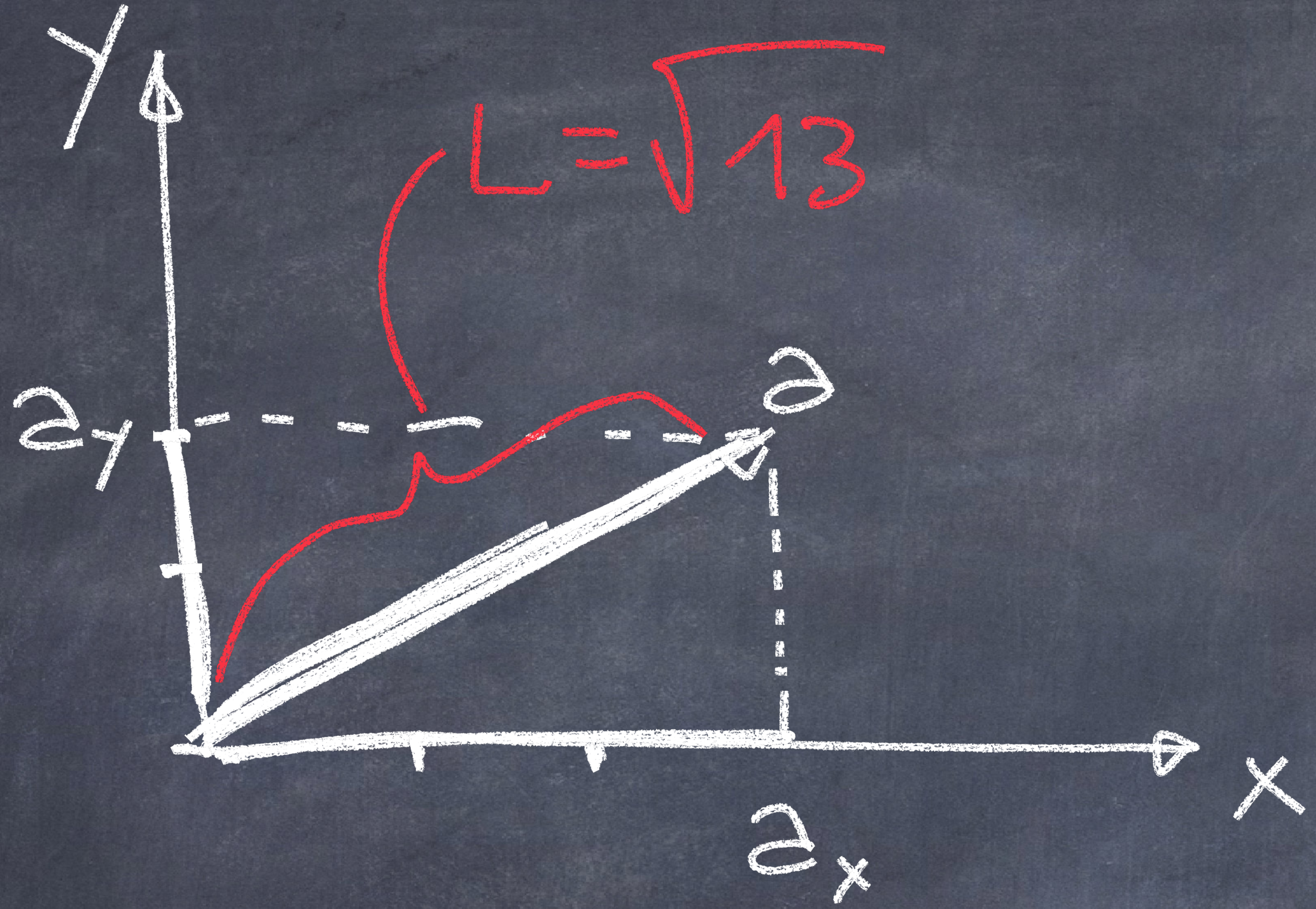


$$\begin{cases} a_x = 3 \\ a_y = 2 \end{cases}$$

$$\langle a|a\rangle = ?$$

$$\langle a| = (a_x, a_y) = (3, 2)$$

$$|a\rangle = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\langle a|a\rangle = (a_x, a_y)\cdot\begin{pmatrix} a_x \\ a_y \end{pmatrix} = a_x a_x + \underline{a_y a_y} = a_x^2 + a_y^2 = 3^2 + 2^2 =$$

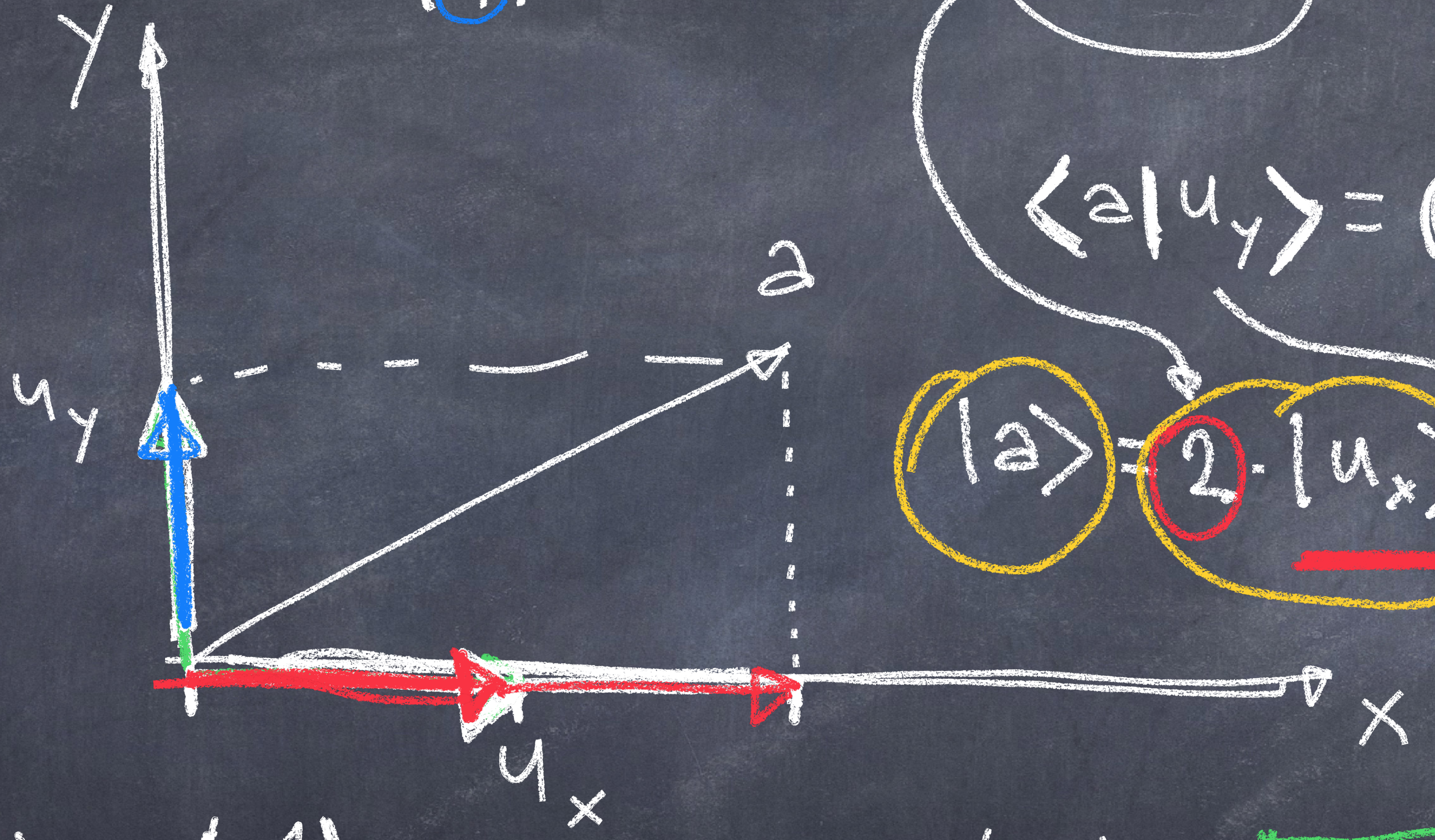
$$= \sum_{k=x,y} a_k a_k$$

$$= 9 + 4 = 13$$

$\boxed{nD}$

$$\langle a|a\rangle = (a_1, a_2, a_3, \ldots, a_n) \cdot \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = a_1 a_1 + a_2 a_2 + \ldots + a_n a_n$$

$$|a\rangle = \begin{pmatrix}2\\1\end{pmatrix}$$



$$\langle a|u_x\rangle = (2,1)\cdot\begin{pmatrix}1\\0\end{pmatrix} = 2$$

$$\langle a|u_y\rangle = (2,1)\cdot\begin{pmatrix}0\\1\end{pmatrix} = 1$$

$$|a\rangle = 2\cdot|u_x\rangle + 1\cdot|u_y\rangle$$

$$|u_x\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$|u_x| = \sqrt{1^2+0^2} = 1$$

$$|u_y\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$|u_y| = \sqrt{0^2+1^2} = 1$$

$$\boxed{\langle u_x|u_y\rangle} = (1,0)\cdot\begin{pmatrix}0\\1\end{pmatrix}$$

$$= 1\cdot 0 + 0\cdot 1 = 0$$

example Real Numbers:

$$2+3-0+1.5+\pi+\sqrt{2} \overset{?}{=} 6+\pi+\sqrt{2} \simeq 10.55$$

5

5

6.5

$$6+\pi+\sqrt{2} \simeq \underbrace{6+3.14}_{9.14}+1.41 = 10.55$$

$\simeq 3.14$ $\quad \simeq 1.41$

① example Complex Numbers: $z = a + ib$

$$\boxed{i^2 = -1}$$

$$\boxed{1 + i2 + 3 - i4 + 0 + i6 \overset{?}{=}} = \underbrace{(1 + 3 + 0)}_{4} + i\underbrace{(2 - 4 + 6)}_{4} = \boxed{4 + i4}$$

