Quantum simulation of an extra dimension

Alessio Celi

based on PRL 108, 133001 (2012), with O. Boada, J.I. Latorre, M. Lewenstein,

Quantum Simulation Workshop

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- Idea of Quantum Simulator (Feynman)

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- Prospects

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$\rightarrow$ Good candidate: Cold atoms

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Note that once interactions are included the above problems become not calculable

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- (Historical) Prototype: Simulation of Synthetic gauge field in optical lattices
  $\longrightarrow$ We borrow some ideas from there

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Quickly developing research area

Gauging the Hubbard model

Example: Electron (without spin) in a 2d crystal, Hubbard model (small filling)

$$H = -J_x \sum_{m,n=-\infty}^{\infty} a^\dagger_{m+1,n} a_{m,n} - J_y \sum_{m,n=-\infty}^{\infty} a^\dagger_{m,n+1} a_{m,n} + h.c.$$

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Generalization to non abelian theory in D dim.:

- $a_{m,n} \to a^\sigma_{\mathbf{r}}$, $\mathbf{r} \in \mathbb{R}^D$, ex. SU$(N)$, $\sigma = 1, \dots N$
- $A_\mu \equiv A^I_\mu T_I$, T_I gauge g. generators, hopping phase $\to$ hopping matrix

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Final result

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For $\mathbf{k}_e - \mathbf{k}_g \,//\, \hat{x} \rightarrow A_y(m,n) = 2\pi\Phi m$, Constant magnetic field in Landau gauge

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Observations:

- classical background gauge field configuration (no dynamics)
- always gauge-fixed Hamiltonian
- relativistic matter & extradimension simulations require synthetic gauge field

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(during the workshop) in OL 3D Hubbard model $\longrightarrow$ 1D,2D by tuning optical potential

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$a^{(\sigma)}_{\mathbf{r}}$,$a^{(\sigma)\dagger}_{\mathbf{r}}$ as Fock generators of different species on same site, extra-dim $\rightarrow$ internal d.o.f
Hopping in extra-dim $\rightarrow$ transmutation between “contiguous” species,

Modelling extra-dimensions (Boada,AC,Latorre,Lewenstein, PRL 2012)

(during the workshop) in OL 3D Hubbard model $\rightarrow$ 1D,2D by tuning optical potential
and > 3D? In a lattice Dimensionality $\equiv$ Connectivity
Basic idea: just kinetic term in D+1 hypercubic lattice

$$H = -J \sum_{\mathbf{q}} \sum_{j=1}^{D+1} a^{\dagger}_{\mathbf{q}+\mathbf{u}_j} a_{\mathbf{q}} + H.c.\,,$$

$\mathbf{q} = (\mathbf{r}, \sigma)$, $D+1$-space in D-layers

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Hopping in extra-dim $\rightarrow$ transmutation between “contiguous” species, 2 ways:

- State-dependent lattice (cf JZ proposal)
- On-site dressed lattice (cf relativistic fermions Toolbox– M.Rizzi talk)

Extra-dimension implementation & detection

State-dependent & On-site dressed, common features

- Open or periodic boundary cond. in the extra-dim $\rightarrow$ Compactification
- Complex hopping on the S^1 (p.b.c.) $\rightarrow$ Flux compactification
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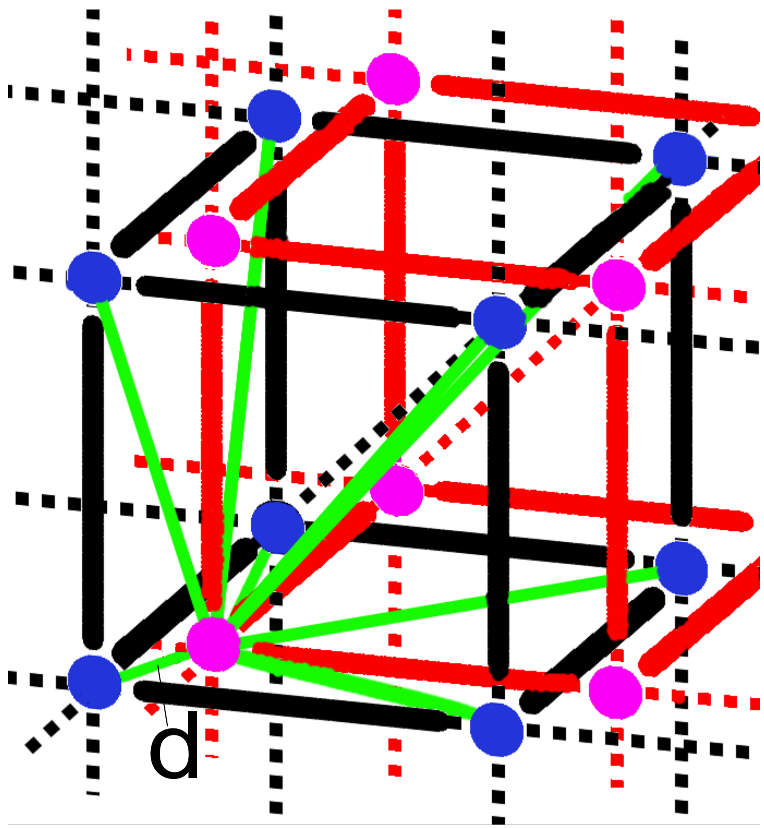
- State-dependent lattice: easy realization

