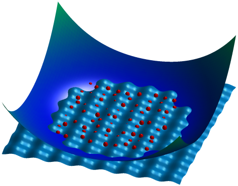
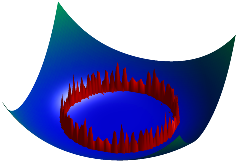
Identifying Topological Edge States in 2D Optical Lattices Using Light Scattering

Nathan Goldman

with Jérôme Beugnon and Fabrice Gerbier (LKB, ENS, Paris)

Phys. Rev. Lett. **108**, 255303 (2012)





Trieste, July 16th 2012

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Collaborations

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Jérome Beugnon (LKB, ENS, Paris) + Discussions with Jean Dalibard

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Predrag Nikolic (NIST, USA)

Cristiane Morais Smith (Univ. Utrecht)
Wouter Beugeling (Univ. Utrecht)

Patrik Ohberg (Heriot–Watt, Edinburgh)
Zhihao Lan (Heriot–Watt, Edinburgh; Southampton Univ.)

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Leonardo Mazza (M. Planck Inst. Garching)

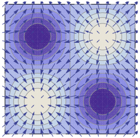
Dario Bercioux (Univ. Freiburg)
Daniel Urban (Univ. Freiburg)

C. H. Oh (National University Singapore)
Feng Mei (National University Singapore)

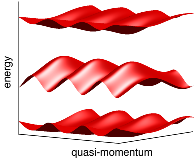
Shi–Liang Zhu (South China Normal Univ. Guangzhou)
Zhi–Ming Zhang (South China Normal Univ. Guangzhou)

Nigel Cooper's talk

Optical Flux Lattice



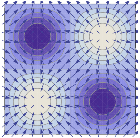
Topological band structure



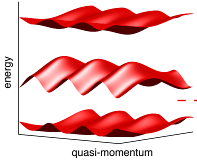
$\} \; N_{\rm chern} \neq 0$

Nigel Cooper's talk

Optical Flux Lattice



Topological band structure



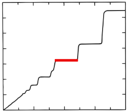
$\} \; N_{\text{chern}} \neq 0$

Cond-mat physics:
Quantum Hall effect

E_{Fermi}

$\sigma_H = 1/\rho_H = N_{\text{chern}} \times (e^2/h)$

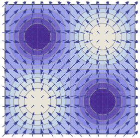
ρ_H



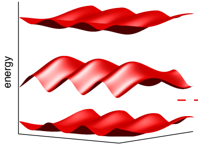
"σ_H *: protected by topology*"

Nigel Cooper's talk

Optical Flux Lattice



Topological band structure



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$E_{\rm Fermi}$

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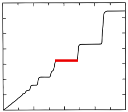
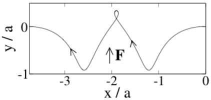


"σ_H *: protected by topology*"

Measuring the Chern number through Bloch oscillations:

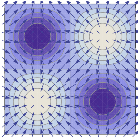


$\mathbf{F} = (0, 0.1 E_R/a)$

Net transverse drift:
nonzero mean Berry curvature

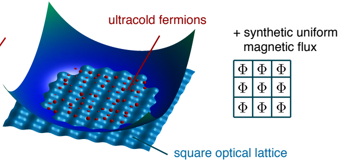
Nigel Cooper's talk

Optical Flux Lattice

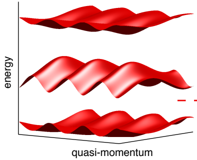


This talk

Hofstadter Optical Lattice



Topological band structure



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Cond-mat physics:
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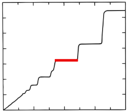

"σ_H *: protected by topology"*

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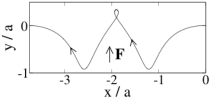
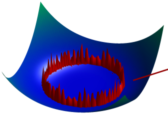


$\mathbf{F} = (0, 0.1 E_R/a)$

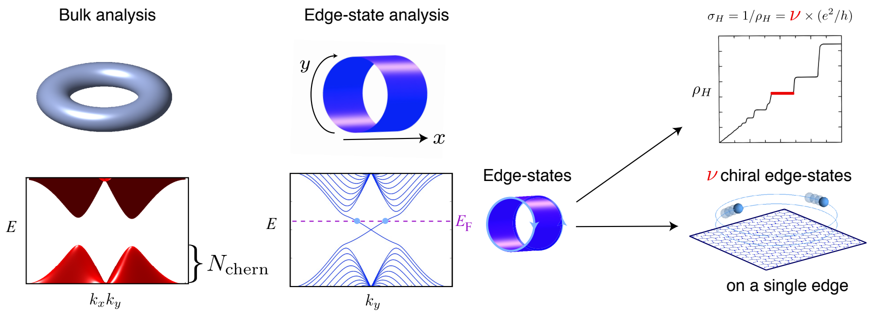
Net transverse drift:
nonzero mean Berry curvature