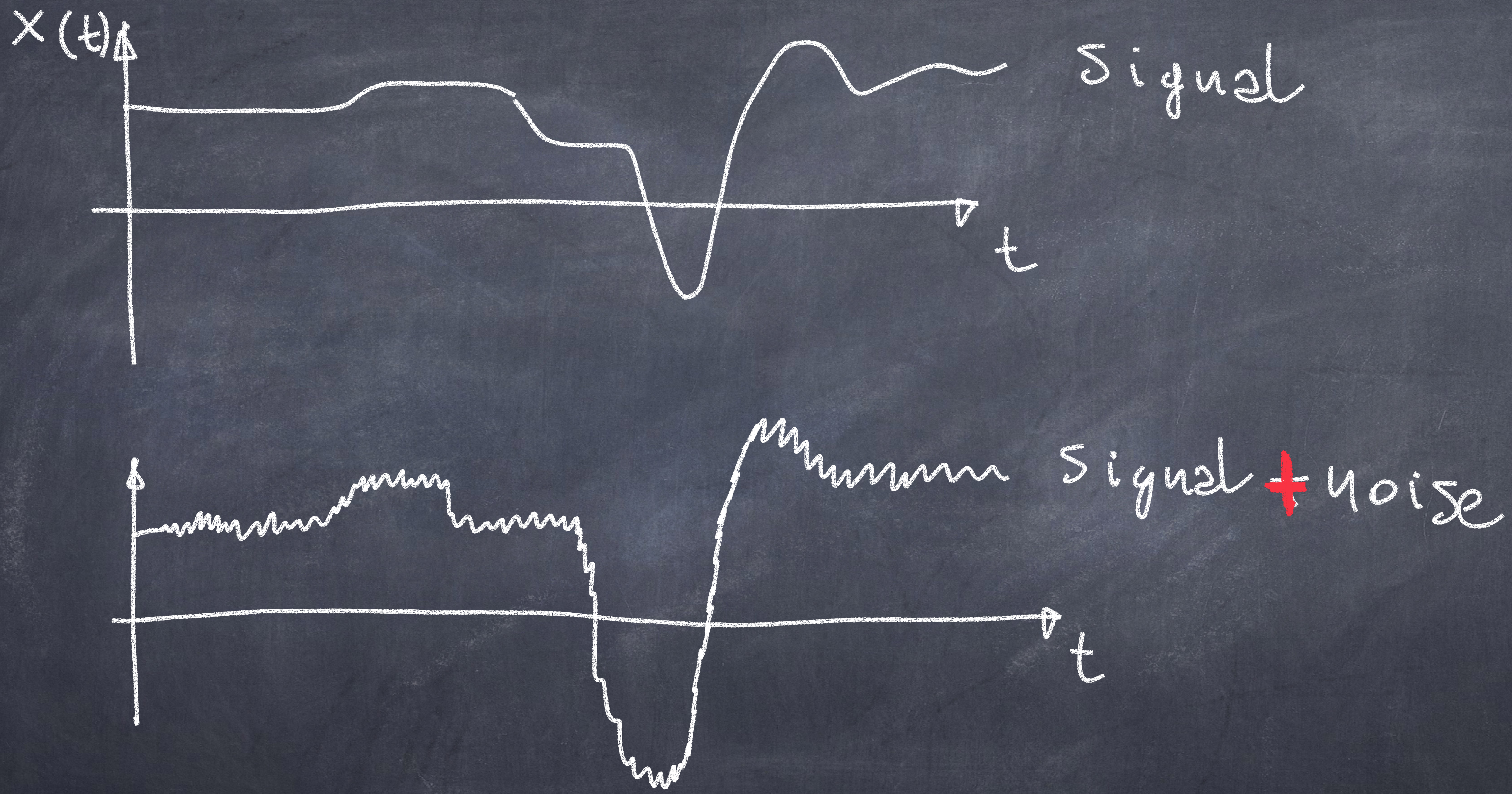
---

② $(1 + i2) \cdot (2 + i3) \overset{?}{=} = 1 \cdot 2 + 1 \cdot i3 + i2 \cdot 2 + i2 \cdot i3 =$

$i^2 - 1$

$$= 2 + i3 + i4 + i^2 6 = \overbrace{2 + i7 - 6}^{Re} = -4 + i7$$

-6


x(t)
t
Signal
t
Signal + noise

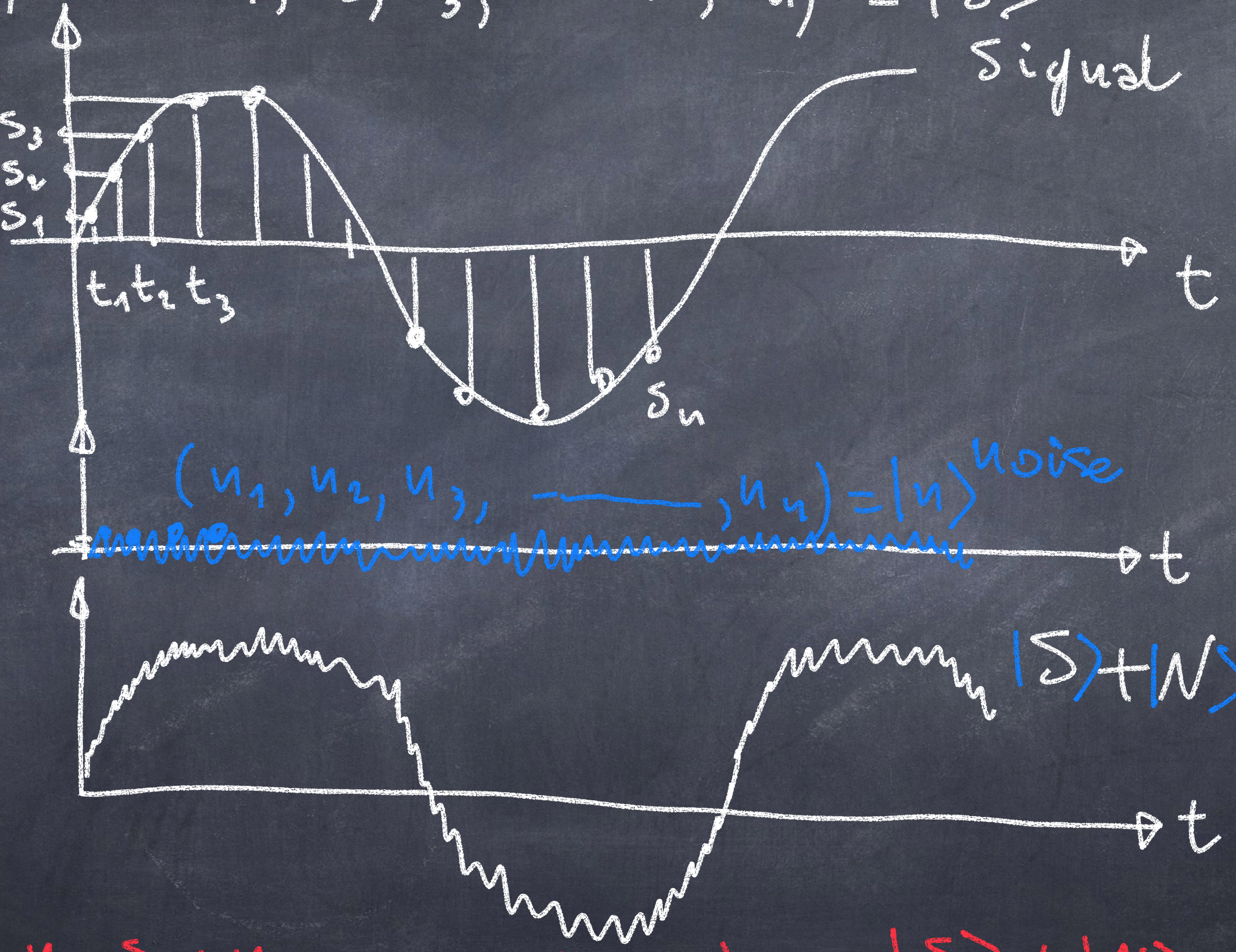
$x(t)$ $\quad (s_1, s_2, s_3, \text{------}, s_n) = |s\rangle$

vectors

nD

$\Updownarrow$

signals



$(n_1, n_2, n_3, \text{------}, n_n) = |n\rangle$ noise

$|S\rangle + |N\rangle = |a\rangle$

$(s_1 + n_1, s_2 + n_2, s_3 + n_3, \text{----}, s_n + n_n) = |S\rangle + |N\rangle$

$a + b + c + d + \cdots\cdots$

$$\underbrace{a_0 + a_1 + a_2 + a_3 + \cdots\cdots + a_n}_{n\text{ - terms}} = \sum_{k=0}^{n \to +\infty ?} a_k = ?$$

k: $0, 1, 2, \cdots\cdots, n$

$a_k = 1$ for each k

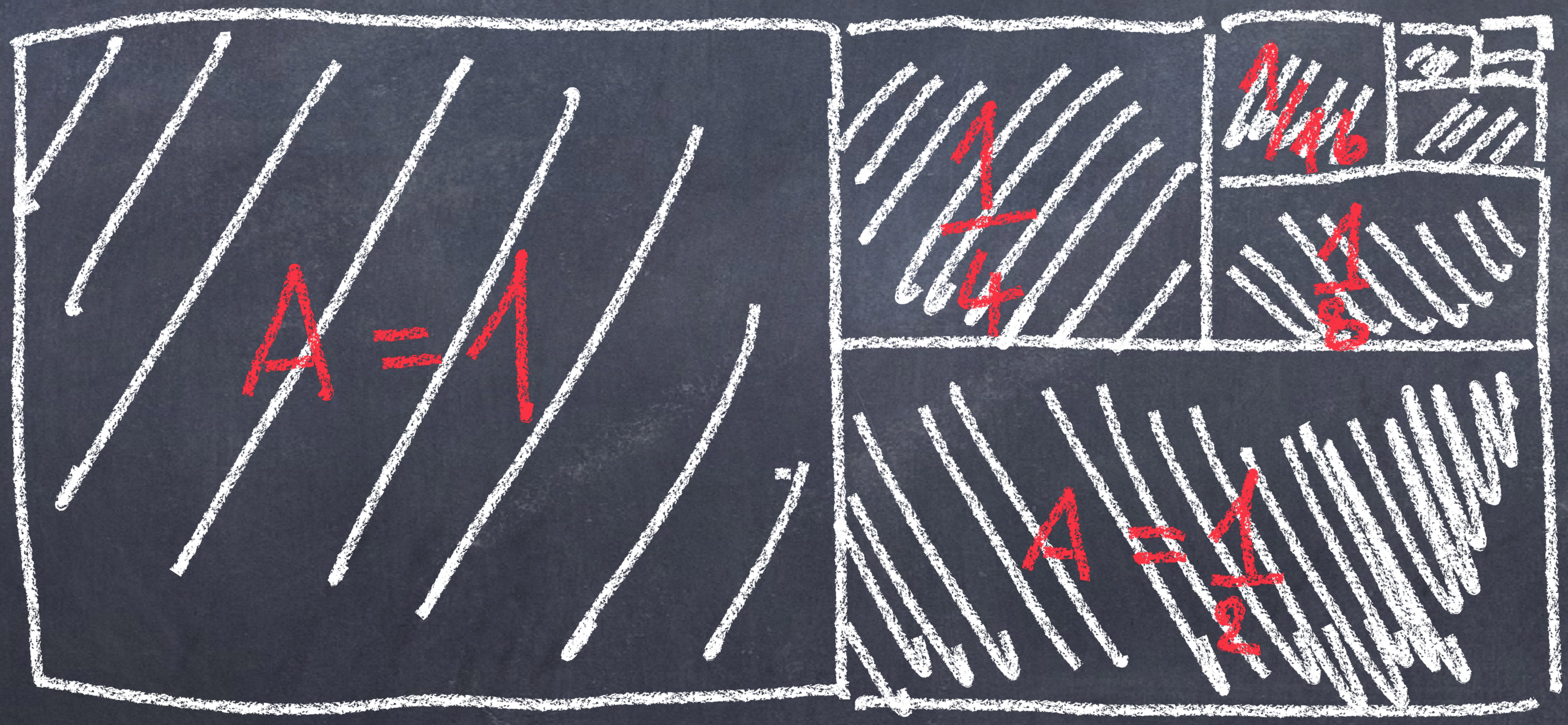
$$\underbrace{1 + 1 + 1 \cdots\cdots + 1}_{n\text{ times}} = n \to +\infty \quad \boxed{\text{DIVERGES}}$$

