Testi del Syllabus

Resp. Did.	**MASPERO ALBERTO**	**Matricola: 029273**

Docente	**MASPERO ALBERTO, 6 CFU**

Anno offerta:	**2021/2022**
Insegnamento:	**528SM - FUNCTIONAL ANALYSIS**
Corso di studio:	**SM34 - MATEMATICA**
Anno regolamento:	**2021**
CFU:	**6**
Settore:	**MAT/05**
Tipo Attività:	**C - Affine/Integrativa**
Anno corso:	**1**
Periodo:	**Secondo Semestre**
Sede:	**TRIESTE**

Testi in italiano

Lingua insegnamento	English
Contenuti (Dipl.Sup.)	Spectrum of linear operators. Spectral theorem for compact and self-adjoint bounded operators in Hilbert spaces. Unbounded linear operators in Hilbert spaces. Applications: Sturm-Liouville type problems and the Schrödinger equation. Differential calculus in Banach spaces. The implicit function theorem. Lyapunov–Schmidt reduction and bifurcation. Nash-Moser iteration. Theory of topological degree and fix point theorems: Brouwer, Schauder.
Testi di riferimento	A. Ambrosetti, G. Prodi: A primer of nonlinear analysis, Cambridge, Cambridge University Press 1993. H. Brezis: Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer-Verlag New York, 2011. K. Deimling: Nonlinear Function Analysis, Springer-Verlag Berlin Heidelberg, 1984. B. Helffer: Spectral Theory and its Applications, Cambridge University Press, 2013. E. Kreyszig: Introductory Functional Analysis with Applications, Wiley Classics Library, 1989 M. Reed, B. Simon: Methods of Modern Mathematical Physics I - Functional analysis, Elsevier, 1972.
Obiettivi formativi	KNOWLEDGE AND UNDERSTANDING At the end of the course the student will have to demonstrate knowledge of the fundamental object of functional analysis, both linear and nonlinear. CAPACITY TO APPLY KNOWLEDGE AND UNDERSTANDING At the end of the course the student must know how to apply the knowledge of basic functional analysis acquired to solve problems and exercises of medium difficulty. The exercises can also be proposed as easy theoretical results. JUDGMENT AUTONOMY At the end of the course the student will be able to recognize and apply

	the most basic techniques of functional analysis and also to recognize the situations and problems in which these techniques can be used advantageously. COMMUNICATIVE SKILLS At the end of the course the student will be able to express himself appropriately on the topics of the course. LEARNING CAPACITY At the end of the course the student should be able to consult the standard texts of functional analysis, both linear and nonlinear.
Prerequisiti	Basic functional spaces: continuous functions and Lebesgue space; weak topologies and a basic knowledge of the theory of Sobolev spaces.
Metodi didattici	Lectures and problem sessions. During the course some exercises will be discussed in class.
Altre informazioni	Information about the progress of the program and teaching materials will be posted on the site https://people.sissa.it/~amaspero/teaching.html ********** Any changes to the methods described here, which become necessary to ensure the application of the security protocols related to the COVID19 emergency, will be communicated on the website of the Department, the Study Program and the teaching.
Modalità di verifica dell'apprendimento	The exam program coincides with the arguments of the lectures. The exam will be oral consisting in verifying the comprehension of the contents (definitions and proofs) and the ability in explaining the subject and to correctly apply the theory.
Programma esteso	Part 1) Linear Analysis. Linear bounded operators. Compactness in infinite dimensional spaces, characterization of compactness in spaces of sequences and in Lp spaces. Compact operators. Fredholm's alternative Theorem. Definition of resolvent and of spectrum of a linear operator, proof that the spectrum is non-empty and closed. Spectrum of self-adjoint operators. Spectral theory for compact and self-adjoint operators. Min-max Fisher-Courant Theorem. Spectral theorem for self-adjoint non-compact operators. Functional calculus. Applications to Sturm-Liouville problems. Linear ODEs in Banach spaces. The stationary and time dependent Schrödinger equation. Part 2) Nonlinear Analysis. Differential Calculus in Banach spaces. Gateaux and Frechet differentiable functions, examples. Mean value Theorem. Higher order derivatives. Differentiability of Nemiski operators. Inverse function Theorem and Implicit function Theorem. Applications to non-linear problems. Constant rank theorems. Lagrange multiplier Theorem and applications. Bifurcation Theorem, necessary condition of bifurcation, Lyapunov-Schmidt reduction. Bifurcation of the simple eigenvalue. Nash-Moser iteration. ODE in Banach spaces, differentiable dependence on initial data. Brower fixed point Theorem, Borsuk Ulam Theorem, Leray Schauder degree, Schauder fixed point Theorem.

