\nu}{\Omega_m} \]
N-body simulations – IV: mesh method

Computing the neutrino gravitational potential on the PM grid and summing up its contribution to the total matter gravitational potential

COMPARISON GRID VS PARTICLES

M $\nu = 0.6$ eV

M $\nu = 1.2$ eV

Brandbyge et al 08b
N-body simulations – V: a hybrid approach

After neutrino decoupling CBE

\[ f = f_0 + \frac{\partial f_0}{\partial T} \delta T = f_0(1 + \Psi) \]

\[ f_0(q) = \frac{1}{e^{q/T} + 1} \]

\[ \frac{df}{dt} + \frac{dx^i}{dt} \frac{\partial f}{\partial x^i} + \frac{dq}{dt} \frac{\partial f}{\partial q} + \frac{dn_i}{dt} \frac{\partial f}{\partial n_i} = 0 \]

\[ \delta \rho_r(k) = 4\pi a^{-4} \int q^2 dq \epsilon f_0 \Psi_0 \]

\[ \epsilon = (q^2 + a^2 m^2)^{1/2} \]

\[ \dot{\Psi}_0 = -qk \frac{3\epsilon}{\Psi_1} \psi_0 - \phi \frac{d\ln f_0}{d\ln q}, \]

\[ \dot{\psi}_1 = \frac{qk}{\epsilon} \left( \psi_0 - \frac{2}{5} \psi_2 \right) - \frac{ek}{q} \frac{d\ln f_0}{d\ln q}, \]

\[ \dot{\psi}_l = \frac{qk}{\epsilon} \left( \frac{l}{2l-1} \psi_{l-1} - \frac{l+1}{2l+3} \psi_{l+1} \right), \quad l \geq 2 \]

\[ \Psi_0(k, q, z) = T(k, q, z) \psi_0^0(k, q) \]

Brandbyge & Hannestad 09
PARTICLES: accurate non-linear sampling but prone to shot-noise errors

GRID: fast and accurate but no phase mixing (i.e. non-linear regime suppression maybe it is less than it should be)
N-body + Hydro simulations – I: slices

TreeSPH code Gadget-III follows DM, neutrinos, gas and star particles in a cosmological volume

Viel, Haehnelt & Springel 2010, JCAP, 06, 15
Hydro simulations – II: redshift/scale dependence of non-linear power

Full hydro simulations: gas physics does impact at the <10 % level at scales $k < 10 \, h/\text{Mpc}$

Viel, Haehnelt & Springel 2010, JCAP, 06 ,15
Hydro simulations – III: halo mass functions

Marulli, Carbone, MV, Moscardini e Cimatti 2011 arxiv: 1103.0278
Hydro simulations – IV: matter and halo clustering

Marulli, Carbone, MV, Moscardini e Cimatti 2011 arxiv: 1103.0278
N-body simulations – V: halo density profile

Brandbyge et al. 2010
Hydro simulations – VI: redshift space distortions

\[
\xi(s_{\perp},s_{\parallel}) = \int_{-\infty}^{\infty} dv f(v) \xi(s_{\perp},s_{\parallel} - v/H(z)/a(z))
\]

\[
f_{\text{exp}}(v) = \frac{1}{\sigma_{12} \sqrt{2}} \exp \left( -\frac{\sqrt{2}|v|}{\sigma_{12}} \right)
\]

\[
P(k) = (1 + \beta \mu^2)^2 P_{\text{lin}}(k)
\]

Marulli, Carbone, MV, Moscardini e Cimatti 2011 arxiv: 1103.0278
Hydro simulations – VII: very non-linear regime comparison with halofit

Bird, MV et al 011
IGM

Ordinary baryonic matter that fills the space between galaxies
Dark matter evolution and baryon evolution – I

linear theory of density perturbation +

Jeans length $L_J \sim \sqrt{T/\rho}$ + mildly non linear evolution

$$x_b \equiv \frac{1}{H_0} \left[ \frac{2\gamma k T_m}{3 \mu m_p \Omega (1 + z)} \right]^{1/2}$$

Jeans length: scale at which gravitational forces and pressure forces are equal

$$\delta_0(x) \equiv \frac{1}{4 \pi x_b^2} \int \frac{\delta_{DM}(x_1)}{|x - x_1|} e^{-|x-x_1|/x_b} dx_1$$

Density contrast in real and Fourier space

$$\delta_0(k) = \frac{\delta_{DM}(k)}{1 + x_b^2 k^2},$$

Non linear evolution lognormal model

$$n(x) = n_0 \exp \left[ \delta_0(x) - \frac{\langle \delta_0^2 \rangle}{2} \right]$$

Dark matter evolution and baryon evolution –II

\[ \text{Cumulative Mass Fraction} \]

\[ \text{Volume Filling Factor} \]

80% of the baryons at $z=3$ are in the Lyman-α forest.

