# Mapping the Berry Curvature of Optical Lattices

Nigel Cooper
Cavendish Laboratory, University of Cambridge

Quantum Simulations with Ultracold Atoms
ICTP, Trieste, 16 July 2012

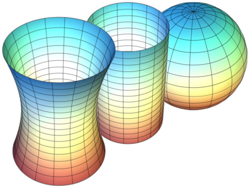
Hannah Price & NRC, PRA **85**, 033620 (2012)

NRC, PRL **106**, 175301 (2011)
NRC & Jean Dalibard, EPL **95**, 66004 (2011)

# Outline

# Topological Invariants

Gaussian curvature $\kappa = \frac{1}{R_1 R_2}$



negative, zero and positive $\kappa$

$$\frac{1}{2\pi}\int_{\text{closed surface}} \kappa \, dA = (2-2g)$$ Gauss-Bonnet Theorem

genus $g = 0, 1, 2, \ldots$ for sphere, torus, 2-hole torus...

Topological invariant: $g$ cannot change under smooth deformations

# Topological Features of 2D Bands

[Thouless, Kohmoto, Nightingale & den Nijs (1982)]

Chern number $\mathcal{C} = \frac{1}{2\pi}\int_{\mathrm{BZ}} d^2\mathbf{k}\ \Omega_{\mathbf{k}}$

Berry curvature $\Omega_{\mathbf{k}} = -i\nabla_{\mathbf{k}} \times \langle u|\nabla_{\mathbf{k}} u\rangle \cdot \hat{\mathbf{z}}$

Crystal momentum $\mathbf{k}$, Bloch state $|u_{\mathbf{k}}\rangle$

Topological invariant:

$\mathcal{C}$ cannot change under smooth variations of the band

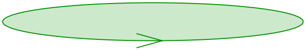
$\mathcal{C}$ can be non-zero if time-reversal symmetry is broken

# Topological Bands: Physical Consequences

- Integer quantum Hall effect [Thouless, Kohmoto, Nightingale & den Nijs (1982)]

$$\sigma_{xy} = \mathcal{C}\frac{e^2}{h} \qquad \text{i.e.} \qquad j_x = \mathcal{C}\frac{F_y}{h}$$

- Gapless chiral edge state



  Bragg spectroscopy [Goldman, Beugnon & Gerbier, arXiv:1203.1246]

- Expansion imaging [Zhao *et al.*, PRA **84**, 063629 (2011); Alba *et al.*, PRL **107**, 235301 (2011)]

  In simple cases one can reconstruct $|u_{\mathbf{k}}\rangle$

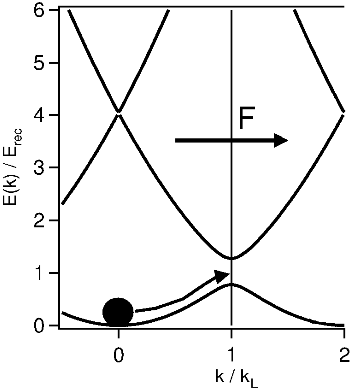
▷ Bloch oscillations [Hannah Price & NRC, PRA **85**, 033620 (2012)]

  Measure $\Omega_{\mathbf{k}}$

# Bloch Oscillations (1D)

Wavepacket centered on momentum $k$ and position $x$

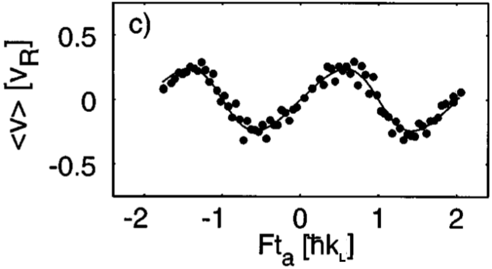
$$\begin{aligned} \hbar\dot{k} &= F \\ \dot{x} &= \frac{1}{\hbar}\frac{d\varepsilon_k}{dk} \end{aligned}$$