$a_k = 0$ $\qquad 0 + 0 + 0 + \text{------ to} = 0$ CONVERGES

$$\sum_{k=0}^{n} \frac{1}{2^k} = ?$$

$k=0 \Rightarrow \frac{1}{2^0} = \frac{1}{1} = 1$

$2 =$



$k=1 \Rightarrow \frac{1}{2^1} = \frac{1}{2}$

$k=2 \Rightarrow \frac{1}{2^2} = \frac{1}{4}$

GEOMETRIC SERIES

$$\frac{d^k}{dx^k} e^{ix} \qquad \frac{d^0}{dx^0} e^{ix} = e^{ix}$$

$$e^u$$

$$\frac{d}{du} e^u = e^u \cdot \overbrace{\frac{du}{du}}^{=1}$$

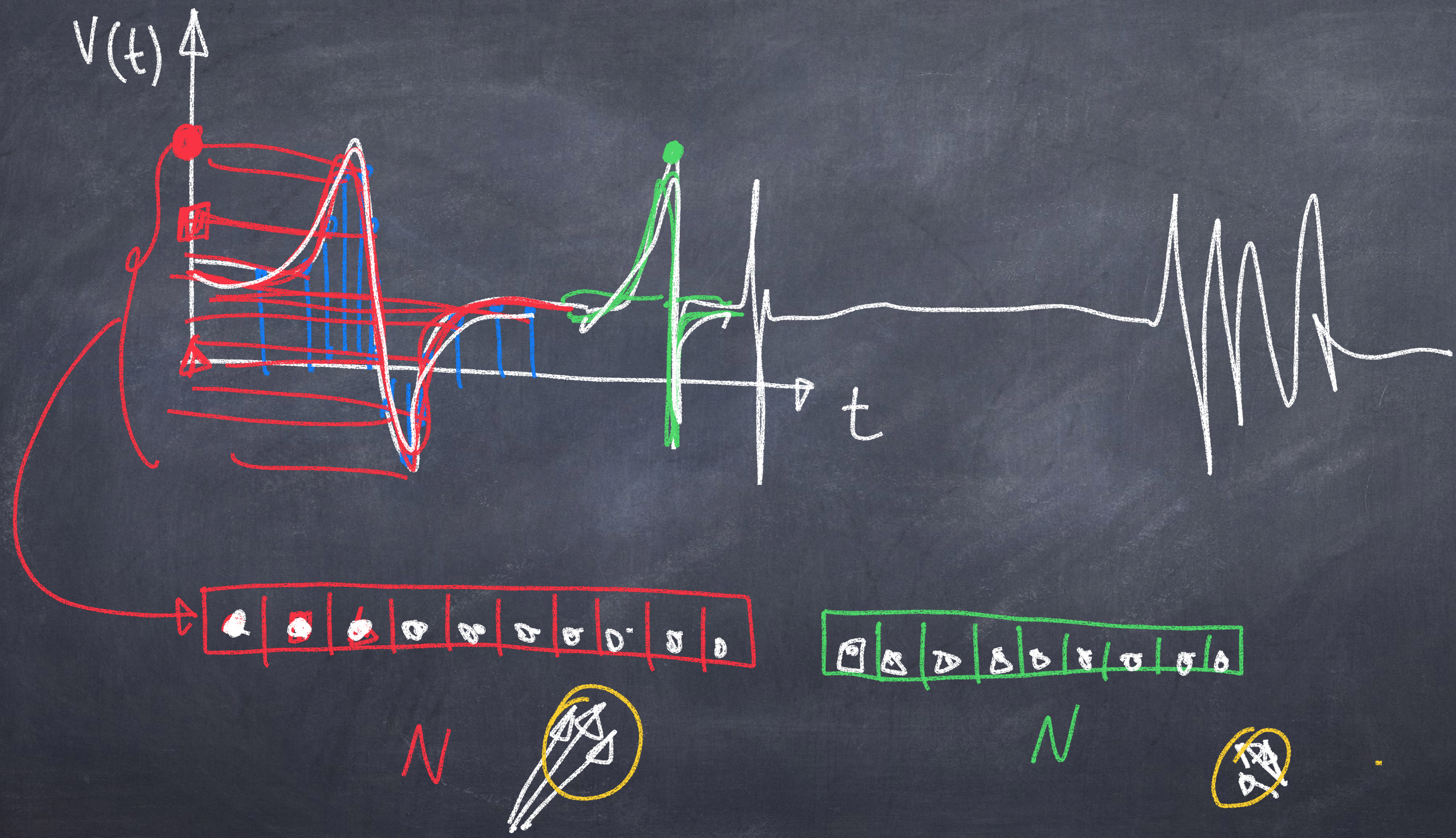
$$e^u$$

$$\frac{d}{dx} e^{ix} = e^{ix} \cdot \frac{d}{dx}(ix) = i \cdot e^{ix}$$

$$\frac{d^2}{dx^2} e^{ix} = \frac{d}{dx}\left[\frac{d}{dx} e^{ix}\right] = \frac{d}{dx}\left(i \cdot e^{ix}\right) = i \frac{d}{dx} e^{ix} =$$