Baryons as tracer of the dark matter density field

$\delta_{\text{IGM}} \sim \delta_{\text{DM}}$ at scales larger than the Jeans length $\sim 1 \text{ com Mpc}$

$\text{flux} = \exp(-\tau) \sim \exp(-(\delta_{\text{IGM}})^{1.6} T^{-0.7})$

Meiksin’s review (2007)
The data sets

SDSS vs UVES

**SDSS**
3035 LOW RESOLUTION LOW S/N

**LUQAS**
30 HIGH RESOLUTION HIGH S/N
The interpretation: full grid of sims - I

SDSS power analysed by forward modelling motivated by the huge amount of data with small statistical errors

CMB: Spergel et al. (05)  Galaxy P(k): Sanchez & Cole (07)  Flux Power: McDonald (05)

Cosmological parameters + e.g. bias + Parameters describing IGM physics

132 data points

z=4.2

z=2.2
Hydro simulations – VI: redshift/scale dependence of flux power

Effect on flux power observables is smaller than matter power

Viel, Haehnelt & Springel 2010, JCAP, 06 ,15
GOAL: the primordial dark matter power spectrum from the observed flux spectrum (filaments)

CMB physics $z = 1100$

dynamics

Lya physics $z < 6$

dynamics +
termodynamics

Temperature, metals, noise

CMB + Lyman $\alpha$

Long lever arm

Constrain spectral index and shape

Relation: $P_{\text{FLUX}}(k) - P_{\text{MATTER}}(k)$ ??

Tegmark & Zaldarriaga 2002
The interpretation: flux derivatives

Analysis of SDSS flux power

The flux power spectrum is a smooth function of $k$ and $z$

$$P_F(k, z; p) = P_F(k, z; p^0) + \sum_{i=1,N} \frac{\partial P_F(k, z; p_i)}{\partial p_i} (p_i - p_i^0)$$

Best fit

$p$: astrophysical and cosmological parameters

but even resolution and/or box size effects if you want to save CPU time
Summary (highlights) of results from the high-res and low-res data

Why Lyman-α? Small scales
- High redshift
- Most of the baryonic mass is in this form
- Quasars sample 75% of the age of the universe

1. Measurement of matter power spectrum McDonald et al. 05,06 Viel et al. 04, 06

2. Tightest constraints to date on neutrino masses and running of the spectral index

Seljak, Slosar, McDonald JCAP (2006) 10 014

3. Tightest constraints to date on the coldness of cold dark matter

Results Lyman-α only: amplitude and slope of matter power

\[ \Delta^2_L(k, z) \approx \left[ \frac{D(z)}{D(z_p)} \right]^2 \Delta^2_L(k, z_p) \times \left[ \frac{k}{k_*(z)} \right]^{3+n_{\text{eff}}(k, z_p) + (1/2) \alpha_{\text{eff}}(k, z_p) \ln[k/k_*(z)]} \]

\( \chi^2 \) likelihood code distributed with COSMOMC

Croft et al. 98,02 40% uncertainty
Croft et al. 02 28% uncertainty
Viel et al. 04 29% uncertainty
McDonald et al. 05 14% uncertainty

Redshift \( z=3 \) and \( k=0.009 \) s/km corresponding to 7 comoving Mpc/h
Fitting SDSS data with GADGET-2
this is SDSS Ly-α only !!

Results Lyman-α only with flux derivatives: correlations

\[ \sigma_8 = 0.91 \pm 0.07 \]
\[ n = 0.97 \pm 0.04 \]
Active neutrinos – I: the effect

\[ k_{nr} \approx 0.018 \Omega_m^{1/2} \left( \frac{m}{1 \text{eV}} \right)^{1/2} h \text{Mpc}^{-1} \]

\[ \Sigma m_\nu = 0.138 \text{eV} \]

\[ \Sigma m_\nu = 1.38 \text{eV} \]

\[ v_{th} = \frac{\langle p \rangle}{m} \approx 3T_\nu \frac{m}{a} \approx 150(1 + z) \left( \frac{1 \text{eV}}{m} \right) \text{km s}^{-1} \]

\[ k_{FS}(t) = \left( 4\pi G \bar{\rho}(t) a^2(t) \right)^{1/2}, \quad \lambda_{FS}(t) = 2\pi \frac{a(t)}{k_{FS}(t)} = 2\pi \sqrt{\frac{2v_{th}(t)}{3H(t)}} \]
Active neutrinos – II: constraints