Testi in inglese

	English

Spectrum of linear operators. Spectral theorem for compact and self-adjoint bounded operators in Hilbert spaces.
Unbounded linear operators in Hilbert spaces. Applications: Sturm-Liouville type problems and the Schrödinger equation.
Differential calculus in Banach spaces. The implicit function theorem. Lyapunov–Schmidt reduction and bifurcation.
Nash-Moser iteration. Theory of topological degree and fix point theorems: Brouwer, Schauder.

A. Ambrosetti, G. Prodi: A primer of nonlinear analysis, Cambridge, Cambridge University Press 1993.
H. Brezis: Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer-Verlag New York, 2011.
K. Deimling: Nonlinear Function Analysis, Springer-Verlag Berlin Heidelberg, 1984.
B. Helffer: Spectral Theory and its Applications, Cambridge University Press, 2013.
E. Kreyszig: Introductory Functional Analysis with Applications, Wiley Classics Library, 1989
M. Reed, B. Simon: Methods of Modern Mathematical Physics I - Functional analysis, Elsevier, 1972.

KNOWLEDGE AND UNDERSTANDING
At the end of the course the student will have to demonstrate knowledge of the fundamental object of functional analysis, both linear and nonlinear.
CAPACITY TO APPLY KNOWLEDGE AND UNDERSTANDING
At the end of the course the student must know how to apply the knowledge of basic functional analysis acquired to solve problems and exercises of medium difficulty. The exercises can also be proposed as easy theoretical results.
JUDGMENT AUTONOMY
At the end of the course the student will be able to recognize and apply the most basic techniques of functional analysis and also to recognize the situations and problems in which these techniques can be used advantageously.
COMMUNICATIVE SKILLS
At the end of the course the student will be able to express himself appropriately on the topics of the course.
LEARNING CAPACITY
At the end of the course the student should be able to consult the standard texts of functional analysis, both linear and nonlinear.

Basic functional spaces: continuous functions and Lebesgue space; weak topologies and a basic knowledge of the theory of Sobolev spaces.

Lectures and problem sessions. During the course some exercises will be discussed in class.

Information about the progress of the program and teaching materials will be posted on the site
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The exam program coincides with the arguments of the lectures. The exam will be oral consisting in verifying the comprehension of the contents (definitions and proofs) and the ability in explaining the subject and to correctly apply the theory.

Part 1) Linear Analysis.
Linear bounded operators. Compactness in infinite dimensional spaces, characterization of compactness in spaces of sequences and in Lp spaces. Compact operators. Fredholm’s alternative Theorem. Definition

of resolvent and of spectrum of a linear operator, proof that the spectrum is non-empty and closed. Spectrum of self-adjoint operators. Spectral theory for compact and self-adjoint operators. Min-max Fisher-Courant Theorem.
Spectral theorem for self-adjoint non-compact operators. Functional calculus.
Applications to Sturm-Liouville problems. Linear ODEs in Banach spaces. The stationary and time dependent Schrödinger equation.

Part 2) Nonlinear Analysis.
Differential Calculus in Banach spaces. Gateaux and Frechet differentiable functions, examples. Mean value Theorem. Higher order derivatives. Differentiability of Nemiski operators. Inverse function Theorem and Implicit function Theorem. Applications to non-linear problems. Constant rank theorems. Lagrange multiplier Theorem and applications. Bifurcation Theorem, necessary condition of bifurcation, Lyapunov-Schmidt reduction. Bifurcation of the simple eigenvalue. Nash-Moser iteration.
ODE in Banach spaces, differentiable dependence on initial data. Brower fixed point Theorem, Borsuk Ulam Theorem, Leray Schauder degree, Schauder fixed point Theorem.