  Example: ^{87}Rb, $F = 1$, $F = 2 \rightarrow$

  Bivolume $\equiv N = 2$ layers



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Observables: dimensionality dependence

- Single-particle, scaling properties, Example: density of states
- Many-body, phase diagram, Example: MI-Superfluid

Exotic Boundary Conditions same people + J.Rodriguez-Laguna

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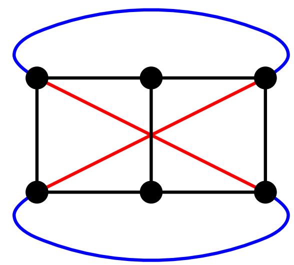

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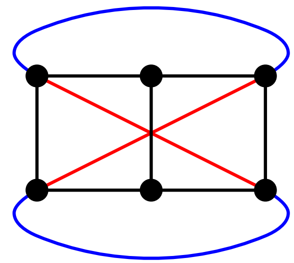

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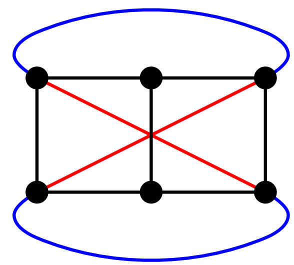

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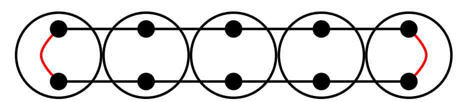
  Idea: Two internal states $+$ Single site addressing
  $\longrightarrow$ Scalable circle



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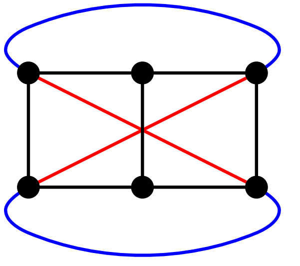

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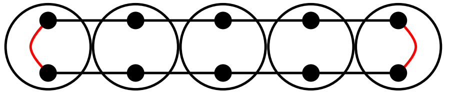

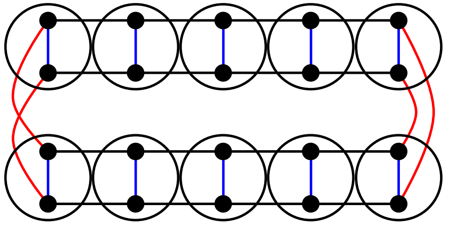

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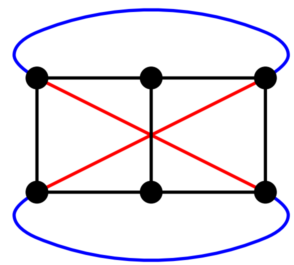

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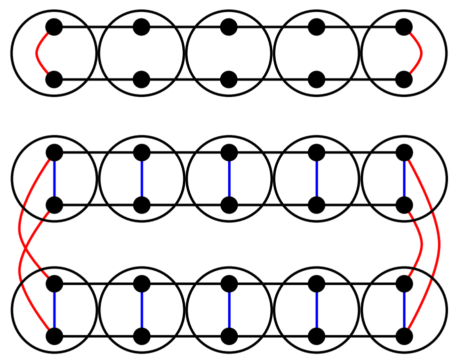

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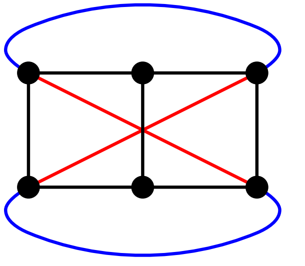

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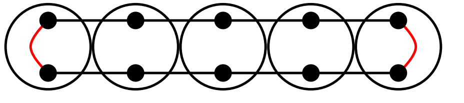

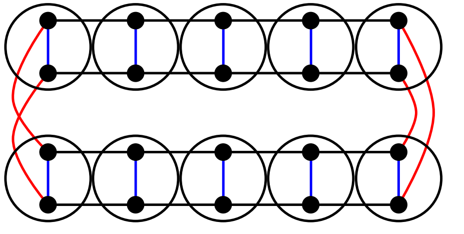

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Additional ingredients: Generalized Twisted B.C. (with phases), Interactions

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