Measuring the Chern number through the *bulk-edge correspondence*

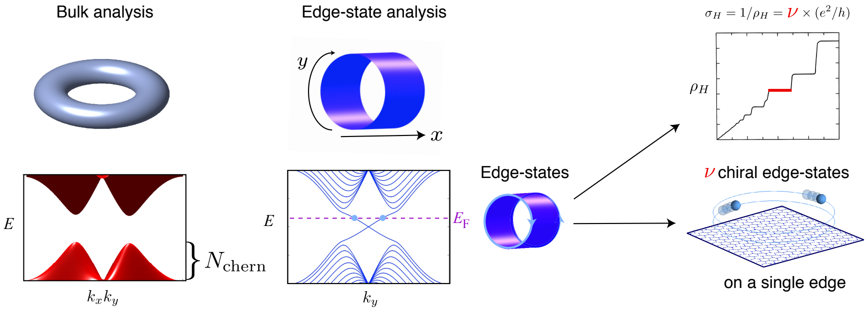


topological current-carrying edge states

Bulk-Edge correspondence



Bulk-Edge correspondence



- Bulk-edge correspondence : the number of edge-states ν is *topologically protected*

$$\nu = N_{\text{chern}} \qquad\qquad \sigma_H = \frac{e^2}{h}\nu$$

- The edge state chirality (orientation of propagation) : sign$(\partial E/\partial k_y) =$ sign(ν)

Chern insulator = an insulator with robust chiral edge states, protected by topology (Chern numbers)

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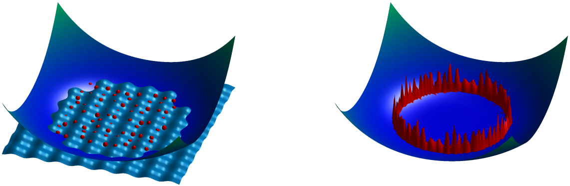

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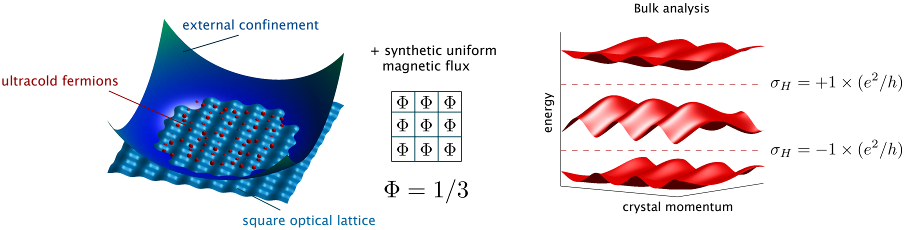
The System: The Hofstadter Optical Lattice

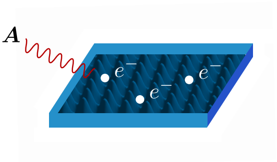
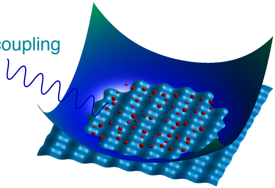


- Current experiments in Paris (Dalibard–Gerbier), Munich (Bloch), NIST (Spielman), Hamburg (Sengstock), ...
- First steps: 2D: Staggered magnetic field (Aidelsburger et al., PRL 2011)

  1D: Lattice Shaking (Struck et al., PRL 2012)

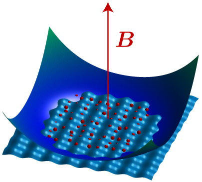
  1D: RF fields and Raman lasers (Jimenez–Garcıa et al., PRL 2012)
- Other schemes: Optical flux lattices (N. R. Cooper, PRL 2011)

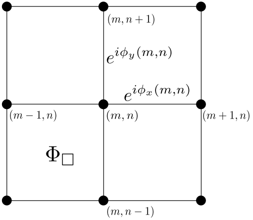
Synthetic gauge potentials in optical lattices

Electrons in a solid	Cold atoms in optical lattices
	
$\hat{H} = \sum_j \frac{1}{2m}(\boldsymbol{p}_j - q\boldsymbol{A}(\boldsymbol{x}_j))^2 + V(\boldsymbol{x}_j) + ...$	$\hat{H} = \sum_j \frac{1}{2m}(\boldsymbol{p}_j - \kappa\boldsymbol{A}(\boldsymbol{x}_j))^2 + V(\boldsymbol{x}_j) + ...$
q : the electron charge $\boldsymbol{A}$: gauge potential	κ : coupling constant $\boldsymbol{A}$: synthetic gauge potential

Magnetic field $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ ⟶ synthetic magnetic field for neutral atoms

J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg, Rev. Mod. Phys. (2011)