$$= i \cdot i \cdot e^{ix} = i^2 \cdot e^{ix} = -e^{ix}$$

$$\boxed{i^2 = -1}$$



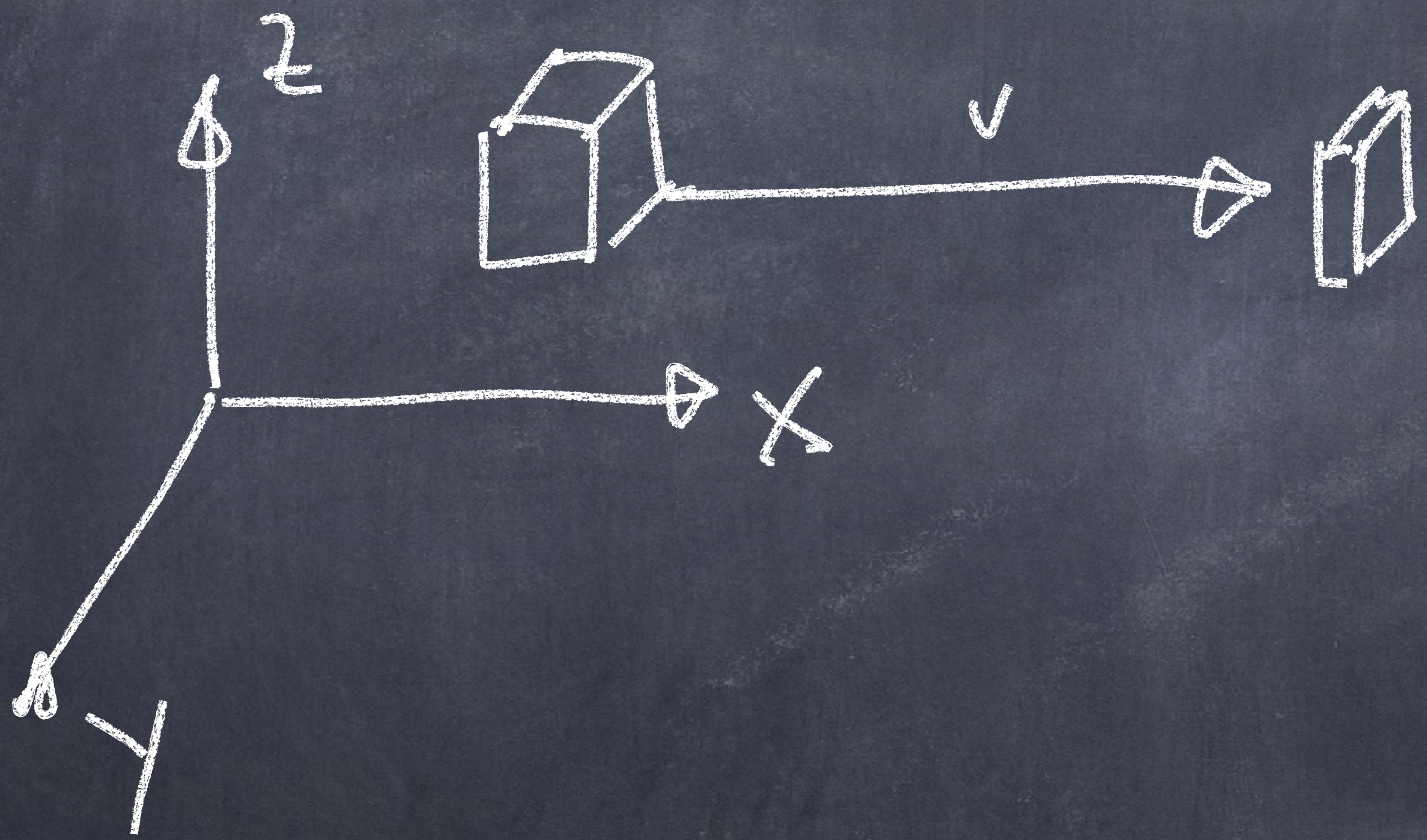
V(t)
t
N
N

$E = \cancel{\boxed{\frac{1}{2} m v^2}}$

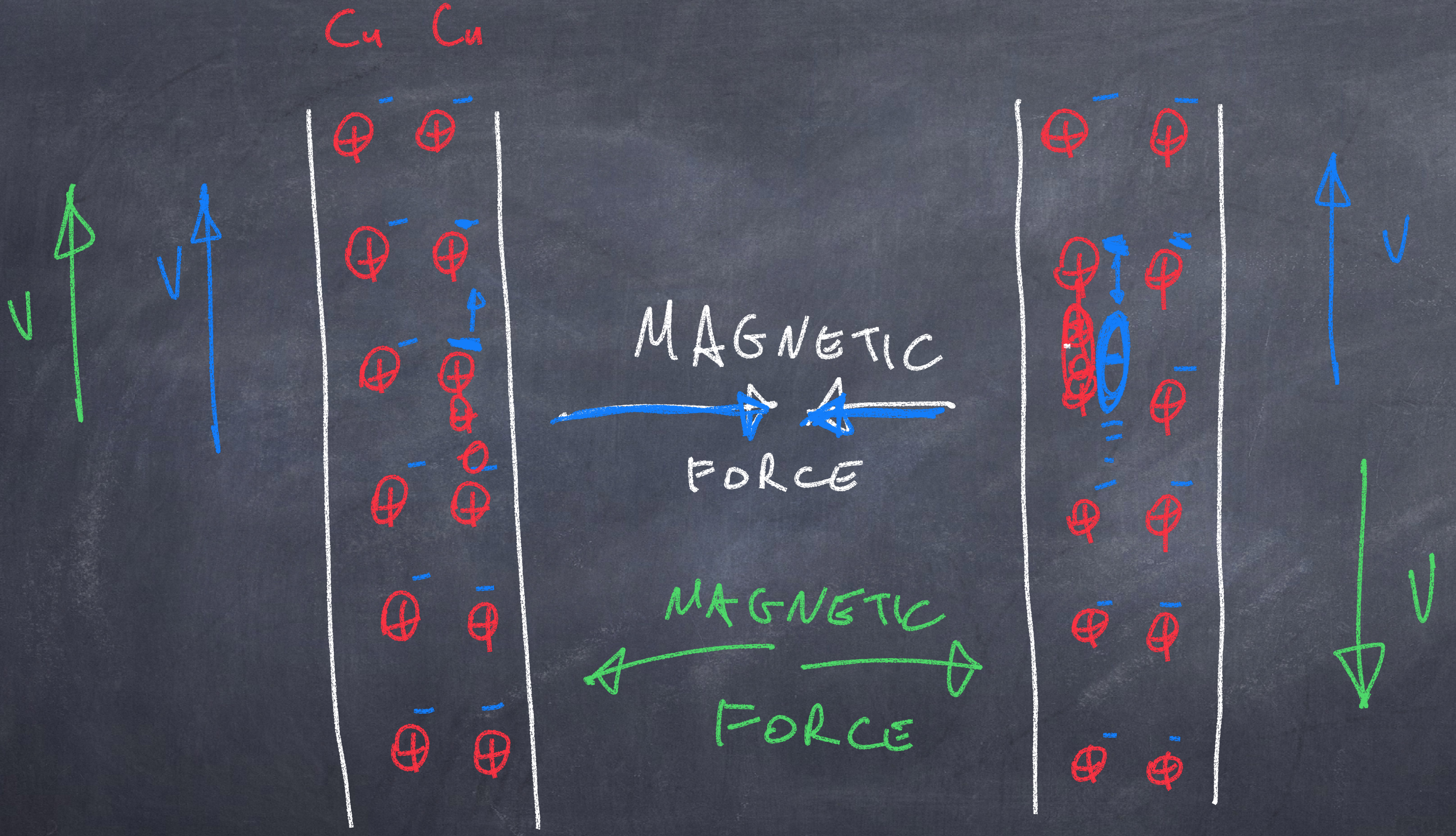
$v \to c$ speed of light

(x, y, z, ct) : $\boxed{4D}$

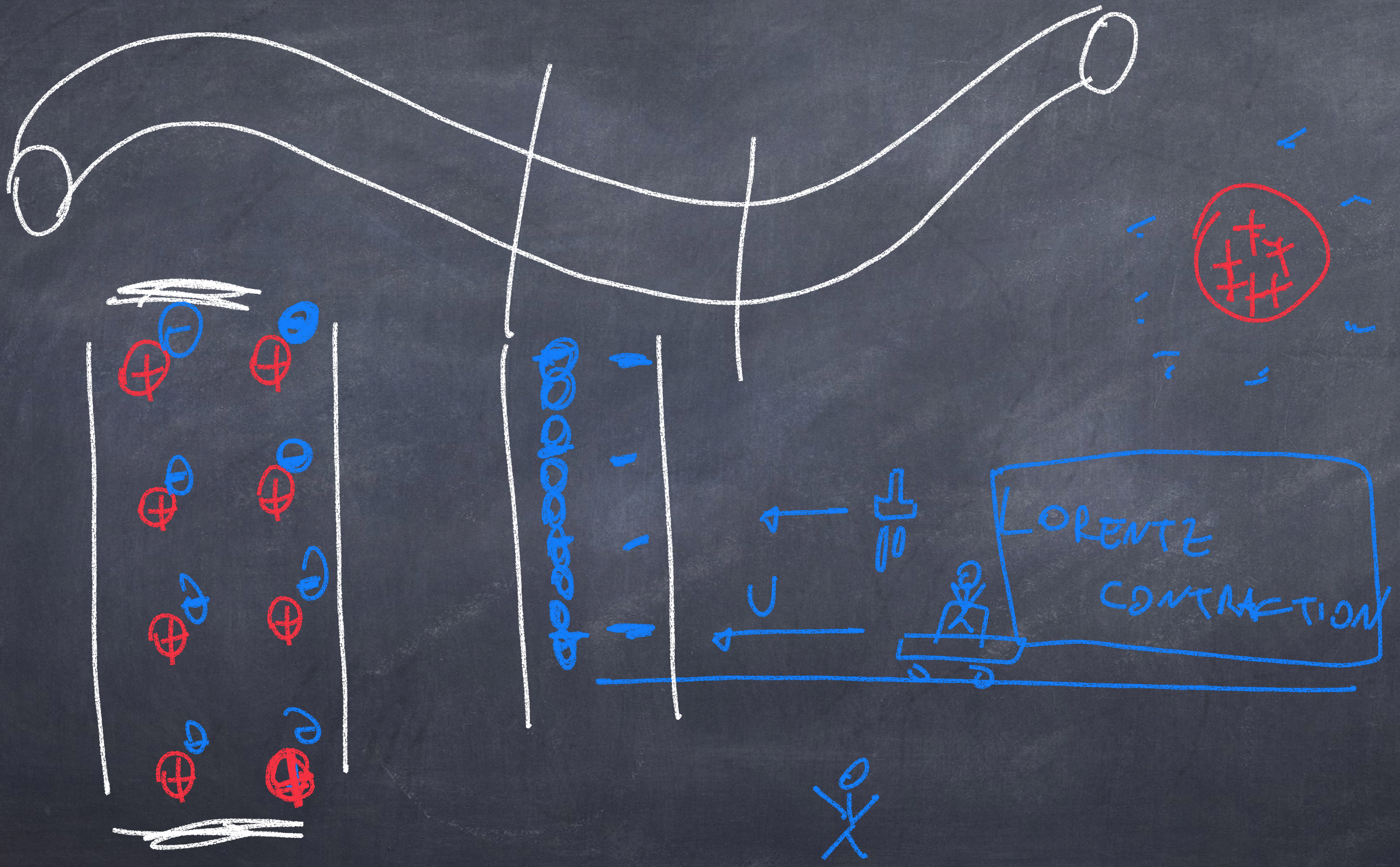
relativity



Lorentz contraction



Cu Cu
V
V
MAGNETIC
FORCE
MAGNETIC
FORCE
V
V


LORENTZ
CONTRACTION

$$\begin{cases} R_1 I_A + R_2 (I_A - I_B) = V_1 \\ R_2 (I_B - I_A) + R_3 I_B + R_4 I_B = 0 \end{cases}$$

$$\begin{cases} R_1 I_A + R_2 I_A - R_2 I_B = V_1 \\ R_2 I_B - R_2 I_A + R_3 I_B + R_4 I_B = 0 \end{cases}$$

$$\begin{cases} [(R_1 + R_2) I_A] [- R_2 I_B] = V_1 \\ [- R_2 I_A] + [(R_2 + R_3 + R_4) I_B] = 0 \end{cases}$$

$$\begin{pmatrix} V_1 \\ 0 \end{pmatrix} = \begin{pmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 + R_4 \end{pmatrix} \cdot \begin{pmatrix} I_A \\ I_B \end{pmatrix}$$

$$\boxed{V = R \cdot I}$$

$$R^{-1} V = \underbrace{R^{-1} R}_{\mathbb{1}} \cdot I$$

$$= I$$

$$Z(\omega) = \frac{V(\omega)}{I(\omega)}$$

[IT DOES NOT DEPEND ON ω]

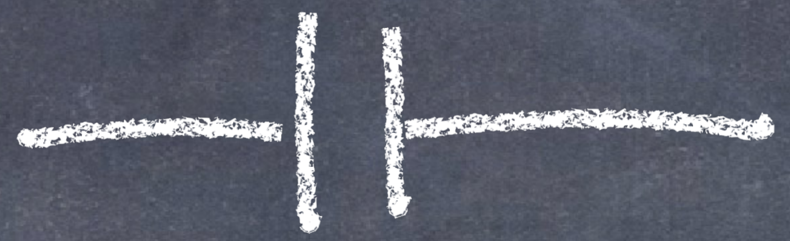
$$\boxed{\begin{aligned} V(\omega) &= V e^{i\omega t} \\ I(\omega) &= I e^{i\omega t} \end{aligned}}$$

PHASORS $\mathbb{C}$

R

$Z(\omega) = R$

C



$Z(\omega) = \frac{1}{i\omega C}$

L

$Z(\omega) = i\omega L$

good for all R, C, L

Z

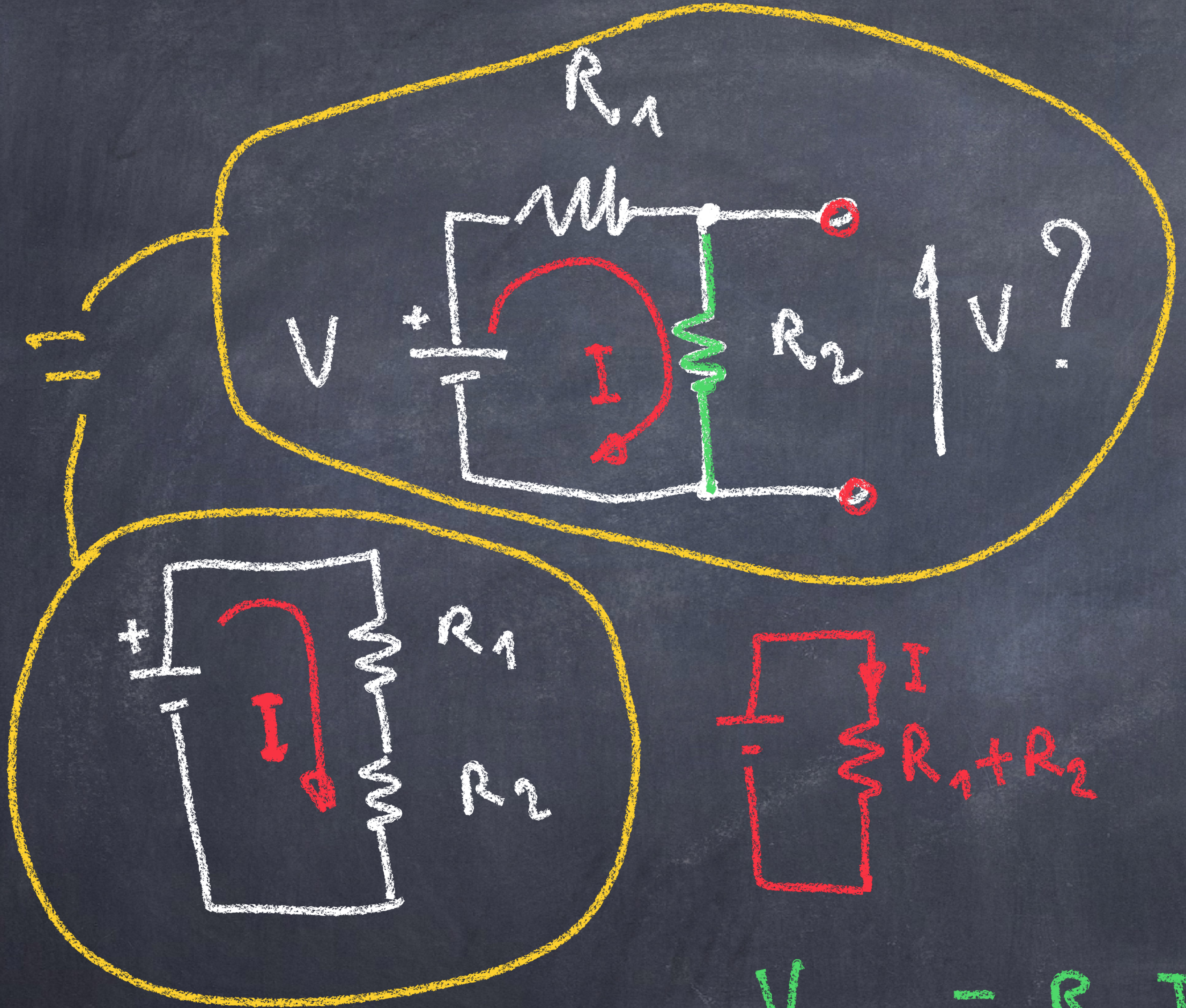
$$[V(\omega) = Z(\omega) \cdot I(\omega)] \quad \boxed{AC}$$