Seljak, Slosar, McDonald, 2006, JCAP, 0610, 014

\[ \Sigma m_\nu (\text{eV}) < 0.17 \ (95 \% \text{C.L.}), \ < 0.19 \text{ eV} \ (\text{Fogli et al. 08}) \]
\[ r < 0.22 \ (95 \% \text{C.L.}) \]
\[ \text{running} \ = \ -0.015 \pm 0.012 \]
\[ \text{Neff} = 5.2 \ (3.2 \text{ without Ly } \alpha) \]
\[ \text{CMB + SN + SDSS gal+ SDSS Ly-}\alpha \]

Tight constraints because data are marginally compatible

Goobar et al. 06 get upper limits 2-3 times larger......

for forecasting see Gratton, Lewis, Efstathiou 2007
Active neutrinos – III: comparison with other constraints

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, Slosar, PhysRevD, 2007, 75, 0533001
RESULTS

WARM DARK MATTER

Or if you prefer.. How cold is cold dark matter?
(Some) Motivations

Some problems for cold dark matter at the small scales: 1- too **cuspy cores**, 2- too many **satellites**, 3- **dwarf galaxies** less clustered than bright ones (e.g. Bode, Ostriker, Turok 2001)

Although be aware that 1- **astrophysical processes** can act as well to alleviate these problems (feedback); 2- number of **observed satellites** is increasing (SDSS data); 3- galaxies along filaments in warm dark matter sims is probably a **numerical artifact**

**Minimal extension of the Standard Model** for particle physics that accommodates neutrino oscillations naturally

Hints of a sterile sector: LSND experiment prefers a sterile neutrino $m_\nu < 1$ eV

but Lyman-$\alpha$ data $m_\nu < 0.26$ eV

and best fit $N_{\text{eff}}$ (active) = 5.3

The LSND result has been rejected by MiniBoone

Although be aware that LSND results are controversial and that Lyman-$\alpha$ data that wish to probe the subeV limits are prone to systematic effects
**Lyman-α and Warm Dark Matter - I**

30 comoving Mpc/h  \( z=3 \)

In general

\[
\kappa_{FS} \sim 5 \frac{T_v}{T_x} \left( \frac{m}{1\text{keV}} \right) \text{ Mpc}^{-1}
\]

Set by relativistic degrees of freedom at decoupling

See Bode, Ostriker, Turok 2001
Abazajian, Fuller, Patel 2001

MV, Lesgourgues, Haehnelt, Matarrese, Riotto, PRD, 2005, 71, 063534
Lyman-α and Warm Dark Matter - II

\[ P(k) = A k^n T^2(k) \]

\[ \left[ P(k)_{\text{WDM}} / P(k)_{\text{CDM}} \right]^{1/2} \]

\[ T_x = 10.75 \]

\[ T_{\nu} \frac{1}{g(T_D)}^{1/3} \]

Light gravitino contributing to a fraction of dark matter

MV, Lesgourgues, Haehnelt, Matarrese, Riotto, PRD, 2005, 71, 063534
Lyman-α and Warm Dark Matter - III

$m_{\text{WDM}} > 550$ eV thermal
$> 2$ keV sterile neutrino
$< 16$ eV gravitino

Viel et al. (2005) from high-res

$\Lambda_{\text{susy}} \approx \left( \sqrt[3]{3} m_{3/2} M_p \right)^{1/2} \lesssim 260$ TeV

$m_{\text{WDM}} > 1.5$-2 keV thermal
$> 10$-14 keV sterile neutrino

Seljak, Makarov, McDonald, Trac, PhysRevLett, 2006, 97, 191303
MV, Lesgourgues, Haehnelt, Matarrese, Riotto, PhysRevLett, 2006, 97, 071301
Lyman-α and Warm Dark Matter - IV


SDSS + HIRES data

Tightest constraints on mass of WDM particles to date:

\[ m_{\text{WDM}} > 4 \text{ keV} \] (early decoupled thermal relics)

\[ m_{\text{sterile}} > 28 \text{ keV} \]
Lyman-\(\alpha\) and sterile neutrinos - V

Boyarsky, Lesgourgues, Ruchayskiy, Viel, 2009, JCAP, 05, 012– REVIEW!
Lyman-α and resonantly produced sterile neutrinos -VI