Lattice Hamiltonian in the presence of a magnetic field

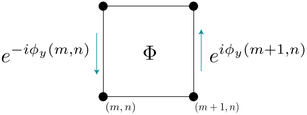
$$\hat{H} = -J \sum_{m,n} e^{i\phi_x(m,n)} \hat{c}^\dagger_{m+1,n} \hat{c}_{m,n} + e^{i\phi_y(m,n)} \hat{c}^\dagger_{m,n+1} \hat{c}_{m,n} + \text{h.c.}$$

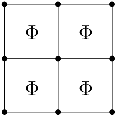
Relation between the magnetic flux and the Peierls phases $e^{i\phi_x(m,n)}$ and $e^{i\phi_y(m,n)}$

$$2\pi\Phi_\square = \sum_\square \phi_{x,y}(m,n)$$

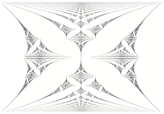
Uniform magnetic flux $\Phi_\square = \Phi$ Landau gauge: $\phi_x = 0$, $\phi_y(m) = 2\pi\Phi m$

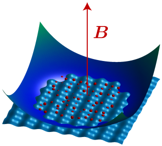
equiv. to $\boldsymbol{A}(\boldsymbol{x}) = (0, Bx)$

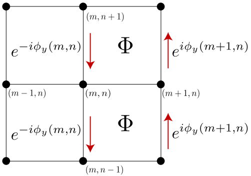




"Hofstadter model"



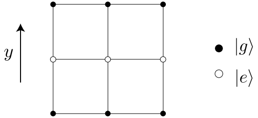




$$\phi_x = 0 \ , \ \phi_y(m) = 2\pi\Phi m$$

How can we induce these phases in a 2D optical lattice?

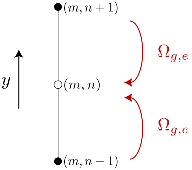
1). Trap atoms in two different internal states



Refs (theory): Jaksch and Zoller, NJP 2003
Gerbier and Dalibard, NJP 2010

2). Prevent the direct tunneling along y

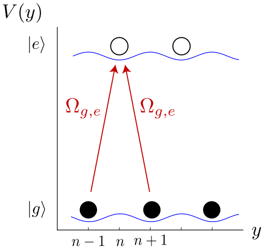
3). Induce the tunneling along y through atom-light coupling

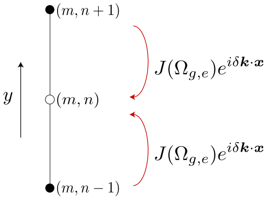


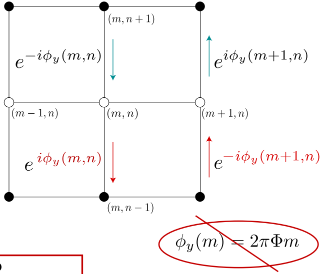
Induced-tunneling:

$$J_{\text{eff}}(g \rightarrow e) = J(\Omega_{g,e}) e^{i\delta \boldsymbol{k}\cdot\boldsymbol{x}}$$

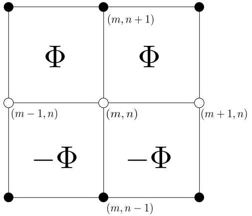
Using a single coupling: the sign problem







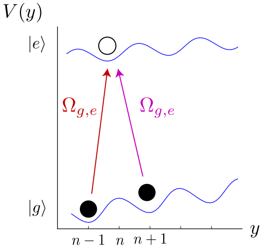
Staggered magnetic field!

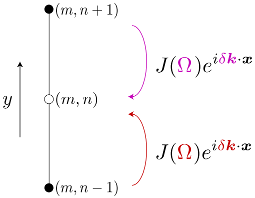


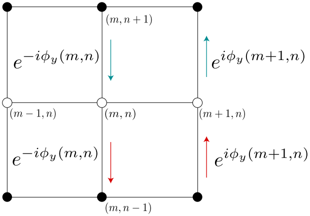
Realized experimentally: Aidelsburger, Atala, Nascimbène, Trotsky, Chen and Bloch, PRL 2011

Refs (theory): Jaksch and Zoller, NJP 2003, Gerbier and Dalibard, NJP 2010

Using two couplings: the flux rectification



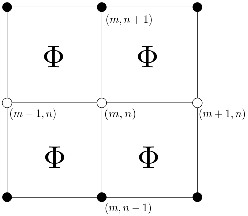




$$\delta\boldsymbol{k}\cdot\boldsymbol{x} = -\delta\boldsymbol{k}\cdot\boldsymbol{x}$$

$$\phi_y(m) = 2\pi\Phi m$$

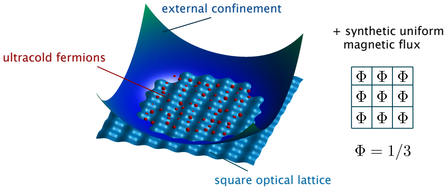
Uniform magnetic field!