$$[V = R \cdot I] \quad \boxed{DC}$$

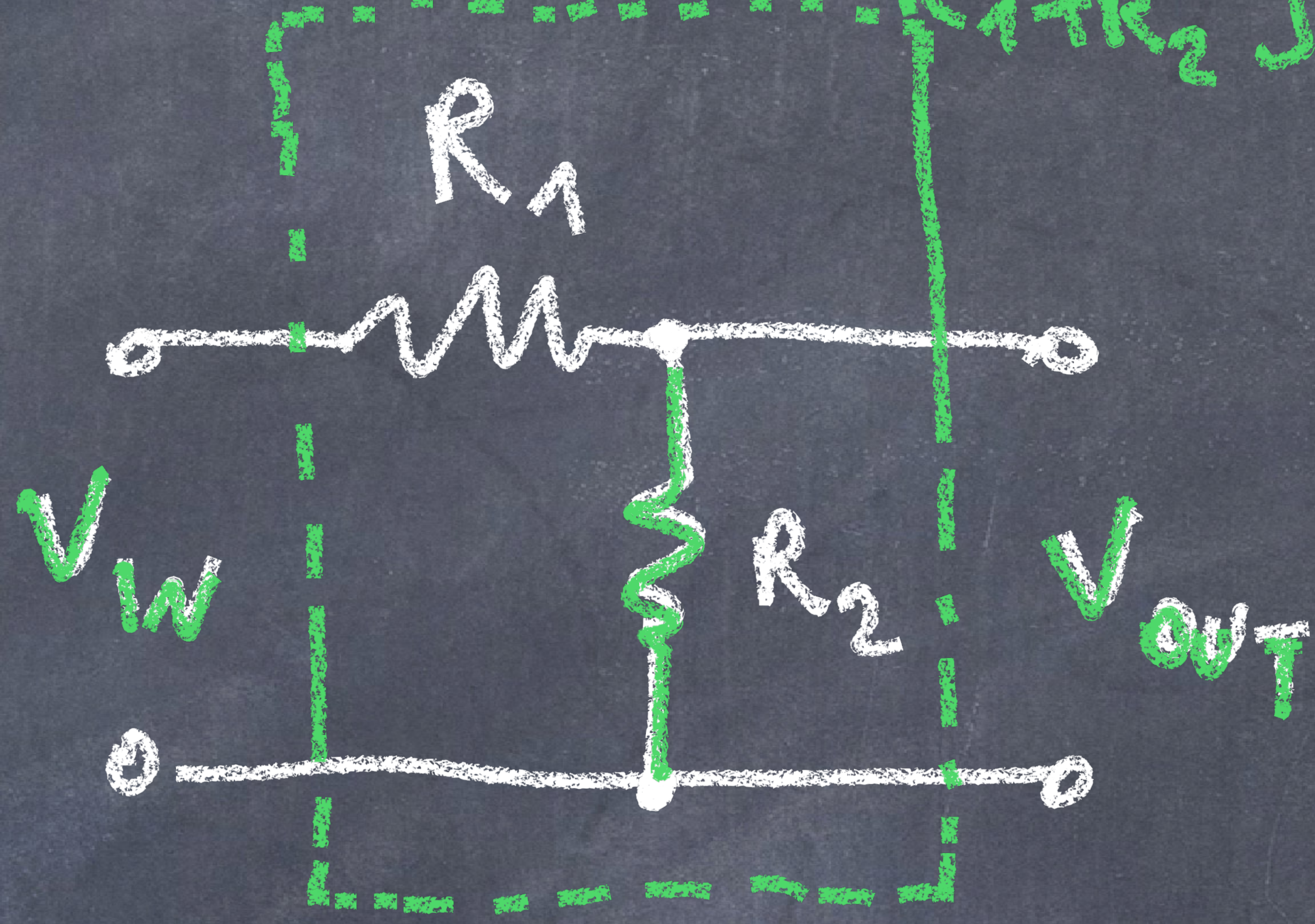
only for resistors

Ohm's Law

VOLTAGE DIVIDER



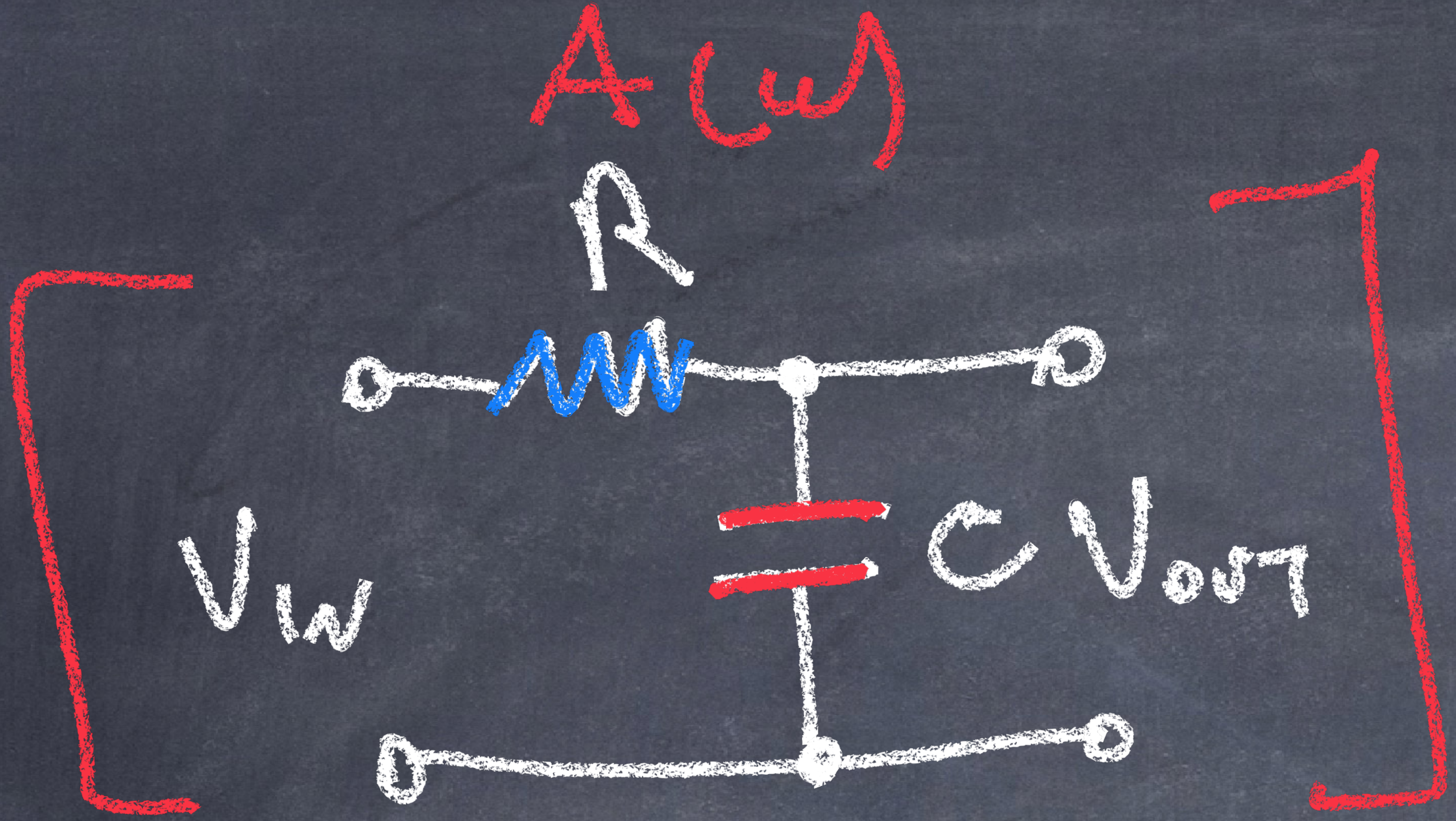
$$\left[A = \frac{R_2}{R_1+R_2} \right]$$