Oscillations in $\dot{x}$ (and $x$) with period $T_{\mathrm{B}} = \frac{2\hbar k_L}{F}$

# Bloch Oscillations (1D)

Accelerated 1D lattice [Ben Dahan, Peik, Reichel, Castin & Salomon, PRL **76**, 4508 (1996)]




$$\langle \hat{v} \rangle = \frac{1}{m} \langle \hat{p} \rangle$$ from expansion images

# Bloch Oscillations in 2D

Modified by the geometry of the Bloch wave functions $|u_{\mathbf{k}}\rangle$

[Chang & Niu, PRL **75**, 1348 (1995)]

$$
\begin{aligned}
\hbar\dot{\mathbf{k}} &= \mathbf{F} \\
\dot{\mathbf{r}} &= \frac{1}{\hbar}\frac{\partial \varepsilon_{\mathbf{k}}}{\partial \mathbf{k}} - (\dot{\mathbf{k}} \times \hat{\mathbf{z}})\Omega_{\mathbf{k}}
\end{aligned}
$$

Berry curvature $\quad \Omega_{\mathbf{k}} = -i\nabla_{\mathbf{k}} \times \langle u|\nabla_{\mathbf{k}} u\rangle \cdot \hat{\mathbf{z}}$

Crystal momentum $\mathbf{k}$, Bloch state $|u_{\mathbf{k}}\rangle$

*The physical properties of a band depend on both $\varepsilon_{\mathbf{k}}$ and $\Omega_{\mathbf{k}}$*

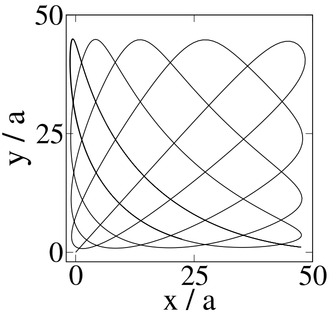
# Bloch Oscillations in 2D

$$\hbar\dot{\mathbf{k}} = \mathbf{F} \qquad\qquad \dot{\mathbf{r}} = \frac{1}{\hbar}\frac{\partial \varepsilon_{\mathbf{k}}}{\partial \mathbf{k}} - (\dot{\mathbf{k}}\times\hat{\mathbf{z}})\Omega_{\mathbf{k}}$$

Complicated trajectories even for $\Omega_{\mathbf{k}} = 0$

e.g. $\varepsilon_{\mathbf{k}} = -2J\left[\cos k_x a + \cos k_y a\right]$

$$\begin{aligned}\dot{\mathbf{r}} &= \frac{2Ja}{\hbar}(\sin k_x a, \sin k_y a) \\ &= \frac{2Ja}{\hbar}\left(\sin\frac{F_x ta}{\hbar}, \sin\frac{F_y ta}{\hbar}\right)\end{aligned}$$




Lissajous figures when $\mathbf{F}$ not along a high-symmetry direction

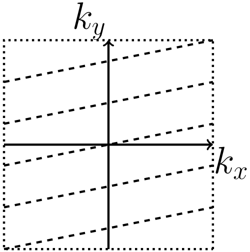
# Time-Reversal Protocol

[Hannah Price & NRC, PRA **85**, 033620 (2012)]

$$\begin{aligned} \hbar\dot{\mathbf{k}} &= \mathbf{F} \\ \dot{\mathbf{r}} &= \frac{1}{\hbar}\frac{\partial \varepsilon_{\mathbf{k}}}{\partial \mathbf{k}} - (\dot{\mathbf{k}} \times \hat{\mathbf{z}})\Omega_{\mathbf{k}} \end{aligned}$$

Measure $v_{\mathbf{k}}(+\mathbf{F})$ and $v_{\mathbf{k}}(-\mathbf{F})$

$$\begin{aligned} \mathbf{v}_{\mathbf{k}}(+\mathbf{F}) + \mathbf{v}_{\mathbf{k}}(-\mathbf{F}) &= \frac{2}{\hbar}\frac{\partial \varepsilon_{\mathbf{k}}}{\partial \mathbf{k}} \\ \mathbf{v}_{\mathbf{k}}(+\mathbf{F}) - \mathbf{v}_{\mathbf{k}}(-\mathbf{F}) &= -\frac{2}{\hbar}(\mathbf{F} \times \hat{\mathbf{z}})\Omega_{\mathbf{k}} \end{aligned}$$