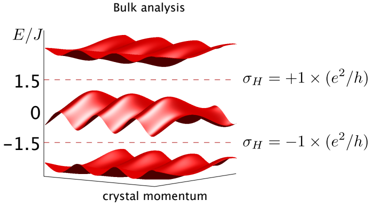


Not yet realized experimentally... Challenge: requires superlattices, subtle couplings,...

Refs: Jaksch and Zoller, NJP 2003, Gerbier and Dalibard, NJP 2010

The Hofstadter Optical Lattice: what can we measure with cold atoms?

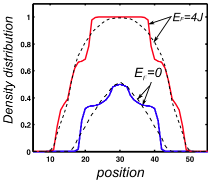




1) General property:

a) The synthetic magnetic field opens bulk gaps

Observation: density measurements
Ref: Gerbier and Dalibard 2010



2) Topological properties when E_{F} is inside a bulk gap:

b) The Chern numbers associated with the bands are non-trivial

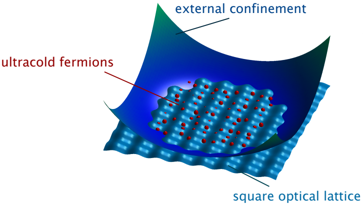
Observation: density or momentum density measurements, Bloch oscillations
Refs: Umucalilar et al. 2008, Alba et al. 2011, Zhao et al. 2011, Price-Cooper 2012

bulk ↕ edge

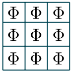
c) A single chiral edge state, with opposite chirality for $E_{\mathrm{F}} = \pm 1.5J$

Could we "see" these edge states and probe their chirality?

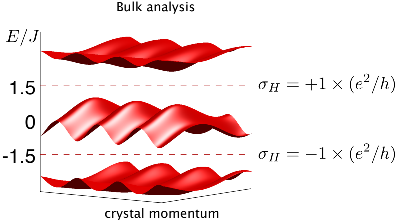
The Hofstadter Optical Lattice: the edge state structure



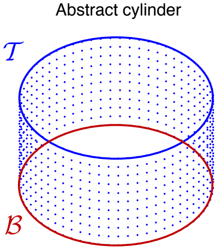
+ synthetic uniform magnetic flux

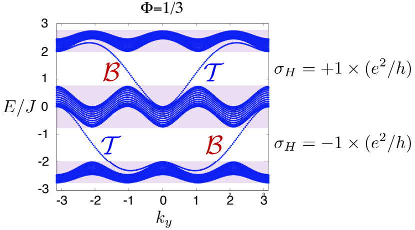


$\Phi = 1/3$

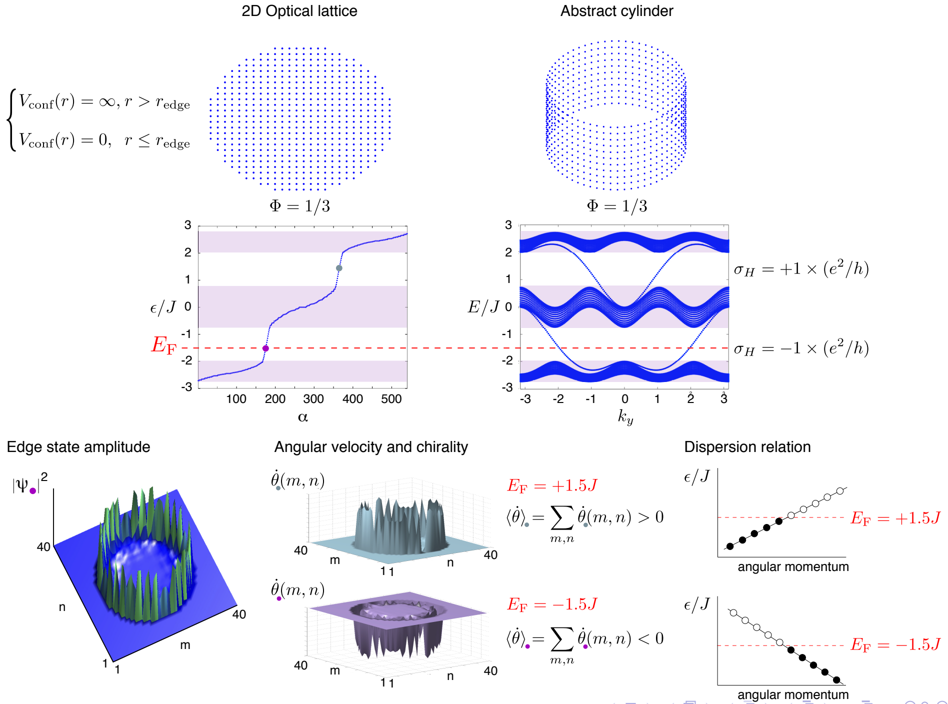


A single chiral edge state, with opposite chirality for $E_{\mathrm{F}} = \pm 1.5J$