$$V_{OUT} = A \cdot V_{IN}$$

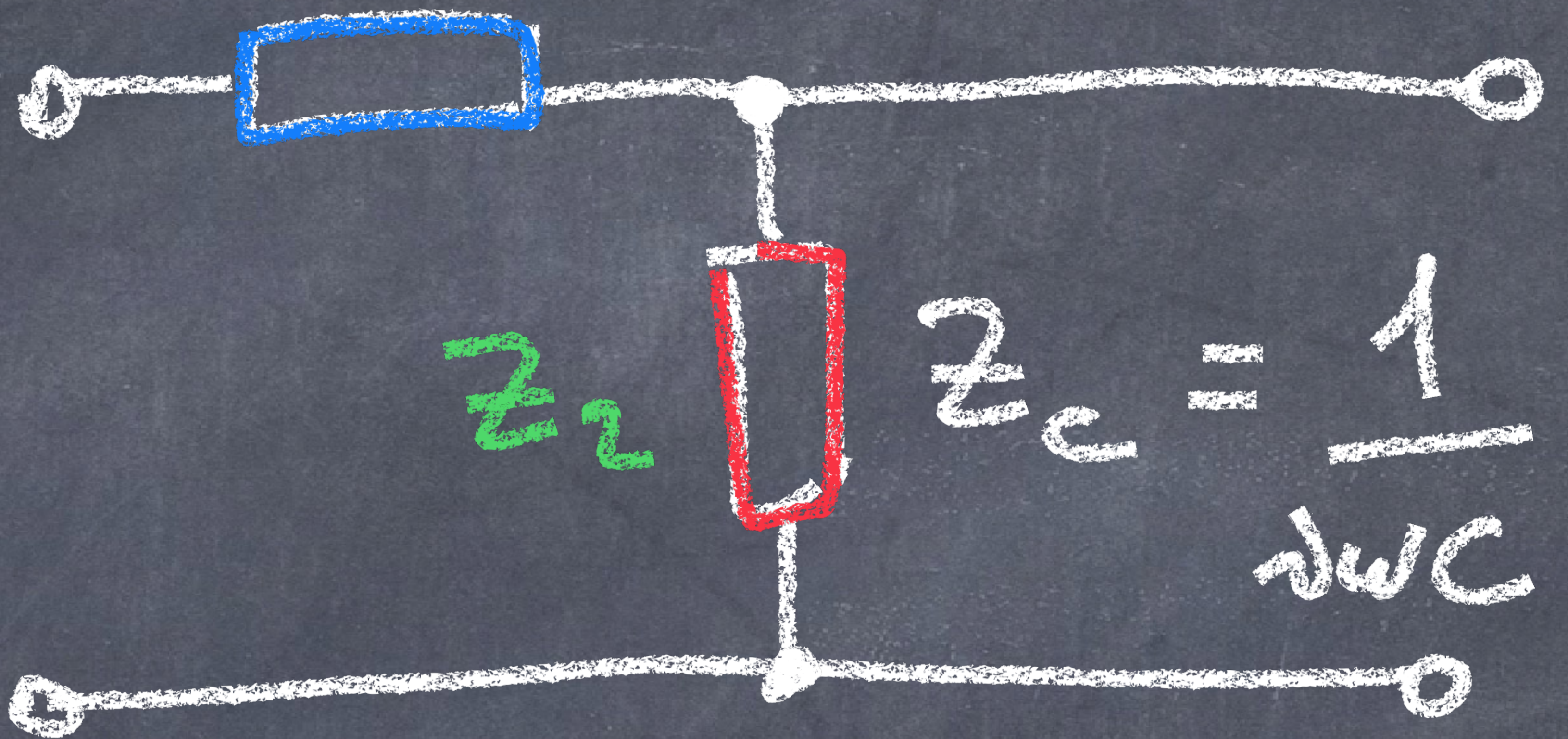
$$I = \frac{V}{R_1+R_2}$$

$$V_{OUT} = R_2 I = V \frac{R_2}{R_1+R_2} = V_{IN} \frac{R_2}{R_1+R_2}$$



=

$Z_1 = Z_R = R$



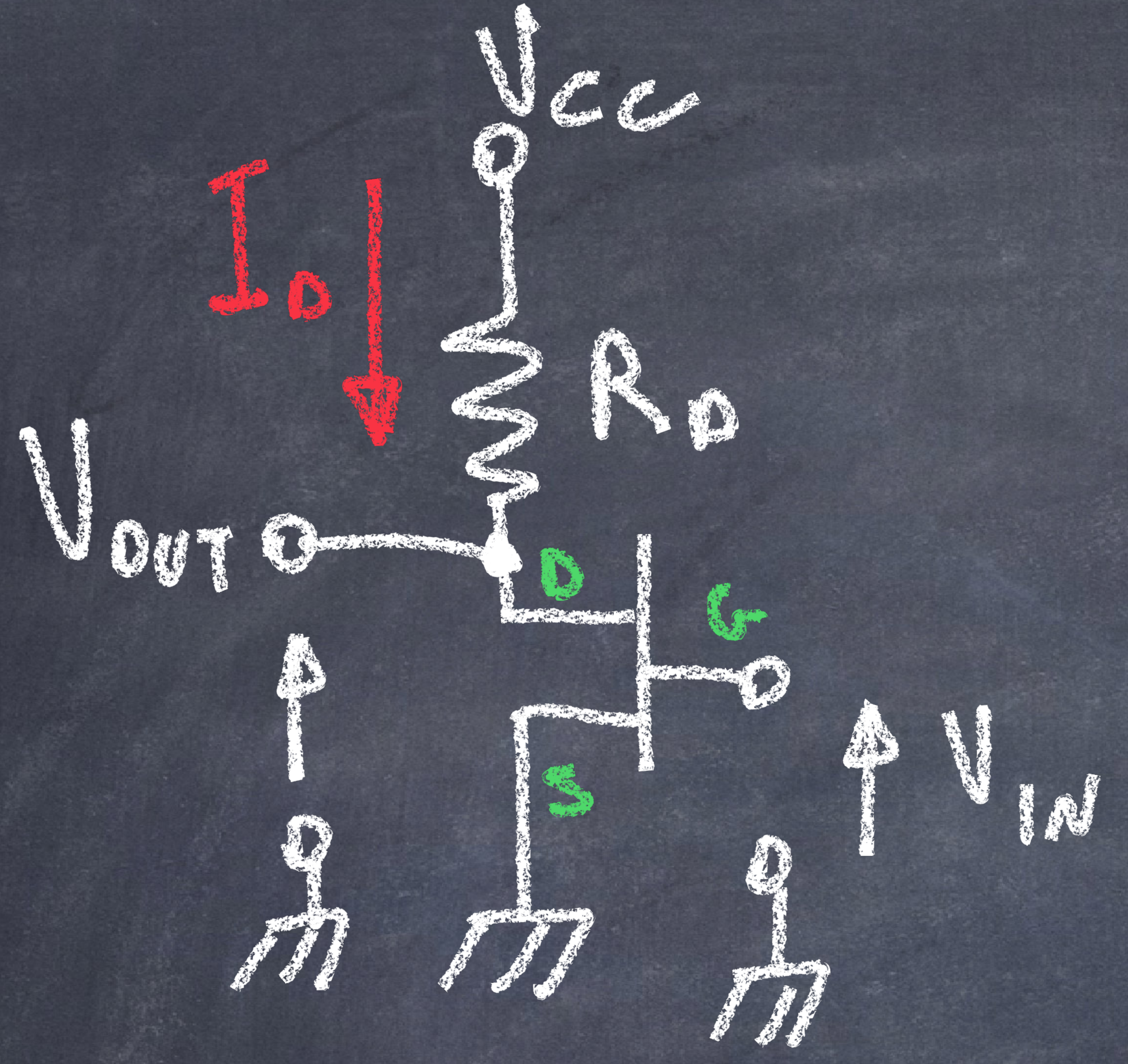
$Z_c = \frac{1}{j\omega C} = \frac{1}{sC}$

$\boxed{s = i\omega}$

$$A = \frac{R_2}{R_1 + R_2}$$

↔

$$A = \frac{Z_2}{Z_1 + Z_2} = \frac{Z_c}{Z_R + Z_c} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{s\underbrace{RC}_{\tau} + 1} = \frac{1}{s\tau + 1}$$



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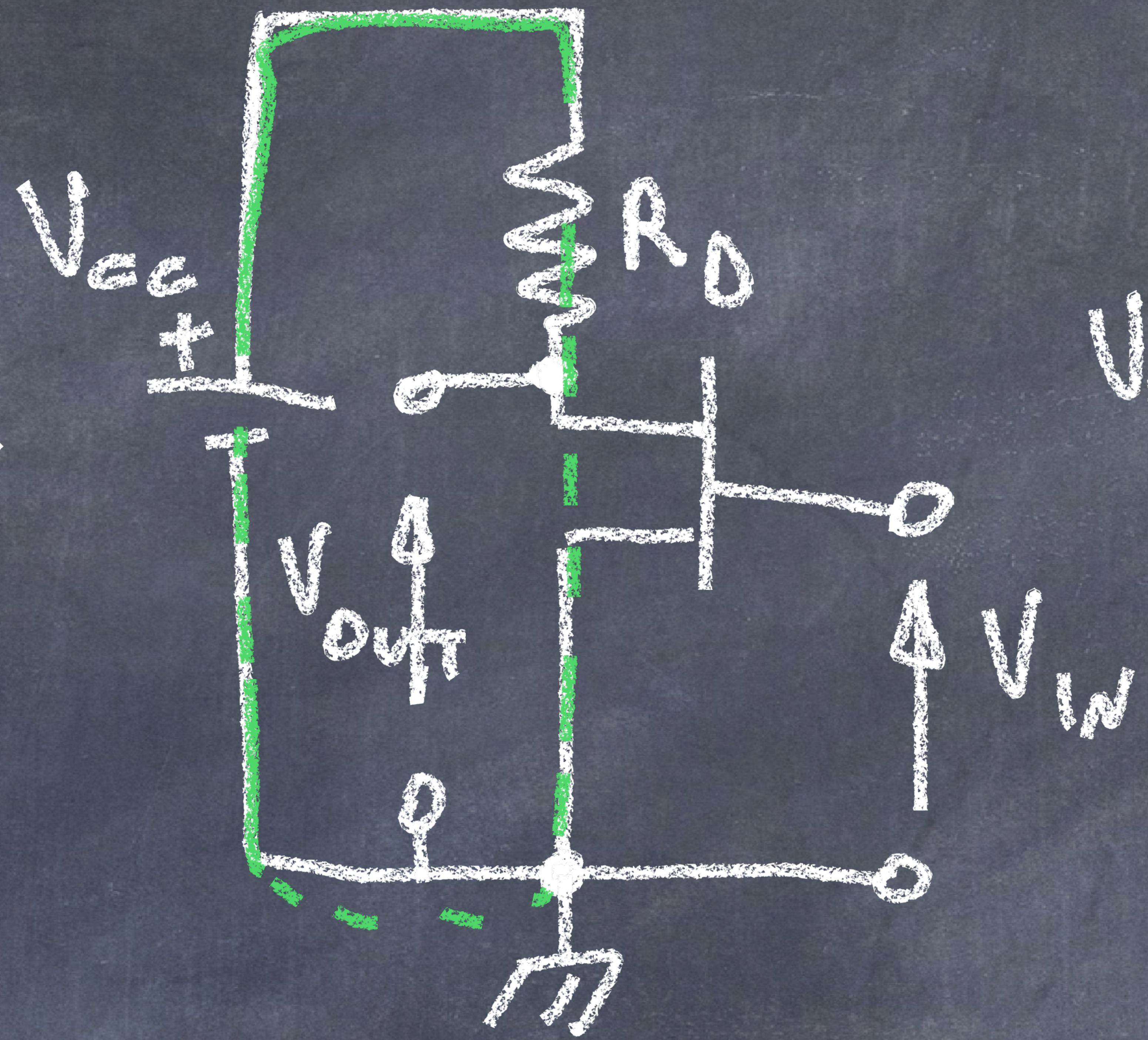
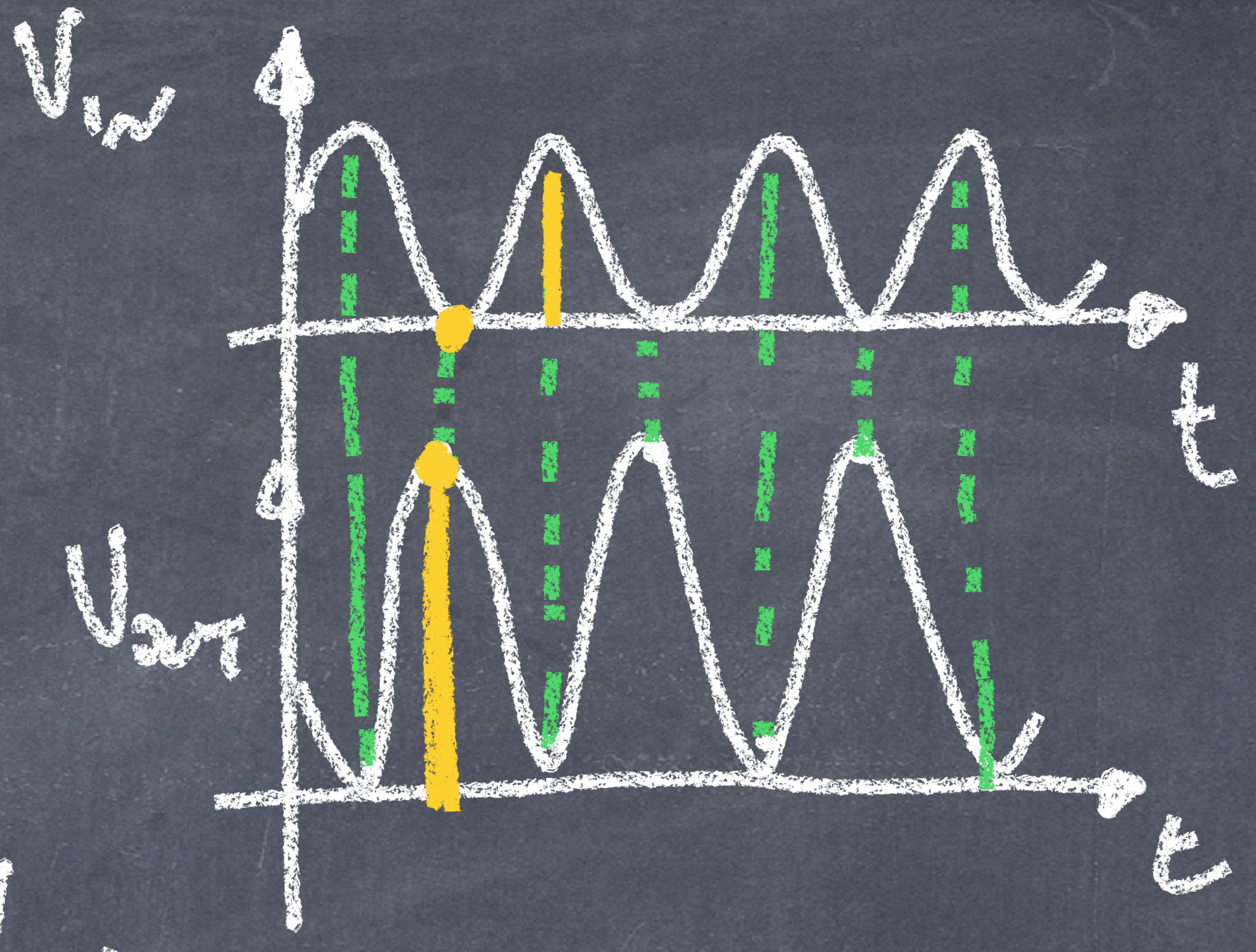




INVERTED

$$V_{R_D} = R_D I_D$$

$$V_{CC} = V_{R_D} + V_{DS} = R_D I_D + V_{OUT} \Rightarrow V_{OUT} = V_{CC} - R_D I_D$$

$$\boxed{I_D = g_m V_{GS}} = g_m V_{IN}$$

const?