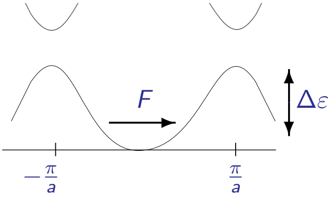


# Size of the effect

- Bloch period $T_{\mathrm{B}} = \dfrac{h}{aF}$

- Group velocity $v_g \sim \dfrac{\Delta\varepsilon a}{h}$

  $\Rightarrow x_g = v_g T_{\mathrm{B}} \sim \dfrac{\Delta\varepsilon}{F}$



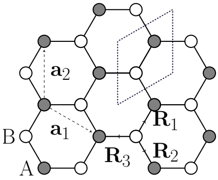

- "Berry curvature" velocity $v_\Omega = \Omega_{\mathbf{k}} \dfrac{F}{\hbar} \sim a^2 \dfrac{F}{\hbar} \quad \Rightarrow \quad x_\Omega \sim a$

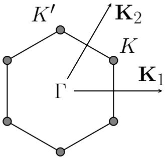
$$\frac{v_\Omega}{v_g} = \frac{x_\Omega}{x_g} \sim \frac{Fa}{\Delta\varepsilon}$$

For $Fa \simeq \Delta\varepsilon$ the effects of Berry curvature are of the same scale as conventional Bloch oscillations.

# Outline

# Honeycomb Lattice



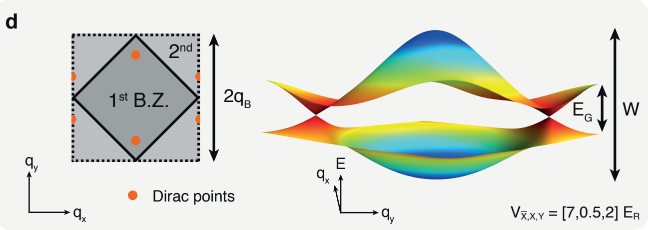




$$\varepsilon_{\mathbf{k}} = \pm |V_{\mathbf{k}}|$$

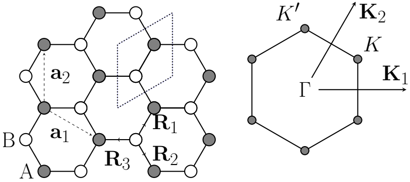
$$V_{\mathbf{k}} = J[e^{i\mathbf{k}\cdot\mathbf{R}_1} + e^{i\mathbf{k}\cdot\mathbf{R}_2} + e^{i\mathbf{k}\cdot\mathbf{R}_3}]$$

Dirac points at $K$, $K'$

[Tarruell, Greif, Uehlinger, Jotzu & Esslinger, Nature **483**, 302 (2012)]