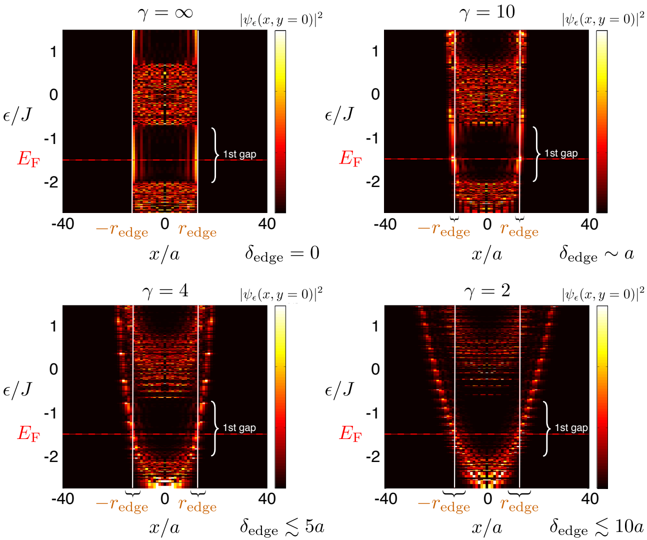
Linear dispersion relation for the edge modes $E/\hbar \approx v_e k_y$



2D Optical lattice
Abstract cylinder
$\begin{cases} V_{\text{conf}}(r) = \infty, r > r_{\text{edge}} \\ V_{\text{conf}}(r) = 0, \quad r \le r_{\text{edge}} \end{cases}$
$\Phi = 1/3$
$\Phi = 1/3$
ϵ/J
E_F
α
E/J
k_y
$\sigma_H = +1 \times (e^2/h)$
$\sigma_H = -1 \times (e^2/h)$
Edge state amplitude
Angular velocity and chirality
Dispersion relation
$|\psi|^2$
$\dot{\theta}(m,n)$
$E_F = +1.5J$
$\langle\dot{\theta}\rangle = \sum_{m,n} \dot{\theta}(m,n) > 0$
$E_F = -1.5J$
$\langle\dot{\theta}\rangle = \sum_{m,n} \dot{\theta}(m,n) < 0$
ϵ/J
$E_F = +1.5J$
$E_F = -1.5J$
angular momentum
angular momentum

The confinement and the robust edge states

$V(r) = J(r/r_{\rm edge})^\gamma$ $\quad$ $r_{\rm edge} = 13a$



see also Buchhold, Cocks and Hofstetter (2012)

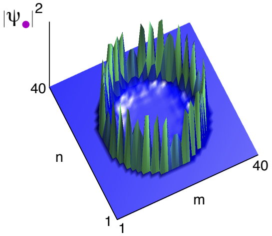
Goal

Find a probe to demonstrate the presence of topological edge states:

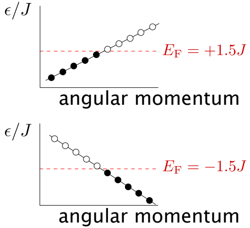
– Localized states with energy inside the bulk gaps

– Dispersion relation dictated by the Chern numbers

Edge state amplitude



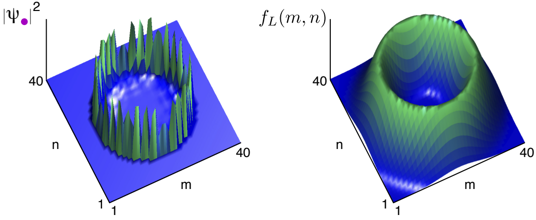
Dispersion relation



Light Bragg spectroscopy

- We probe the edge states and their chirality with a time-dependent perturbation

$$\hat{H}_{\text{Bragg}}(t) = \frac{\hbar\Omega}{2} \int \mathrm{d}\boldsymbol{x}\, \hat{\psi}^{\dagger}(\boldsymbol{x})\hat{\psi}(\boldsymbol{x}) f_L(r) e^{iq\theta} e^{-i\omega_L t} + \text{h.c.}, \tag{1}$$



- Probe = two lasers in Laguerre-Gaussian modes

$$E_{1,2}(r) \sim (r/r_0)^{|l_{1,2}|} e^{-r^2/2r_0^2} \exp(-il_{1,2}\theta - i\omega_{1,2}t) \tag{2}$$

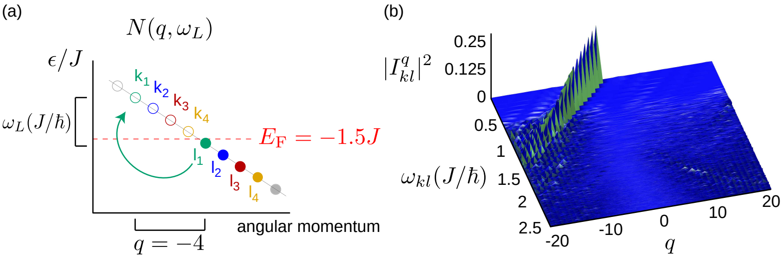
- The probe transfers angular momentum $\hbar q = \hbar(l_2 - l_1)$ and energy $\hbar\omega_L = \hbar(\omega_1 - \omega_2)$ to the system