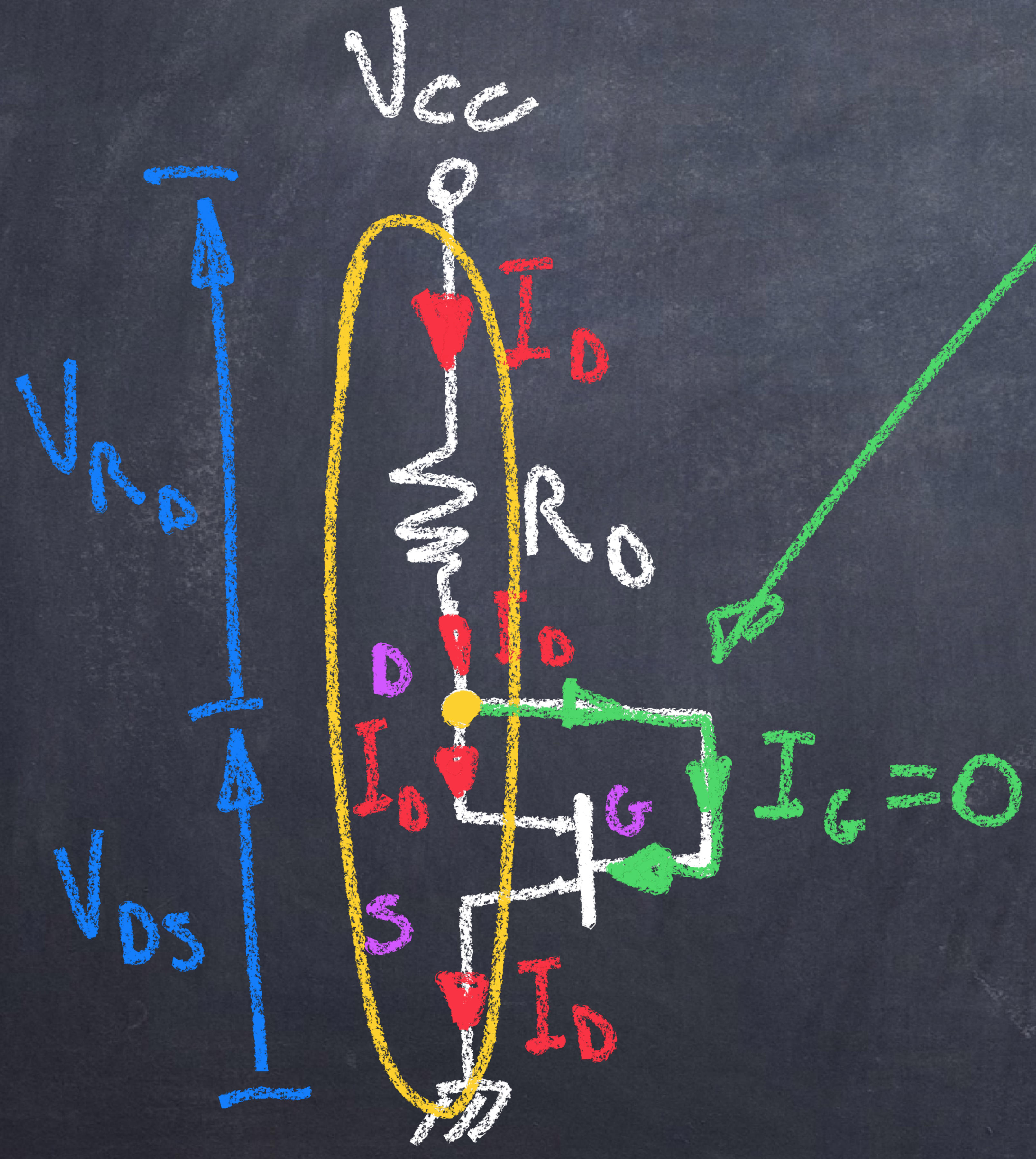
$$\boxed{V_{OUT} = V_{CC} - R_D g_m V_{IN}}$$

$$\begin{cases} R_D I_D + V_{DS} = V_{cc} \\ I_D = g_m V_{GS} \Rightarrow V_{GS} = \dfrac{I_D}{g_m} = V_{DS} \\ \boxed{V_{DS} = V_{GS}} \end{cases}$$

CURRENT MIRROR

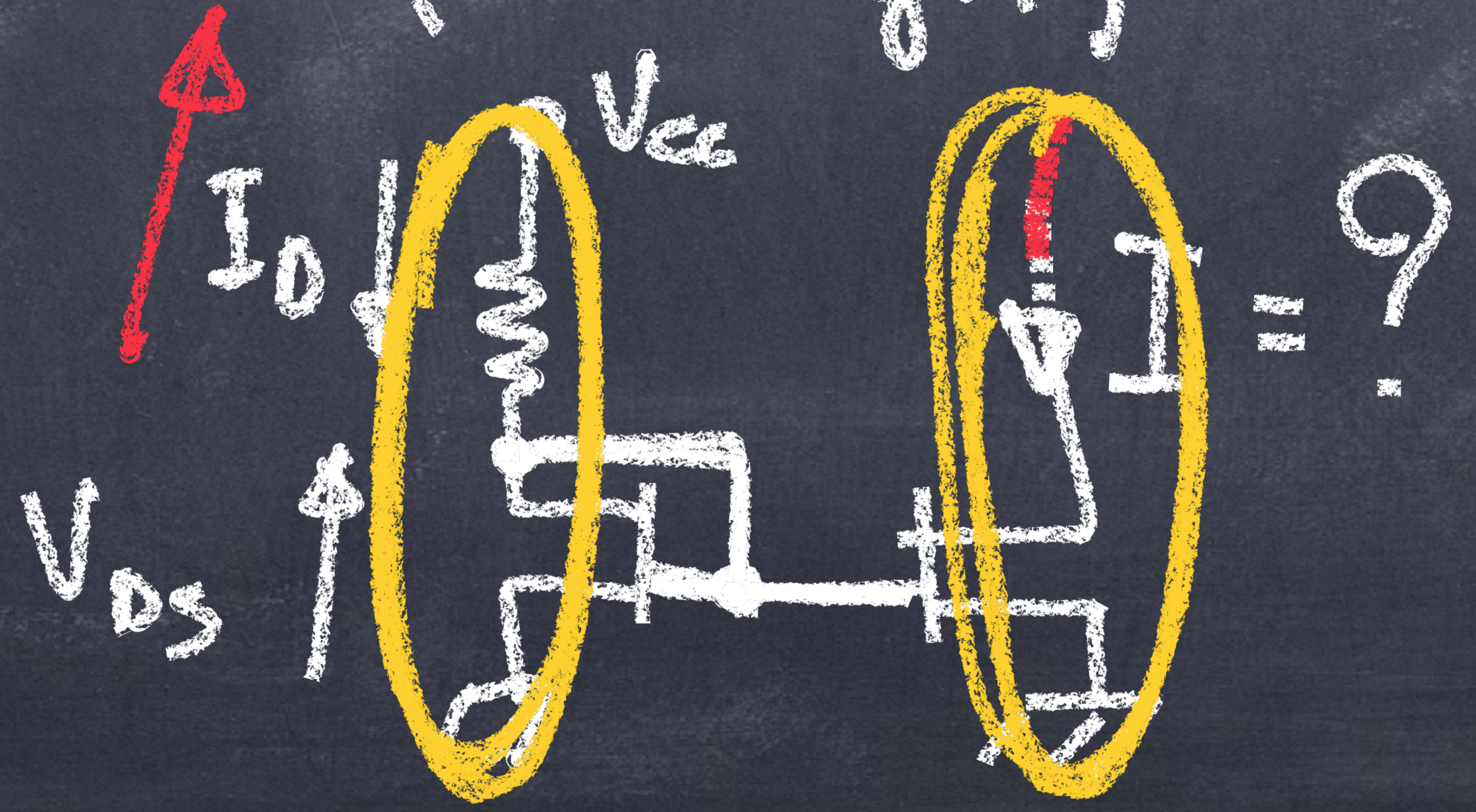
CURRENT GENERATOR

I_D

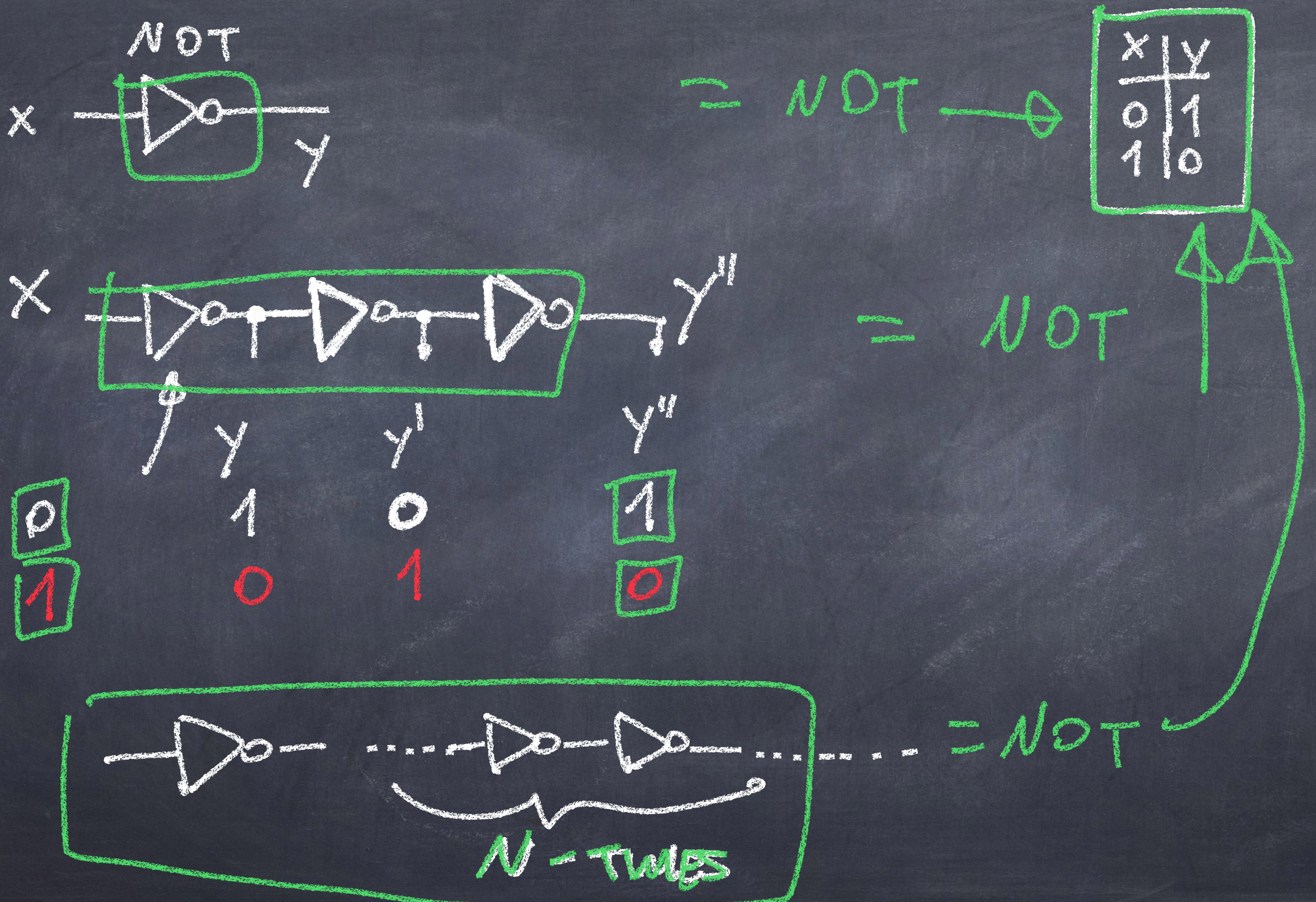


$$\boxed{R_D I_D + \frac{I_D}{g_m} = V_{cc}}$$

$$I_D\left(R_D + \frac{1}{g_m}\right) = V_{cc} \Rightarrow \boxed{I_D = \frac{V_{cc}}{R_D + \frac{1}{g_m}}}$$