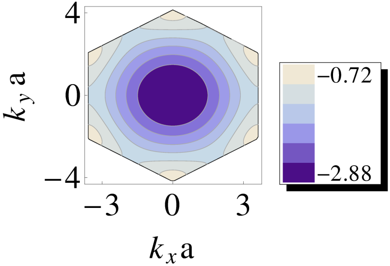
# Asymmetric Honeycomb Lattice



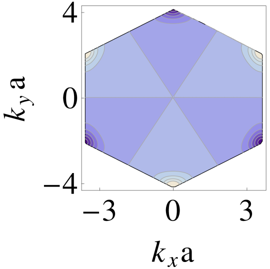

Asymmetric, $V_{\mathrm{A}} = -V_{\mathrm{B}} = W$

$$\varepsilon_{\mathbf{k}} = \pm\sqrt{W^2 + |V_{\mathbf{k}}|^2}$$

Fully gapped

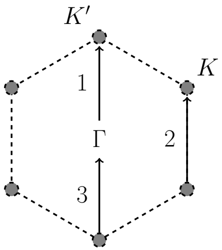


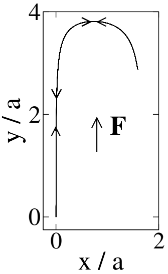

$\Omega_{\mathbf{k}} \neq 0$

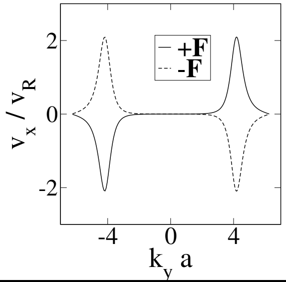


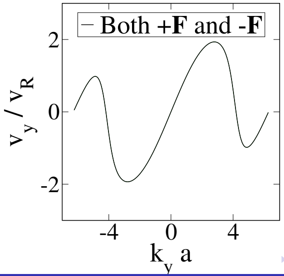
# Bloch Oscillations in Asymmetric Honeycomb Lattice

$$\mathbf{F} = \left(0, \tfrac{2}{3}\right) \frac{J}{a}$$
$$W = \tfrac{1}{2} J$$



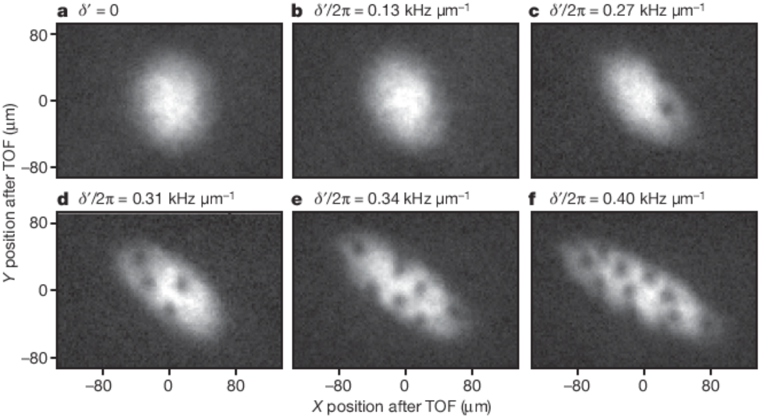



# Outline

# Optically Induced Gauge Fields

[J. Dalibard, F. Gerbier, G. Juzeliūnas & P. Öhberg, RMP **83**, 1523 (2011)]

[Y.-J. Lin, R.L. Compton, K. Jiménez-García, J.V. Porto & I.B. Spielman, Nature **462**, 628 (2009)]



# Optical Flux Lattices

[NRC, PRL **106**, 175301 (2011); NRC & Jean Dalibard, EPL **95**, 66004 (2011)]

- Coherent coupling of internal states of the atom

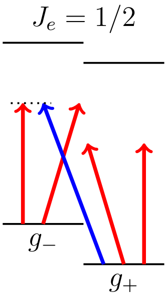
$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

- Simple laser configurations, shallow lattices, $\mathcal{V} \sim E_R$

- Landau levels: Narrow bands with $\mathcal{C} = 1$
  ($n_\phi \simeq 10^9\mathrm{cm}^{-2}$ $\Rightarrow$FQH states at high particle densities)

# Two-Photon Dressed States

[NRC & Jean Dalibard, EPL **95**, 66004 (2011)]

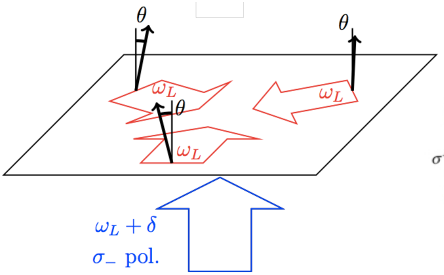


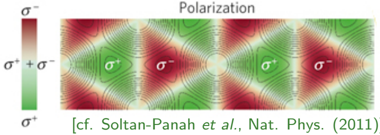

Light at two frequencies:

- $\omega_L$ with Rabi freqs. $\kappa_m$ $(m = 0, \pm 1)$
- $\omega_L + \delta$ with Rabi freq. $E$ in $\sigma_-$

$$\hat{V} = \frac{\hbar \kappa_{\rm tot}^2}{3\Delta}\hat{\mathbb{I}} + \frac{\hbar}{3\Delta}\begin{pmatrix} |\kappa_-|^2 - |\kappa_+|^2 & E\kappa_0 \\ E\kappa_0^* & |\kappa_+|^2 - |\kappa_-|^2 \end{pmatrix}$$

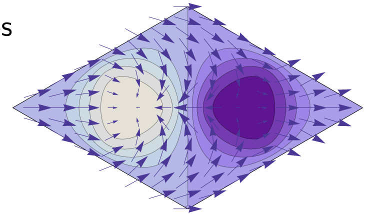
[NRC & Jean Dalibard, EPL **95**, 66004 (2011)]




[cf. Soltan-Panah *et al.*, Nat. Phys. (2011)]

Bloch vector $\langle\psi_0(\mathbf{r})|\hat{\vec{\sigma}}|\psi_0(\mathbf{r})\rangle$ wraps the sphere once within the unit cell

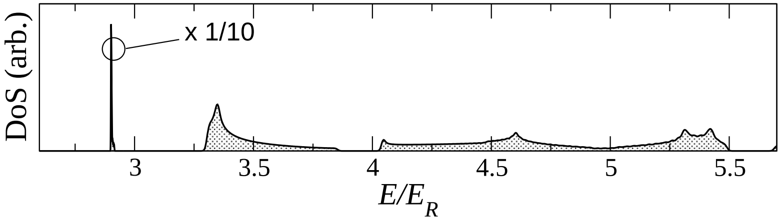


Berry phase $2\pi$ (2-level system)
$\Rightarrow N_\phi = 1$ flux quantum

# Bandstructure

$J_g = 1/2$ (e.g. $^6$Li, $^{171}$Yb, $^{199}$Hg)

$\mathcal{V} = 2E_{\mathrm{R}}$, $\theta = \pi/4$, $\epsilon = 1.3$



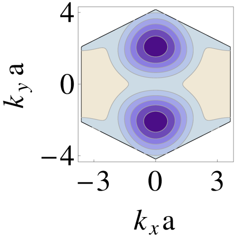

Narrow lowest energy band, with Chern number $\mathcal{C} = 1$
Optical flux lattice analogue of the lowest Landau level

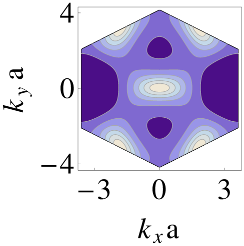
# Bloch Oscilllations

$[\mathcal{V} = 1.8E_R,\ \theta = 0.3,\ \epsilon = 0.4]$

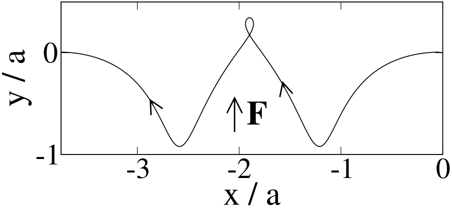
[Hannah Price & NRC, PRA **85**, 033620 (2012)]

$\varepsilon_{\mathbf{k}} \rightarrow$




$\leftarrow \Omega_{\mathbf{k}}$




$\mathbf{F} = (0, 0.1E_R/a)$

Net transverse drift:
nonzero mean Berry curvature

# Summary

- Two dimensional bands can have a topological character, encoded in the Berry curvature $\Omega_{\mathbf{k}}$.
- The Berry curvature modifies the Bloch oscillations of an atomic wave packet.
- A "time-reversal" protocol cleanly extracts the effects of the Berry curvature.
- For non-zero Chern number, there is a net drift of the wave packet transverse to the force. This drift is the lattice analogue of the edge state of the IQH effect.