- The number of scattered particles is given by the Fermi golden rule

$$N(q,\omega_L) = 2\pi\Omega^2 t \sum_{k>E_F, l\leq E_F} |I^q_{kl}|^2 \delta(\omega_{kl} - \omega_L), \quad (3)$$

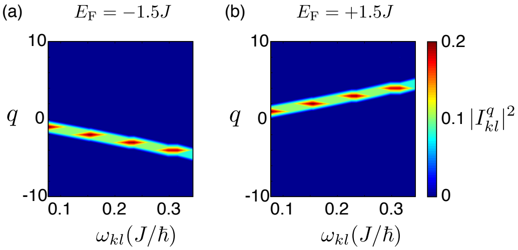
where

$$I^q_{kl} = \frac{1}{2}\int d\boldsymbol{x}\, \phi^*_k(\boldsymbol{x})\phi_l(\boldsymbol{x}) f_L(r) e^{iq\theta} \quad (4)$$



- Probes the angular velocity : $\omega_L^{res} \approx \omega_{kl}^{res}(q) \approx q\,\langle\dot{\theta}\rangle_{edge}$ for $\omega_L^{res} \ll$

- Probes the chirality : $\omega_L^{\text{res}} \approx q \, \langle \dot{\theta} \rangle_{\text{edge}}$, at $\omega_L \ll J/\hbar$

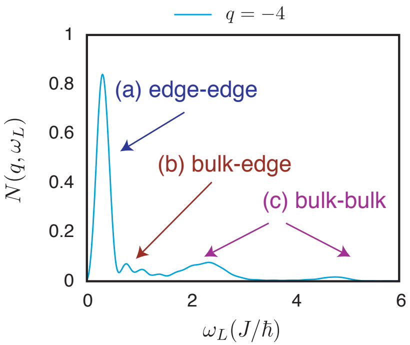


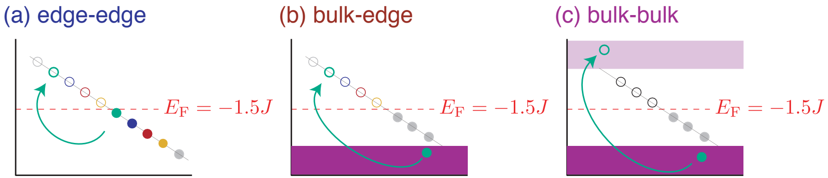
$$N(q, \omega_L) = 2\pi\Omega^2 t \sum_{k > E_{\mathrm{F}}, l \leq E_{\mathrm{F}}} |I^q_{kl}|^2 \delta(\omega_{kl} - \omega_L), \qquad (5)$$

- When $E_{\text{Fermi}} = -1.5J$: signal for $q < 0$ (negative slope)

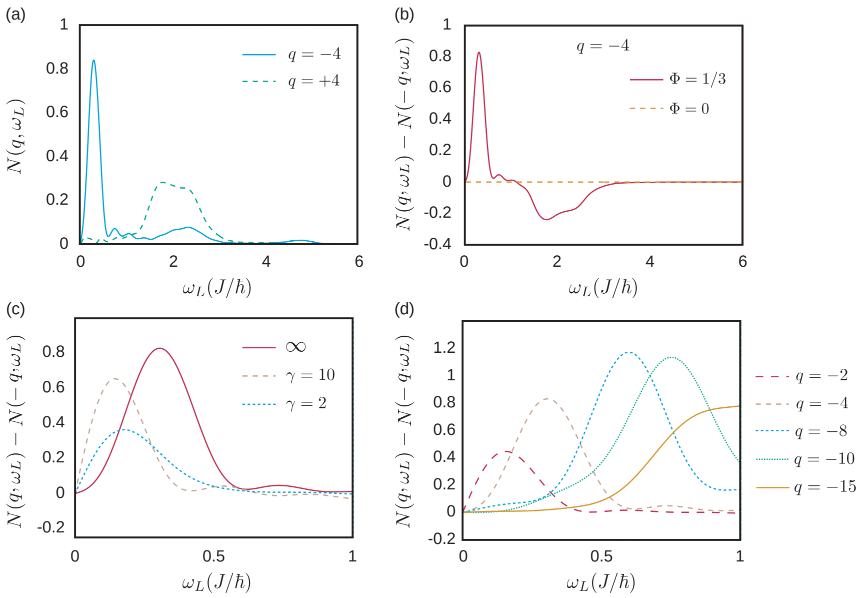
- When $E_{\text{Fermi}} = +1.5J$: signal for $q > 0$ (opposite chirality : positive slope !)