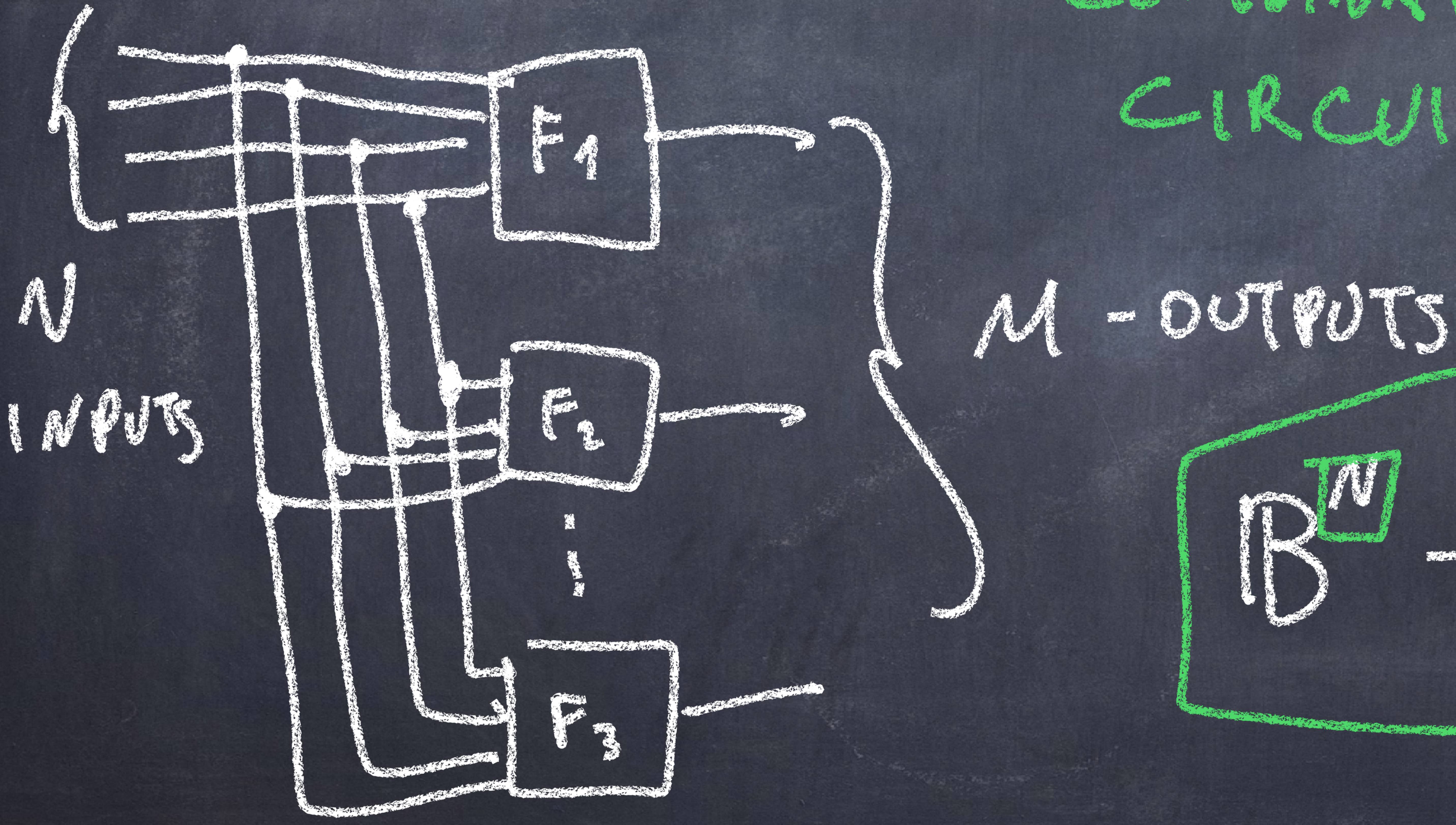


$$\boxed{I = g_m V_{GS} = I_D}$$


NOT
x
y
= NOT
x | y
0 | 1
1 | 0
x
y''
= NOT
y
y'
y''
0 1 0 1
1 0 1 0
= NOT
N - TIMES

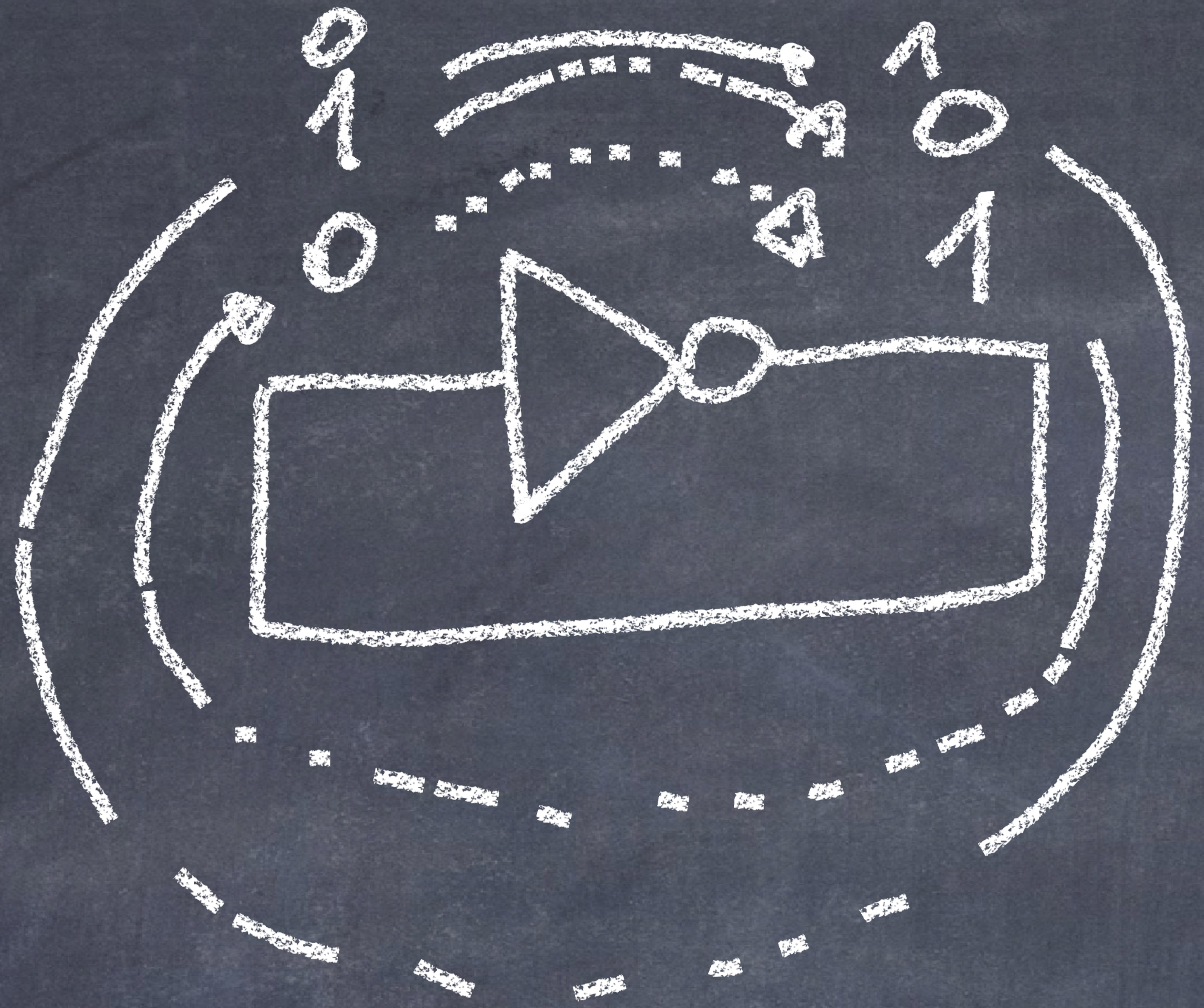
$$\underset{\text{1 OUTPUT}}{Y} = F\underbrace{(x_1, x_2, \ldots, x_N)}_{\text{N-INPUTS}}$$

COMBINATIONAL LOGIC CIRCUITS



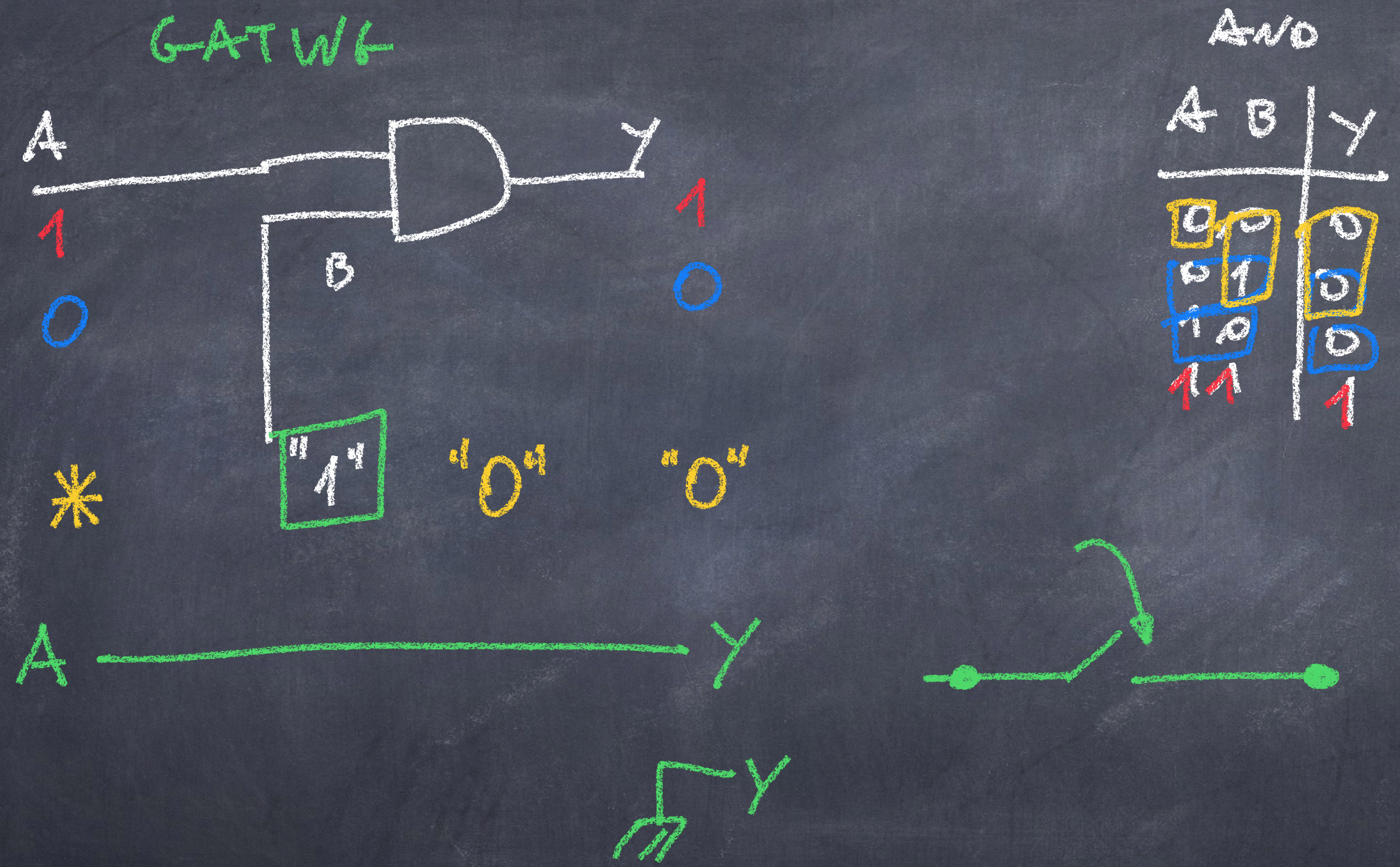
TRUTH TABLES

$$\mathbb{B}^{\boxed{N}} \xrightarrow{F} \mathbb{B}^{\boxed{M}}$$



? UNDEFINED

[NON-LOGICAL]


AND
A B Y
0 0 0
0 1 0
1 0 0
1 1 1
A
Y
B
1
0
"1"
"0"
"0"
A
Y
Y