Boyarsky, Lesgourgues. Ruchayski, Viel, 2009, PRL, 102, 201304
Little room for warm dark matter...... at least in the standard DW scenario...the cosmic web is likely to be quite “cold”
To constrain the sterile neutrino particle we need two parameters:

1) Neutrino mass $m_s$

2) Mixing angle $\theta$ that describes the interaction between active and sterile neutrino families
Lyα-WDM VII: analysis with flux derivatives

\[ \nu = 0 \]
Leptonic number is conserved

Radiative decay channel at a rate that depends on its mass and the mixing angle with active neutrinos
\[ \Gamma \sim \sin^2 2\theta m_{\text{sterile}}^5 \]

\Delta E_{\text{line}} = \nu_{\text{virial}} \frac{E}{c} \sim 50 \text{ eV for a galaxy cluster} \ 5 \text{ eV for a galaxy for } E=5 \text{ keV}
Decaying channel into photons and active neutrinos line with $E = m_s/2$ (X-band)

$5.66\text{ keV}$

$2 \times 10^{-13} \text{ erg/cm}^2/\text{s}$

Line flux $\sim 5 \times 10^{-18} \text{ erg cm}^{-2} \text{s}^{-1} (D_L/1\text{Mpc})^{-2} (M_{DM}/10^{11} \text{M}_{\odot}) (\sin^2 2\theta/10^{-10}) (m_s/1\text{keV})^5$
XMM-Newton reveals the origin of elements in galaxy clusters

10 May 2006

These X-ray images of the clusters of galaxies 'Sersic 1384.3' (right) and '2A 0335+096' (left) were taken by the European Photon Imaging Camera (EPIC) on-board ESA's XMM-Newton, in November 2002 and August 2003 respectively. Thanks to these observations, astronomers could determine the abundances of nine chemical elements in the clusters' plasma – a gas containing charged particles such as ions and electrons. These elements include oxygen, iron, neon, magnesium, silicon, argon, calcium, nickel, and chromium. The distribution of silicon (produced by type II and 'core collapse' supernova types) relative to iron (mainly produced by 'type Ia' supernovae) in these two clusters is very different, showing that they had a different evolution.

Credits: ESA and the XMM-Newton EPIC consortium

5.66 keV !!!
Satellites of the Milky Way and Warm Dark Matter

Lovell et al. 11
CONCLUSIONS

- Neutrinos do impact on the LSS at a level which is very much constrained by present data sets. The effect is small and **systematic effects** should be addressed at an unprecedented level of precision. Modelling the power spectrum at the 1 % level at small scales is difficult: **relevant physical processes and numerics** should be modelled and under control.

- Among different observables I outlined the important role of the **IGM**, which is currently providing the tightest constraints on the mass (0.17 eV – 2σ upper limit); **weak lensing** and **galaxy redshift surveys** are likely to provide interesting results.

- Coldness of cold dark matter at small scales is a fundamental observable since possible deviations from the standard model can be measured or a candidate can show up. At present the constraints on the **sterile neutrinos** are tight (especially from IGM data) and are 14 keV (2σ lower bound) in the non-resonantly production mechanism or about 2 keV (2σ lower bound) in the resonant production scenario.

- Tools to investigate these topics beyond the linear regime are **N-body simulations** (and others)
Science [http://adlibitum.oats.inaf.it/viel/cosmoIGM](http://adlibitum.oats.inaf.it/viel/cosmoIGM) → Postdoctoral positions in Trieste

**COSMOLOGY**

- IGM as a tracer of the large scale structure of the universe: tomography of IGM structures; systematic/statistical errors; synergies with other probes – IGM unique in redshift and scales

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**cosmoIGM**

**PARTICLE PHYSICS**

- IGM as a probe of fundamental physics: dark matter at small scale; neutrinos; coldness of dark matter; fundamental constants; cosmic expansion

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**GALAXY FORMATION**

- Galaxy/IGM interplay: metal enrichment and galactic feedback; impact on the cosmic web and metal species; the UV background; the temperature of the IGM
55 HIRES spectra QSOs $z=2-6.4$ from Becker, Rauch, Sargent (2006)

Masking of DLAs and metal lines associated to the DLAs, or identified from other lines outside the forest (so there could be still some metal contamination)

Unexplored part of the flux power spectrum which is very sensitive to:

Temperature, Metals, Noise, Galactic winds, Ionizing fluctuations, Damping wings.... ...and maybe more