- Excited fractions $N(q, \omega_L)$ at finite times ($t = 20\hbar/J$)





- Excited fractions $N(q, \omega_L)$ at finite times ($t = 20\hbar/J$)



First conclusions and Drawbacks

- Chiral edge states lead to unambiguous signatures in the Bragg spectra (excited fraction $N(q, \omega)$)

  But...

- The excited fraction $N(q, \omega)$ is not directly measured in cold-atom experiments,

  And unfortunately...

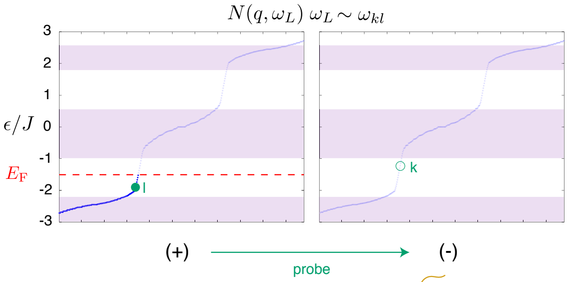
- Excitations only slightly modify the particle and momentum densities (dominated by the bulk states !)

  $\rightarrow$ the effects of the probe are not observable through *in situ* or TOF measurements !

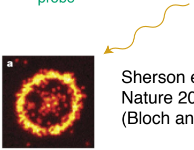
The Shelving Method

- We transfer the excited states into an empty copy of the system

$$\hat{H}_{\text{Shelving}}(t) = \frac{\hbar\Omega}{2} \int \mathrm{d}\boldsymbol{x}\, \hat{\psi}^{\dagger}_{(-)}(\boldsymbol{x}) \hat{\psi}_{(+)}(\boldsymbol{x}) f_L(r) e^{iq\theta} e^{-i\omega_L t} + \text{h.c.}, \qquad (6)$$



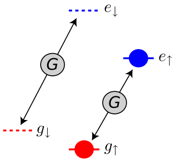
Single-atom-resolved
In situ imaging
(dark background)



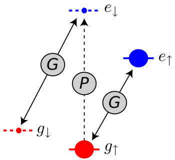
Sherson et al.
Nature 2010
(Bloch and Kuhr's group)

The Shelving Method in 5 steps

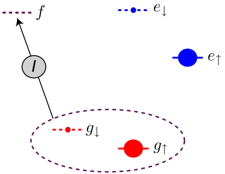
(1)



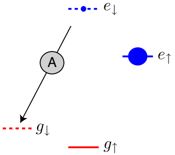
(2)



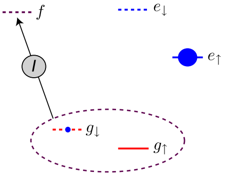
(3)

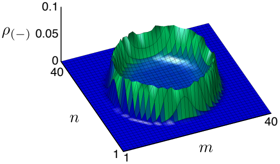


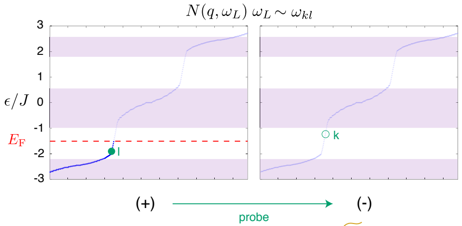
(4)



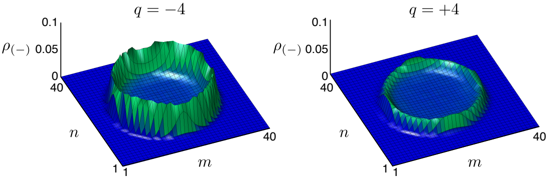
(5)

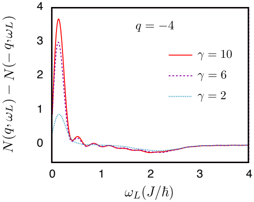






Density for $E_{\rm F} = -1.5J$ and $\omega_L > 0$





- The integrated density (excited fraction) shows a clear resonance peak

Signature of the edge state chirality inside the first bulk gap

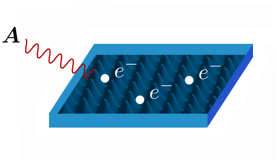
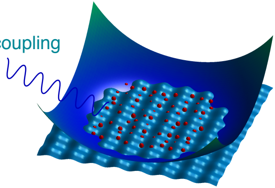
Conclusions

- Our system and method to detect topological edge states
  - Synthetic magnetic fields for cold atoms produce quantum Hall edge states
  - Using the Shelving method, one can directly *see* the topological edge states, using available imaging technics
  - The Bragg spectra $N(q, \omega_L)$ give the dispersion relations (chirality) of the edge states
  - Our method does not rely on the lattice or on the setup which generates the synthetic magnetic field (laser-induced, lattice shaking, atom-chip, rotation, ...)
  - Method applies for all cold-atom realization of 2D topological phases (with circular geometry)
  - Our method applies in the presence of interactions (i.e. fractional Hall regime) or disorder

- Quantum simulation perspectives
  - Our method is complementary to transport measurements (cond-mat framework)
  - Detect, manipulate, image topological edge states in a highly controllable and clean system
  - Obtain dispersion relations of the edge states in the fractional regime : in the interacting regime, edge physics is still intriguing

"*Detecting Chiral Edge States in the Hofstadter Optical Lattice*"
N. G., J. Beugnon and F. Gerbier
Phys. Rev. Lett. 108, 255303 (2012)

Synthetic gauge potentials in optical lattices

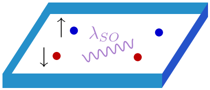
Electrons in a solid	Cold atoms in optical lattices
	
$\hat{H} = \sum_j \frac{1}{2m}(\boldsymbol{p}_j - q\boldsymbol{A}(\boldsymbol{x}_j))^2 + V(\boldsymbol{x}_j) + ...$	$\hat{H} = \sum_j \frac{1}{2m}(\boldsymbol{p}_j - \kappa\boldsymbol{A}(\boldsymbol{x}_j))^2 + V(\boldsymbol{x}_j) + ...$
q : the electron charge $\boldsymbol{A}$: gauge potential	κ : coupling constant $\boldsymbol{A}$: synthetic gauge potential

Ex: Magnetic field $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ ⟶ synthetic magnetic field for neutral atoms

Spin-orbit coupling $A_\mu \sim \hat{\sigma}_{x,y,z} \in \mathfrak{su}(2)$ ⟶ synthetic spin-orbit coupling for neutral atoms

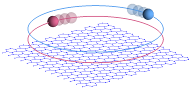
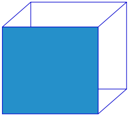
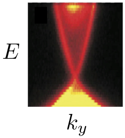
J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg, Rev. Mod. Phys. (2011)

Synthetic spin-orbit coupling: Topological insulators physics



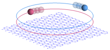
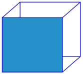
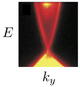
λ_{SO} : spin-orbit coupling

- *intrinsic*
- *induced* (e.g. external electric field)

2D: the quantum spin Hall effect	3D: topological insulators
 Helical edge states	 Helical Dirac fermion (on the surface)  Xia et al. Nat. Phys. 2009

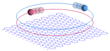
Exquisite properties: - Robust spin transport *protected by topology*
- Dissipationless transport
- Helical Dirac fermions (on the surface of a 3D TI)
- Charge fractionalization, Spin-Charge separation
- Majorana fermions (proximity to a superconductor)

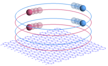
Synthetic spin-orbit coupling: Topological insulators physics

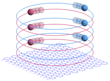
2D: the quantum spin Hall effect	3D: topological insulators
 Helical edge states	 Helical Dirac fermion (on the surface)  Xia et al. Nat. Phys. 2009

Cold-atom simulator for topological insulators: Why?

- Test the $\mathbb{Z}_2$ classification $(\nu = 0, 1)$







. . .

$\nu = 1$ $\nu = 0$ $\nu = 1$

destroyed by disorder

- Test the robustness against TRS-breaking perturbations
- Study the effects of interactions (Helical liquids on the edges)
- Direct imaging of the helical edge states
- Probe the physics of Majorana fermions

References :

- Detection method for 2D cold-atom topological phases
  - *Detecting Chiral Edge States in the Hofstadter Optical Lattice*
    N. G., J. Beugnon and F. Gerbier
    Phys. Rev. Lett. **108**, 255303 (2012)

- Z_2 topological phases in 2D and 3D cold-atom lattice systems
  - *Realistic Time-Reversal Invariant Topological Insulators With Neutral Atoms*,
    N. G., I. Satija, P. Nikolic, A. Bermudez, M.A. Martin-Delgado, M. Lewenstein and I. B. Spielman
    Phys. Rev. Lett. **105**, 255302 (2010)
  - *Wilson Fermions and Axion Electrodynamics in Optical Lattices*,
    A. Bermudez, L. Mazza, M. Rizzi, N. G., M. Lewenstein and M.A. Martin-Delgado,
    Phys. Rev. Lett. **105** 190404 (2010)
  - *An optical-lattice-based quantum simulator for relativistic field theories and topological insulators*,
    L. Mazza, A. Bermudez, N.G., M. Rizzi, M.A. Martin-Delgado and M. Lewenstein,
    New Journal of Physics **14** 015007 (2012)

Downloads @ homepages.ulb.ac.be/ngoldman

Slides @ homepages.ulb.ac.be/ngoldman/Bragg2012.pdf